CMSC 421 – Exam 2 Practice Problems

1. Probability & Bayes' Rule

Problem 1: Spam Filter
You know 20% of all emails are spam. A spam filter flags 95% of spam emails and 10% of legitimate emails.
What's the probability an email is actually spam if it was flagged?

Problem 2: Chain Rule
Given:
P(A) = 0.6, P(B|A) = 0.5, P(C|A,B) = 0.8
Find P(A, B, C)

2. Markov Chains

Problem 3: Predict the Weather
Transition matrix:
Sunny -> Sunny = 0.7, Sunny -> Rainy = 0.3
Rainy -> Sunny = 0.4, Rainy -> Rainy = 0.6
Start: Sunny
What's the probability it's Sunny in 2 days?

3. Hidden Markov Models

Problem 4: Simple HMM
States: Rainy, Sunny
Emissions: Walk, Shop, Clean
Initial: Rainy, Transitions: R->R=0.7, R->S=0.3
Emissions from Rainy: Walk=0.1, Shop=0.4, Clean=0.5
What's the probability of seeing "Shop" on Day 1 if we start in Rainy?

4. Filtering

Problem 5: HMM Filtering
Belief: Awake=0.6, Sleep=0.4
Transition: Sleep->Awake=0.2, Awake->Sleep=0.1
Emission: Awake emits 'quiet'=0.7, Sleep emits 'quiet'=0.2
After hearing 'quiet', what is your updated belief?

5. Game Theory

Problem 6: Dominant Strategy
Payoff matrix:
Up: (4,4), (1,5)
Down: (5,1), (2,2)
Does either player have a dominant strategy?

Problem 7: Mixed Strategy
Find B's mixed strategy that makes A indifferent.

Problem 8: Pareto Optimal?
Which outcomes in the matrix are Pareto optimal?

6. Incomplete Info Games

Problem 9: Auction Beliefs
Two types: value=$100 (60%), value=$150 (40%)
Your value = $120, second-price auction.
What should you bid?

7. Voting Theory

Problem 10: May's Theorem
Does majority rule satisfy anonymity, neutrality, and monotonicity?

Problem 11: Arrow's Theorem
Votes: 2 voters A>B>C, 2 voters B>C>A, 1 voter C>A>B
Is there a Condorcet winner?

Problem 12: Create a Condorcet cycle
Design 3 voter rankings so A>B, B>C, C>A

8. Challenge Problem

Problem 13: Mixed Strategy Nash Equilibrium
Payoff matrix:
Top: (2,3), (4,1)
Bottom: (1,2), (3,4)
Solve for both players' mixed strategies.