Q1. a. $B_0 = 1.5T$ $\quad G_{max} = 25\,mT/m$ $\quad \tau_{RF} = 1ms$

$$TBW = 2 = \tau_{RF} \cdot BW = 10^{-3}s \cdot BW$$

$BW = 2\times10^{3}Hz$ $\quad \Delta z$ = Slice thickness = 3.0mm

Thus, based on Larmour equation, we know

$$BW = \gamma G_{ss} \cdot \Delta z \Rightarrow G_{ss} = \frac{2\times10^{3}Hz}{3\times10^{-3}m \cdot 42.58\times10^{6}Hz/T} = 0.0157\,T/m = 15.7\,mT/m$$

The slice selective gradiant has an amplitude

$$G_{ss} = 15.7\,mT/m$$

The duration of the gradiant assuming max slew rate $180T/m/s$

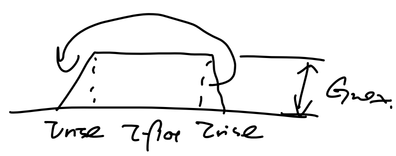
$$\text{Total time} = 2\tau_{ss}^{rise} + \tau_{ss} \qquad \tau_{ss} = \tau_{RF} = 1ms$$

$$= 2 \cdot \frac{G_{ss}}{\text{Slew rate}} + 1\times10^{-3}s$$

$$= 2\times\frac{15.7\times10^{-3}T/m}{180T/m/s} + 10^{-3}s$$

$$= 0.174\,ms + 1ms = 1.174ms$$

Q1. b $\quad \Delta y = \frac{1}{2k_{y\,max}} = 1.2\,mm \Rightarrow k_{y,max} = \frac{1}{2.4}\,mm^{-1}$



$$k_{y,max} = \int_0^{\tau_{PE}} \gamma G_{PE}\, t\, dt$$

Since we have ramping up and ramping down, we can decompose into three parts

$$k_{y,max} = 2\int_0^{\tau_{PE}^{rise}} \gamma G_{PE}(t)\,dt + \gamma G_{PE}\,\tau_{PE\text{-}flat} \qquad \tau_{PE}^{rise} = \frac{G_{max}}{\text{slew rate}} = \frac{25mT/m}{180T/m/s} = 1.39\times10^{-4}s$$

$$= \tau_{PE}^{rise} \cdot \gamma G_{ymax} + \tau_{PE\text{-}flat}\,\gamma G_{ymax}$$

Solve for $\tau_{PE\text{-}flat}$:

$$\frac{1}{2.4\,mm} = 1.39\times10^{-4}\; 42.58\times10^{6} \times 25\times10^{-3} + \tau_{PE\text{-}flat} \cdot 42.58\times10^{6} \cdot 25\times10^{-3}$$

$$\Rightarrow \tau_{PE\text{-}flat} = 2.52\times10^{-4}\,s$$

Thus the shortest possible duration of PE gradiant is

$$\tau_{PE} = 2\times\tau_{PE\text{-}rise} + \tau_{PE\text{-}flat} = 5.3\times10^{-4}s = 0.53\,ms$$

Q1.c. The receiver bandwidth is given by

$$rBW = \gamma \cdot G_{read} \cdot FOV_x = 42.58\times10^{6}\ Hz/T \cdot G_x \cdot 256\times1.2\times10^{-3} m$$

$$\Rightarrow rBW/pixel = 42.58\times10^{6}\ Hz/T \cdot G_x \cdot 1.2\times10^{-3}\ m = 750\ Hz/pixel$$

$$\Rightarrow G_x = 14.7\ mT/m$$

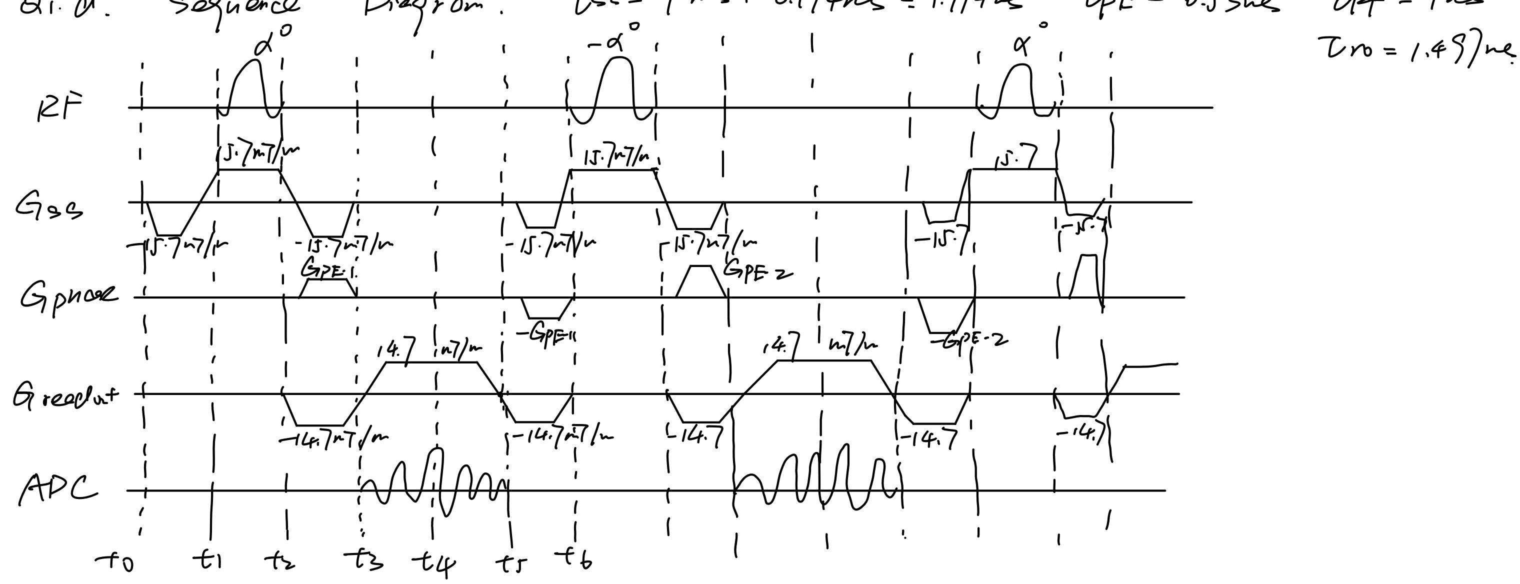
The total time duration is

$$T_{readout} = 2\cdot T_{ro\text{-}rise} + T_{sampling} = 2\cdot\frac{14.7\ mT/m}{180\ T/m/s} + \frac{1}{rBW/pixel}$$

$$= 1.63\times10^{-4}\ s + \frac{1}{750}\ s = 0.163\times10^{-3}\ s + 1.333\times10^{-3}\ s$$

$$= 1.497\ ms$$

Q1.d. Sequence Diagram. $T_{ss} = 1\ ms + 0.174\ ms = 1.174\ ms$ $T_{PE} = 0.53\ ms$ $T_{RF} = 1\ ms$ $T_{ro} = 1.497\ ms$



We know $T_{ro} = 1.497\ s$ thus the time duration for it's prephasing gradient $T_{ro\text{-}pre} = \frac{1.497}{2}\ s > \frac{1.178}{2} + \frac{0.178}{2}\ s$ = the rephasing gradient of Gss + ramping down time of Gss $= t_3 - t_2$

Similarly, between t5 and t6, since Gro should not be overlapped, thus t5 should be the start time of Gro dephasing Gradient, whose time is bigger than T_{PE} and $\frac{T_{ss}}{2} + T_{ss\text{-}rise}$

thus t6 should be the time of the end of Gro dephasing and also the start of next RF pulse.

All the Gradient amplitude are marked on the diagram except phase-encoding gradient. We can calculate here

For the first phase-encoding line. (assume τ_{PE} is all the same)

$$k_y = \frac{k_{y\,max}}{128} = \frac{\frac{1}{2.4}\,mm^{-1}}{128} = \int_0^{\tau_{PE}} \gamma\, G_{PE1} t\, dt.$$

$$\Leftrightarrow \frac{1}{2.4\times128}\,mm^{-1} = \underbrace{2.52\times10^{-4}s}_{\tau_{PE-flat}} \times G_y \times 42.58\times10^{6}\,Hz/T + \underbrace{2\times1.39\times10^{-4}}_{2\times\tau_{PE-rise}} \times G_y \times \gamma \times \frac{1}{2}$$

$$\Rightarrow G_{PE-1} = \frac{G_{y\,max}}{128} = 1.46\times10^{-4}\,T/m = 0.196\,mT/m.$$

Similarly $G_{PE-2} = G_{y\,max} \times \frac{2}{128} = 0.39\,mT/m$.

Here is the list for all the timings ($t_0 - t_6$) relevant to first TR:

$t_0 = 0s$ $\quad$ $t_1 = \frac{\tau_{ss-total}}{2} + \tau_{ss-rise} = \frac{1.178}{2} + \frac{0.178}{2} = 0.678\,ms.$

$t_2 = t_1 + \tau_{PE} = 1.678\,ms$ $\quad$ $t_3 = t_2 + (t_1 - t_0) = 1.678 + 0.678 = 2.356\,ms$

$t_4 = t_2 + \frac{\tau_{ro}}{2} + \frac{\tau_{ro}}{2} = 1.678 + 1.497 = 3.175\,ms$

$t_5 = t_4 + \frac{\tau_{ro}}{2} = 3.175 + \frac{1.497}{2} = 3.924\,ms$

$t_6 = t_4 + \tau_{ro} = 4.672\,ms$

Thus, the shortest possible TE and TR are.

$TR = t_6 - t_1 = 4.672 - 0.678 = 3.993\,ms.$

$TE = t_4 - \frac{t_2 + t_1}{2} = 3.175 - 1.178 = 1.997\,ms$

We can also see $TE = \frac{TR}{2}$.

Q1. e. 1. Change from Cartesian Sapling to other sapling pattern like radial sapling.

2. Use a shorter RF pulse.

3. Use method like partial Fourier in readout direction to minimize TR and TE.

Q2. a. See the codes and plots.

b. i. Max magnitude : 0.065765.

ii See the codes and plots.

iii See the codes and plots.

Based on the simulation, the best RF phase is about 59°.

Q3.1. TBW = 8 $G_{ss-max} = 25$ mT/m Max slew rate = 180 mT/m/ms.

$\Delta z = 5$ mm $T_{RF} = 2$ ms

Thus bandwidth of the pulse (main lobe)

$$BW = \frac{TBW}{T_{RF}} = 4 \text{ kHz}.$$

number of zero-crossings should be equal to TBW.

For Gss,

$$BW = \gamma \, G_{ss} \cdot \Delta z.$$

$$4 \times 10^{3} = 42.58 \times 10^{6} \cdot G_{ss} \cdot 5 \times 10^{-3}$$

$$\Rightarrow G_{ss} = 18.8 \text{ mT/m}$$

Take the slew rate = 180 mT/m/ms.

$$T_{rise} = \frac{G_{ss}}{\text{Slew rate}} = \frac{18.8 \text{ mT/m}}{180 \text{ mT/m/ms}} = 0.104 \text{ ms}$$

$$T_{total} = 2 \text{ ms} + 2 \times 0.104 \text{ ms} = 2.208 \text{ ms}$$

For the rephasing Gradient, it's essentially flip the sign of Gss and make sure the area of it is half of the Slice-selective gradient.

Q3.2 $FA = \frac{\pi}{2} = \int_0^{T_{RF}} \gamma \, B_1(t) \, dt$

Results see codes and plots.

Q3.3. Results of Slice profile in Plots folder.

The Bloch Simulation and FT have Similar magnitude in Slice profile. but the slice profile of Bloch simulation is wider.

For $T_2 = 2$ ms, the FT have much higher magnetization amplitude, as T_2 is very short, the T_2 relaxation happened very quickly => $|M_{xy}|$ decay rapidly.

Q3.4. Plot see plot folder Q3

From the plot we can see. if we don't use slice rephasing gradient, although M_{xy} and M_z don't change. in transverse plane, M_x and M_y dephased very badly due to T_2.

The purpose of slice rephasing gradient is to rephase the transverse magnetization.

Q3.5. To do SMS, we just need to add a phase modulation for each slice.

Slice thickness = 5 mm Gap = 20 mm

For 5 mm slice. BW = 4×10^3 Hz.

Slice periodicity = 5 + 20 = 25 mm.

Thus the carrier frequency is

$\Delta f = \gamma G_{ss} \cdot$ periodicity $= 2 \times 10^4$ Hz.

Thus the phase modulation is given by

$$\{ e^{i2\pi \Delta f_1 t} \quad e^{i2\pi \Delta f_2 t} \quad e^{i2\pi \Delta f_3 t} \quad e^{i2\pi \Delta f_4 t} \quad e^{i2\pi \Delta f_5 t} \}$$

$\Delta f_1 = -2\Delta f \quad \Delta f_2 = -\Delta f \quad \Delta f_3 = 0 \quad \Delta f_4 = \Delta f \quad \Delta f_5 = 2\Delta f.$

Simulation see plots and codes.