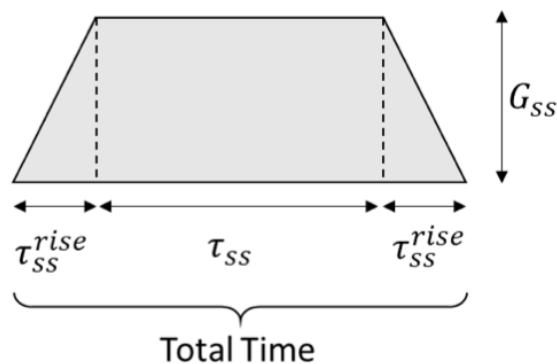


Problem 1: Spatial Encoding

Suppose you want to collect a 2D bSSFP image at a spatial resolution of $1.2 \times 1.2 \times 3.0 \text{ mm}^3$ with 256×256 matrix size. The receiver bandwidth is 750 Hz / pixel . You are imaging on a 1.5T scanner with maximum gradient amplitude 25 mT/m (for one axis) and maximum slew rate of 180 T/m/s . In the following problems, you will design gradients for slice-selection, phase encoding, and frequency encoding and calculate the shortest possible TR and TE.

1a. You decide to use a sinc-shaped RF pulse with time bandwidth product 2 and a duration of 1 ms . Calculate the amplitude and total duration of the slice-select gradient. Note that the total duration includes both the flat-top time (τ_{ss}) and the time needed to ramp-up and ramp-down the gradient (τ_{ss}^{rise}), as shown in the figure below.



$$1a) \quad TBW = 2$$

$$T_{rf} = 1 \text{ ms}$$

$$BW = 750 \text{ Hz / pixel}$$

$$N_{\text{pix}} = 256$$

$$TBW = T_{rf} (BW) = T_{rf} (\Delta f_{tx})$$

$$\Delta f_{tx} = \gamma G \Delta z$$

$$G = \frac{\Delta f_{tx}}{\gamma \Delta z} = \frac{TBW}{T_{rf} \gamma \Delta z}$$

$$= \frac{2}{0.001 \text{ s} \left(\frac{42.577 \text{ MHz}}{T} \right) (3 \text{ mm})}$$

$$= \frac{2}{0.001 \cancel{\text{s}} \left(\frac{42.577 (10^6) \cancel{\text{Hz}}}{T} \right) (0.003 \text{ m})}$$

$$= 0.0157 \text{ T/m}$$

$$= \boxed{15.7 \text{ mT/m}} \quad \text{gradient amplitude}$$

need T_{ss} and $T_{ss \text{ rise}}$

slew rate 180 T/m/s

want get up to 0.0157 T/m

$$\frac{0.0157 \text{ T/m}}{180 \text{ T/m/s}} = 8.72 (10^{-5}) \text{ s}$$

$$= 0.0000872 \text{ s}$$

$$= 0.0872 \text{ ms} = T_{ss \text{ rise}}$$

$$1 \text{ ms} + 2 (0.0872 \text{ ms}) = \boxed{1.1744 \text{ ms}}$$

total duration of SS gradient

1b. Calculate the shortest possible duration of the phase encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

$$K_{max} = \frac{1}{2\Delta y} = \frac{1}{2(1.2 \text{ mm})} = \frac{1}{2.4 \text{ mm}} = 0.4167 \text{ mm}^{-1}$$

$$K_y = \gamma \int \frac{1}{2} (\gamma_{pe} + 2\gamma_{pe \text{ rise}}) G_{pe} dt$$

$$K_y = \gamma G_{pe} (\gamma_{pe_{top}} + \gamma_{pe \text{ rise}}) T_{pe}$$



slew rate 180 T/m/s
want get up to 25 mT/m

$$\frac{0.025 \text{ T/m}}{180 \text{ T/m/s}} = 1.3889(10^{-4}) \text{ s} = T_{pe \text{ rise}}$$

$$K_y = \gamma G_{pe} T_{pe_{top}} + \gamma G_{pe} T_{pe \text{ rise}}$$

$$\gamma G_{pe} T_{pe} = K_y - \gamma G_{pe} T_{pe \text{ rise}}$$

$$T_{pe_{top}} = \frac{K_y - \gamma G_{pe} T_{pe \text{ rise}}}{\gamma G_{pe}}$$

$$T_{pe_{top}} = \frac{K_y}{\gamma G_{pe}} - T_{pe \text{ rise}}$$

$$\gamma_{pe_{\text{top}}} = \frac{(10.0024) \text{ m}}{42.577 (10^6) \frac{\text{Hz}}{\text{T}} (0.025 \frac{\text{T}}{\text{m}})} - \gamma_{\text{penre}}$$

$$\gamma_{pe_{\text{top}}} = 2.5 (10^{-4}) \text{ s}$$

$$= 0.25 \text{ ms (flat top)}$$

$$\gamma_{pe \text{ total}} = 0.25 + (2)(0.139) = \boxed{0.53 \text{ ms}} \rightarrow \text{shortest possible duration for pe gradient}$$

1c. Calculate the amplitude and total duration of the frequency encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

$$N = 256$$

$$\text{FOV} \rightarrow 256 (1.2 \text{ mm}) = 307.2 \text{ mm}$$

$$\text{FOV} = 307.2 \times 307.2 \text{ mm}^2$$

$$r_{\text{BW pix}} = 750 \text{ Hz/pix}$$

$$r_{\text{BW}} = 750 \frac{\text{Hz}}{\text{pix}} (256 \text{ pix}) = 192000 \text{ Hz}$$

$$G_{\text{read}} = \frac{r_{\text{BW}}}{\gamma \text{FOV}_x} = \frac{192000 \text{ Hz}}{(42.577 (10^6) \frac{\text{Hz}}{\text{T}}) (0.307 \text{ m})}$$

$$= 0.0147 \text{ T/m} = \boxed{14.7 \text{ mT/m}} \quad G_{\text{se amp}}$$

$$r_{\text{BW pix}} = \frac{1}{T_s} = \frac{750 \text{ Hz}}{\text{pix}} \quad T_s = 0.0013 \frac{\text{sec}}{\text{pix}} = 1.3 \frac{\text{ms}}{\text{pix}}$$

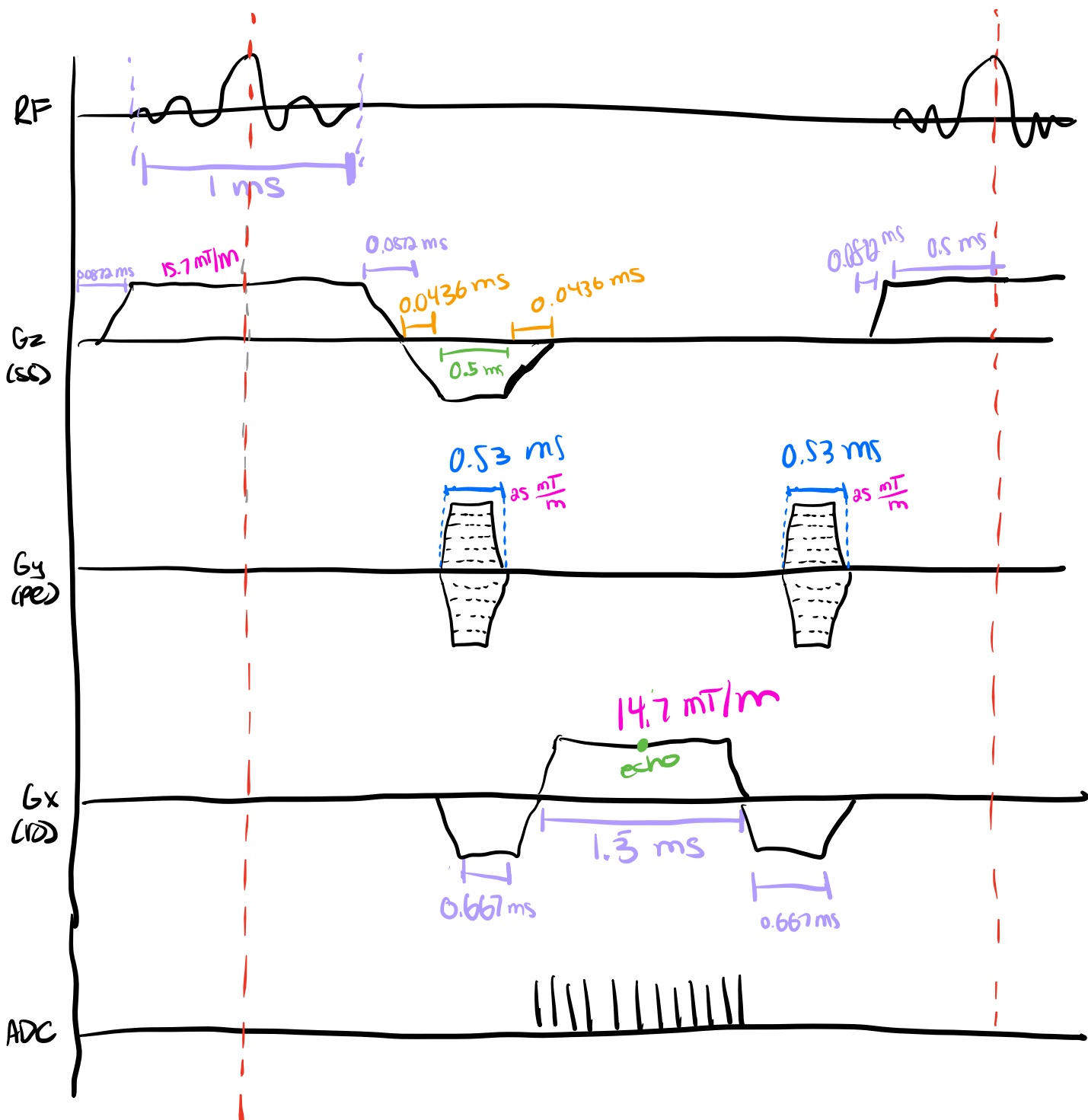
sampling time

$$\boxed{1.3 \text{ ms}}$$

total gradient duration
(mult by 256 so units work)?

1d. Based on your results in parts a-d, calculate the shortest possible TE and TR. Please draw a sequence diagram and label the timings and amplitudes of all RF and gradient events. It is okay to submit a handwritten diagram.

- HINT #1: You can assume that the duration of the slice-rephasing gradient equals half the duration of the slice-selection gradient, and likewise that the duration of the read pre-phasing gradient is half that of the readout gradient.
- HINT #2: You can let certain gradients overlap to minimize the TE and TR.



Min TR: $0.0872 \text{ ms} + 1 \text{ ms} + 0.0872 \text{ ms} + 0.667 \text{ ms} + 1.333 \text{ ms} + 0.667 \text{ ms} = \boxed{3.84 \text{ ms}}$ min TR

• echo
min TE: (assume TR starts at the peak of the RF Sinc pulse and ends at center of echo)

$$0.5 \text{ ms} + 0.0872 \text{ ms} + 0.667 \text{ ms} + \frac{1.33}{2} \text{ ms} = \boxed{1.92 \text{ ms}}$$

min TE
*(1/2 TR)

1e. List three ways you could further decrease the TE and TR by maintaining the same spatial resolution and FoV.

- 1) make RF pulse shorter (which means increasing slice select gradient amplitude)
- 2) increase slew rate so faster ramp up
- 3) increase amplitude of rephasing and prephasing gradients to make them take less time
(Not assume is half the duration)

2a) see plots folder

2bi) 0.0757 (and plots folder)

2bii) see plots folder

2biii) should be around 117° , not
sure how to get from the plot
(and plots folder)

Problem 3: Slice Profile Simulation

- Design an RF pulse and its corresponding slice selective gradient to excite a slice thickness of 5 mm

- RF: 2-msec truncated Sinc pulse with a time bandwidth product of 8
- Slice selective and rephasing gradients with the following gradient specifications:
 - The maximum gradient strength: 25 mT/m
 - The maximum gradient slew rate: 180 mT/(m*ms)

num of lobes
zero crossings

$$TBW = T(BW) = \text{num zero crossings} = 8$$

$$2(\text{ms}) BW = 8$$

$$BW = 4 \text{ KHz}$$

$$TBW = T_{rf}(BW) = T_{rf}(\Delta f_{tx})$$

$$\Delta f_{tx} = \gamma G \Delta z = BW$$

$$G = \frac{\Delta f_{tx}}{\gamma \Delta z} = \frac{TBW}{T_{rf} \gamma \Delta z}$$

$$G = \frac{8}{0.002 \text{ s} \left(\frac{42.577 (10^6) \text{ Hz}}{T} \right) (0.005 \text{ m})}$$

$$G = 0.0188 \text{ T/m} = 18.8 \text{ mT/m}$$

$$10^4 G = 1 \text{ T}$$

need γ_{ss} and γ_{ss} rise

slew rate 180 T/m/s

want get up to 0.0157 T/m

$$\begin{aligned}\frac{0.0188 \text{ T/m}}{180 \text{ T/m/s}} &= 1.04 (10^{-4}) \text{ s} \\ &= 0.000104 \\ &= 0.104 \text{ ms} \quad \text{rise time}\end{aligned}$$

total gradient duration:

$$2 + 2(0.104 \text{ ms}) = 2.208 \text{ ms}$$

convert units:

$$\begin{aligned}\frac{18.8 \cancel{\text{ mT}}}{\cancel{\text{ m}}} &\times \frac{\cancel{10^4} \text{ G}}{1 \cancel{\text{ T}}} \times \frac{1 \cancel{\text{ T}}}{\cancel{10^3 \text{ mT}}} \times \frac{1 \cancel{\text{ m}}}{\cancel{100 \text{ cm}}} \\ &= 1.88 \text{ G/cm}\end{aligned}$$

some notes

- integral of sinc pulse times γ is the flip angle

$$\alpha = \gamma \int_0^{trf} b(T) dT$$

rf amp

$$\alpha = \gamma B_1 (\text{sum})$$

$$B_1 = \frac{\alpha}{\gamma (\text{sum})}$$

3.1.2 → see plots folder

3.1.3 fft version has higher magnitude but lower (incorrect) slice thickness the magnitude difference is more apparent for 90° pulse than 10° pulse

shorter T_2 - signal decays faster so lower magnitude slice profile

(and see plots folder)

3.1.4 see plots folder
purpose of slice rephasing gradient is so not have linear phase across slice.

phase accumulates when
apply slice select gradient
and need make the phase
constant