The Many Identities Involving π
Recreational Mathematics

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$$\sum_{n=1}^{\infty} \frac{1}{n\binom{2n}{n}} = \frac{\pi}{3\sqrt{3}}$$

$$\sum_{n=1}^{\infty} \frac{2n^{-2}}{\binom{2n}{n}} = \frac{\pi^2}{9}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(2n-1)(4n-3)} = \frac{\pi}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{(4n-1)(4n-3)} = \frac{\pi}{8}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(4n-1)(4n-3)} = \frac{\pi}{3} - \ln 2$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)(n+2)} = \frac{\pi^2}{12} - \frac{5}{8}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)^2} = \frac{\pi^2}{3} - 3$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3} = 10 - \pi^2$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)} = \frac{\pi^2}{6} - 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3(n+1)} = \zeta(3) + 1 - \frac{\pi^2}{6}$$

1 Generalization

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)^2} = 2 - \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)^3} = -\zeta(3) + 3 - \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)^4} = -\zeta(3) + 4 - \frac{\pi^2}{6} - \zeta(4)$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)^5} = -\zeta(3) + 5 - \frac{\pi^2}{6} - \zeta(4) - \zeta(5)$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)^m} = -\zeta(3) + m - \frac{\pi^2}{6} - \zeta(4) - \zeta(5) - \cdots - \zeta(m)$$