Neutrino Physics
A Brief Overview

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Majorana suggested that the neutrino could be a spin-$\frac{1}{2}$ particle that was its own antiparticle. At the time, the neutrino had not been seen experimentally. When it was discovered, its properties seemed to disprove Majorana's suggestion as there was a clear difference between neutrinos and their anti-neutrino partners. However, further developments in neutrino physics reignited interest in the neutrino being a Majorana fermion.

1 Lepton Number Conservation

One major problem with the neutrino as a Majorana fermion is the fact that the Standard Model (SM) uses *Lepton Number* to distinguish ν from $\overline{\nu}$ which would mean $\nu \neq \overline{\nu}$. To illustrate this distinction, we will consider the pion decay. When a π^+ decays a μ^+ is emitted along with a ν_μ

$$\pi^+ \to \mu^+ + \nu_\mu.$$

This muon neutrino interacts with matter to produce only a μ^-

$$\nu_\mu N \to \mu^- X.$$

For π^- decay, a similar process takes place. A $\overline{\nu}_\mu$ is produced which only interacts with matter to produce μ^+. This is normally explained by saying that $\overline{\nu}_\mu$ and ν_μ are distinct particles, and there is a conserved quantum number for these interactions.

This conserved quantum number, known as the lepton number, is the number of leptons (electron, muon or tau) minus the number of anti-leptons. The SM relies on the conservation of this lepton number in interactions. As an example, the interaction below shows how the lepton number is conserved in muon decay.

$$\begin{aligned} \mu^- &\to \nu_\mu + e^- + \overline{\nu}_e \\ L: 1 &= 1 + 1 - 1 \end{aligned}$$

The lepton number became a useful tool in describing interactions. It led to the discovery of the different flavors of neutrino corresponding to each of the three different leptons: the electron neutrino; muon neutrino and tau neutrino. These properties showed there is a distinction between neutrinos and anti-neutrinos implying that they are not Majorana fermions.

However, this need not be the case. Another explanation for these interactions comes when considering the helicity of the particles. In π^+ decay the neutrino produced i left-handed. while in π^- decay the anti-neutrino produced is right-handed. Supposing that every left-handed neutrino interacts giving a μ^- and every right-handed neutrino interacts giving a μ^+ then there is no need to appeal to lepton number conservation to describe these interactions. The neutral particle produced in both the π^+ and π^- decays would then be the same particle just with different helicities. In other words, the neutrino is the same as the anti-neutrino i.e. it is a Majorana fermion.

Therefore, we have two explanations for the same process. If the lepton number is conserved then $\nu \neq \overline{\nu}$ and the neutrino is the normal Dirac neutrino. On the other hand, if the lepton number is not conserved $\nu = \overline{\nu}$ and the neutrino is the Majorana neutrino.

2 Massive Neutrinos and the Standard Model

The SM describes neutrinos as being massless as there are no gauge invariant interactions that can deliver neutrinos with non-zero mass. However, when trying to unify the weak, the strong and the electromagnetic interactions in Grand Unified Theories (GUTs), we construct a large multiplet containing the neutrino ν, a lepton l and a positively and a negatively charged quark q^+ and q^-:

$$\begin{pmatrix} q^+ \\ q^- \\ l \\ \nu \end{pmatrix}$$

In this construction, neutrinos are put on the same footing as the massive leptons and quarks and therefore it would seem reasonable to think that neutrinos are themselves massive. Many experiments have revealed interesting aspects of the neutrino which seem to contradict what is said in the SM giving validity to this idea of neutrino mass.

2.1 Neutrino Oscillations

The Super-Kamiokande Experiment detects and observes solar, atmospheric and man-made neutrinos. This has shown atmospheric muon neutrinos and solar neutrinos vanishing from their detectors. As a neutrino's momentum and energy is unlikely to disappear, an explanation could be that the neutrino is changing into another type of neutrino which we can not detect. The idea is that muon neutrinos from the atmosphere would oscillate into tau neutrinos which could be picked up experimentally - hence the vanishing. These neutrino oscillations mean that a neutrino of one flavor can change into a different one. However, is all three neutrinos have zero mass (or the same non-zero mass) this would not be possible.

As the neutrino travels, it oscillates between the different flavors - having a probability of being a particular type of neutrino at any particular time. To set up neutrino oscillations there has to be a mixing between neutrino mass and weak interaction eigenstates. This can be written as

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

There also needs to be a mass difference between the neutrino mass eigenstates i.e. $m_1 \neq m_2$. So, we have neutrinos of masses m_1 and m_2 which are allowed to propagate as matter waves of differing frequencies. From the time-dependent Schrödinger equation we can construct the following

$$\begin{pmatrix} \nu_1(\mathbf{x},t) \\ \nu_2(\mathbf{x},t) \end{pmatrix} = e^{i\mathbf{p}\cdot\mathbf{x}} \begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix} \begin{pmatrix} |\nu_1(0)\rangle \\ |\nu_2(0)\rangle \end{pmatrix}.$$

Using

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

we have that

$$\begin{pmatrix} |\nu_\mu(\mathbf{x},t)\rangle \\ |\nu_\tau(\mathbf{x},t)\rangle \end{pmatrix} = e^{i\mathbf{p}\cdot\mathbf{x}} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_\mu(0)\rangle \\ |\nu_\tau(0)\rangle \end{pmatrix}.$$

Then $|\nu_\mu(0)\rangle = 1$ and $|\nu_\tau(0)\rangle = 0$ and so

$$||\nu_\mu(\mathbf{x},t)\rangle|^2 = \sin^2 2\theta \sin^2 \frac{(E_2 - E_1)t}{2} \equiv P(\nu_\mu \to \nu_\tau)$$

which gives the probability of a muon neutrino turning into a tau neutrino. Experimentally, we know that if the neutrino has mass it must be much smaller that its kinetic energy $E_1 E_2 \gg m_1 m_2$ and therefore we can write

$$E_2 - E_1 \approx \frac{m_2^2 - m_1^2}{2p}.$$

We can then make the approximations that $t \approx |\mathbf{x}| \equiv L$ where L is the distance travelled by the neutrino and $p \approx E$. We then obtain

$$P(\nu_\mu \to \nu_\tau) \approx \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right)$$

where Δm^2 is the difference in the masses squared. From this result, we see that if the probability of a muon neutrino changing into a tau neutrino is non-zero, then $\Delta m^2 \neq 0$. To put it in another way, if neutrino oscillations correctly describe the experimental vanishing of atmospheric and solar neutrinos then neutrinos would have some non-zero mass.

2.2 The Seesaw Mechanism

So where does Majorana fermion fit in all this? When constructing a theory for neutrino mass you can have a Dirac mass term that conserves the lepton number and involves transitions between two different neutrinos ν_L and N_R. The L and R subscripts denote the left-handed and the right-handed states respectively. So we can write the Lagrangian for the Dirac mass term as

$$-\mathcal{L}_{Dirac} = m_D(\overline{\nu}_L N_R + \overline{N}_R \nu_L) = m_D \overline{\nu}\nu$$

where we have defined the Dirac field as $\nu \equiv \nu_L + N_R$. So the Dirac neutrino has four components ν_L and N_R and their CPT partners ν_R^c and N_L^c. In this case, the ν_L is in an $SU(2)$ doublet and N_R resides in the $SU(2)$ singlet. The mass is generated by the Yukawa coupling with $SU(2)$ symmetry breaking. However, there is a problem with this. The neutrino mass is thought to be of the order of 10 eV and this would require a very small Yukawa coupling of the order of 10^{-10} which does not seem physical.

However, this is not the only mass term we can have in the Lagrangian. If the neutrinos were a Majorana fermion, we would have $N_R \equiv \nu_R^c$. We can then combine our neutrino fields to give a Majorana mass term

$$-\mathcal{L}_{Majorana} = \frac{1}{2} m_M^L (\overline{\nu}_L \nu_L^c + \overline{\nu}_L^c \nu_L) + \frac{1}{2} m_M^R (\overline{\nu}_R \nu_R^c + \overline{\nu}_R^c \nu_R).$$

If both the Majorana and Dirac mass terms are present then the matrix notation can be used to represent the terms

$$-\mathcal{L} = \frac{1}{2} \overline{\nu}_L \overline{\nu}_L^c \mathbf{M} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

where $\mathbf{M}$ is known as the seesaw matrix

$$\mathbf{M} = \begin{pmatrix} m_M^L & m_D \\ m_D & m_M^R \end{pmatrix}.$$

We can assume that the elements of the matrix some hierarchy

$$m_M^R = M \gg m_D \gg m_M^L = \mu$$

with $\mu \ll 1$. Following this hierarchy $\mathbf{M}$ would have the eigenvalues

$$\lambda_\pm \approx \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2}$$

and $\lambda_+ \approx M$

$$\lambda_\approx - \frac{m_D^2}{M}$$

so the determinant is

$$\det \mathbf{M} \approx -m_D^2.$$

Therefore, if λ_+ goes up then λ_- goes down and vice versa. Hence the name, seesaw mechanism. $\mathbf{M}$ therefore gives the mass matrix for the right-handed neutrino with the Majorana mass M at the GUT scale and the Dirac mass m_D being of the electroweak scale.

The seesaw mechanism allows for the neutrino to exist both as Majorana and Dirac fermions. In this scenario, the small neutrino mass that has already been detected is the Dirac fermion with the larger mass being the Majorana fermion - indicating the the lepton number is violated at the GUT scale.

3 Neutrinoless Beta Decay

Currently, many processes are being used to delve into the true nature of the neutrino. One example is the *Double Beta Decay* which releases two beta rays in a single process. This can appear in two different modes

$$\begin{aligned} A(Z,N) &\rightarrow A(Z+2,N-2)+2e^-+2\overline{\nu}_e \\ A(Z,N) &\rightarrow A(Z+2,N-2)+2e^- \end{aligned}$$

The first mode emits two anti-neutrinos and is allowed by the SM. However, the second mode violates the lepton number conservation by two units. As no neutrinos are produced this is known as the *Neutrinoless Double Beta Decay* or simply $0\nu2\beta$. With its violations of lepton number conservation, the discovery of $0\nu2\beta$ would imply that the neutrino is a Majorana fermion.

3.1 $0\nu2\beta$

$0\nu2\beta$ not only provides proof for Majorana fermions but also underpins many other aspects of physics, so experiments are trying to catch a glimpse of this elusive interaction. Isotopes that are good candidates for $0\nu2\beta$ are also liable to decay as $2\nu2\beta$ so the difficulty lies in distinguishing between the two decay processes.

3.2 Theory vs. Experiment

The discussion surrounding neutrino physics gives a concrete example of the interplay between theoretical and experimental physics. The success brought from the theoretical view of the SM led to the neutrino being thought of as massless. This was put in doubt with the observations made of atmospheric neutrinos. It was then necessary to find a new theoretical framework which is possibly given by neutrino oscillations and the seesaw mechanism. This in turn reopened the possibility of the Majorana neutrino leading to the need for more experimental data.