

# CMSC 421 – Exam 2 Practice Problems

## 1. Probability & Bayes' Rule

Problem 1: Spam Filter

You know 20% of all emails are spam. A spam filter flags 95% of spam emails and 10% of legitimate emails. What's the probability an email is actually spam if it was flagged?

Problem 2: Chain Rule

Given:

$P(A) = 0.6$ ,  $P(B|A) = 0.5$ ,  $P(C|A,B) = 0.8$

Find  $P(A, B, C)$

## 2. Markov Chains

Problem 3: Predict the Weather

Transition matrix:

Sunny  $\rightarrow$  Sunny = 0.7, Sunny  $\rightarrow$  Rainy = 0.3

Rainy  $\rightarrow$  Sunny = 0.4, Rainy  $\rightarrow$  Rainy = 0.6

Start: Sunny

What's the probability it's Sunny in 2 days?

## 3. Hidden Markov Models

Problem 4: Simple HMM

States: Rainy, Sunny

Emissions: Walk, Shop, Clean

Initial: Rainy, Transitions:  $R \rightarrow R = 0.7$ ,  $R \rightarrow S = 0.3$

Emissions from Rainy: Walk=0.1, Shop=0.4, Clean=0.5

What's the probability of seeing "Shop" on Day 1 if we start in Rainy?

## 4. Filtering

Problem 5: HMM Filtering

Belief: Awake=0.6, Sleep=0.4

Transition: Sleep $\rightarrow$ Awake=0.2, Awake $\rightarrow$ Sleep=0.1

Emission: Awake emits 'quiet'=0.7, Sleep emits 'quiet'=0.2

After hearing 'quiet', what is your updated belief?

## 5. Game Theory

Problem 6: Dominant Strategy

Payoff matrix:

Up: (4,4), (1,5)

Down: (5,1), (2,2)

Does either player have a dominant strategy?

Problem 7: Mixed Strategy

Find B's mixed strategy that makes A indifferent.

Problem 8: Pareto Optimal?

Which outcomes in the matrix are Pareto optimal?

## 6. Incomplete Info Games

Problem 9: Auction Beliefs

Two types: value=\$100 (60%), value=\$150 (40%)

Your value = \$120, second-price auction.

What should you bid?

## 7. Voting Theory

Problem 10: May's Theorem

Does majority rule satisfy anonymity, neutrality, and monotonicity?

Problem 11: Arrow's Theorem

Votes: 2 voters  $A > B > C$ , 2 voters  $B > C > A$ , 1 voter  $C > A > B$

Is there a Condorcet winner?

Problem 12: Create a Condorcet cycle

Design 3 voter rankings so  $A > B$ ,  $B > C$ ,  $C > A$

## 8. Challenge Problem

Problem 13: Mixed Strategy Nash Equilibrium

Payoff matrix:

Top: (2,3), (4,1)

Bottom: (1,2), (3,4)

Solve for both players' mixed strategies.