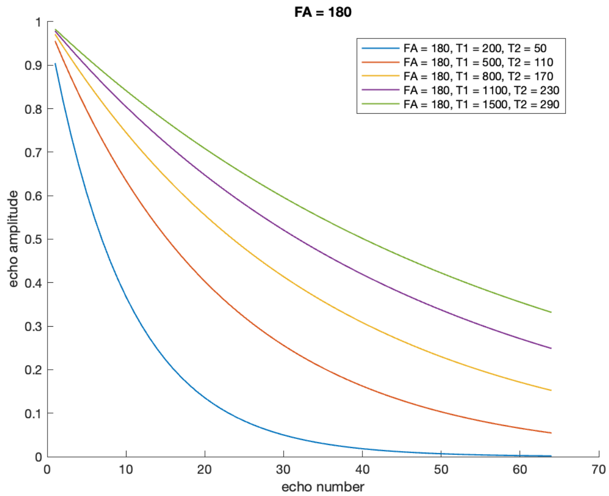
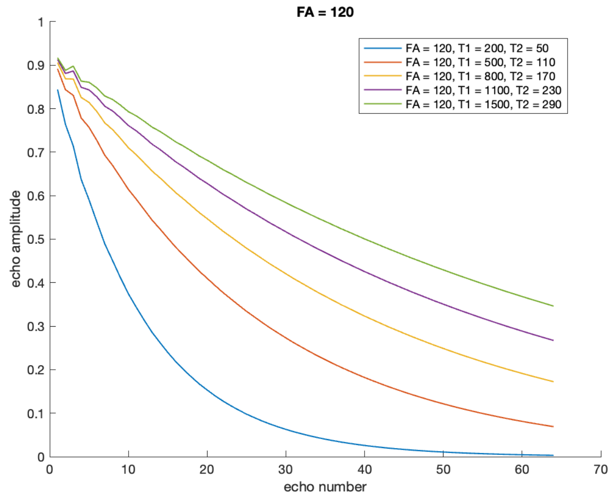
Pr.1

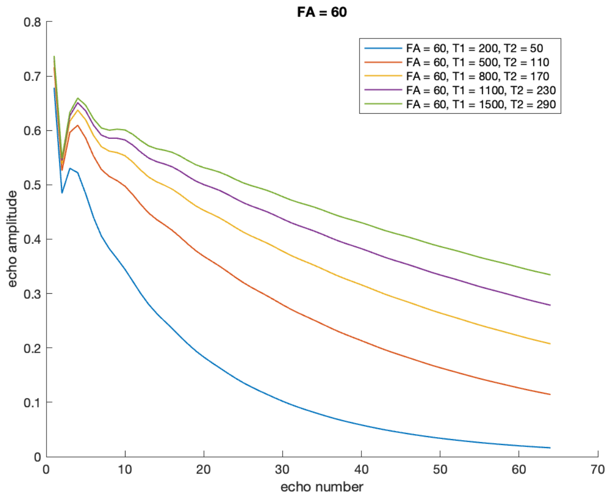
(a) i. For alpha = 180 degree, T1,T2 pair are chosen in a ramping up way from (200,50)ms to (1500, 290)ms





ii. For alpha = 120 degree, same relaxation parameters are chosen (above right hand side)

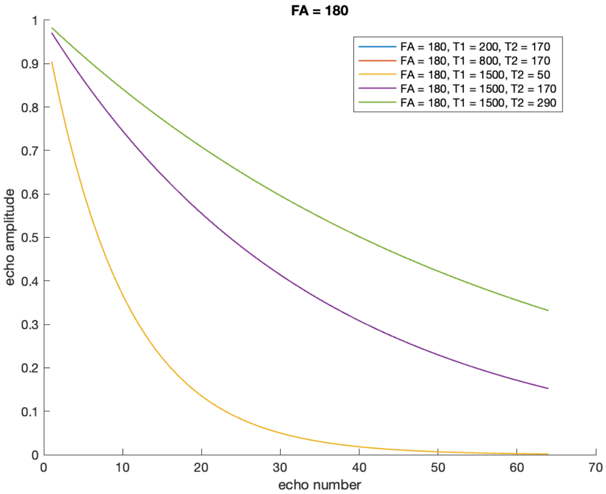
iii. For alpha = 60 degree, same relaxations parameters are chosen

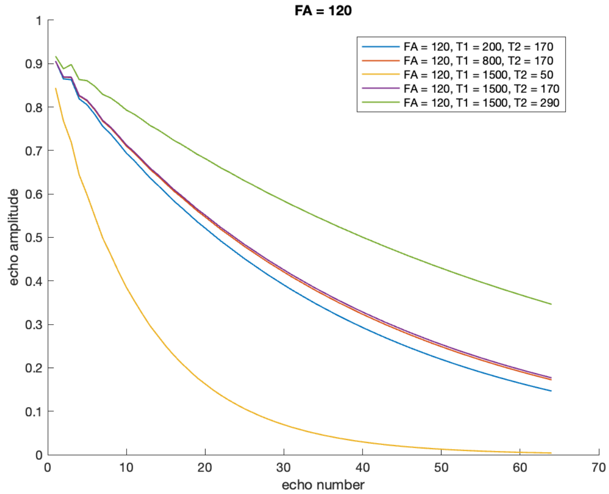


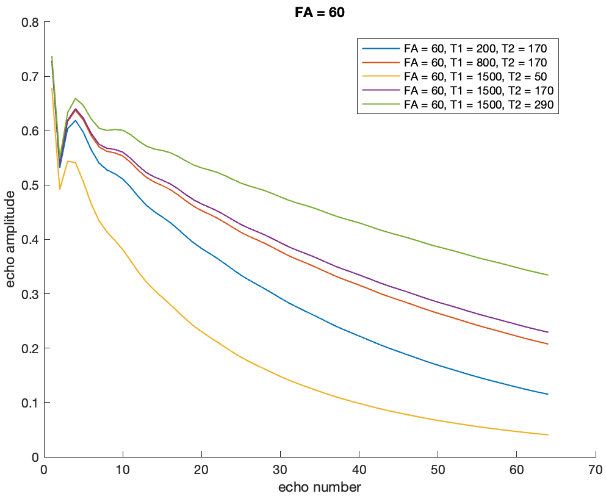
For i, ii, iii, it seems the echo amplitude increases along T1 and T2. We need to do another set of simulation to fix T1 and T2 separately and see their effects in the next page.

Codes used for plots here: HW2pr1.m, EPG_RF.m, EPG_grad.m, EPG_relax.m
uncomment for the T1 T2 value indexing for plots in pr1(a) section

For FA = 180, 120, 60 degree, choose a different set of T1, T2 value
(T1,T2) = [(200,170) (800,170) (1500,50) (1500,170) (1500,290)]ms
pair 1,2,4 are used for comparing T1 effect with constant T2, pair 3,4,5 are used for comparing T2 effect with constant T1







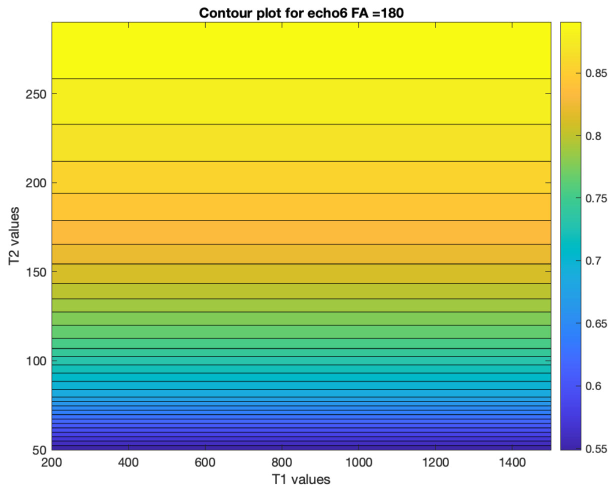
For FA = 180, the amplitude is only affected by T2 value as the lines of (T1,T2) = (200, 170), (800, 170), (1500, 170) coincides. And the amplitude increases as T2 increases as expected.

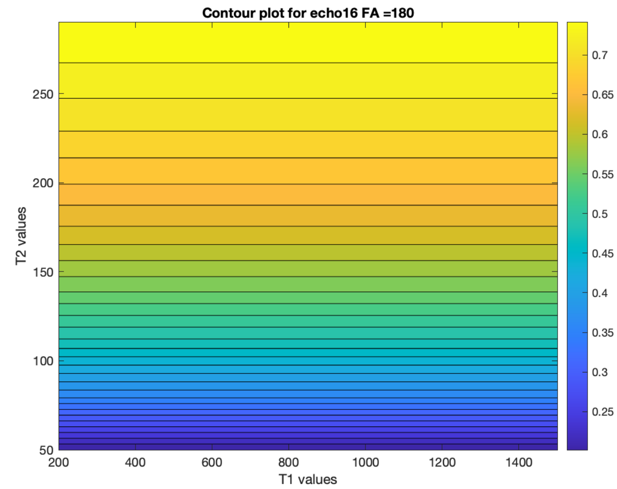
From FA = 120, 60, we can also see that apart from the T2 effect, for the same T2, the signal for the same echo is actually increasing with T1 value. This is because for echo train in TSE, within one TR, when the T1 is longer, the longitudinal magnetization is recovering more slowly, which means that more transverse magnetization (abs(F0)) is left at the same time point. Hence, there will be a larger signal amplitude for the same echo as T1 value increases.

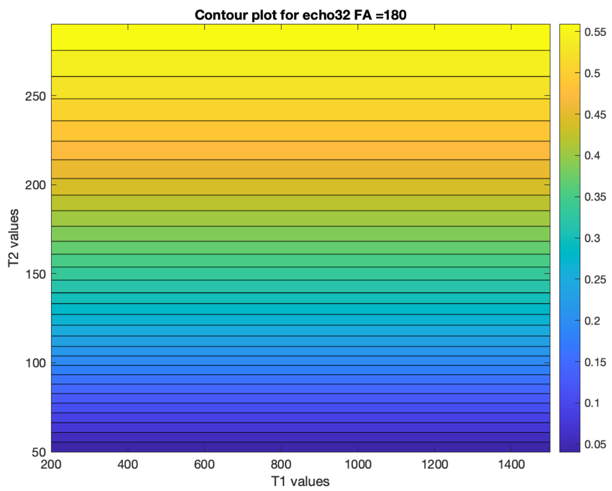
Files for generating plots: HW2pr1.m, EPG_RF.m, EPG_grad.m, EPG_relax.m

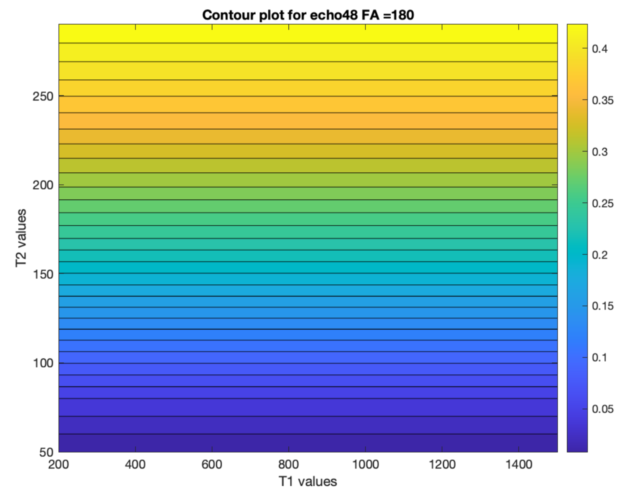
Pr1.(b)

Contour plots for FA = 180 degree, colorbar representing the echo amplitude





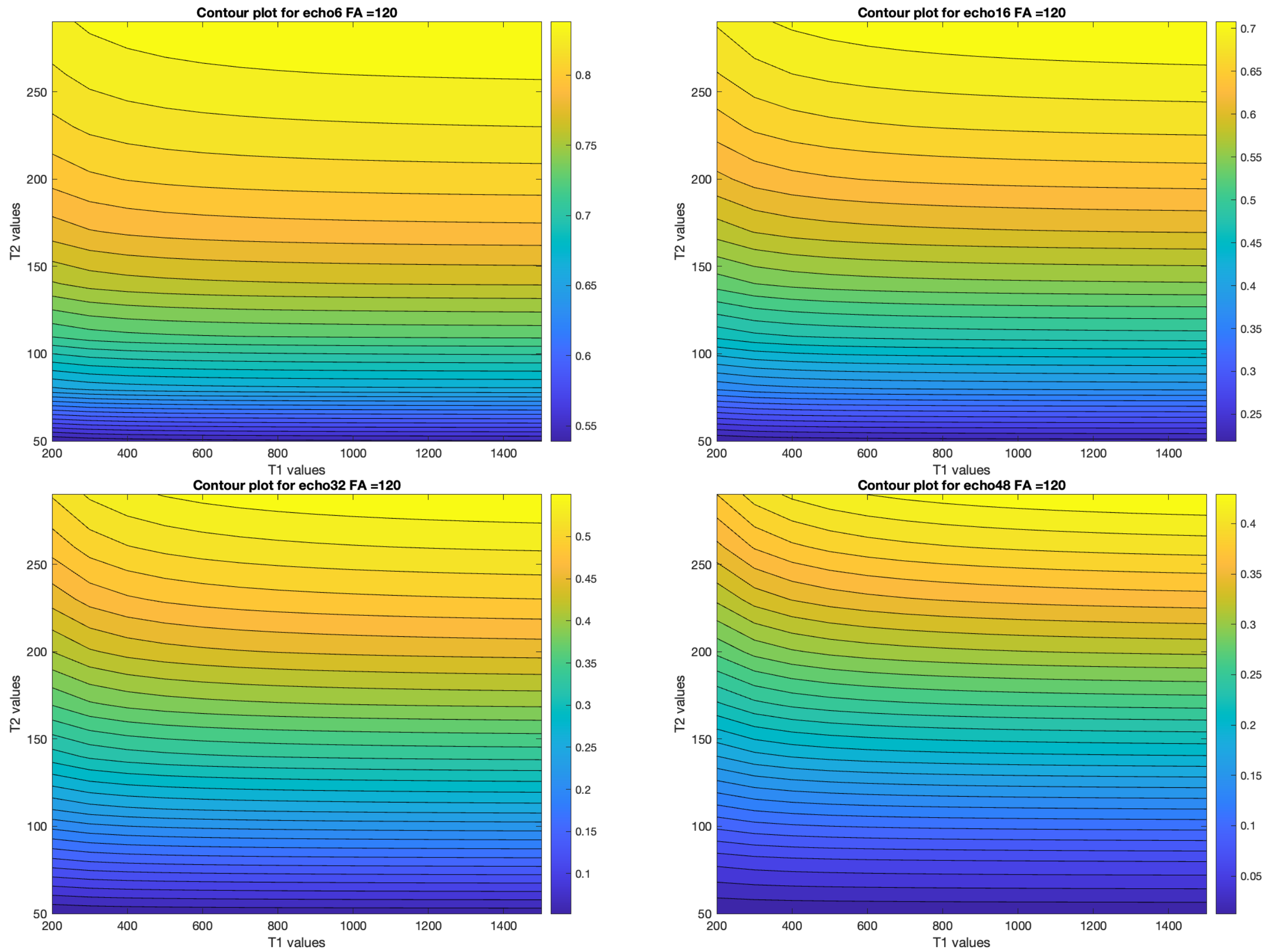




we can observe that for perfect 180 refocusing pulse, it's purely T2 weighted. And as the echo number increases, the echo amplitude is decreasing generally.

Files for generating plots: HW2pr1.m, EPG_RF.m, EPG_grad.m, EPG_relax.m

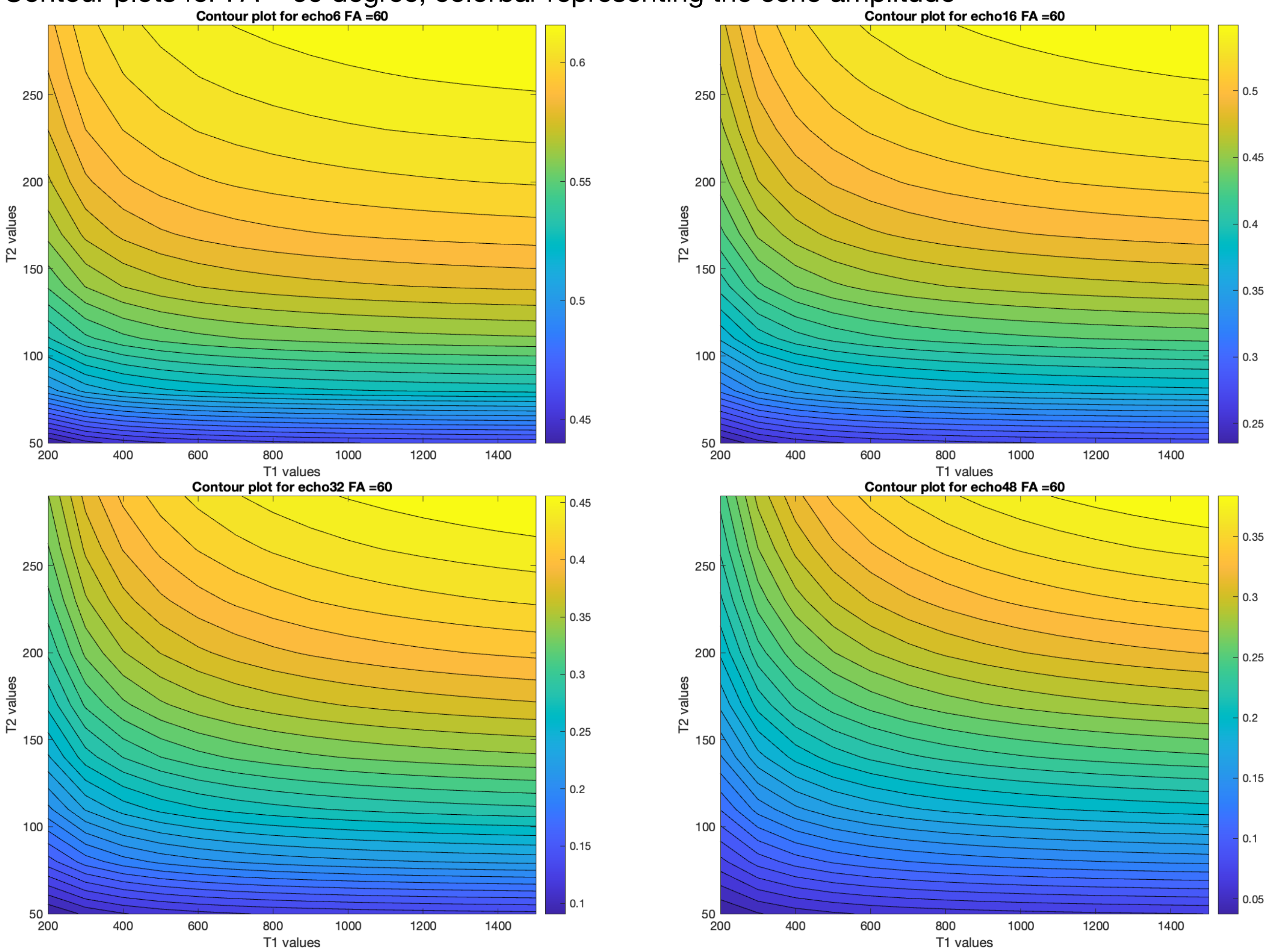
Contour plots for FA = 120 degree, colorbar representing the echo amplitude



Compared with perfect 180, the contrast here is not pure T2 now, but still predominately T2 weighted. Meanwhile, we can see the echo amplitude is slightly smaller compared to FA = 180

Files for generating plots: HW2pr1.m, EPG_RF.m, EPG_grad.m, EPG_relax.m

Contour plots for FA = 60 degree, colorbar representing the echo amplitude



We can see the contrast is even more both T1 and T2 weighted here. Meanwhile, the echo amplitude is the smallest among three flip angle generally.

Files for generating plots: HW2pr1.m, EPG_RF.m, EPG_grad.m, EPG_relax.m

Pr1.(c) From the plots of (a) and (b) we can see that Fast Spin Echo is indeed T2 weighted sequence. If perfect refocusing (FA = 180 degree) is achieved, in the contour plots we can see the contour lines are perpendicular to the axis of T2 and parallel to the axis of T1. It means its echo amplitude is only affected by T2.

As the flip angles decreases, the contrast is gradually showing more T1 effect but still dominated by T2.

Pr2.(a)
Simulation of single-echo spin echo sequence:

Since we acquire one k-space line each echo (TR) usually, to generate contrast weighted image from these maps, we need to simulate 256 echoes (as the image size is 256x256), and for each echo simulation, we need to simulate the magnetization for each pixel, and use FT to transform into k-space, then acquire the nth k-space line from the nth echo.

After that, we combine the 256 k-space line from 256 echoes together form a full acquired k-space, and use IFT to transform it back to image domain and get the contrast weighted image.

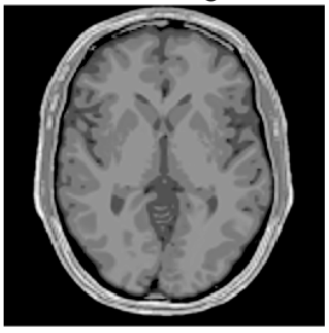
And for different contrast weighted images, we just need to choose proper TE and TR.

For T1 weighted image, use short TR and short TE since we want to enhance T1 difference by short TR and minimize T2 difference by short TE.
TR = 500ms, TE = 15ms

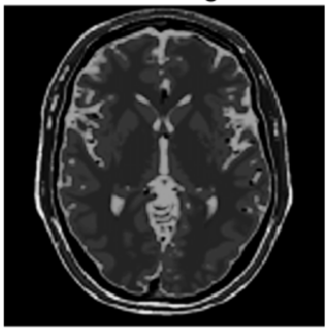
For T2 weighted image, use long TR and long TE since we want to enhance T2 difference by long TE and minimize T1 difference by long TR.
TR = 3000ms, TE = 100ms

For PD weighted image, use long TR and short TE since we want to minimize T1 difference by long TR and minimize T2 difference by short TE.

T1w image

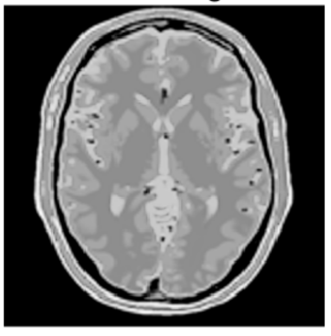


T2w image

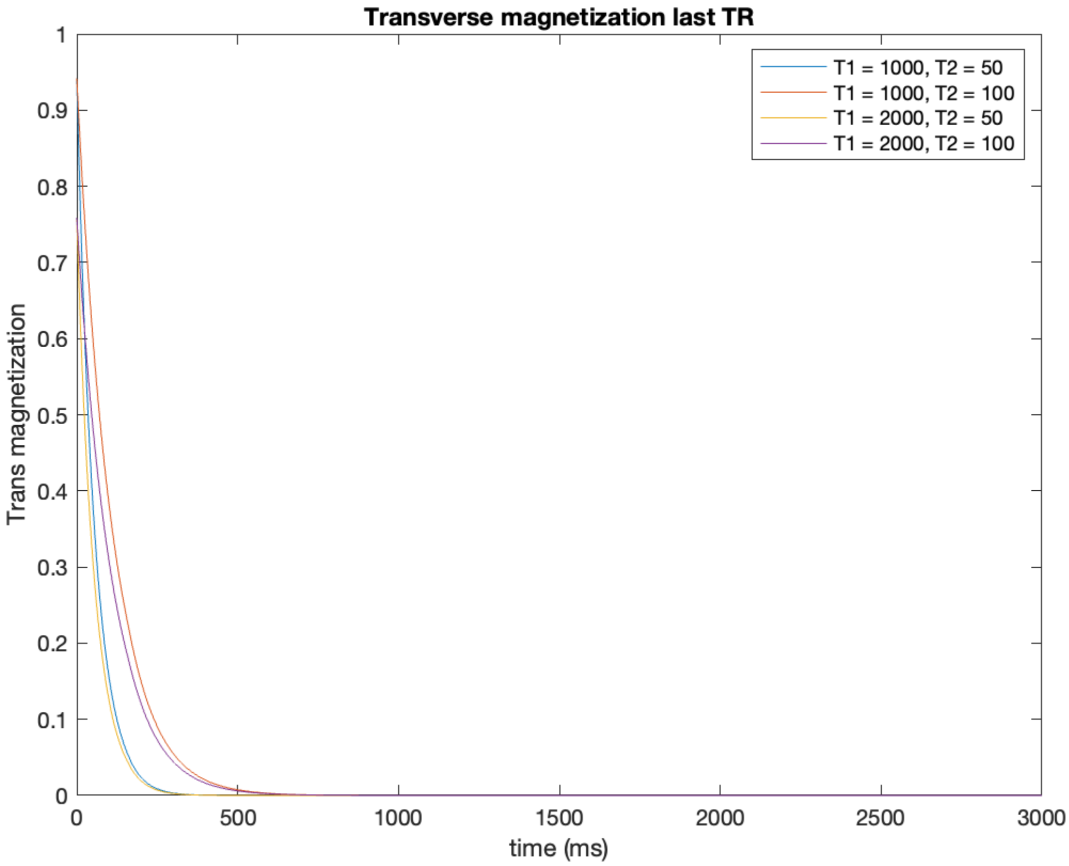


Code used for this part:
HW2pr2.m, EPG_single_echo_SE.m
EPG_grad/relax/RF.m

PDw image

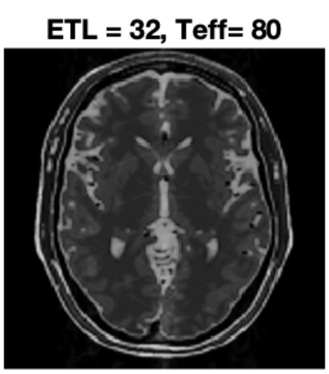


Pr2.(b) i. Code used for generate this plot: HW2pr2.m, EPG_FSE.m, EPG_grad/relax/RF.m



From the plots, it is observed that for the same T1, smaller T2 makes decay faster.
For the same T2, smaller T1 leads to larger signal magnitude.

Pr2.(b).ii. The image:



Total acquisition time:

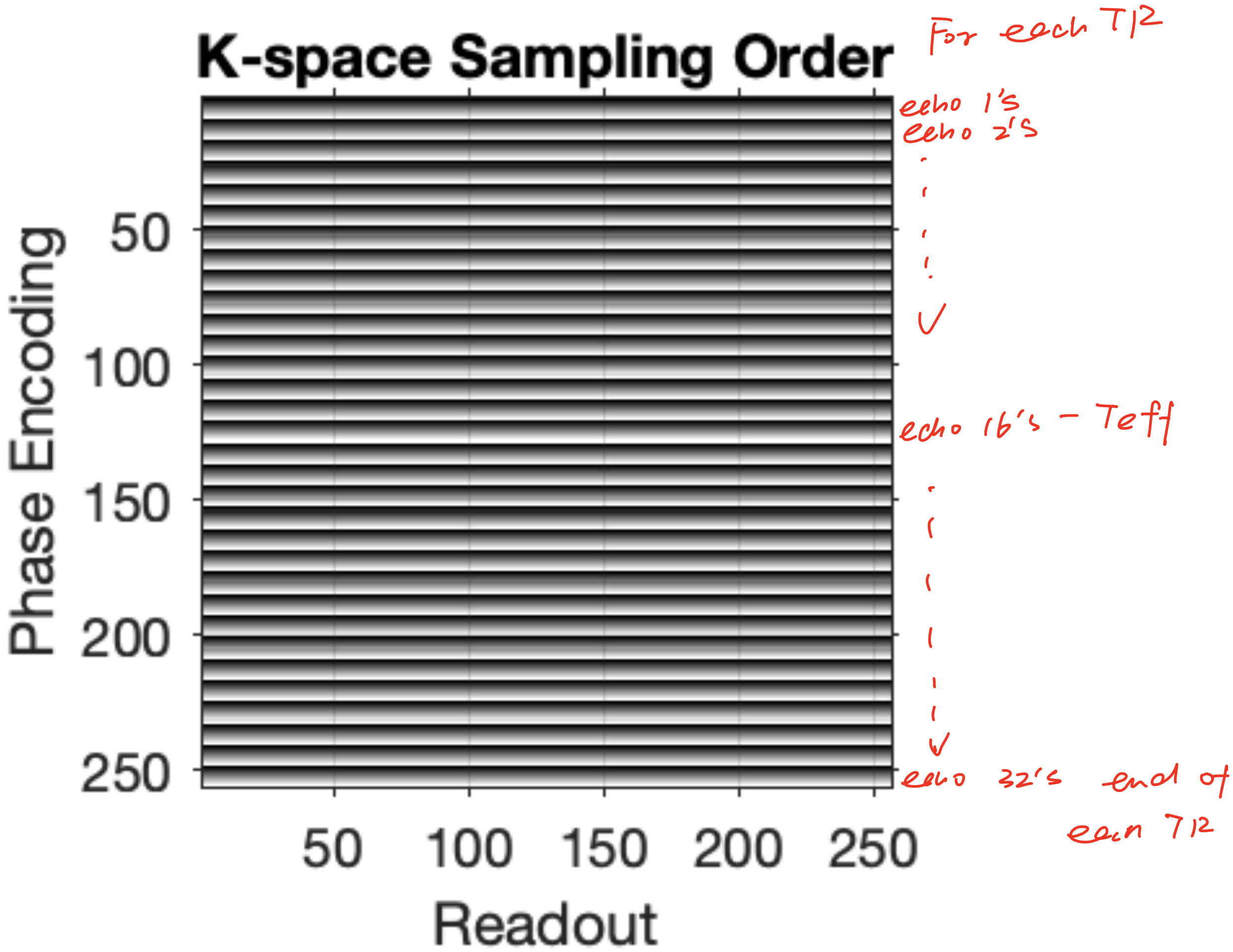
TR x number of TR = 3 x (256/32) = 24s

Acquisition time for single-echo spin-echo:

3 x 256 = 768s

The FSE is way faster compared to single echo spin echo, this is controlled by the ETL.

K-space sampling order is plotted in the gray scale image:



With the lines of the same gray scale sampled at same TR. (line n, n+8, n+16, ... at the same TR)

As we have 8 TR (256/ETL), and since Teff = 80ms, which means the 16th echo should be sampling the k-space enter, we perform phase-encoding interleaving here.

For example, for the first TR, line 1:8:256 is acquired, and the 16th echo is line 121 which are at the center region of the k-space.

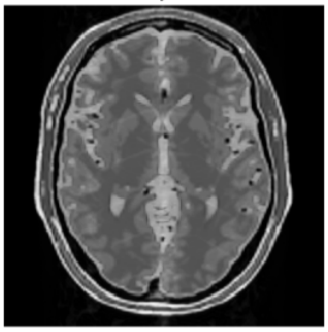
Thus, for the nth TR, line n:8:256 is acquired, make sure the Teff echo is at the k-space center.

And the first echo of each TR always start from the top group (line 1-8).

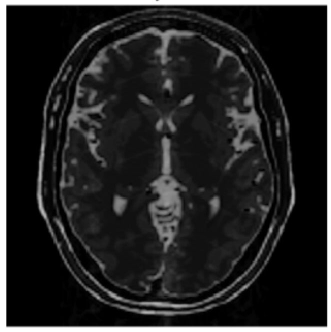
Codes used for this part: HW2pr2.m, EPG_FSE.m

Pr2.(b) iii.

ETL = 32, Teff= 40



ETL = 32, Teff= 120



For the k-space filling order, it's just applying circular shift of

Teff / ESP - ETL / 2 (e.g. (40 / 5 - 16 = -8)

times. Negative means left circular shift, positive means right circular shift.

For example, at Teff = 80ms, for the first TR, we are sampling line 1, 9, 17, ..., 121 (16th echo), ..., 249.

Now at Teff = 40ms, we want the 8th echo to be at the k-space center where the 16th echo is located at Teff = 80ms, we just need to left circular shift the original line sampling order for each TR with 8 times. Hence, now we are sampling line 65, 73, ..., 121 (8th echo), ..., 249, 1, 9, ..., 57.

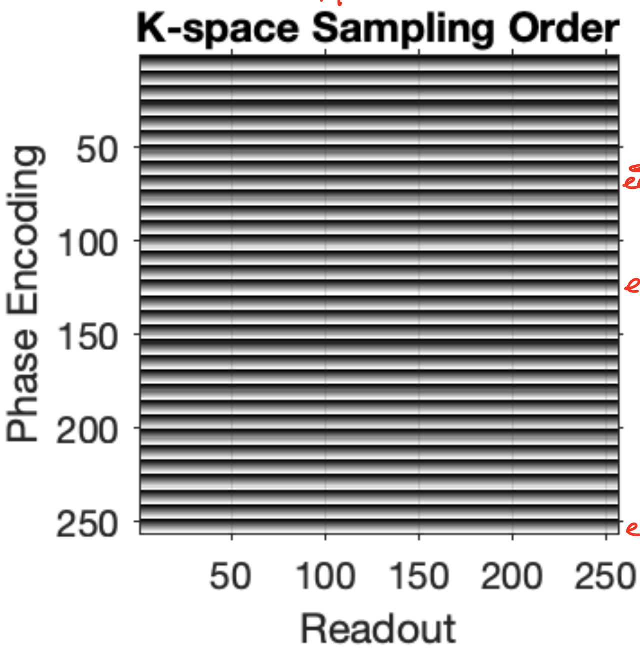
Similar for Teff = 120ms, the only change is that now we right circular shift for 8 times.

As for the k-space filling diagram, it's still the same, except the start echo for each TR is located at the 9th group (Teff=40ms) or the 25th group (Teff = 120ms).

The codes used to generate this part: HW2pr2.m
For different Teff, just change line 241 in the script to the Teff we want.
As for the k-space filling order plot, I'm just copy-pasting the plots from the last question and do annotation by hand :).

Teff = 40 ms   the 8th echo at center

for each TR



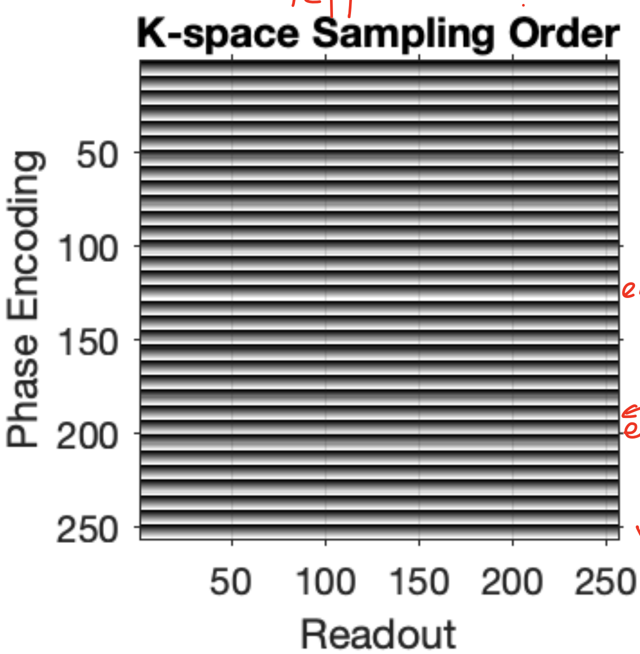
echo 32's last sampled in each TR.

echo 1's

echo 8's — Teff at k-space center

echo 24's

Teff = 120 ms.   The 24th echo at center.



echo 24's

echo 32's end of each TR

echo 1's ← At begining of each TR.

Pr2.(b).iv.
As Teff = 80ms, thus the 16th echo is always the center of k-space.

By doing phase-encoding interleaving, no matter how long ETL is, we can always define lines to acquire at each TR, given by

for the nth TR: lines_to_acquire = n:(256/ETL):256
total_num_of_TR: 256/ETL

Note: echo order is fixed, we are shifting the vector of lines.

The shift numbers of the lines_to_acquire vector is given by

Teff / ESP - ETL / 2 = 16 - ETL / 2

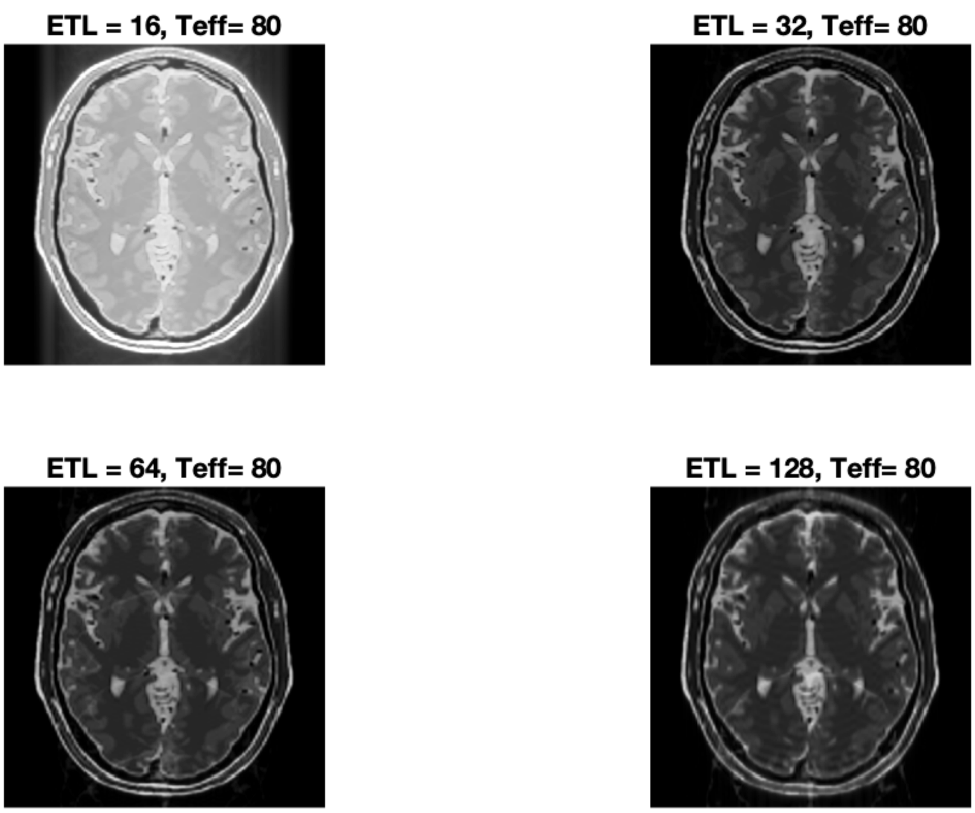
Explanation: we want the k-space center to be acquired at the 16th echo, and within each TR, we acquired ETL number of k-space line. Thus, the (ETL/2)th k-space line will need to be move to the 16th echo, which gives this equation.

For ETL = 16, the 16th echo is the k-space center: circshift(lines_to_acquire, 8)

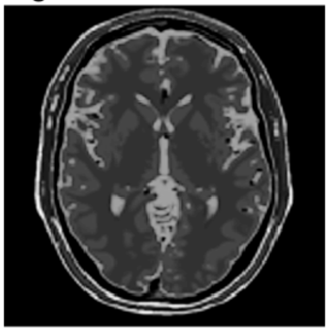
For ETL = 64, the 16th echo is the k-space center: circshift(lines_to_acquire, -16)

For ETL = 128, the 16th echo is the k-space center: circshift(lines_to_acquire, -48)

Codes used to generate plot here: HW2pr2.m, EPG_FSE.m
For different ETL, just change ETL value on line 283.



Single echo SE TE = 80 ms



When ETL get longer, the image get more blurry the signal magnitude is getting smaller overall. This is because as ETL increases, more and more k-space lines are acquired at echoes with more decay. For instance, the center of k-space is always acquired at the 16th echo due to fixed Teff. But as ETL increases, more lines are distributed to echoes larger than 32 or even 64, which have been decayed vastly as shown in part i.

Pr2.c. With very long ETL, we can perform various flip angle to compensate T2 decay.