New Identities Involving π
Recreational Mathematics

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1 Part 1

$$\sum_{n=1}^{\infty} \frac{1}{n^2(2n+1)(2n-1)} = 2 - \frac{\pi^2}{6}$$

2 Part 2

$$\sum_{n=1}^{\infty} \frac{1}{n(2n+1)(4n+1)} = 6 - \pi - 4\ln 2$$

3 Part 3

$$\sum_{n=1}^{\infty} \frac{1}{n(4n-1)} = 3\ln 2 - \frac{\pi}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(4n-3)} = \ln 2 + \frac{\pi}{6}$$

4 Part 4

$$\sum_{n=1}^{\infty} \frac{1}{(4n+1)(2n-1)} = \frac{2}{3} - \frac{\pi}{12} - \frac{\ln 2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(4n-1)} = \frac{1}{3} - \frac{\pi}{12} + \frac{\ln 2}{6}$$

5 Part 5

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)(4n+1)(4n-1)} = \frac{\pi}{6} - \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{(6n+1)(6n-1)} = \frac{1}{2} - \frac{\sqrt{3}\pi}{12}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(6n-3)} = \frac{\pi^2}{24}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(4n-3)} = \frac{\pi}{4} + \frac{\ln 2}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{(4n-3)(4n+3)} = \frac{1}{24} + \frac{\pi}{24}$$