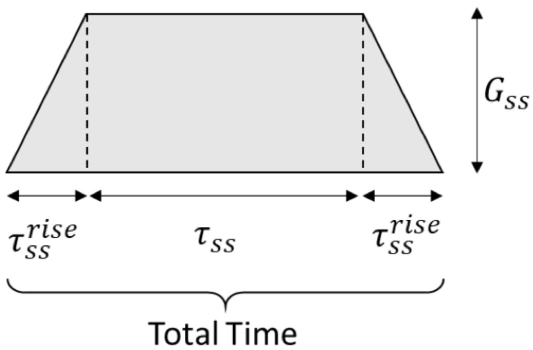
Problem 1: Spatial Encoding

Suppose you want to collect a 2D bSSFP image at a spatial resolution of 1.2 x 1.2 x 3.0 mm^3 with 256 x 256 matrix size. The receiver bandwidth is 750 Hz / pixel. You are imaging on a 1.5T scanner with maximum gradient amplitude 25 mT/m (for one axis) and maximum slew rate of 180 T/m/s. In the following problems, you will design gradients for slice-selection, phase encoding, and frequency encoding and calculate the shortest possible TR and TE.

1a. You decide to use a sinc-shaped RF pulse with time bandwidth product 4 and a duration of 1 ms. Calculate the amplitude and total duration of the slice-select gradient. Note that the total duration includes both the flat-top time (τ_{ss}) and the time needed to ramp-up and ramp-down the gradient (τ_{ss}^{rise}), as shown in the figure below.



TBW = 2
τ_{RF} = 1 ms

Δz = 3 mm
γ = 42.58 MHz/T

$\tau_{ss} = \tau_{RF}$

@ 1.5T, γ = 42.58 MHz/T

$$TBW = \tau_{RF} \times BW$$

$$TBW = \tau_{RF} \times (\Delta z \cdot \gamma \cdot G_{ss})$$

$$2 = (1\,ms) \cdot (3\,mm)(42.58\,MHz/T) \cdot G_{ss}$$

$$G_{ss} = \frac{2}{(1\times10^{-3}s)(3\times10^{-3}m)(42.58\cdot10^{6}\,Hz)} \cdot T$$

$$G_{ss} = \frac{2}{(3)\cdot(42.58)} \cdot \frac{T}{m}$$

$$G_{ss} = 0.0156\ T/m$$

Slew rate: S = 180 T/m/s

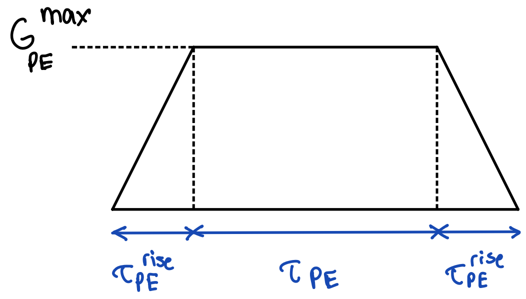
$$S = \frac{G_{ss}}{\tau_{ss}^{rise}}$$

$$\tau_{ss}^{rise} = \frac{0.0156}{180}\ \frac{\cancel{T/m}}{\cancel{T/m}/s} = 0.087\ ms$$

Total duration = $2(\tau_{ss}^{rise}) + (\tau_{ss})$ = 1.174 ms

1b. Calculate the shortest possible duration of the phase encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

- $\tau_{sr} = \frac{\tau_{ss}}{2} = 0.587$ ms



$N = 256$

$\Delta y = 1.2$ mm

$G_{PE}^{max} = 25$ mT/m

$\tau_{PE} = ?$

@ 1.5T, $\gamma = 42.58$ MHz/T

$K^{max} = \frac{1}{2 \cdot \Delta y}$

$K = \gamma \cdot G \cdot \tau$

$\frac{1}{2\Delta y} = \gamma \cdot G_{PE}^{max} \cdot \tau_{PE}$

$$\tau_{PE} = \frac{1}{2 \cdot \gamma \cdot G_{PE}^{max} \cdot \Delta y} = \frac{1}{2 \cdot (42.58 \text{ MHz}/\cancel{T}) \cdot (25 \times \cancel{10^{-3}} \cancel{T}) \cdot (1.2 \times \cancel{10^{-3}} m)}$$

$\tau_{PE} = 0.3914$ ms

Slew rate: $S = 180$ T/m/s $= \frac{G_{PE}^{max}}{\tau_{PE}^{rise}}$

$$\tau_{PE}^{rise} = \frac{0.025 \text{ T/m}}{180 \cdot \text{T/m/s}} = 0.1389 \text{ ms}$$

PE duration $= 2(\tau_{PE}^{rise}) + \tau_{PE} = 2(0.139) + 0.3914 = 0.6694$ ms

1c. Calculate the amplitude and total duration of the frequency encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

rBW = 750 Hz/pixel , $\Delta x = 1.2$ mm

total rBW $= 750 \frac{Hz}{pixel} \cdot 256$ pixel $= 192\,000$ Hz

total rBW $= \gamma \cdot G_{read} \cdot FOV$

$$G_{read} = \frac{\text{total rBW}}{\gamma \cdot FOV} = \frac{192\,000 \cancel{Hz}}{42.58 \cdot 10^{\cancel{6}\,3} \frac{\cancel{Hz}}{T} \cdot 307.2 \cdot \cancel{10^{-3}} m}$$

- $K_{x,max} = \frac{1}{2 \cdot \Delta x} = 0.4167$ mm^{-1}

$$= \frac{192\,\cancel{000}}{42.58 \cdot \cancel{10^3} \cdot 308.4} \cdot T/m$$

- $K_x = 2 \cdot (0.42) = 0.84\ mm^{-1}$
- $\Delta K_x = \frac{0.83\ mm^{-1}}{256} = 0.0033\ mm^{-1}$
- $FOV_x = \frac{1}{\Delta K_x} = 307.2\ mm$
- $T_s = \frac{1}{rBW} = 1.3\ ms$

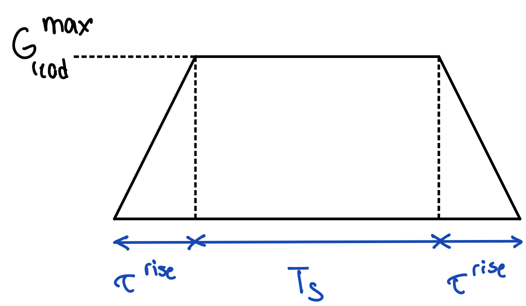
$= 0.01467\ T/m$

$= 14.7\ mT/m$

- $2A = \frac{\cancel{2}\tau^{rise} \cdot G_{read}}{\cancel{2}} + G_{read}^{max} \cdot T_s$

$2A = 2.0768 \times 10^{-5}\ s \cdot mT/m$

$A = 1.038 \times 10^{-4}\ s \cdot mT/m$



$G_{pre\text{-}ro} = 0.0139\ T/m$

- $S = 180\ T/m/s$, $S = \frac{G_{read}}{\tau^{rise}}$

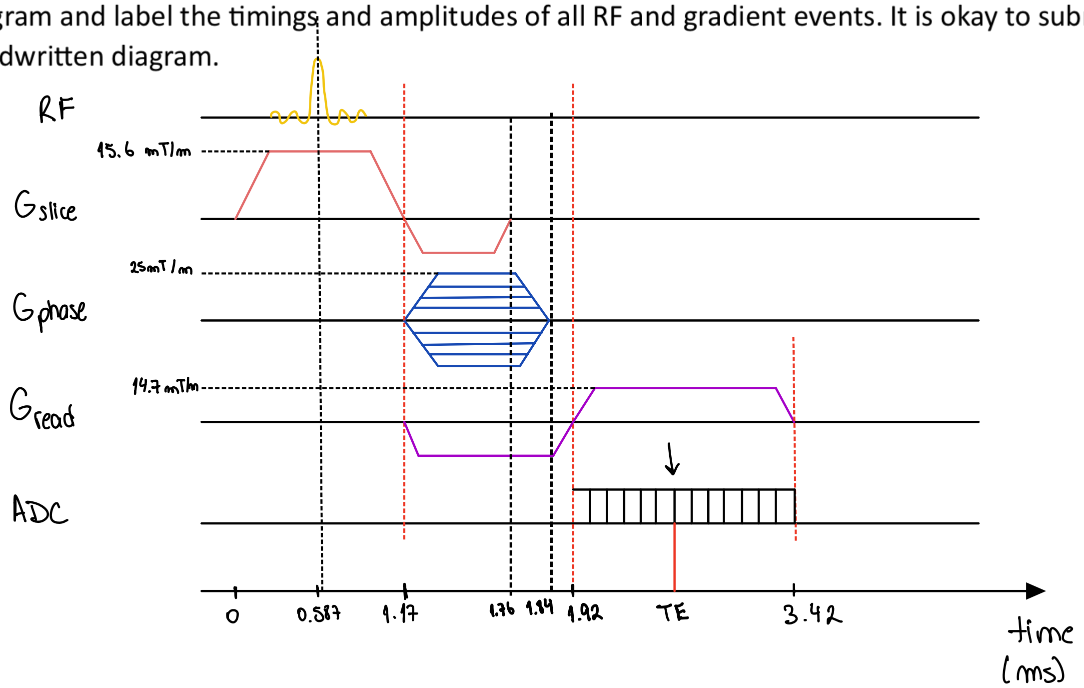
$\tau^{rise} = \frac{14.7 \cdot 10^{-3}\ T/m}{180\ \ T/m/s} = 0.081\ ms$

→ Readout time : $2\tau^{rise} + T_s = 2 \cdot (0.081) + 1.3 = 1.49\ ms$

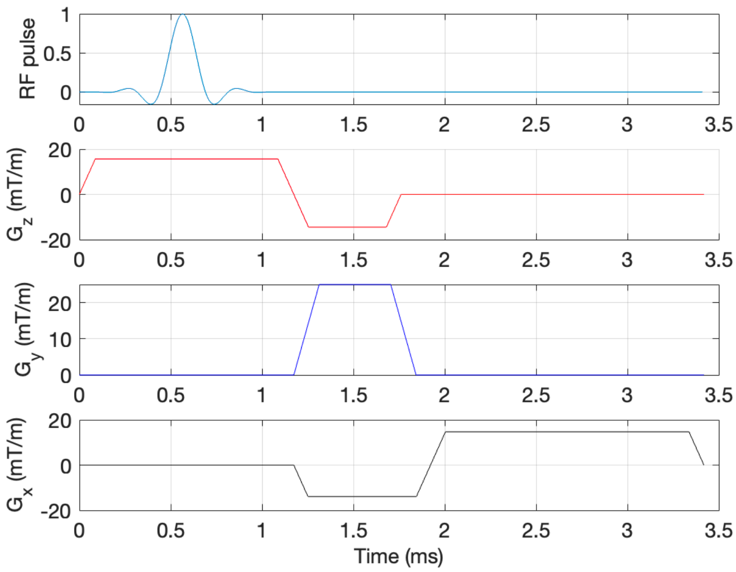
→ Pre-phasing time: (Readout time) / 2 = 0.746 ms

∴ Total duration of the freq. encoding gradient: $1.49 + 0.7482 = \underline{2.24\ ms}$

1d. Based on your results in parts a-d, calculate the shortest possible TE and TR. Please draw a sequence diagram and label the timings and amplitudes of all RF and gradient events. It is okay to submit a handwritten diagram.



⇒ TR = 3.42 ms , TE = 2.67 ms



1e. List three ways you could further decrease the TE and TR by maintaining the same spatial resolution and FoV.

- Acquire half of the K-space
- Use a smaller flip angle
- Use parallel imaging

Problem 2: Balanced and Spoiled Steady-State Sequences

2a. Simulate the steady-state frequency response of a bSSFP sequence with a flip angle of 60° using Bloch equation simulations. Show results for (1) TR=5ms and TE=2.5ms, (2) TR=10ms and TE=5ms, and (3) TR=20ms and TE=10ms.

Relaxation : $E_1 = e^{-t/T_1}$, $E_2 = e^{-t/T_2}$

$$m(0) = R_y(\alpha) \cdot R_z(180° + \phi) \cdot E \cdot m(0) + R_{-y}(\alpha) \begin{bmatrix} 0 \\ 0 \\ 1-E_1 \end{bmatrix}$$

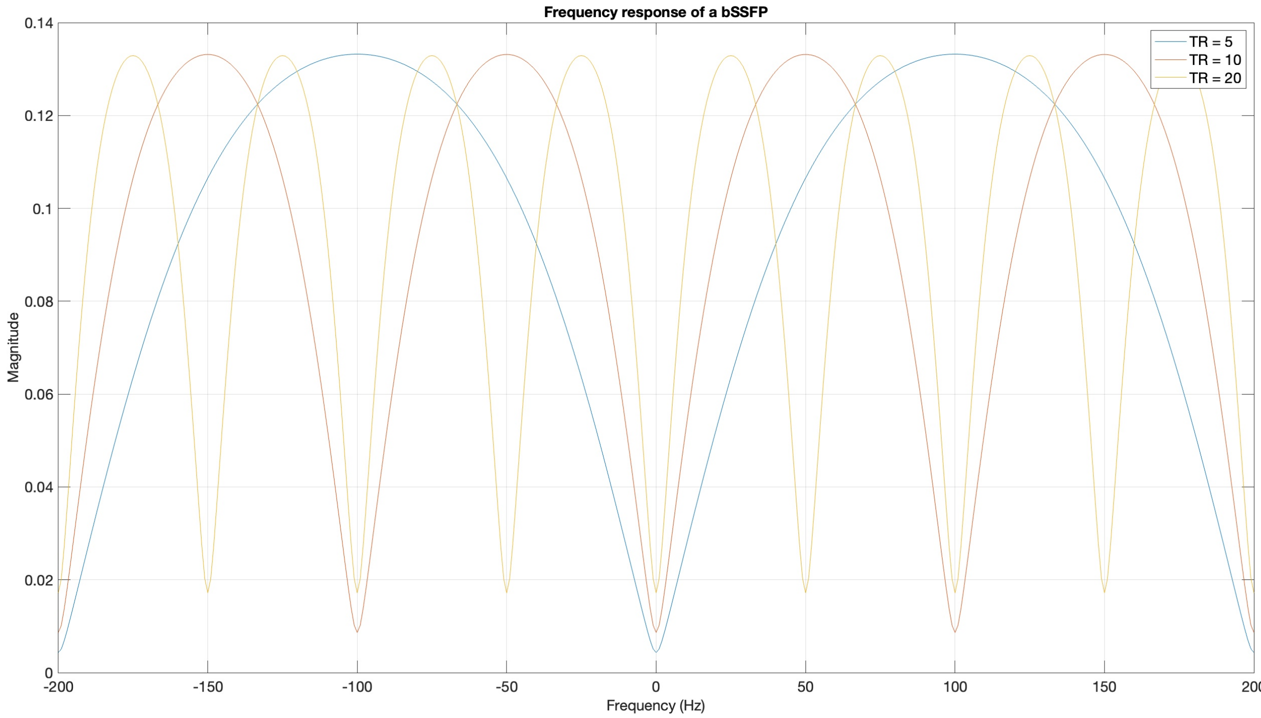
• If $\phi = 0$:

$$m(0) = \begin{bmatrix} -\cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & -1 & 0 \\ \sin(\alpha) & 0 & -\cos(\alpha) \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} E_2 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{bmatrix} \cdot m(0) + \begin{bmatrix} -\cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & -1 & 0 \\ -\sin(\alpha) & 0 & -\cos(\alpha) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1-E_1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} E_2 \cdot \cos(\alpha) & 0 & -E_1 \sin(\alpha) \\ 0 & E_2 & 0 \\ -E_2 \cdot \sin(\alpha) & 0 & -E_1 \cdot \cos(\alpha) \end{bmatrix}}_{A} \cdot m(0) + \underbrace{\begin{bmatrix} -\sin(\alpha) \cdot (1-E_1) \\ 0 \\ -\cos\alpha\,(1-E_1) \end{bmatrix}}_{B} = 0$$

$$m(0) = A^{-1} \cdot B$$

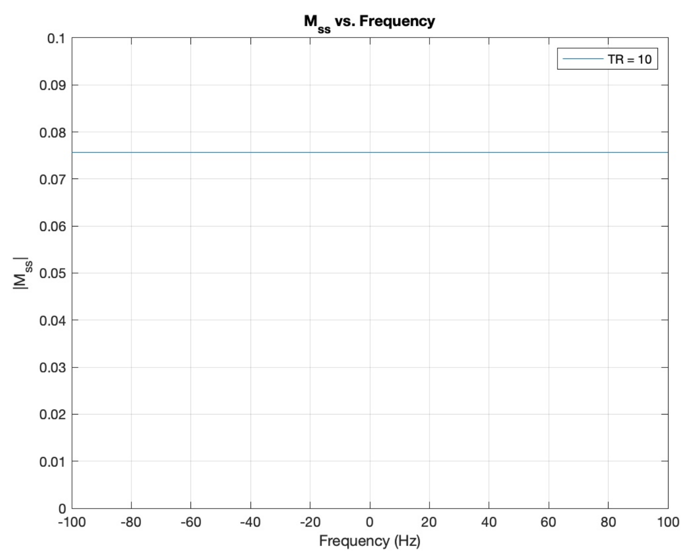
$$m(0) = \frac{1 - E_1}{(1 + \cos\alpha \cdot (E_2 - E_1) - E_1 \cdot E_2)} \cdot \begin{bmatrix} \sin\alpha \\ 0 \\ E_2 + \cos\alpha \end{bmatrix}$$



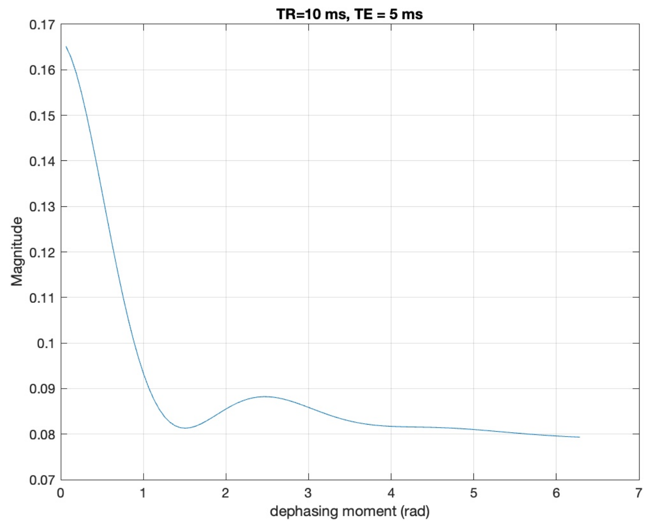
- Amplitude agrees with 1/TR and it's flat at TE/2.

2b. Modify the bSSFP sequence to generate a FLASH sequence by adding a gradient spoiler along the slice selection direction. You can assume TR=10ms and TE=5ms.

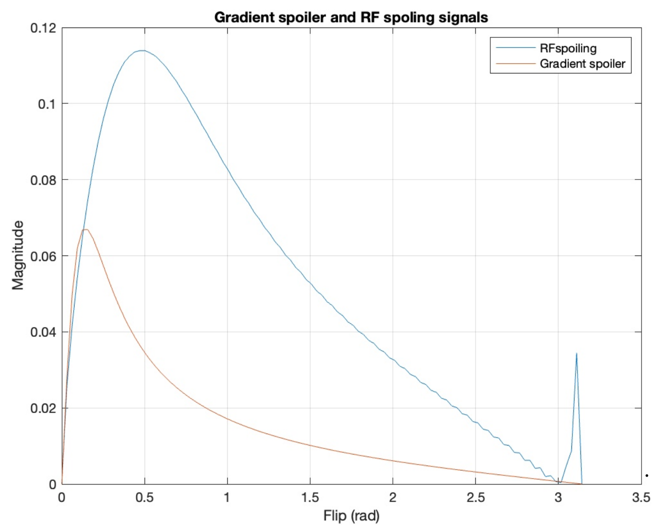
i) Spoiler gradient as a "perfect spoiler"



ii) Assuming ideal spoiling



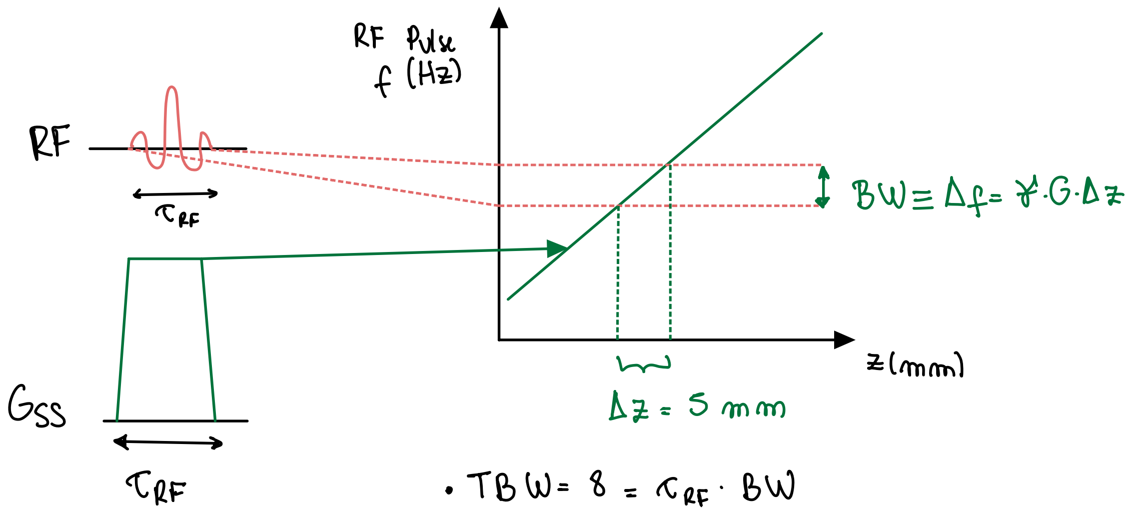
iii) Gradient spoiler and RF spoiling



Optimal phase that eliminates transverse magnetization (minimum steady state signal) is 3.079 rad

Problem 3: Slice Profile Simulation

1. Design an RF pulse and its corresponding slice selective gradient to excite a slice thickness of 5 mm
   - RF: 2-msec truncated Sinc pulse with a time bandwidth product of 8 (TBW = 8)
   - Slice selective and rephasing gradients with the following gradient specifications:
     - The maximum gradient strength: 25 mT/m
     - The maximum gradient slew rate: 180 mT/(m*ms)



$\bullet\ TBW = 8 = \tau_{RF} \cdot BW$

$B_0 = 1.5\ T$

$BW = \dfrac{8}{2ms} = 4\ KHz$

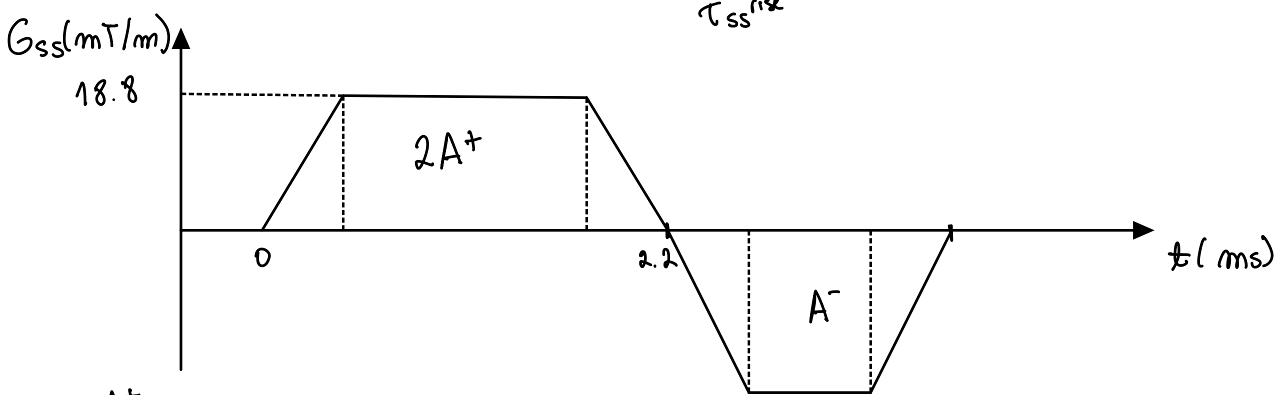
$\tau_{RF} = 2 \cdot \tau_{ss}^{rise} + \tau_{ss}$

$\bullet\ G_{ss} = \dfrac{BW}{\gamma \cdot \Delta z} = \dfrac{4 \cdot \cancel{10^3}}{42.58\ \cancel{MHz}/T \cdot 5 \cdot \cancel{10^{-3}}}$

$G_{ss} = 18.8\ mT/m$

Slew rate : $S = 180\ T/m/s$

$S = \dfrac{G_{ss}}{\tau_{ss}^{rise}} \Rightarrow \tau_{ss}^{rise} = 0.1044\ ms$



$A^+ = \dfrac{(0.1044 \times 18.8)\ \cancel{2}}{\cancel{2}} + (1.7912)(18.8)$

$A^+ = 36.637 \longrightarrow A^- = A^+/2$

$A^- = (\tau_{ss}' \cdot G_{ss}) + G_{ss} \cdot \tau_A^-$

$\tau_{ss}' = \dfrac{G_{ss}}{180}$

$G_{ss}^{max} = 25\ mT/m$

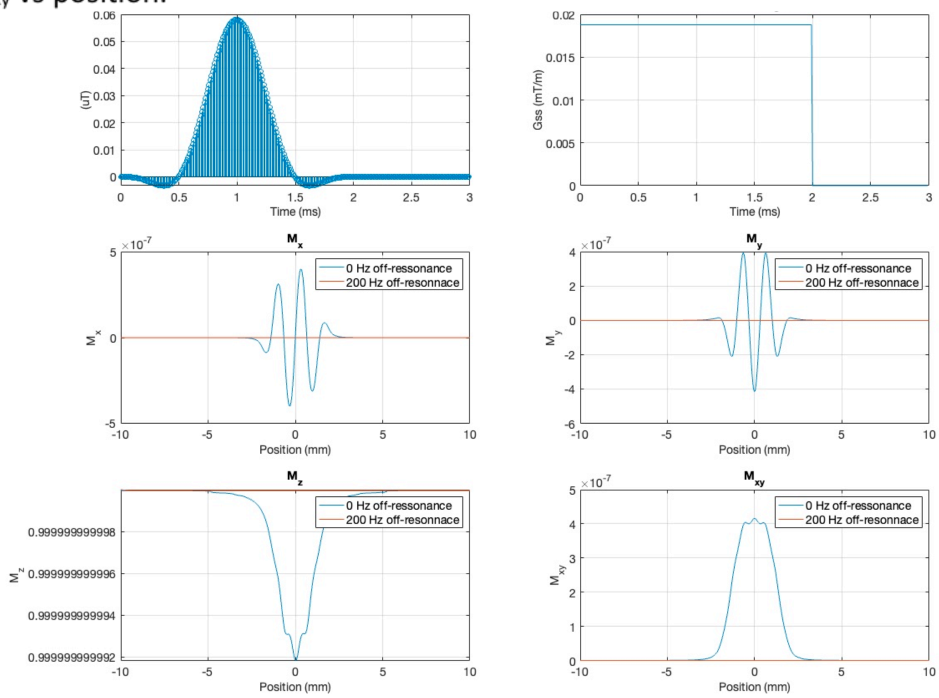
$\tau_{ss}' = 0.139\ ms$

$\dfrac{36.637}{2} = (0.139)(25) + (25) \cdot \tau_A^-$

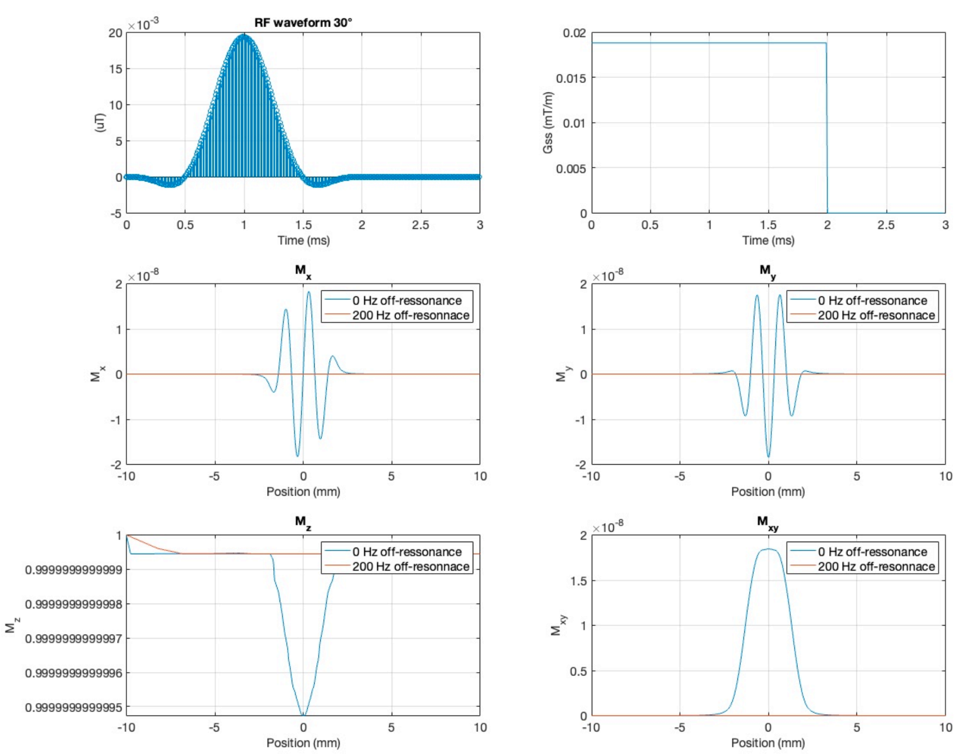
$18.32 = 25(0.139 + \tau_A^-)$

$\tau_A^- = 0.5937\ ms$

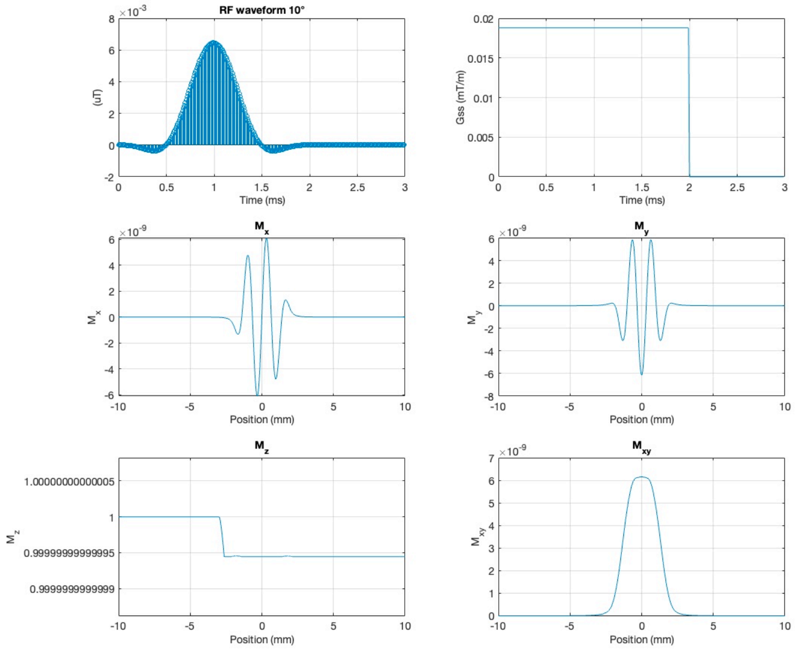
2. Simulate the slice profile of your RF pulse with a flip angle of 90° (i.e., Mz is tipped into Mxy) at 0 Hz and 200 Hz off-resonance using a Bloch equation simulation with a T1 of 1000 msec and a T2 of 100 msec. Plot the RF waveform (in μT), corresponding gradient waveform (in mT),and m_x, m_y, m_z and m_{xy} vs position.



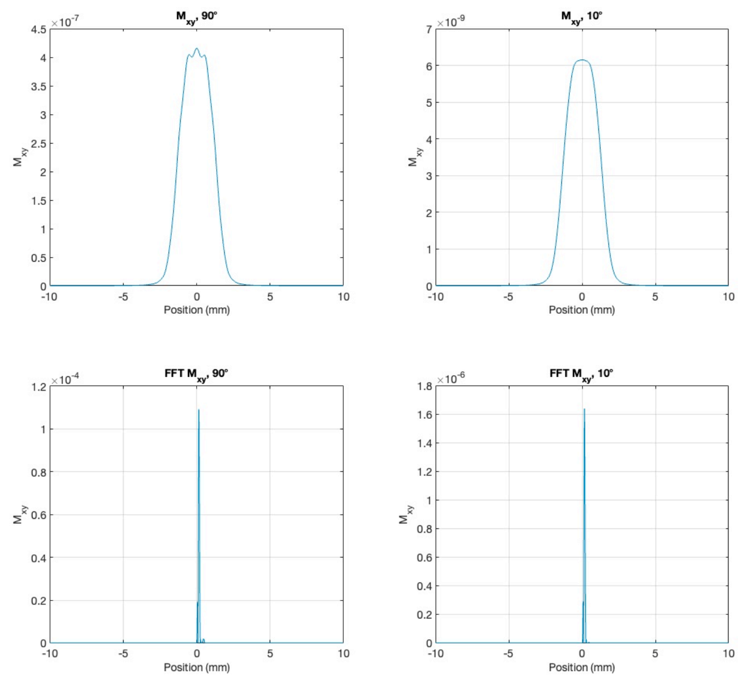
3. Now that you have a 90° pulse, we will look at producing different flip angles.
   - Generate an RF pulse for a flip angle of 30° and simulate its slice profile.



- Generate a 10° RF pulse and simulate its slice profile.

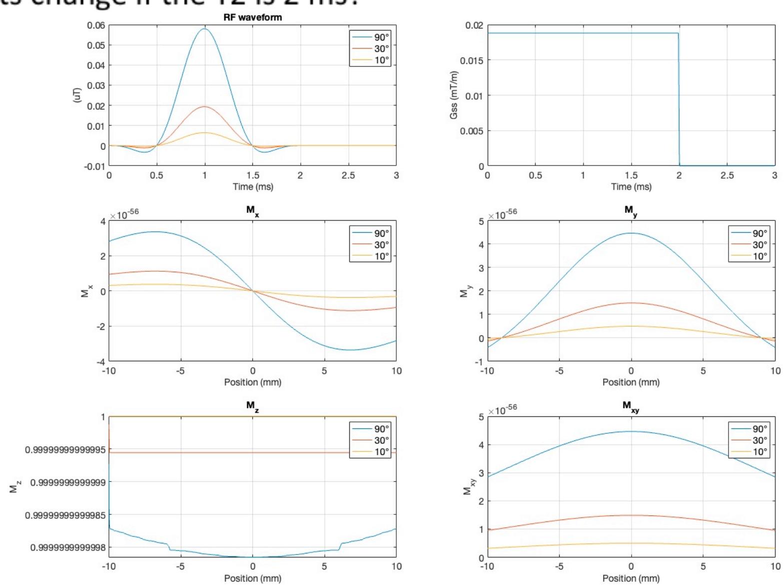


- Now take your results from the 90° and 10° pulses, and on the same plot at the slice profile show the slice profile approximation you would obtain by simply Fourier transforming the RF pulse. Note the differences and similarities between the Fourier transform and Bloch equation simulation at the small and large flip angles.



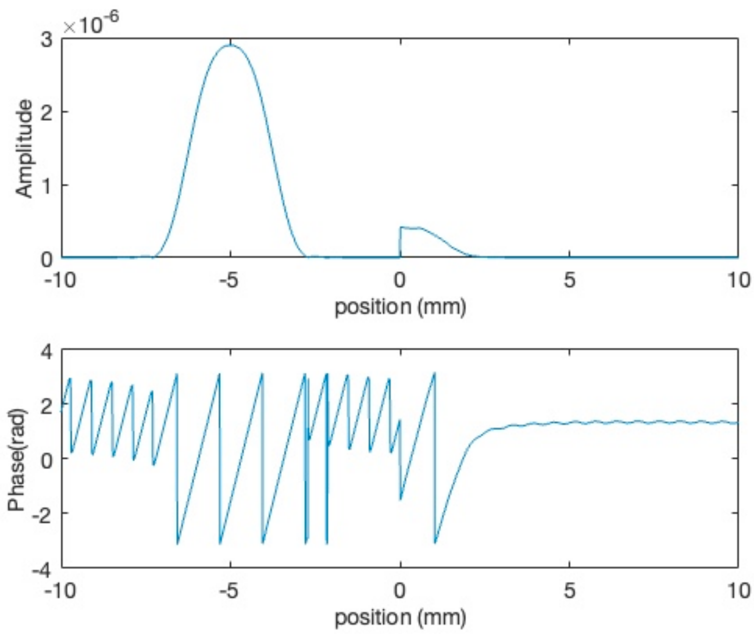
The amplitude changes. It is higher with larger flip angles. The bandwidth keeps the same.

- How do your results change if the T2 is 2 ms?



There is more signal decay due to shorter T2, and the slice profile gets distorted.

4. For one of the pulses you generated, simulate m_x, m_y, m_z and m_{xy} at the point in time between the slice selective gradient and the slice rephasing gradient. Explain the purpose of the slice rephasing gradient.



To ensure a coherent signal, and bring back spins that have different frequencies.

5. Simultaneous multi-slice acquisition

$$RF_{MB}(t) = A(t) \cdot \sum_{N} e^{i\Delta\omega_n t + \varphi_n}.$$