BME 599 HW 1

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Problem 1: Spatial Encoding

1a. The sinc shaped rf pulse has time bandwidth 2 and duration 1ms, then

$$T \cdot BW = 2, \quad \Longrightarrow BW = 2/1ms = 2000Hz$$

from the image resolution, the selected slice thickness should be 3mm, suppose the amplitude of slice selective gradient is G_s, then

$$BW = \bar{\gamma} G_{ss} \Delta z,$$

$$\Longrightarrow G_{ss} = \frac{BW}{\bar{\gamma}\Delta z} = \frac{2000Hz}{42.48MHz/T \times 3mm} = 15.6568mT/m < 25mT/m$$

The duration that τ_{ss} should be the same as the duration of rf pulse, thus $\tau_{ss} = 1ms$, then

$$\tau_{ss}^{rise} = G_{ss}/s_{grad} \approx 0.087ms$$

then the total duration of the slice-select gradient is

$$\tau = 2\tau_{ss}^{rise} + \tau_{ss} \approx 1.174ms$$

In addition, it needs the rephasing gradient of at least a duration of

$$25mT/m \times t' + \frac{25mT/m}{180mT/m/ms} \times 25mT/m \times \frac{1}{2} \times 2 = \frac{1}{2}(\tau_{ss}^{rise} G_{ss} \times \frac{1}{2} \times 2 + G_{ss}\tau_{ss})$$

$$\Longrightarrow t' = 0.201ms$$

$$t_{rephasing} = t' + 2 \times \frac{25mT/m}{180mT/m/ms} = 0.479ms$$

1b. for the phase encoding, we only move to one pixel distance in k-space,

$$FOV_y = \frac{1}{\Delta k_y} \Longrightarrow \Delta k_y = \frac{1}{FOV_y} = \frac{1}{1.2mm \times 256} = 3.255m^{-1}$$

and consider the maximum phase encoding gradient we need to use is

$$k_{y,\max} = \Delta k_y \times 256/2 = 416.667m^{-1}$$

$$k_y = \bar{\gamma} \cdot \text{Area}_{G_p} \Longrightarrow \text{Area}_{G_p} = \frac{k_y}{\bar{\gamma}} = 9.7855 \times 10^{-6} T/Hz/m$$

first compare with the gradient only has ramp-up and ramp-down shape, with slew rate 180T/m/s, then the maximum area it can achieve is $\frac{25mT/m}{180T/m/s} \times 25mT/m = 3.472 \times 10^{-6} T/m \cdot s$ which is smaller than Area_{G_p} we need. Thus we need the phase encoding gradient contains a flat area and use the maximum amplitude for it, suppose the time for flat-top is t_{flat}, then

$$\frac{25mT/m}{180T/m/s} \times 25mT/m \times \frac{1}{2} \times 2 + 25mT/m \times t_{flat} = \text{Area}_{G_p}$$

$$\Longrightarrow t_{flat} \approx 0.25253ms$$

then the shortest possible duration of the phase encoding gradient is

$$\frac{25mT/m}{180T/m/s} \times 2 + t_{flat} \approx 0.530ms$$

1c. similar to the phase encoding direction, for frequency encoding

$$FOV_x = \frac{1}{\Delta k_x} \implies \Delta k_x = \frac{1}{FOV_x} = \frac{1}{1.2mm \times 256} = 3.255m^{-1}$$

suppose the amplitude of the frequency encoding is G_f, and one frequency encoding area (ignore the gradient rise up and down parts) is

$$\Delta k_x \times 256 = \bar{\gamma} G_f \times T_{adc}$$

the total time of adc is

$$T_{adc} = \frac{1}{750Hz/pixel \times 256} \approx 1.333ms$$

consider that adc is only on when the readout gradient is constant, thus the amplitude of the frequency encoding gradient is

$$G_f = \frac{\Delta k_x \times 256}{\bar{\gamma} \times T_{adc}} \approx 14.678mT/m < 25mT/m$$

and the rise time for the gradient is

$$t_r = \frac{G_f}{180mT/m/ms} \approx 0.0815ms$$

the total duration of frequency encoding gradient is

$$t_{total} = 2t_r + T_{adc} \approx 1.496ms$$

In addition the prephasing of readout gradient needs at least

$$\frac{25mT/m}{180T/m/s} \times 25mT/m \times \frac{1}{2} \times 2 + t' \times 25mT/m = \frac{1}{2}(G_f \times t_r \times \frac{1}{2} \times 2 + T_{adc} \times G_f)$$

$$\implies t'_{flat} = 0.276ms,$$

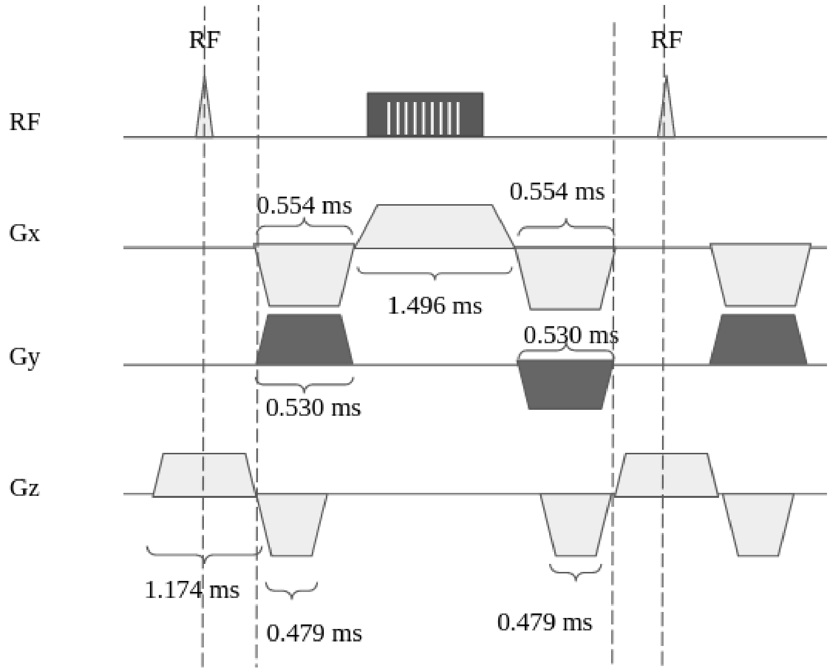
$$t_{prephasing} = t'_{flat} + \frac{25mT/m}{180mT/m/ms} \times 2 = 0.554ms$$

and in order to be a bSSFP sequence, we will also need another same gradient in the end of each TR

$$\implies t_{rephasing} = 0.554ms$$

1d. the shortest possible TE and TR:

draw the sequence diagram and put all event's possible duration in it, then we have a diagram looks like the following



then can see that the shortest TR is

$$TR = 1.174 + 0.554 + 1.496 + 0.554 \approx 3.778ms$$

the shortest TE is half of the TR which is

$$TE = TR/2 \approx 1.889ms$$

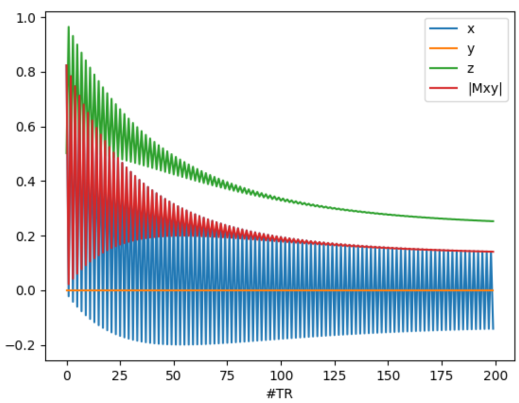
1e. To further decrease the TE and TR by maintaining the same spatial resolution:

(1) increase the receiver bandwidth, then the time for ADC will be shorter.
(2) use the partial Fourier acquisition, then acquire less data but with the same resolution
(3) use of shorter RF pulse

Problem 2: Balanced and Spoiled Steady-State Sequences

2a Simulate the steady-state frequency response of a bSSFP sequence with a flip angle of 60° using Bloch equation simulations.

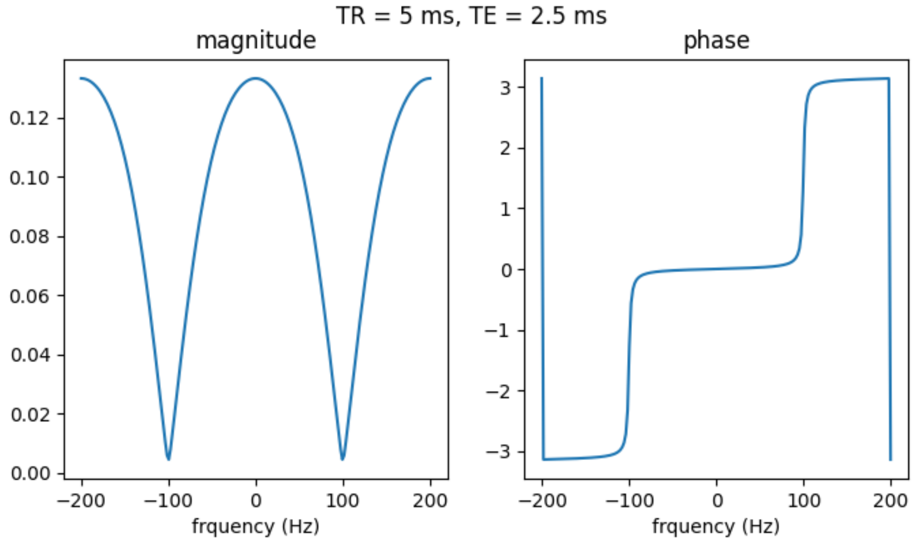
first check how many TR can the signal get steady-state,



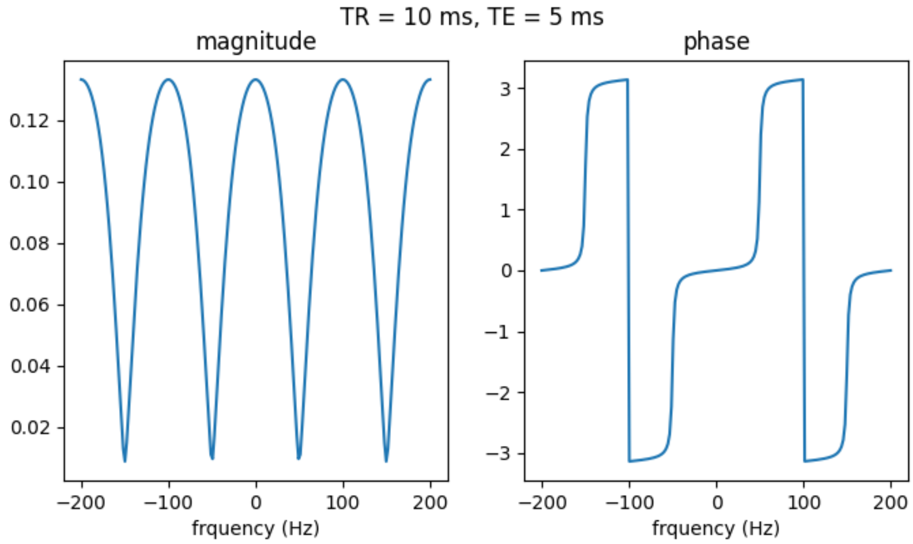
then it seems using TR=500 will absolutely have signal in steady state.

plot of frequency response:
the plot of phase may be different depends on the signal in TR with odd or even number

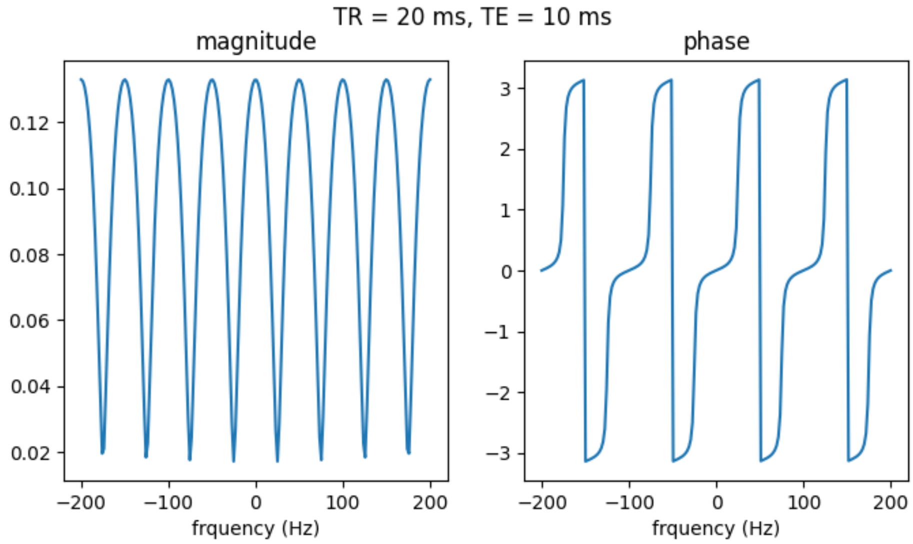
(1) TR=5ms and TE=2.5ms:



(2) TR=10ms and TE=5ms:



(3) TR=20ms and TE=10ms:



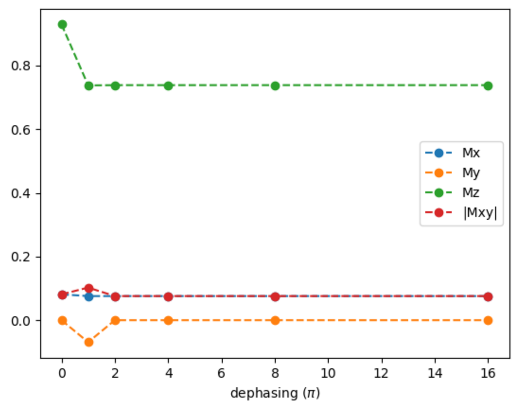
2b Modify the bSSFP sequence to generate a FLASH sequence by adding a gradient spoiler along the slice selection direction.

(i) Assumeing the spoiler gradient as a "perfect spoiler" that completely eliminates transverse magnetization components, please calculate the steady state signal at T1 = 1000 ms, T2 = 100 ms and the flip angle = 10° using a Bloch equation simulation.

The steady state signal from the Bloch simulation is

$$M_{end} = [0.0657,\ 0,\ 0.3951]^T$$

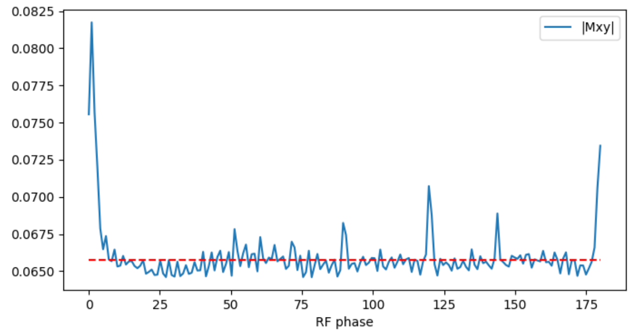
(ii) Rather than assuming ideal spoiling, now let us examine the effect of a gradient spoiler using a Bloch equation simulation. Simulate the steady state signal with the gradient spoiler achieving 2π, 4π, 8π, and 16π dephasing (1 cycle, 2 cycles, 4 cycles, and 8 cycles) per voxel (each with the same TR). Plot the signal as a function of dephasing moment.



the steady-state signal magnetization is approximately $|M_{xy}| = 0.0757$

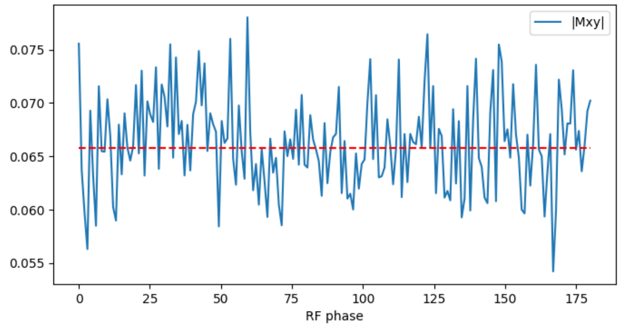
(iii) Now simulate the effects of both the gradient spoiler and RF spoiling. Simulate the steady-state signal by sweeping RF phase from $0° - 180°$ with a reasonable step. Plot the signal as a function of RF phase. Please list your choice of RF phase that best eliminates transverse magnitization.

suppose the ϕ is rf pulse in first TR, in each next TR, the RF phase is linearly increase at each TR, then



if the transverse magnetization is best eliminates the transverse magnetization, then the result signal should match the perfect spoiling, the red line in the plot.

OR if suppose the rf phase is changing quadraticly based on changing as $\frac{n(n+1)}{2}\phi$



Problem 3: Slice Profile Simulation

1. the pulse duration is 2ms,

$$TBW = T \cdot BW = 8, \implies BW = \frac{8}{2ms} = 4000Hz$$

and the slice thickness is $\Delta z = 5mm$, thus

$$BW = \bar{\gamma} G_s \Delta z, \implies G_s = \frac{4000Hz}{42.58MHz/T \cdot 5mm} \approx 18.788mT/m$$

then the rise time of the slice-selective gradient is

$$t_{rise} = \frac{G_s}{180T/m/s} \approx 0.104ms$$

then compute the rephasing gradient for this rf pulse, let $t_{re,flat}$ denote the flat time of the gradient, the shortest rephasing gradient is

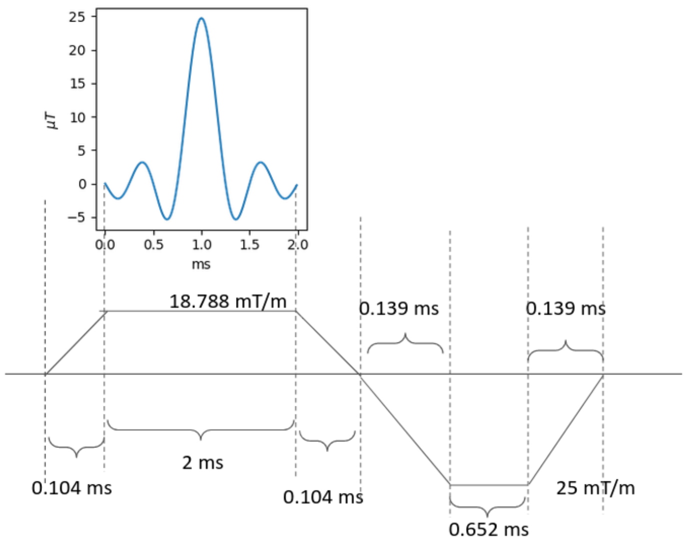
$$t_{re,flat} \times G_{\max} + \frac{G_{\max}}{slewrate} \times G_{\max} = \frac{1}{2}(t_{rise} \times G_s + G_s \times 2ms)$$

$$\implies t_{re,flat} \approx 0.652ms$$

and the rephasing gradient rising time is (same as ramp-down time)

$$t_{re,rise} = \frac{G_{\max}}{slewrate} \approx 0.139ms$$

2. The time-bandwidth of a sinc pulse is equal to the number of points that cross zero in its waveform, the shape of the rf pulse and the slice gradient is



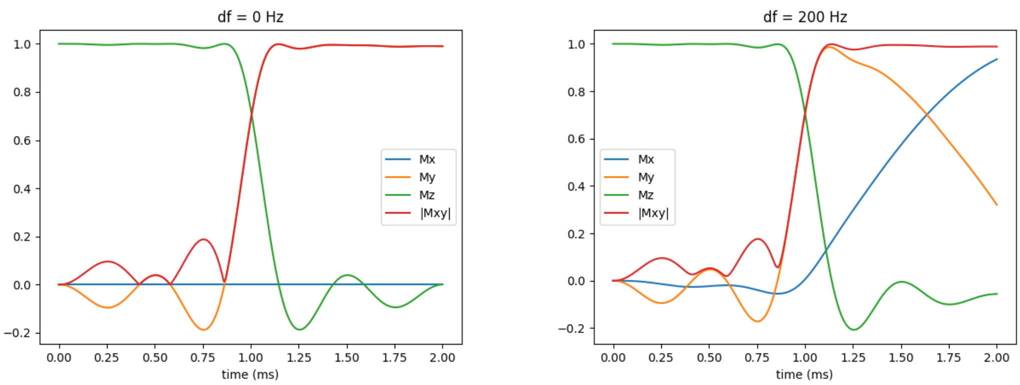
Simulate the slice profile of your RF pulse with a flip angle of 90 degrees:

Consider the spin that is on-resonance, then the rotation of the magnetization given by bloch equation is

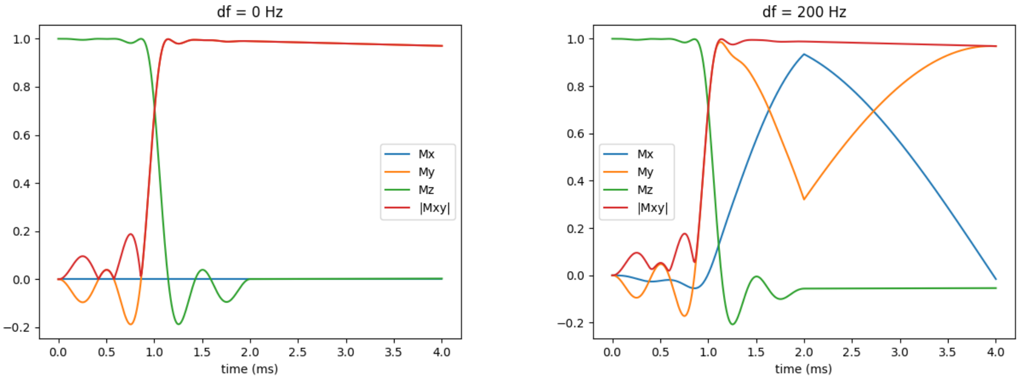
$$\alpha = \int_0^T w(t)dt = \int_0^T \bar{\gamma} B_1(t)dt$$

then use the above sinc rf pulse shape, compute the magnitude of the rf pulse at center (maximum) is 24.72 μT.

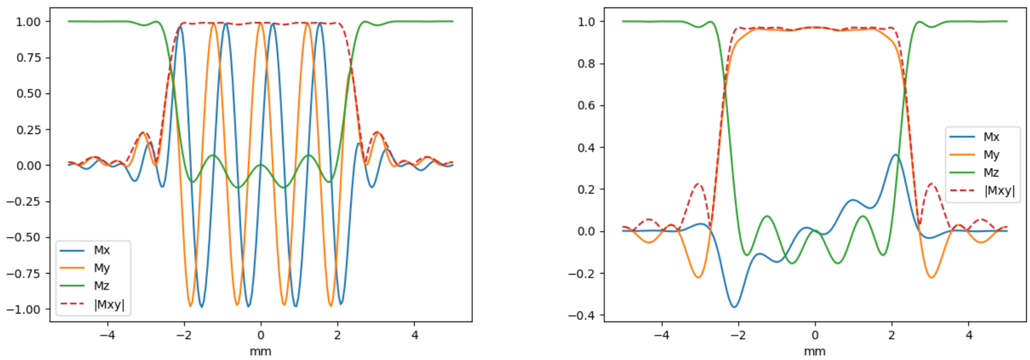
apply the 90° pulse and simulate 0Hz off-resonance, and 200Hz off-resonance:



apply the 90° pulse and simulate 0Hz off-resonance, and 200Hz off-resonance, and adding the rephasing gradient after RF (notice the time of simulation is longer):



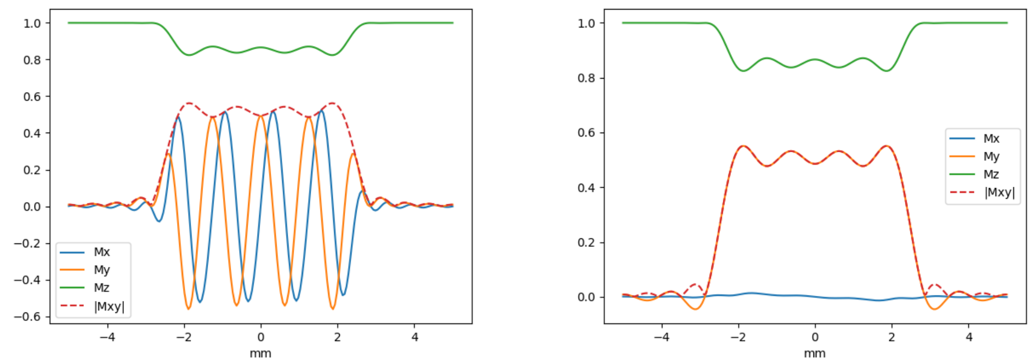
simulate the rf profile of 90° pulse



left: without rephasing gradient right: with rephasing gradient

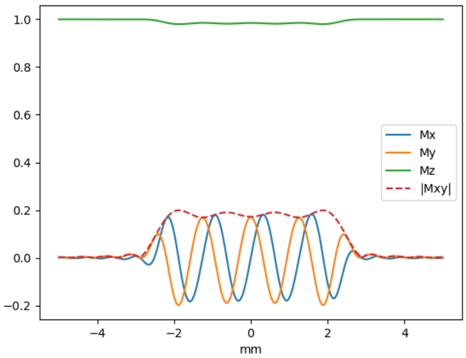
3. now that with a 90° pulse, we can scale it to generate rf pulse with flip angle of 10° and 30°

   simulate the rf pulse with 30° flip angle

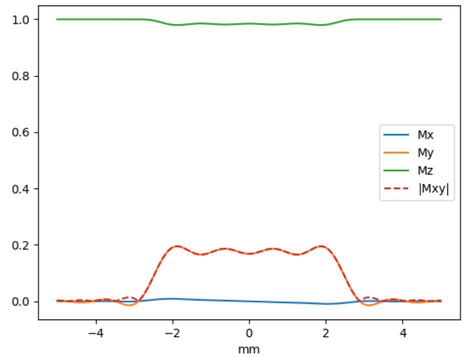


left: without rephasing gradient right: with rephasing gradient

simulate the rf pulse with 10° flip angle

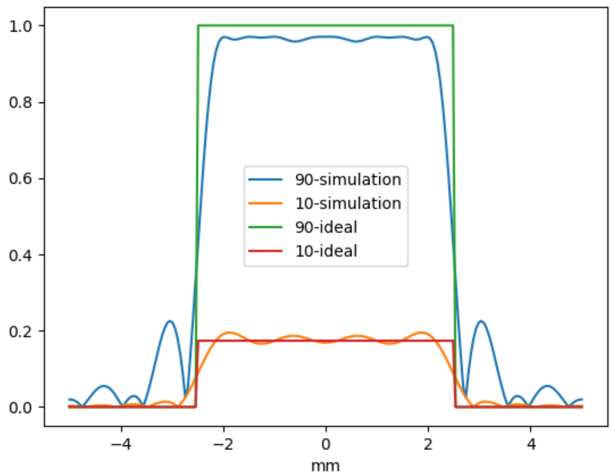


left: without rephasing gradient



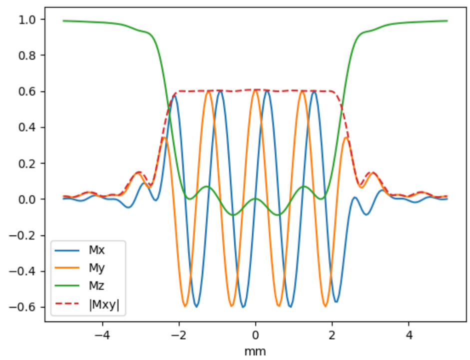
right: with rephasing gradient

plot of 90 degree and 10 degree together with their idea profile:

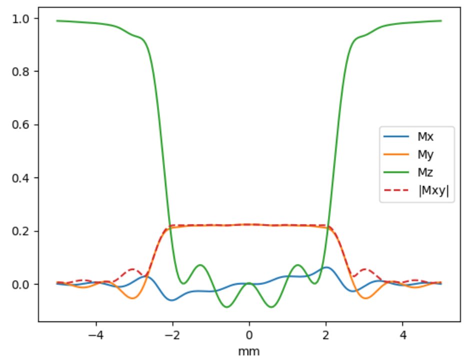


with rephasing gradient

if T2 is 2ms, then the transverse magnetization relax really quickly, and the signal will be much smaller. For the 90 degree pulse simulated with T2=2ms:



left: without rephasing gradient

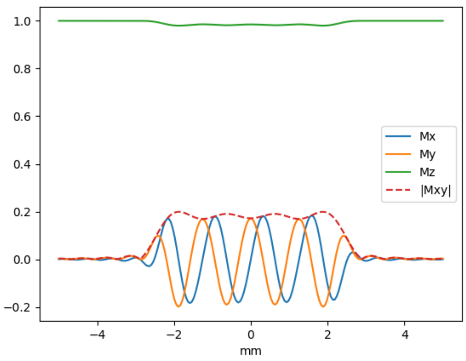


right: with rephasing gradient

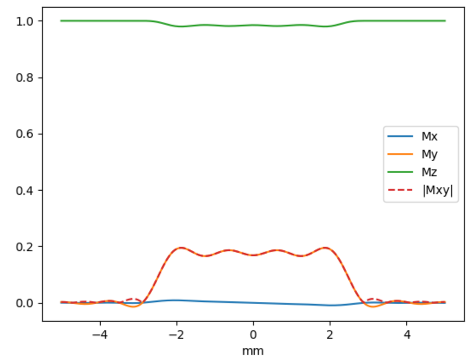
and after the time of rephasing gradient, the signal could get even smaller.

4. simulate mx, my, mz and mxy at the point in time between the slice selective gradient and the slice rephasing gradient.

for the 10° flip angle:



left: without rephasing gradient



right: with rephasing gradient

even we have excited magnetization in the correct slice, but without rephasing gradient, over the slice direction, the phase of the magnetization is not the same, thus could lead to the reduction of the MR signal. The rephasing gradient helps the magnetization at the same phase in the slice, then gives enough signal intensity to do the imaging.

5. design an RF pulse to excite 5 slices for a Simultaneous Multi-slice acquisition.

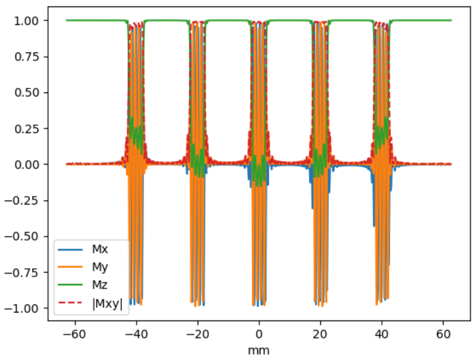
   using the 90 degree pulse, let the gap $\Delta g = 20mm$, then the frequency shift for the RF pulse is

$$\Delta f = \bar{\gamma} G_s \Delta g$$

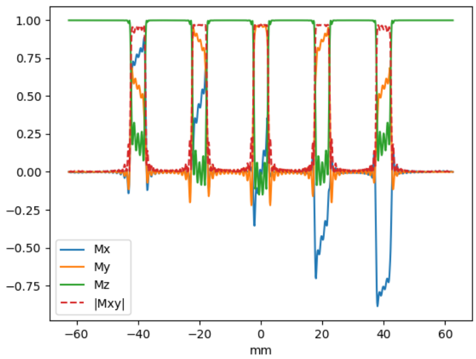
$$\implies \Delta f = 15.99kHz$$

for shifting to a neighbot slice

then modulate the rf pulse and simulate the slice profile



left: without rephasing gradient



right: with rephasing gradient