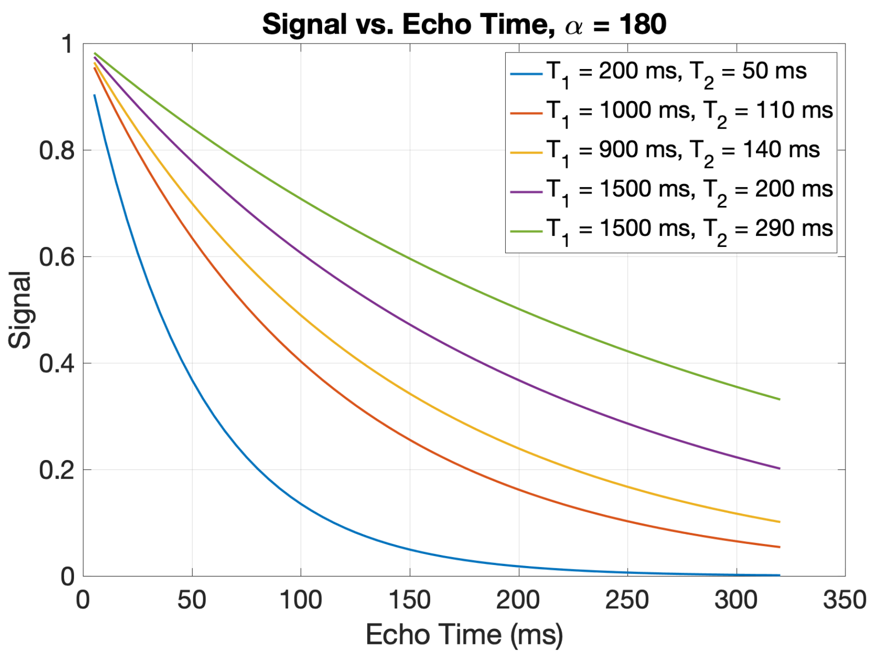
Problem 1 (Extended phase graphs)

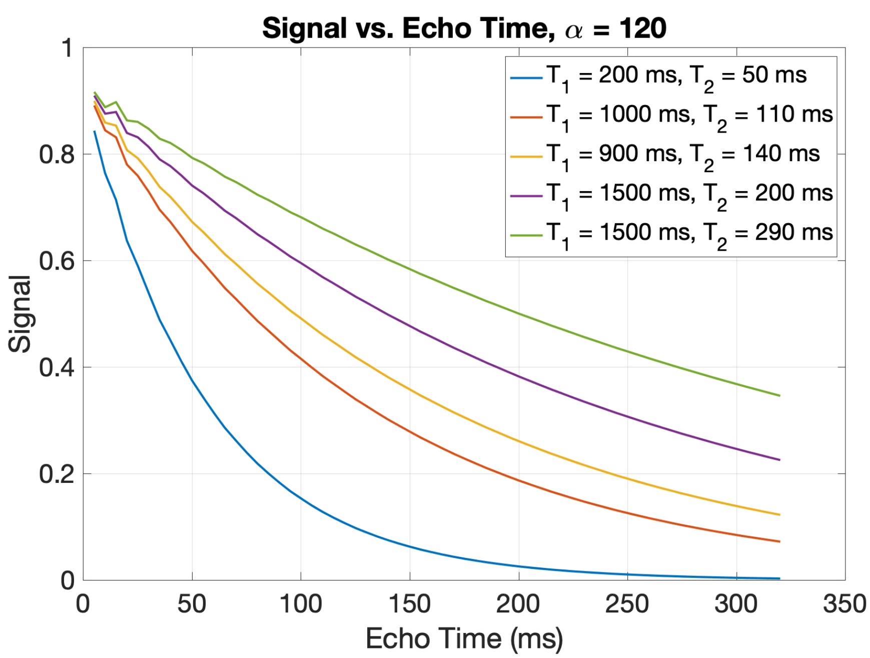
The file used to generate each figure in this problem is `q1.m`.

(a) (i) Five different T1 and T2 combinations with $\alpha = 180°$



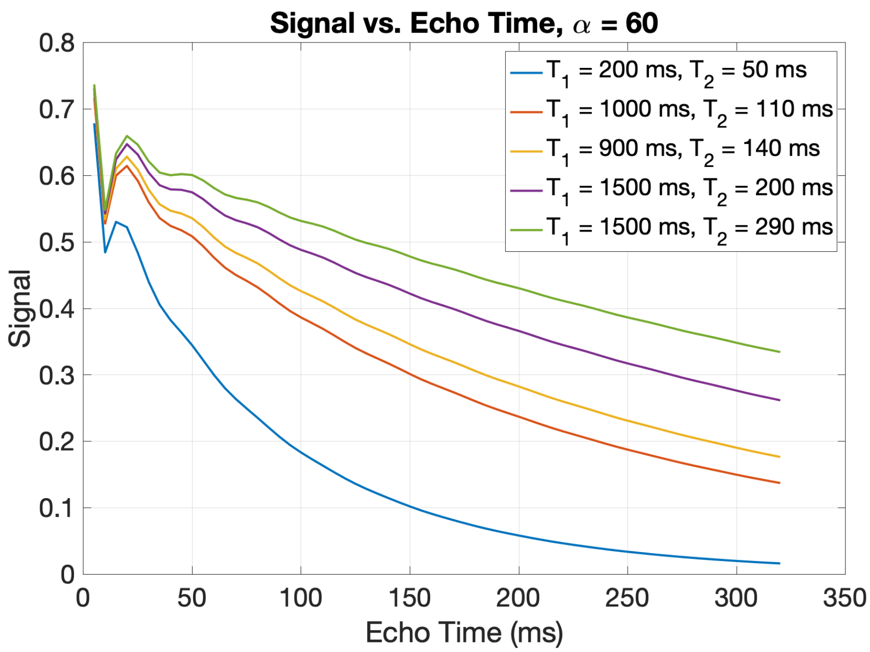
We can see that the amplitude decays faster for small T_2 values, whereas short T_1 values the signal level starts with low intensity than higher T_1 values.

(ii) Five different T1 and T2 combinations with $\alpha = 120°$



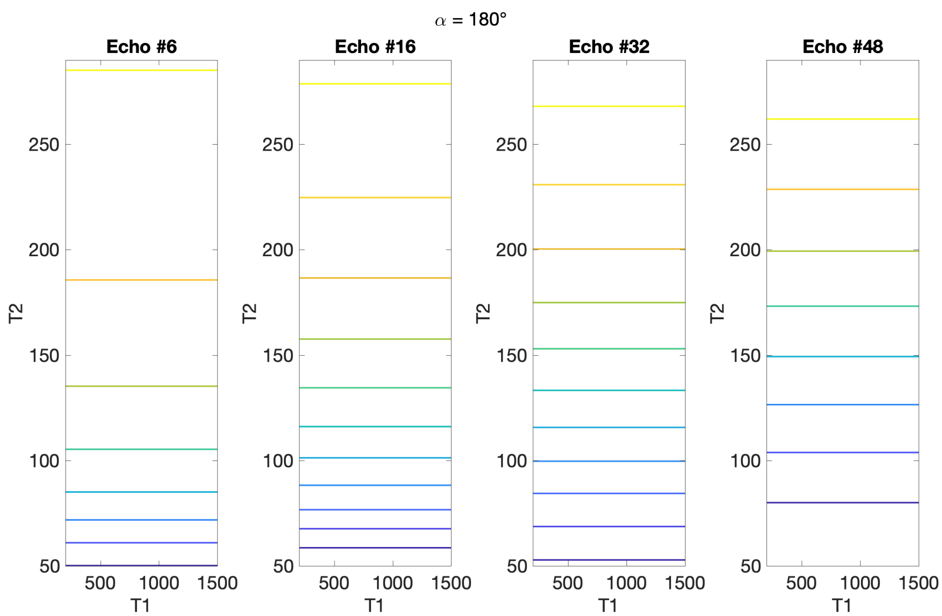
We can see a small peak at the beginning, due to the stabilization pulse ($\alpha = 150$).

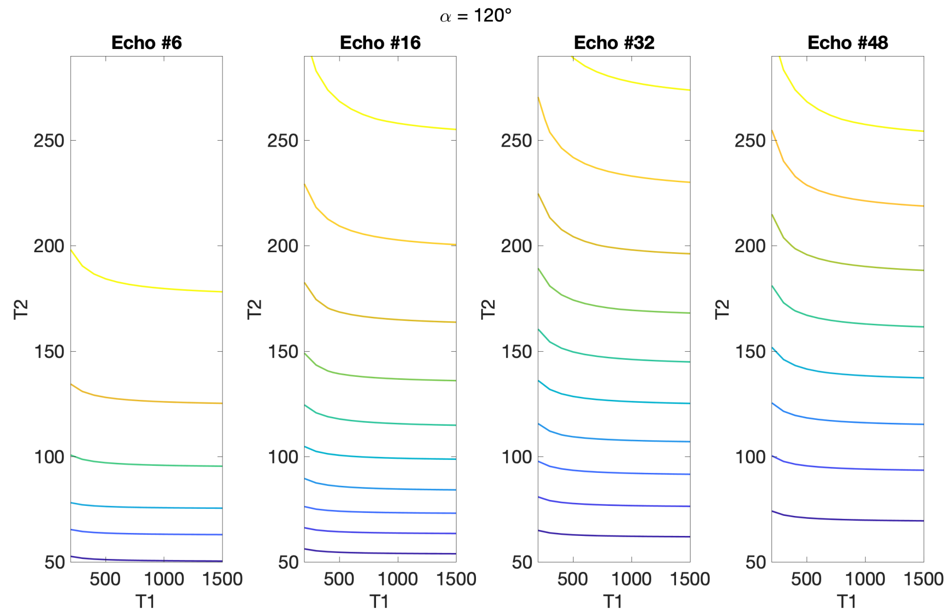
(iii) Five different T1 and T2 combinations with $\alpha = 60°$

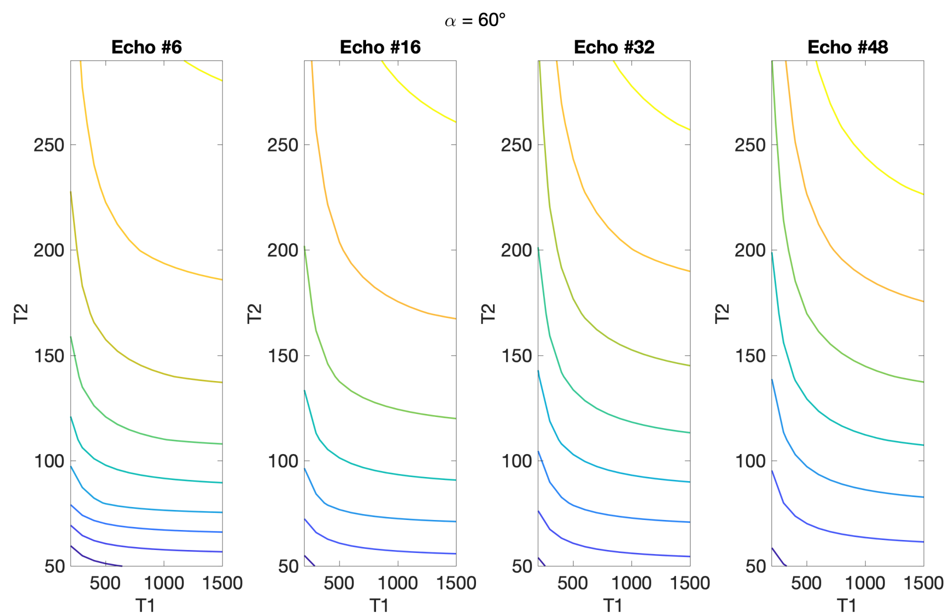


We can also see a peak at the beginning, due to the stabilization pulse ($\alpha = 120$).

(b) Contour plots







(c) For 180°, the contrast is just T_2 weighted along the different echo times. For 120°, the contrast is mainly T_2 weighted at medium/long TRs but mixed at very low T_1 values. For 60°, the contrast is similar but there is more T1 weighting at low and medium T_1 values.

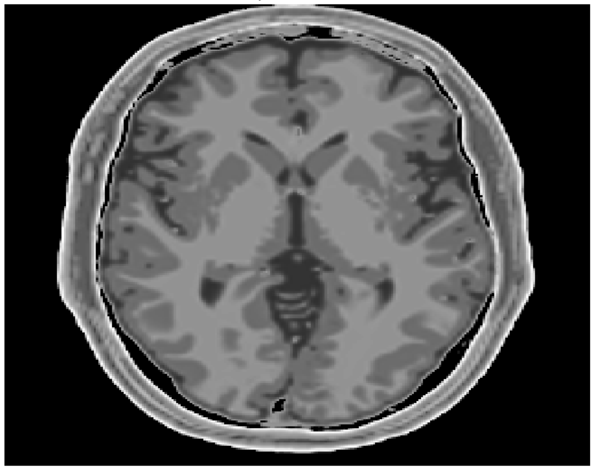
Problem 2 (Single and Multiple Spin Echo Sequences)
The file used to generate each figure in this problem is `q2.m`.

(a) Single-echo spin echo

- T_1-**weighted:** We have $T1_{WM} \approx$ 500ms, $T1_{GM} \approx$ 840ms, and $T1_{CSF} \approx$ 2510ms

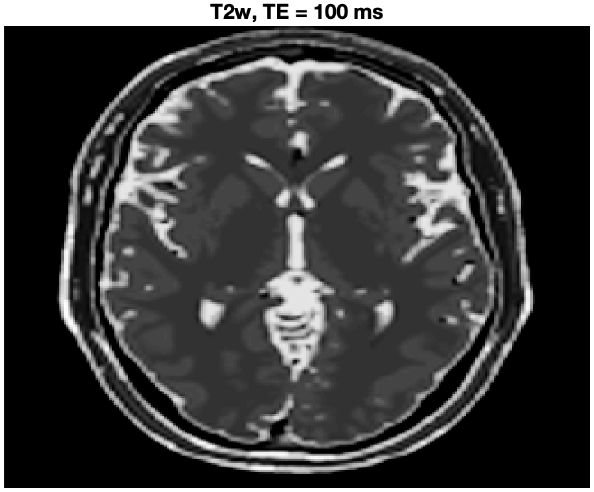
($T1_{WM} < T1_{GM} << T1_{CSF}$). For T_1-weighted, we need short TR and short TE. Then, to minimize T_2 effects, we pick TE = 15 ms and TR =500 ms.

T1w, TE = 15 ms

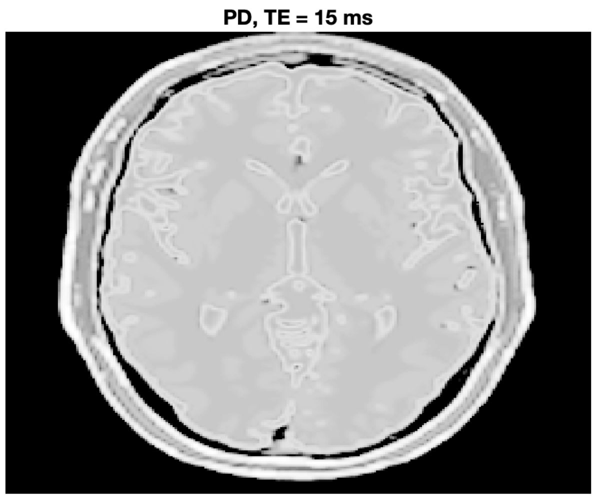


- T_2-**weighted:** We have $T2_{WM} \approx 52$ms, $T2_{GM} \approx 62$ms, and $T2_{CSF} \approx 330$ms ($T2_{WM} < T2_{GM} << T2_{CSF}$). For T_2-weighted, we need long TR and long TE. Then, to minimize T_1 effects, we pick TE = 100 ms and TR =6000 ms.

T2w, TE = 100 ms



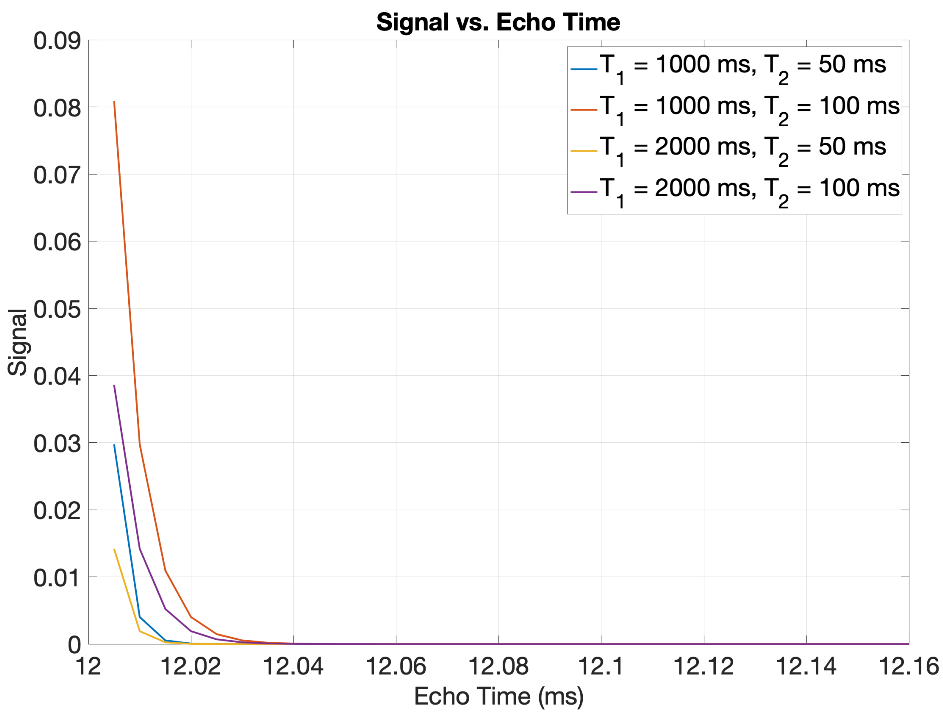
- **Proton density weighted:** We need long TR and short TE, so we pick TR = 4000 ms and TE = 15 ms, to minimize both T_1 and T_2 effects.



(b) Fast spin echo

(i) **Signal behavior for a few T_1 and T_2 combinations**

We can see that for $T_2 = 50$ ms, the signal decays faster, whereas for $T_2 = 100$ ms, it decays slower. Also, the signal magnitude at the beginning of the TR is higher for the smallest T_1 values (1000 ms) compared to 2000 ms, which means that the signals recover faster in the previous TR.



(ii) **FSE to create an image**

As mentioned, the k-space filling affects the contrast weighting; thus, it is important to avoid big spikes or changes in the k-space amplitude (which depends on the echo number) to avoid artifacts. The significant changes in the amplitude can generate a sampling artifact or ripples. The Figure below shows the k-space filling, where the TE_{eff} is in the middle of the k-space and corresponds to echo number 16. I acquired 32 lines of the k-space in each TR, and the dashed lines corresponds to TR after 3 seconds. The number of TR was calculated by $N_{TR} = \frac{N_y}{ETL} = 8$. Then, the total acquisition time will be $TR \times N_{TR} = 3 \times 8 = 24s$. In contrast, the acquisition for a single-echo spin echo scan is 12min 48 sec.

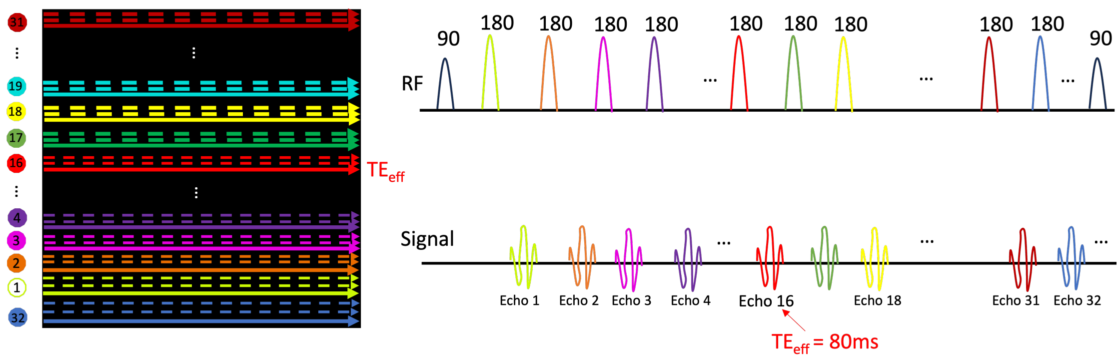


Figure 1: K-space filling.

Figure 2: K-space and reconstructed image for a $TE_{eff} = 80$ ms.

(iii) **FSE images with effective echo times of 40 ms and 120 ms**

- $TE_{eff} = 40$ **ms**
  The k-space was filled by blocks this time. I divided the k-space in 8 blocks, starting from the center to get the TE_{eff} in the 8th echo. The next blocks started with the lines closest to the center of the k-space (e.g. the second blocked started in line 120).

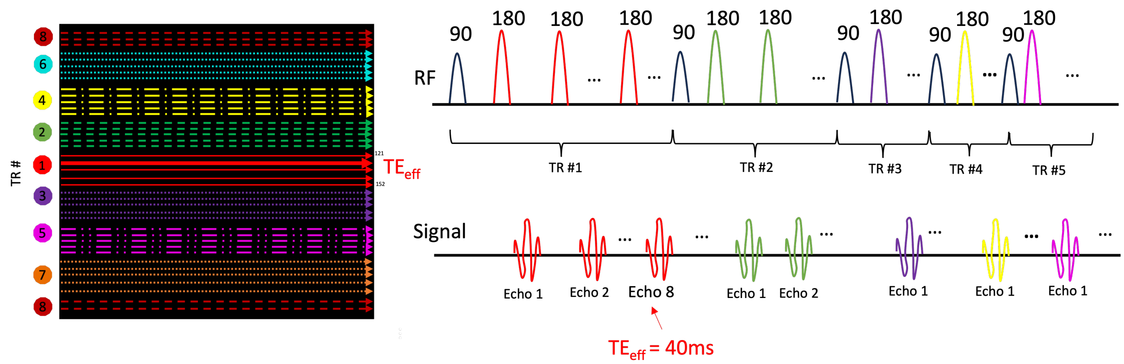


Figure 3: K-space filling for $TE_{eff} = 40$ ms.

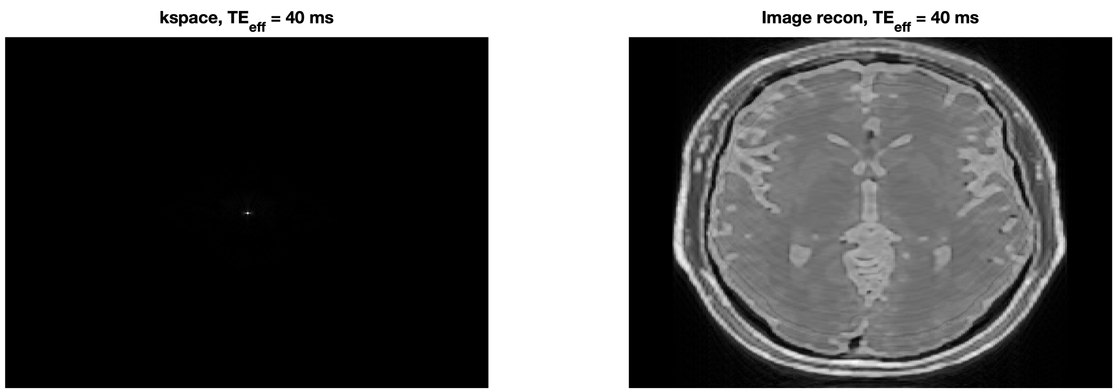


Figure 4: K-space and reconstructed image for a $TE_{eff} = 40$ ms.

- $TE_{eff} = 120$ **ms**
  The k-space was acquired differently this time. I started filling line 220 of the k-space with steps of 4 lines in the next echos. Thus the effective echo was 24. As shown in the Figure, each different echo is shown with a different color.

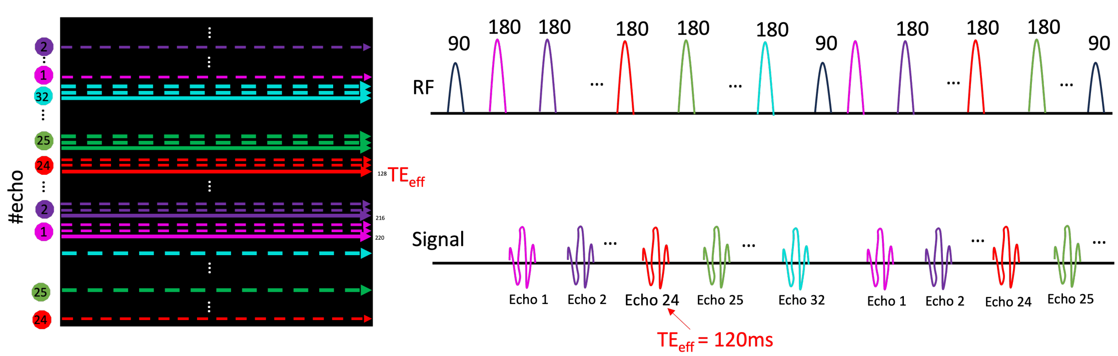
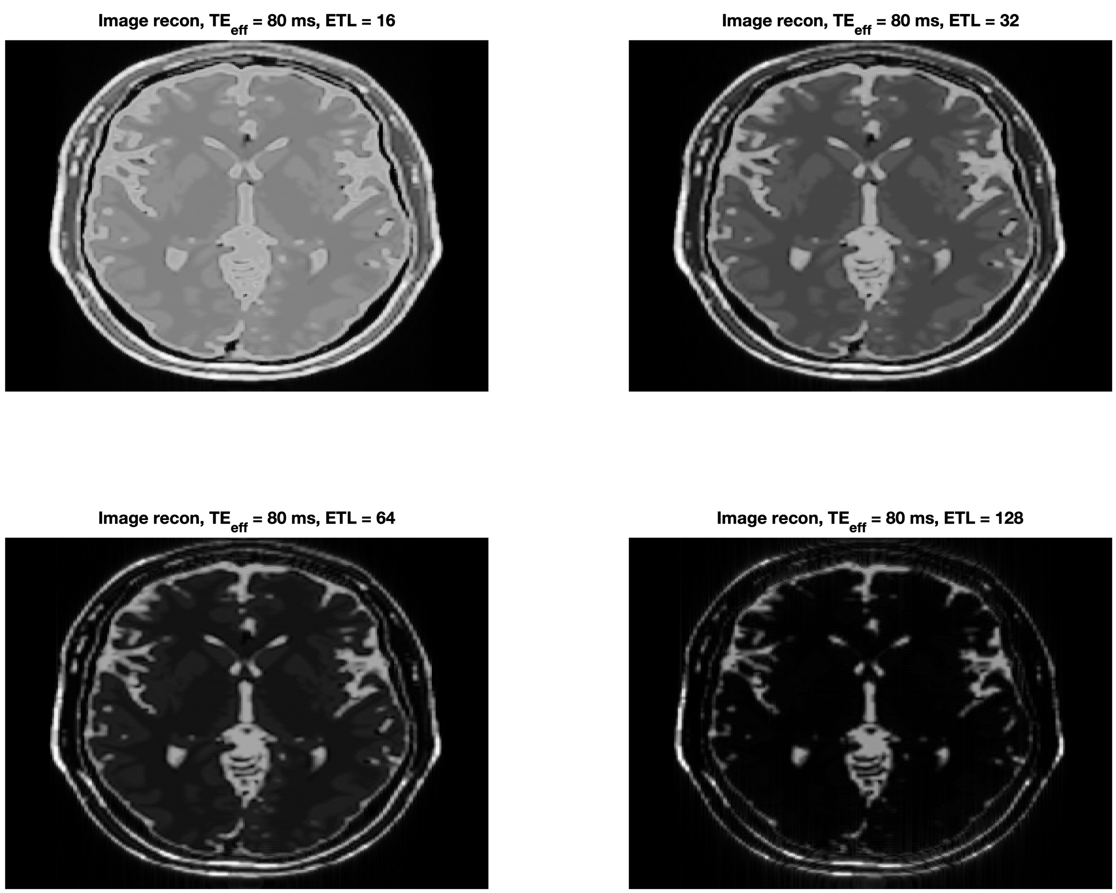


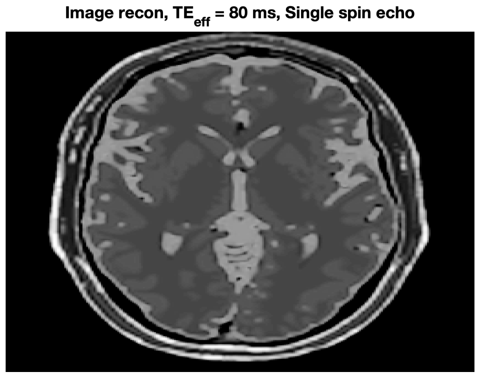
Figure 5: K-space filling for $TE_{eff} = 120$ ms.

Figure 6: K-space and reconstructed image for a $TE_{eff} = 120$ ms.

We can see that we can manipulate the contrast by altering the position in the echo train at which the central portion of the k-space is acquired. Shorter TE_{eff} gives brighter images because the center of the k-space is acquired when the signal has not relaxed significantly, whereas longer TE_{eff} gives darker images because we have small signal at that time.

(iv) **Simulate images with $TE_{eff} = 80$ms and different ETL**





This happens because we are acquiring some lines of the k-space when the signal amplitude is small, so we lose some information from high and low frequencies. When the ETL is longer, the image also gets darker because we have more echoes in one. In addition, we get a little of blur and we lose details.

(c) BONUS

We can improve the image quality by filling slightly more than half of the kspace, instead of filling all the kspace. For a $TE_{eff} = 80$ms, we acquire the center of the kspace in the 16th echo. We acquire the k-space lines when the signal level has not decreased significantly, and set to zero the ones at very high echo numbers. The Figure below shows the partial and full-kspace. The full k-space was filled using the conjugate property of the Fourier Transform.

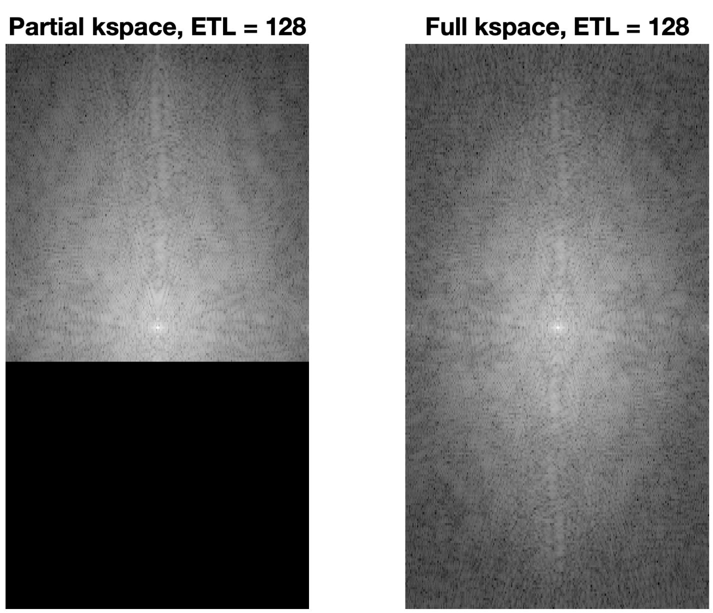


Figure 7: Partial and full k-space.

Zero filling just performs the inverse Fourier transform. In the conjugate synthesis, I used the conjugate symmetry of the kspace, so we can get the missing pixel by mirroring opposite to the origin. After completing all the k-space, I applied the Fourier Transform.

Image recon, zero filling

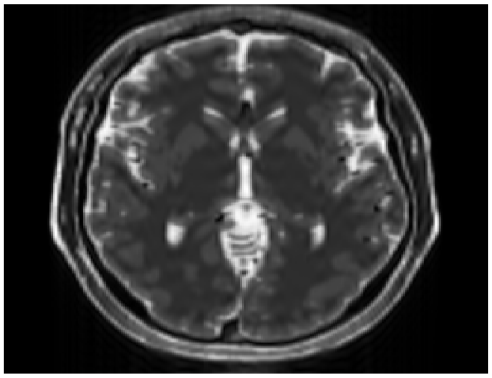
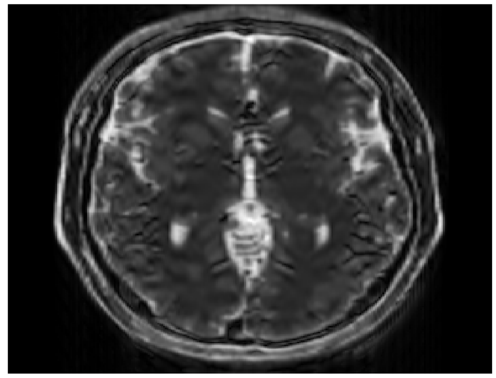
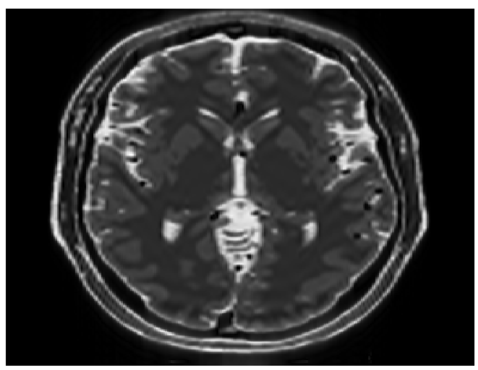


Image recon, conjugate synthesis



Finally, I tried the Projection onto convex sets (POCS) method, which is an iterative method to fill the kspace.

POCS reco



References

- Michael Völker (2023). MRI Partial Fourier reconstruction with POCS, MATLAB Central File Exchange. Retrieved October 29, 2023.
- Bloch Equation Simulation: http://mrsrl.stanford.edu/ brian/bloch/