Traffic accident models and ideas for uncertainty quantification

UM-Bridge Workshop 2023

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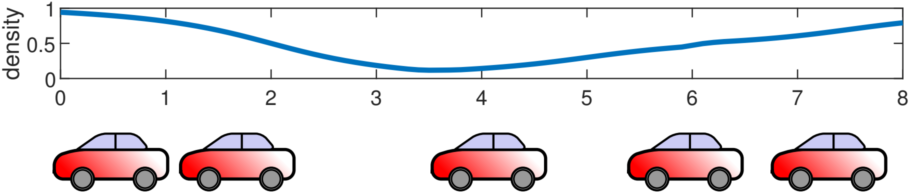
Traffic on a macroscopic scale



The traffic density $\rho : \mathbb{R} \times [0, T] \to [0, 1]$ is given by the solution to the hyperbolic conservation law

$$\rho_t + f(x, \rho)_x = 0, \qquad \rho(x, 0) = \rho_0(x), \tag{1}$$

where the LWR-type flux function f is given by

$$f(x, \rho) = c_{\text{road}}(x)\rho \left(1 - \frac{\rho}{\rho^{\max}}\right).$$

Solutions to the traffic model

We consider weak solution for test functions $\phi \in C_0^1((-\infty, T]) \times \mathbb{R})$

$$\int_0^T \int_{\mathbb{R}} \rho(x,t)\phi_t(x,t) + f(x,\rho)\phi_x(x,t)dxdt = \int_{\mathbb{R}} \rho(x,0)\phi(x,0)dx.$$

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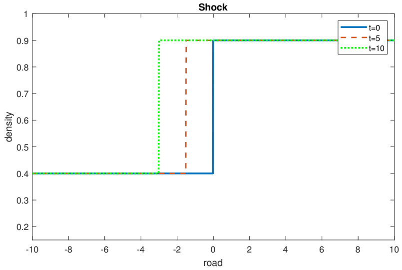
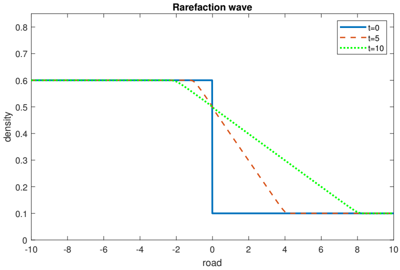
We observe two typical types of solutions:

Shock



Rarefaction wave



Example for an application: Traffic accidents

Model accidents as *capacity reductions* on the road using

- p_j: position of accident j
- s_j: size of accident j
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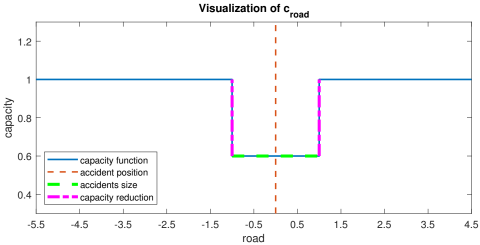
Consider one accident and define the *accident capacity function* c_{road} by

$$c_{\text{road}}(x) = 1 - c_1 \cdot \mathbb{1}_{\left[p_1 - \frac{s_1}{2}, p_1 + \frac{s_1}{2}\right]}(x)$$



How to choose the parameters?

One could assume different approaches for the parameters:

- constant or time-dependent parameters
- traffic flow dependent parameters
- density related parameters
- statistical models for parameters

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In the following we consider the special case in which:

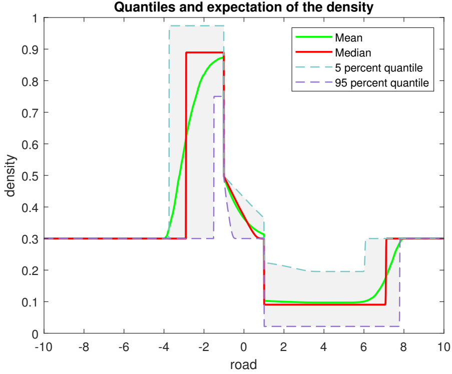
- there is exactly one accident
- with fixed parameters $p_1 = 0, s_1 = 2$
- c_1 is some uncertain quantity

How does the choice of $c = c_1$ affect the density evolution?

Expected density and quantiles of the density

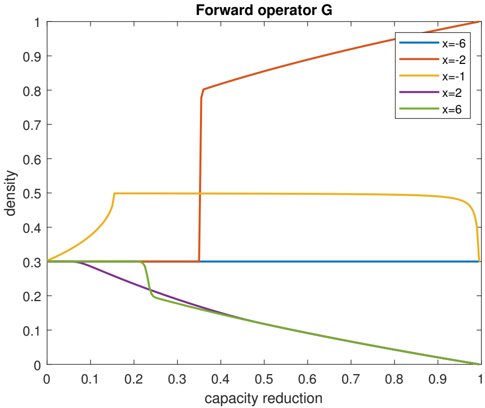
Assume $c \sim Beta(3,2)$. We are interested in

$\mathbb{E}[\rho(x,10)]$ *and* find $z \in [0,1]$ s.t. $P(\rho(x,10) \geq z) = q,\ q \in [0,1]$.



Forward problem

We consider the forward map: $\mathcal{G}_x : [0,1] \to [0,1]$ with $c \mapsto \rho_c(x, 10)$ for different choices of the observation position x:



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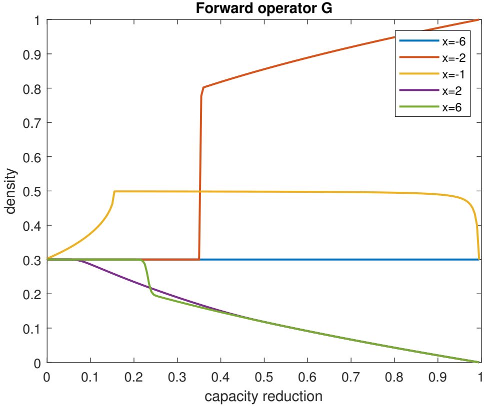

↯ ill-posed inverse problem

↯ forward operator may not even be Lipschitz

Inverse consideration of the problem

We are given information about the density profile at $T = 10$.

Is it possible

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a) to exactly reconstruct the value of c?
b) to provide a likelihood maximizing value of c given some prior information?
c) to provide a posterior distribution for c given some prior information?

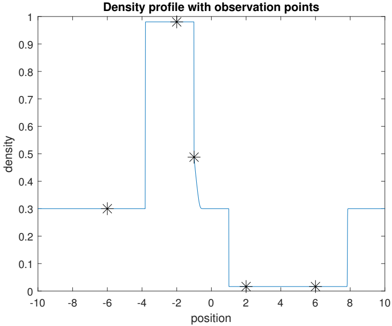

Figure 3: Density profile with $c = 0.92$.

c) A first *easy* problem

We are interested in the posterior measure for c under the following conditions:

- The prior of c is given by $\mu_0 \sim Beta(3, 2)$
- We assume only one observation point $(d = 1)$
- There is an observation noise with standard deviation $\sigma = 0.03$

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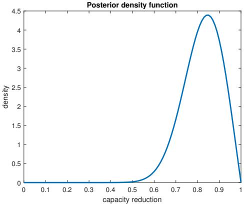
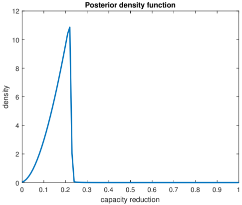
At $x = 6$ and time $t = 10$ there is an observation $y^1 = 0.0162$

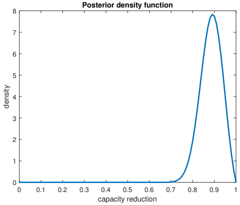


At $x = 6$ and time $t = 10$ there is an observation $y^2 = 0.3$

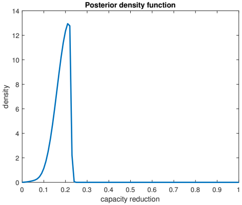


What if we increase the number of observations?

At $x = [-6, -2, -1.5, 2, 6]$ and time $t = 10$ there are observations $y^1 = [0.30, 0.98, 0.98, 0.02, 0.02]$



At $x = [-6, -2, -1.5, 2, 6]$ and time $t = 10$ there are observations $y^2 = [0.30, 0.30, 0.30, 0.24, 0.30]$



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$\rightsquigarrow$ A first hint that we might have done something reasonable.

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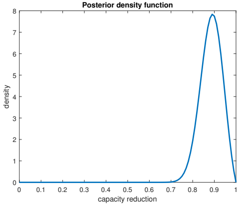

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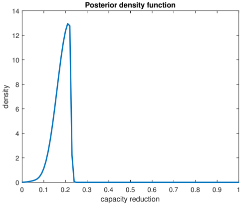

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BUT...

Problems and further ideas

- Interest in more efficient sampling techniques
- What if the dimension of the uncertain parameter space increases (e.g. also further accident parameters are uncertain)?
- How to choose the number and positions of observation locations efficiently?
- Considering a model hierarchy: Is there a connection between ML-estimators or posterior distributions at the different levels?

Thank you for your attention!