ECE 594 V

1. $P(n,k) = {}^{N}C_{k}\, p^{k} (1-p)^{N-k}$

(a) $\sum_{k=0}^{k=N} P(n,k) = \sum_{k=0}^{k=N} {}^{N}C_{k}\, p^{k} (1-p)^{N-k}$ — (1)

We know that binomial expansion of

$(a+b)^{N} = \sum_{k=0}^{N} {}^{N}C_{k}\, a^{k}\, b^{N-k}$ — (2)

Comparing 1 and 2,

$a = p$; $b = 1 - p$

$\therefore$ The summation $= (a+b)^N$

$= (p + 1 - p)^N$

$= 1^N = \underline{\underline{1}}$

(b) Mean of this distribution (Binomial) $= Np$

Variance $= Np(1-p)$ Mean $= Np$

For $N = 10$ Mean $= 5$ Variance $= 2.5$

$N = 100$ Mean $= 50$ Variance $= 25$

$N = 1000$ Mean $= 500$ Variance $= 250$

(c) $N = 10$ Mean $= 4.967$ Var $= 2.268$

$N = 100$ Mean $= 49.895$ Var $= 23.75$

$N = 1000$ Mean $= 499.74$ Var $= 247.72$

Comparing Results with part (b), we understand that our statistical results match with $N = 1000$ analytical calculation, hence confirming the validity of mean and variance formulas.

(Code provided along with this document)

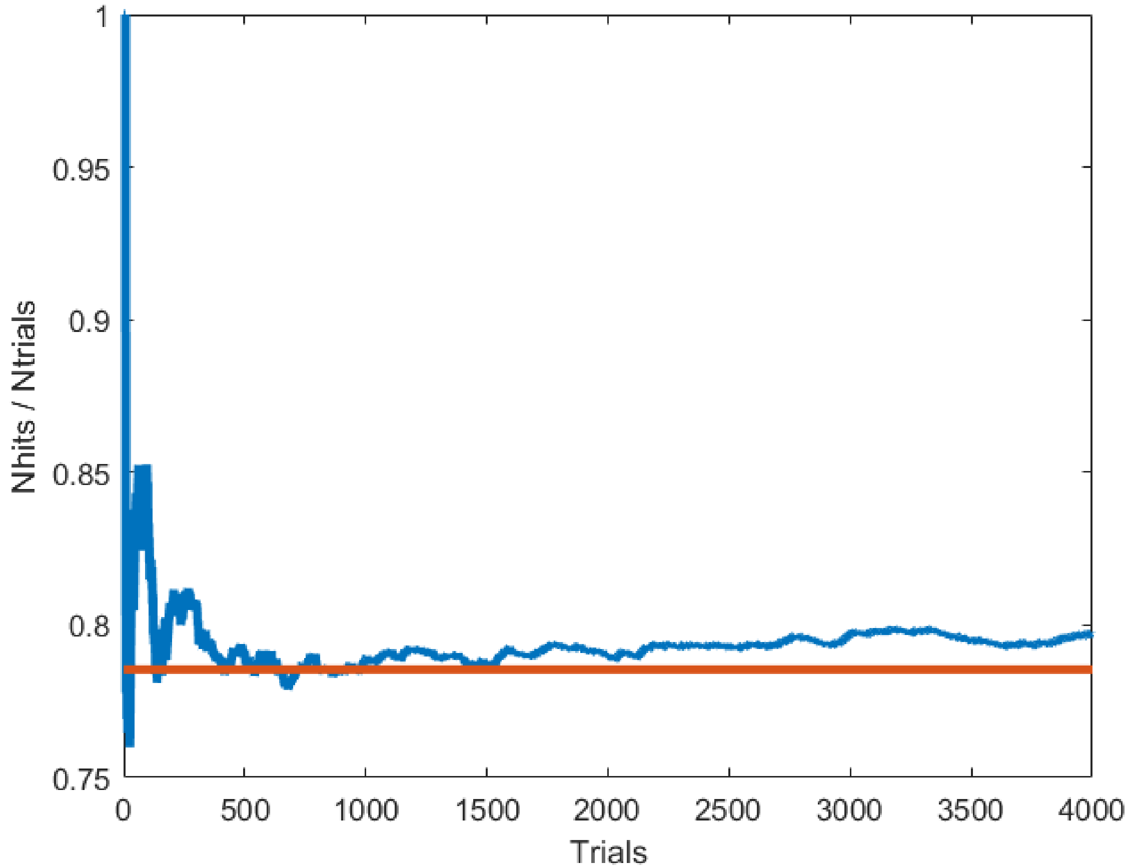
2. (a) Refer to the plot

(b) Mean $= p_{hit} = Np = 4000\left(\pi/4\right) = 3141$

Variance $= Np(1-p) = 4000\left(0.78\right)(1-0.78)$

$= 674.192$

σ (standard deviation) $= \sqrt{Var} = \underline{\underline{26}}$



Solution figure to problem 2 part (a)

(c) The code for the same is provided alongwith the document.

Enclosing	Gaussian	Chebyshev's	Actual
$[-\sigma, \sigma]$	32%	< 100%	30/100
$[-2\sigma, 2\sigma]$	5%	< 25%	7/100
$[3\sigma, 3\sigma]$	0.3%	< 11%	0/100

This indicates that the results are strictly compliant to Chebyshev inequalities as the case should be. On the other hand, it violates the integral confidence defined by Gaussian statistics, specifically the number of runs outside 2σ range exceed the Gaussian limit

Problem 3

(a) The plot is provided at the end

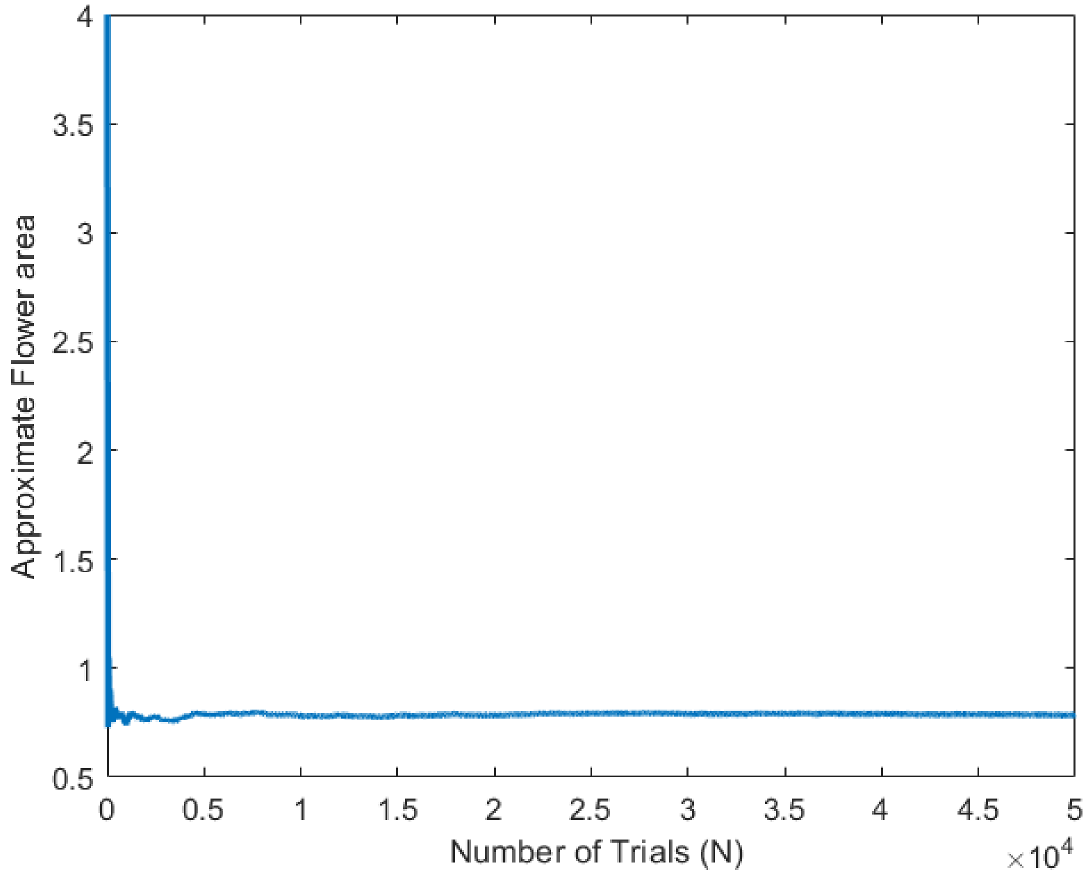
Area = 0.79

(b) Variance from numerical simulations,

$$\mathrm{Var}(N_{hits}) = 772.00 \qquad (N = 5000)$$

As each Bernoulli Trial is independent,

$$\mathrm{Var}(1 \text{ bernoulli trial}) = \frac{\mathrm{Var}(N_{hits})}{N}$$



Solution figure to problem3 part (a)

$$= \frac{772}{5000}$$

$$= 0.1544$$

(d) $\langle p \rangle$

$$Var = \langle p \rangle (1 - \langle p \rangle)$$

$$p - p^2 = 0.1544$$

$$p^2 - p + 0.1544 = 0$$

$$\langle p \rangle = \frac{1 - \sqrt{1 - 4(0.1544)}}{2} = \underline{\underline{0.1837}}$$

As $\langle p \rangle \leq 1$; we only took 1 value.

(e) As seen by the (a)

$$\frac{\text{Area of flower}}{\text{Area of square}} = \frac{0.79}{4} = 0.195$$

The $\langle p \rangle \equiv$ average probability to land in flower region of 1 trial

$$\simeq \frac{\text{Area of flower}}{\text{Area of square}}$$

The results are hence consistent, and equal $\therefore$ validating our model.

Also, with increased number of trials, i.e. 50000, it was checked that the $\langle p \rangle$ approaches 0.19, and the area is close to this value, hence

$N_{trials} \uparrow \implies \langle p \rangle$ better prediction.