Maths of the Day

If α and β are positive integers such that $\alpha\beta = \pi^2$ then

$$\alpha \sum_{m=1}^{\infty} \frac{m}{e^{2m\alpha} - 1} + \beta \sum_{m=1}^{\infty} \frac{m}{e^{2m\beta} - 1} = \frac{\alpha + \beta}{24} - \frac{1}{4}$$

$$\sum_{m=1}^{\infty} \frac{1}{m(e^{2m\alpha} - 1)} - \sum_{m=1}^{\infty} \frac{1}{m(e^{2m\beta} - 1)} = \frac{1}{4} \log \frac{\alpha}{\beta} - \frac{\alpha - \beta}{12}$$