BME 599.020 HW 1 Due Sep 28th, 2023

Q1) Spatial Encoding

a) $TBW = \tau_{RF} \cdot \Delta f$

$2 = 1\,ms \cdot \Delta f \Rightarrow \Delta f = \frac{2}{1\,ms} = 2\,kHz$

$\Delta f = \gamma \cdot G \cdot \Delta z$

$2\,kHz = 42.58\,\frac{MHz}{T} \cdot G_{ss} \cdot 3\,mm$

$G_{ss} = \frac{2\,kHz}{42.58\,MHz} \cdot T \cdot \frac{1}{3\,mm} = 0.0157\,T/m = 15.7\,mT/m$

$\tau_{ss}^{rise} = \frac{G_{ss}}{maxSlew} = \frac{15.7\,mT/m}{180\,mT/m/ms} = 87.2\,\mu s$

$\tau_{ss} = \tau_{RF} = 1\,ms$

$\text{Total time} = \tau_{ss} + 2\tau_{ss}^{rise}$

$= (1000 + 2 \cdot 87.2)\,\mu s$

$= 1174.4\,\mu s$ //

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b) Relative to isocenter $k_{PE\,max} = (1.2\,mm)^{-1}$

$k_{PE\,max} = (1.2\,mm)^{-1} = \gamma G_{PE} \tau_{PE}$, setting $G_{PE} = G_{max} = 25\,mT/m$

$\Rightarrow \tau_{PE} = \frac{(1.2\,mm)^{-1}}{42.58\,MHz/T} \cdot \frac{1}{25\,mT/m} = 7.828 \cdot 10^{-4}\,s = 782.8\,\mu s$

$$\tau_{PE}^{rise} = \frac{G_{max}}{maxSlew} = \frac{25\ mT/m}{180\ mT/m/ms} = 138.9\ \mu s$$

$$\tau_{PE}^{Total} = (782.8 + 2 \cdot 138.9)\ \mu s \approx 1060.6\ \mu s$$

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c) Receiver BW $= 750\ \frac{Hz}{pixel} = \frac{1}{\text{Sampling time}}$

$$\Rightarrow T_s = 1333.\bar{3}\ \mu s = \text{flat top time}$$

Finding G_{read}:

$$rBW = \gamma \cdot G_{read} \cdot \Delta_{read} = 750\ \frac{Hz}{pixel}$$

$$\Rightarrow G_{read} = \frac{750\ Hz/pixel}{42.58\ MHz/T} \cdot \frac{1}{1.2\ mm/pixel} = 0.0147\ T/m$$

$$= 14.7\ mT/m$$

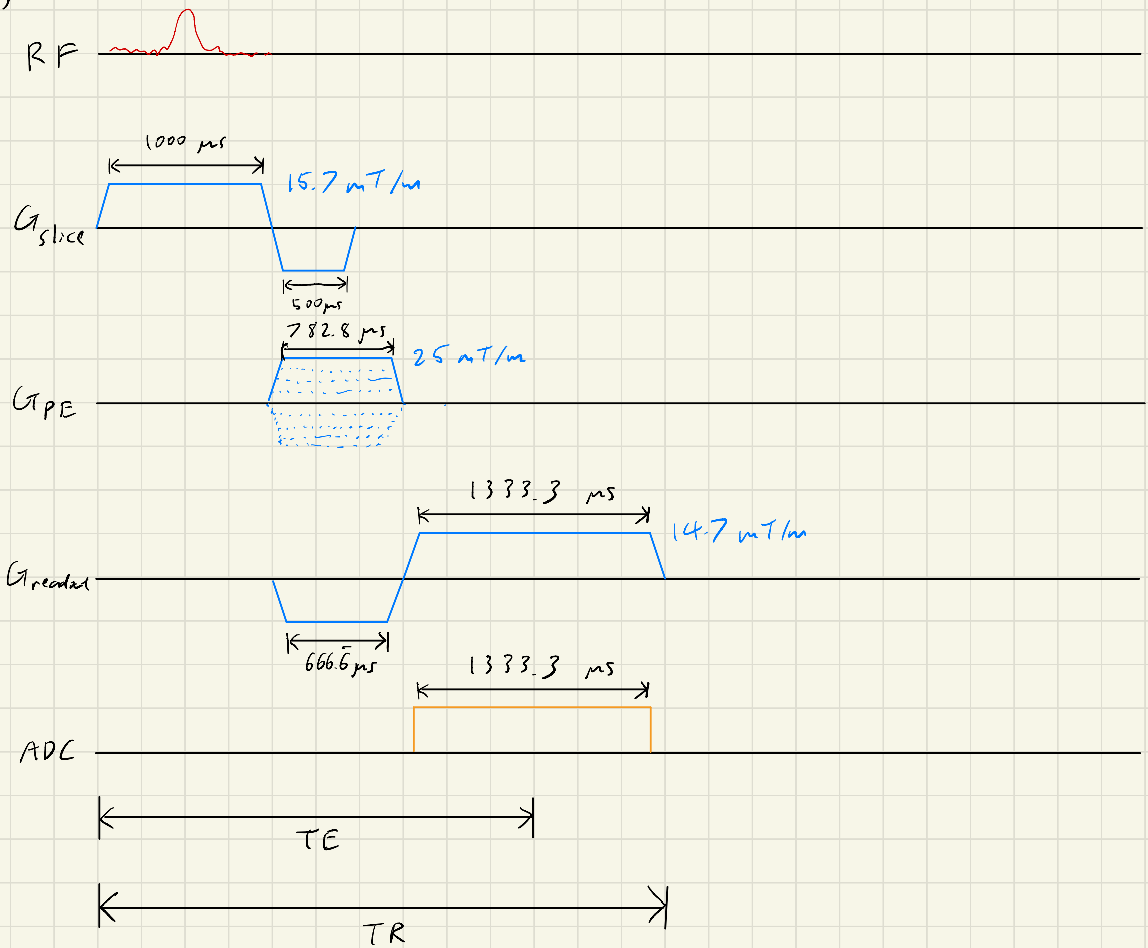
Rise time $\tau_{read}^{rise} = \frac{14.7\ mT/m}{180\ mT/m/ms} = 81.7\ \mu s$

Total duration $= T_s + 2 \cdot \tau_{read}^{rise}$

$$= 1496.7\ \mu s$$

☆ Diagram not to scale

d)



$$TE = \tau_{SS} + \tau_{PE} + \frac{1}{2}\tau_{Read}$$

$$= \left(1174.4 + 1060.6 + \frac{1}{2}\cdot 1496.7\right)\ \mu s = 2983.3\ \mu s$$

$$\text{Shortest possible TR} = \tau_{SS} + \tau_{PE} + \tau_{Read}$$

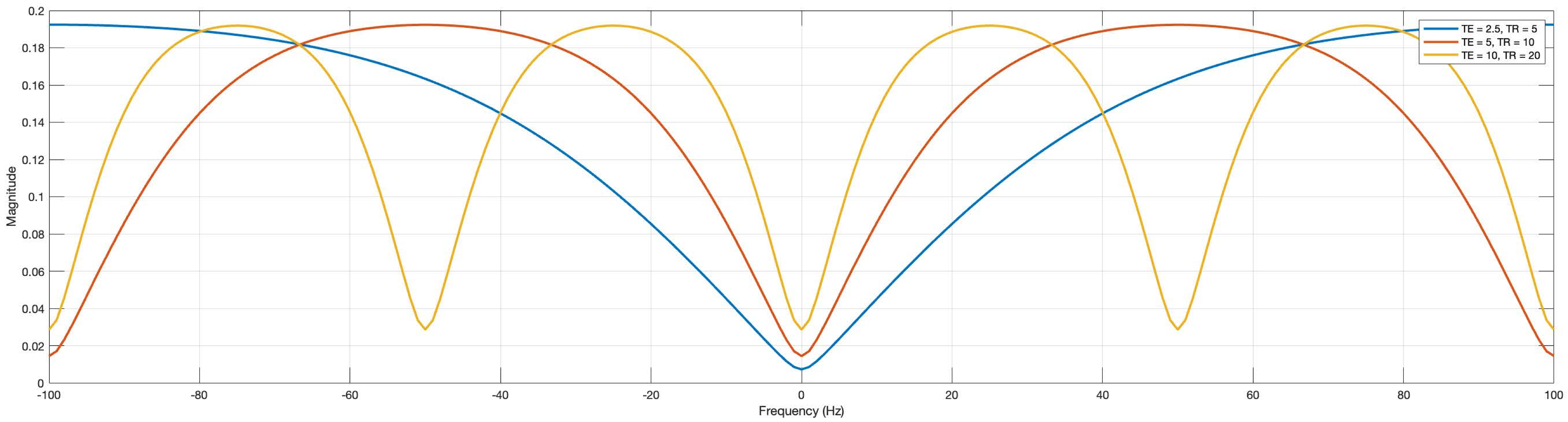
$$= (1174.4 + 1060.6 + 1496.7)\ \mu s$$

$$= 3731.7\ \mu s$$

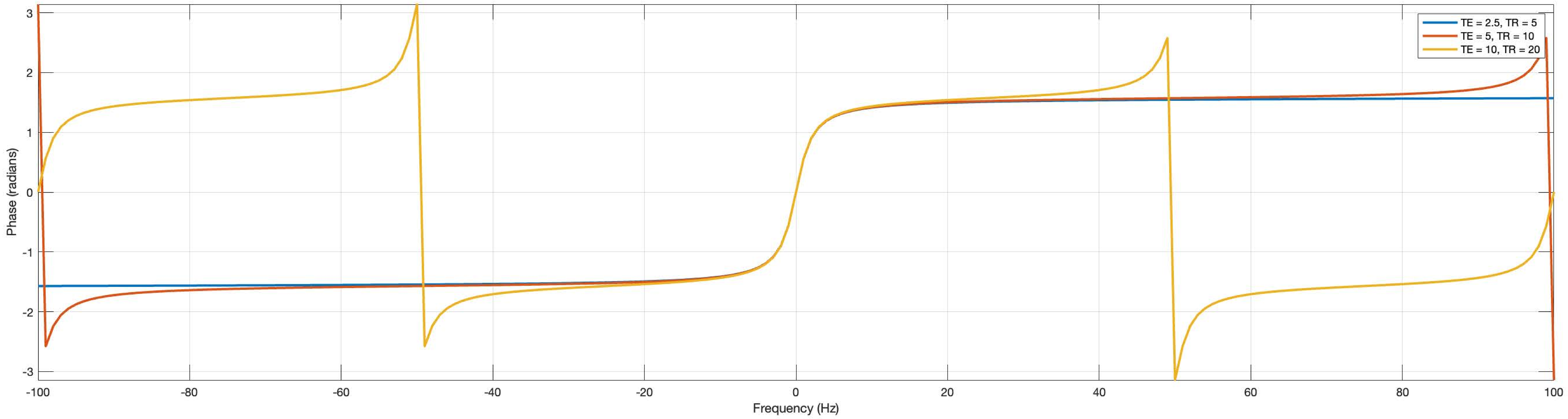
e) 1. Use a scanner with a higher maximum gradient amplitude

2. Use a scanner with a higher maximum slew rate

3. Increase receiver bandwidth



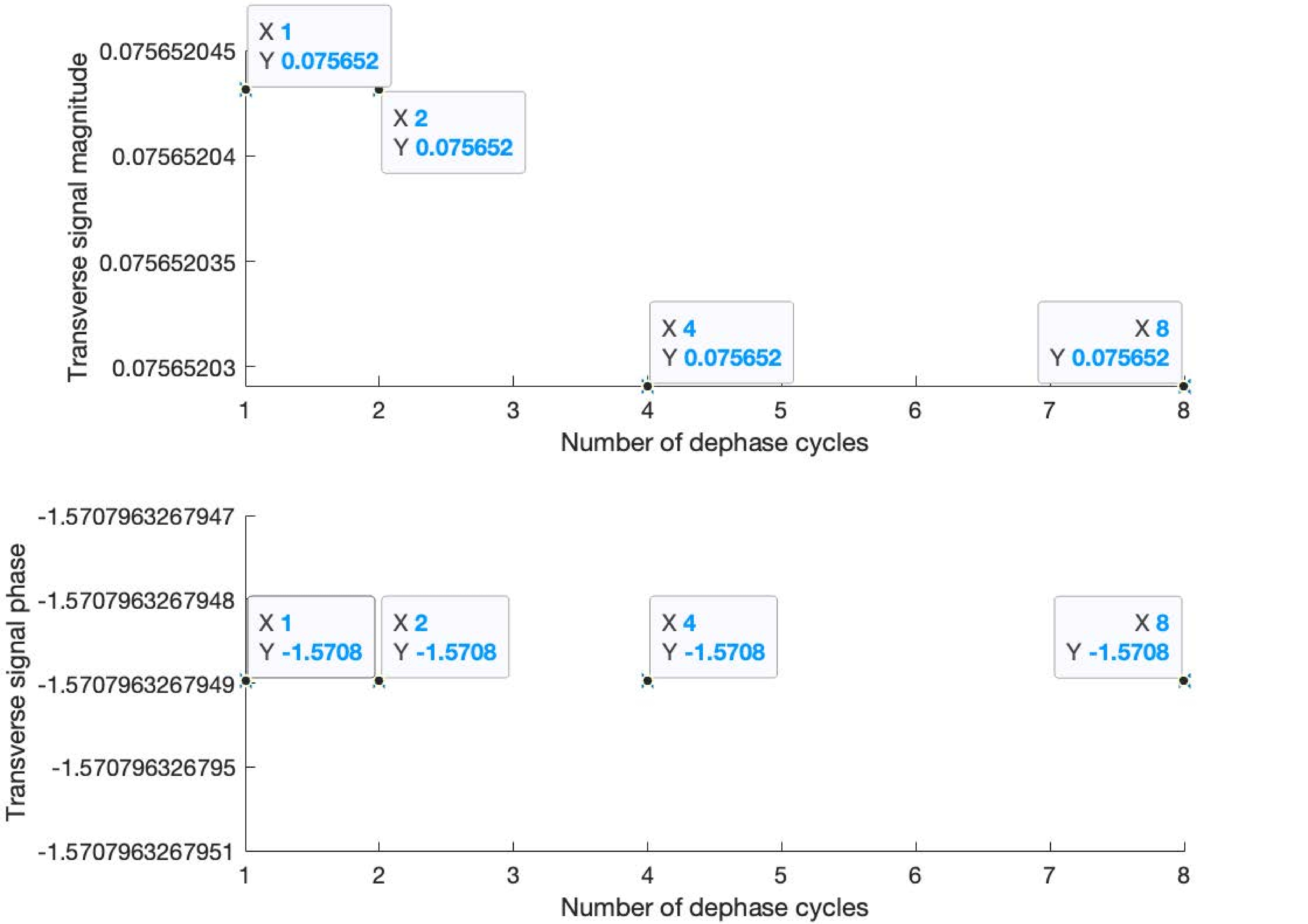
Magnitude
Frequency (Hz)
TE = 2.5, TR = 5
TE = 5, TR = 10
TE = 10, TR = 20
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-100
-80
-60
-40
-20
0
20
40
60
80
100



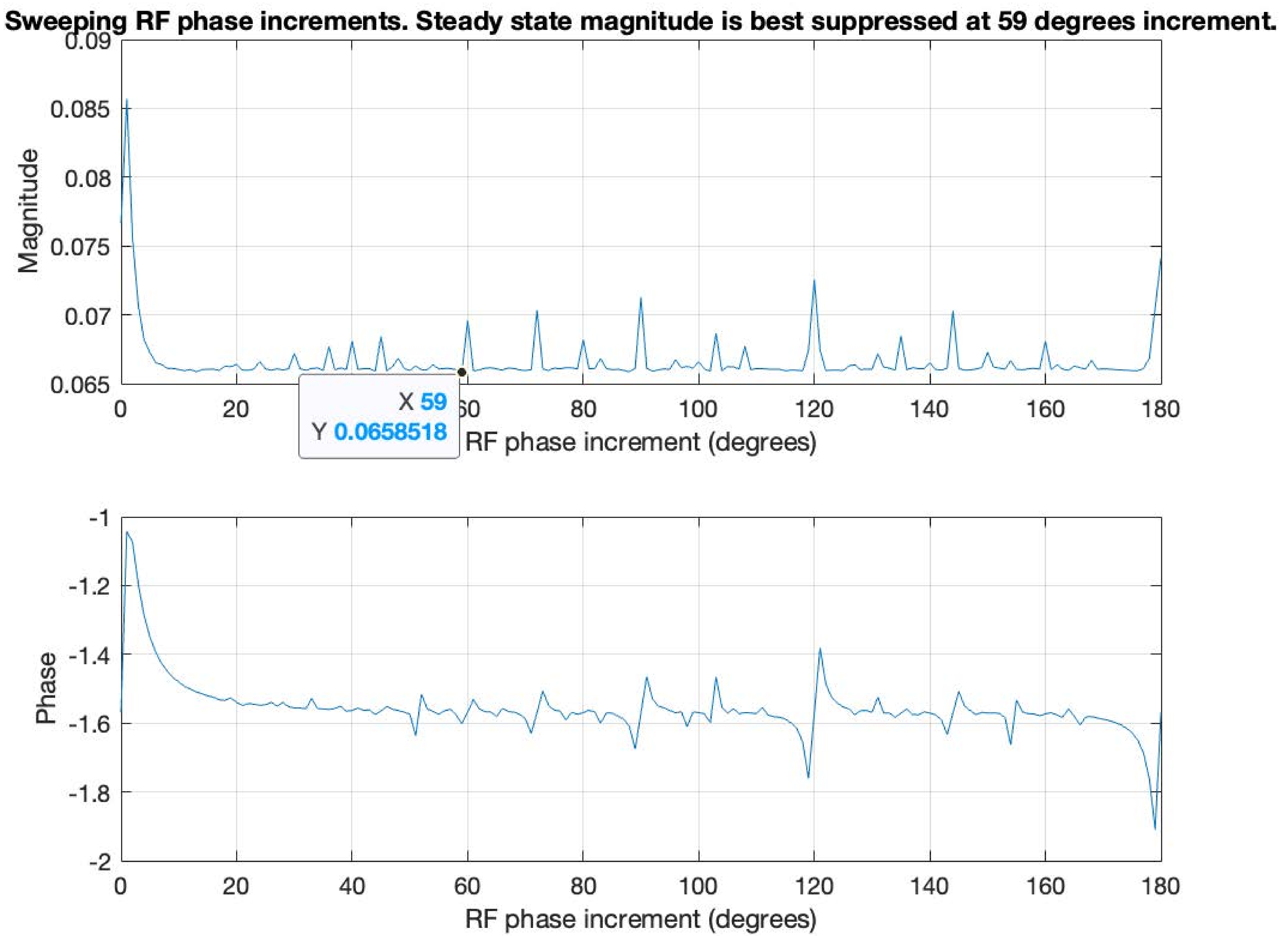
Phase (radians)
Frequency (Hz)
TE = 2.5, TR = 5
TE = 5, TR = 10
TE = 10, TR = 20
3
2
1
0
-1
-2
-3
-100
-80
-60
-40
-20
0
20
40
60
80
100

Q2) Balanced & Spoiled Steady-State sequences

b) i) $$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 0.0757 \\ 0 \\ 0.7377 \end{bmatrix}$$



Transverse signal magnitude
0.075652045
0.07565204
0.075652035
0.07565203
X 1
Y 0.075652
X 2
Y 0.075652
X 4
Y 0.075652
X 8
Y 0.075652
1
2
3
4
5
6
7
8
Number of dephase cycles
Transverse signal phase
-1.5707963267947
-1.5707963267948
-1.5707963267949
-1.570796326795
-1.5707963267951
X 1
Y -1.5708
X 2
Y -1.5708
X 4
Y -1.5708
X 8
Y -1.5708
1
2
3
4
5
6
7
8
Number of dephase cycles



Sweeping RF phase increments. Steady state magnitude is best suppressed at 59 degrees increment.
Magnitude
0.09
0.085
0.08
0.075
0.07
0.065
0
20
60
80
100
120
140
160
180
X 59
Y 0.0658518
RF phase increment (degrees)
Phase
-1
-1.2
-1.4
-1.6
-1.8
-2
0
20
40
60
80
100
120
140
160
180
RF phase increment (degrees)

Q3) Slice Profile Simulation

1) TBW $= 8$, $\Delta slice = 5\,mm$, $\tau_{RF} = 2\,ms$

$$\Delta f = \frac{8}{2ms} = 4kHz$$

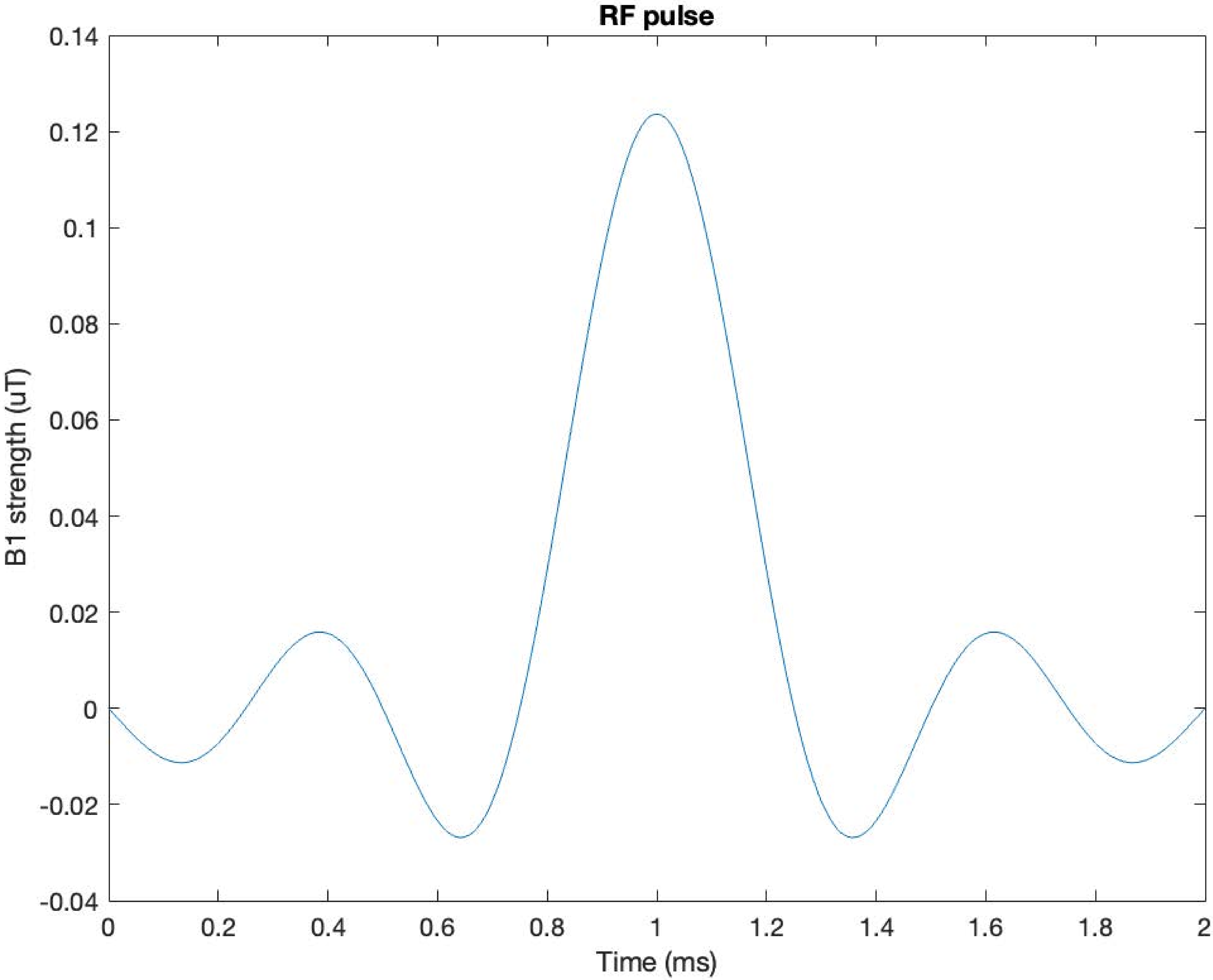
$$G_{slice} = \frac{\Delta f}{\gamma \Delta slice} = \frac{4\,kHz}{42.58\,MHz/T} \cdot \frac{1}{5\,mm} = 18.8\,mT/m$$

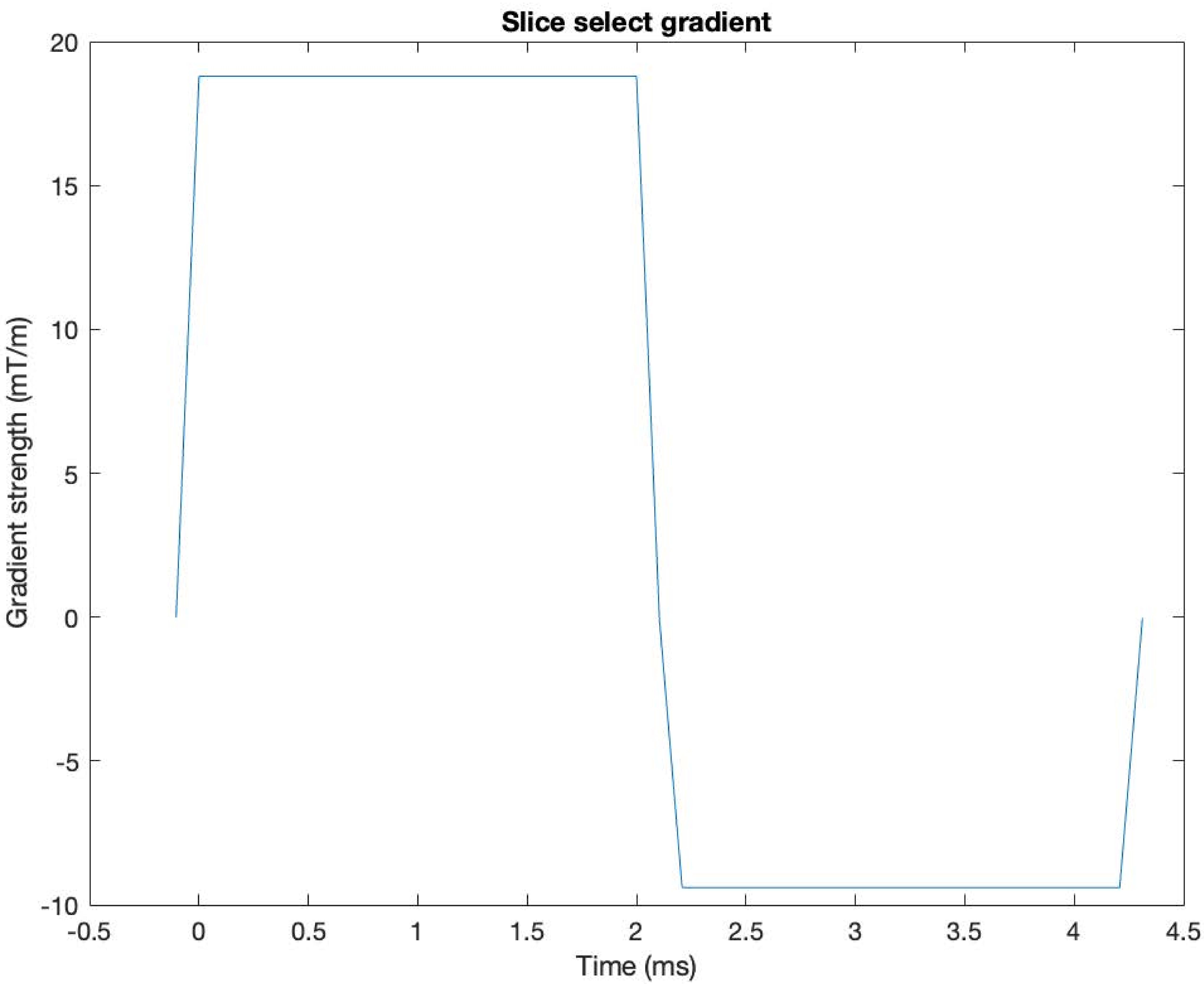
$$\text{ramp time} = \frac{18.8\,mT/m}{180\,mT/m/ms} = 104.4\,\mu s$$

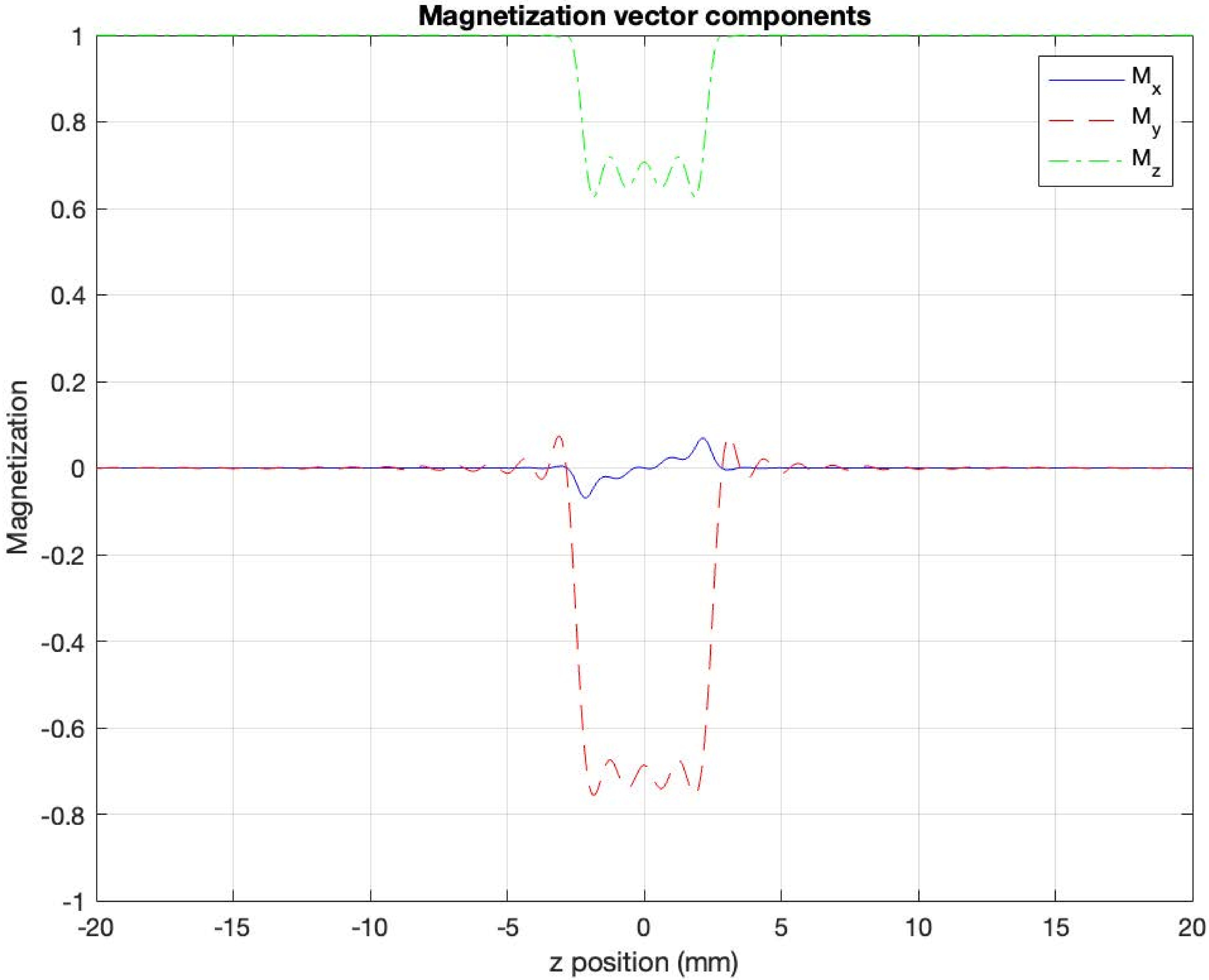
$$\text{Total duration of } G_{slice} = 2000 + 2 \cdot 104.4$$
$$= 2208.8\,\mu s$$

Amplitude of rephasing gradient $= 18.8\,mT/m$

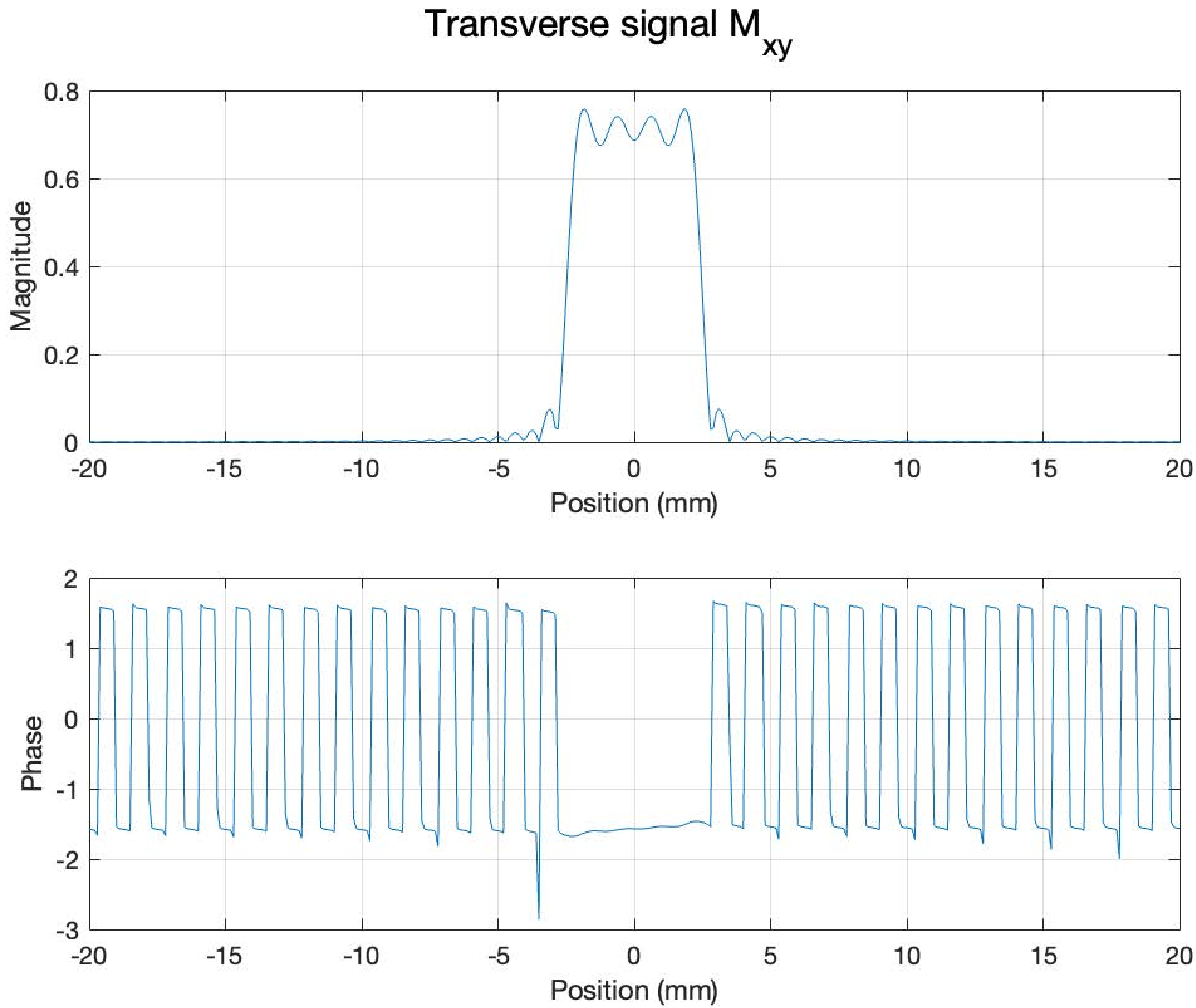
$$\text{Duration of rephasing gradient} = 1000 + 2 \cdot 104.4$$
$$= 1208.8\,\mu s$$


RF pulse
B1 strength (uT)
Time (ms)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

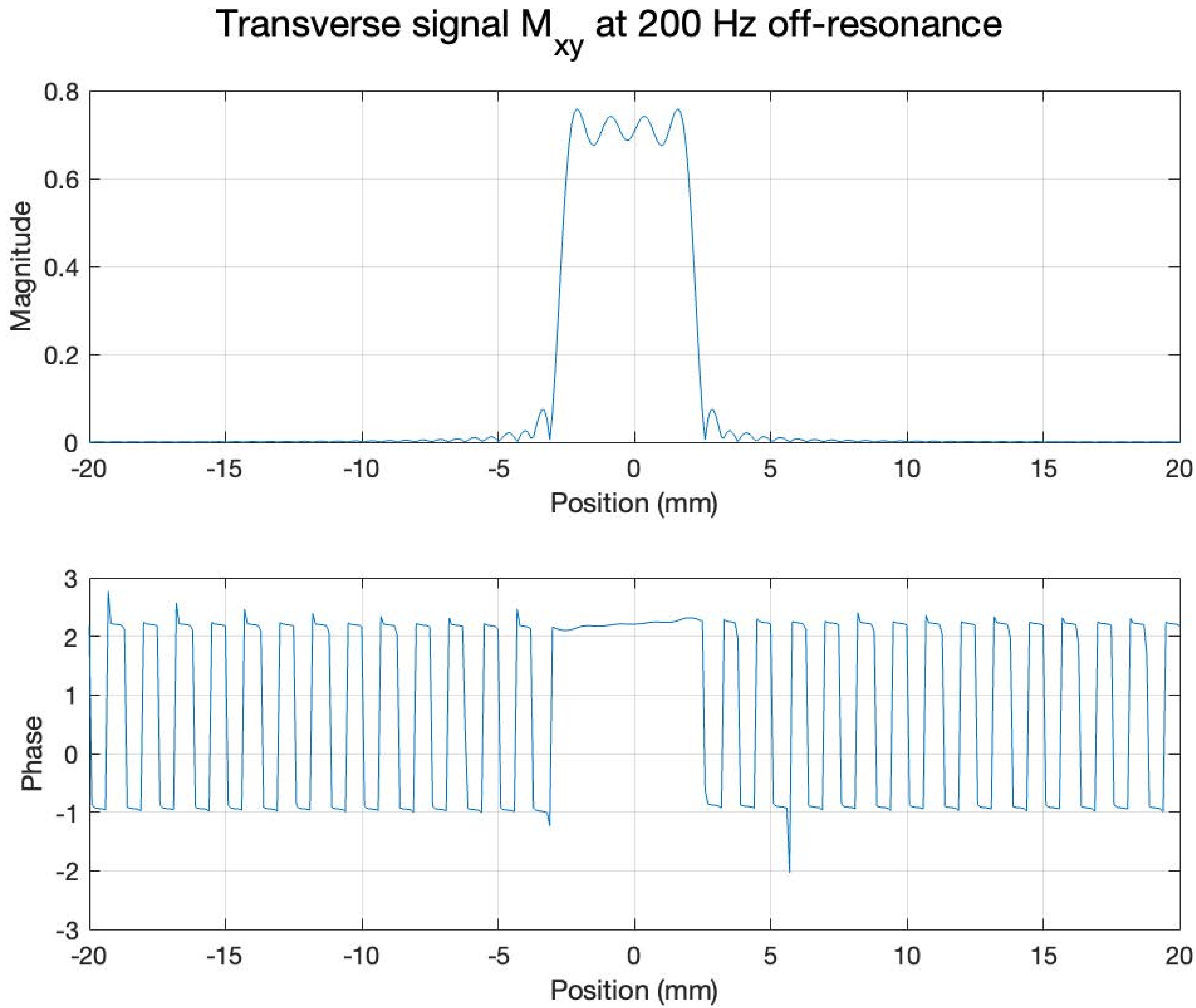

Slice select gradient
Gradient strength (mT/m)
Time (ms)
20
15
10
5
0
-5
-10
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

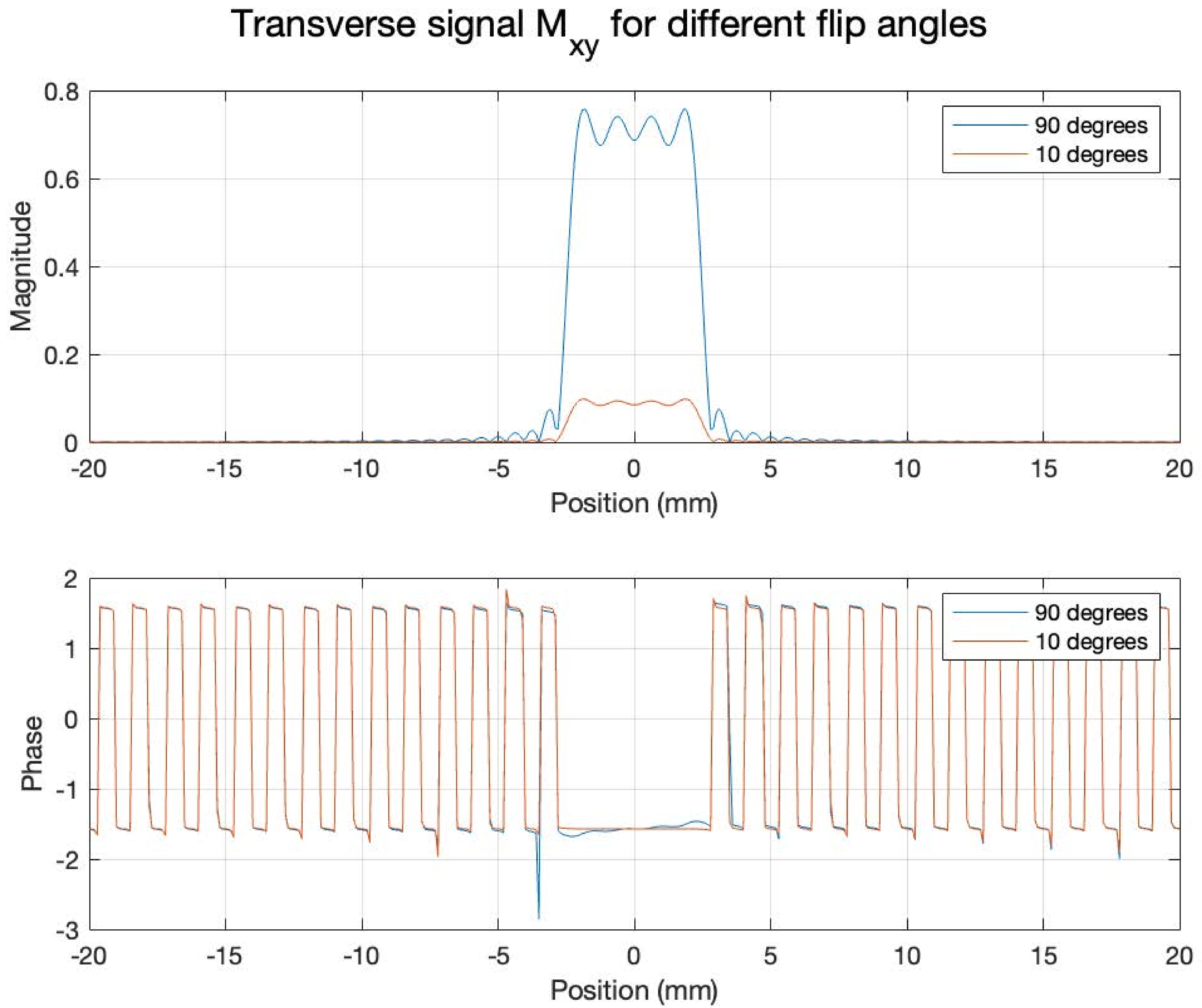


Magnetization vector components
Magnetization
z position (mm)
M_x
M_y
M_z
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-20
-15
-10
-5
0
5
10
15
20

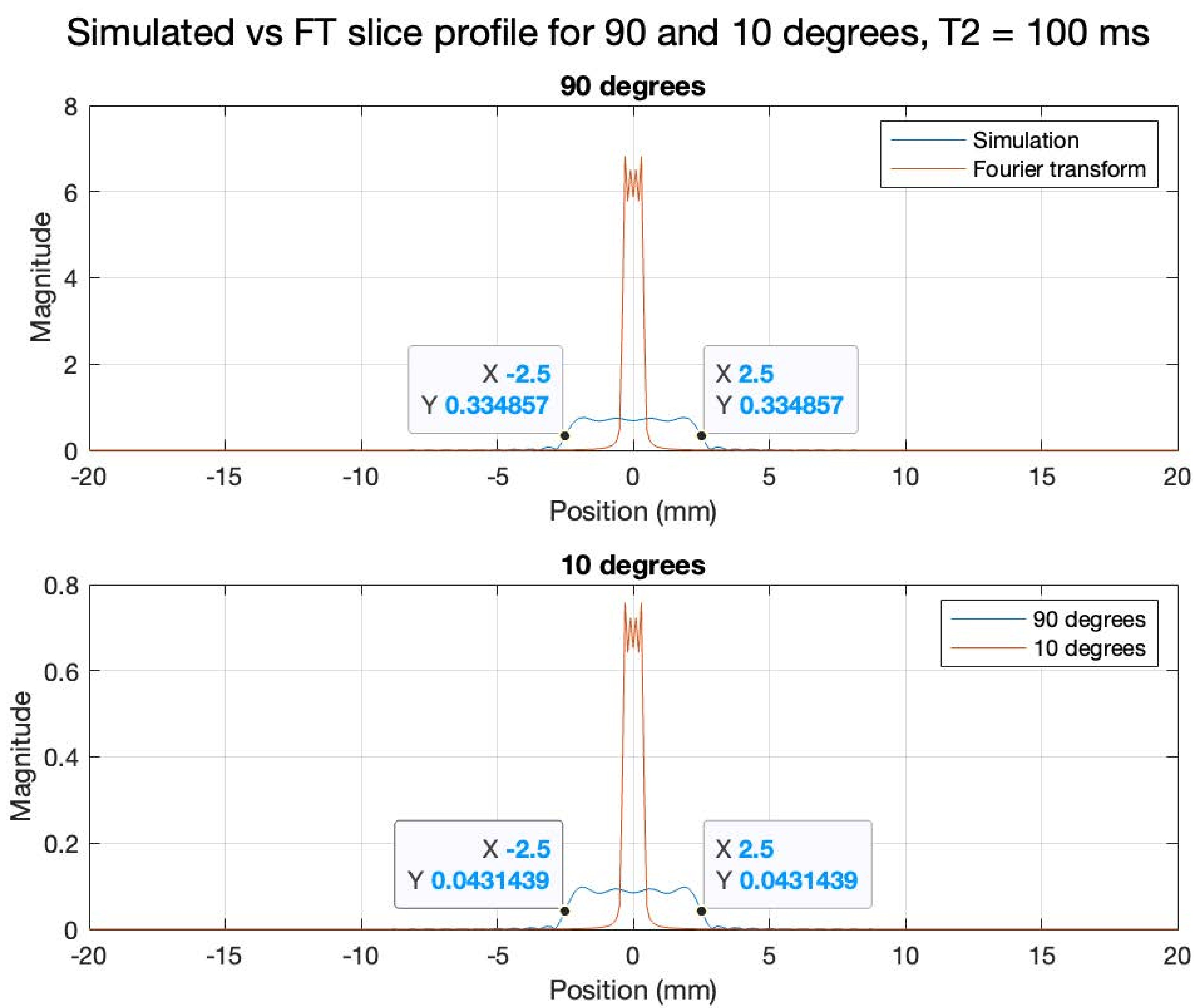


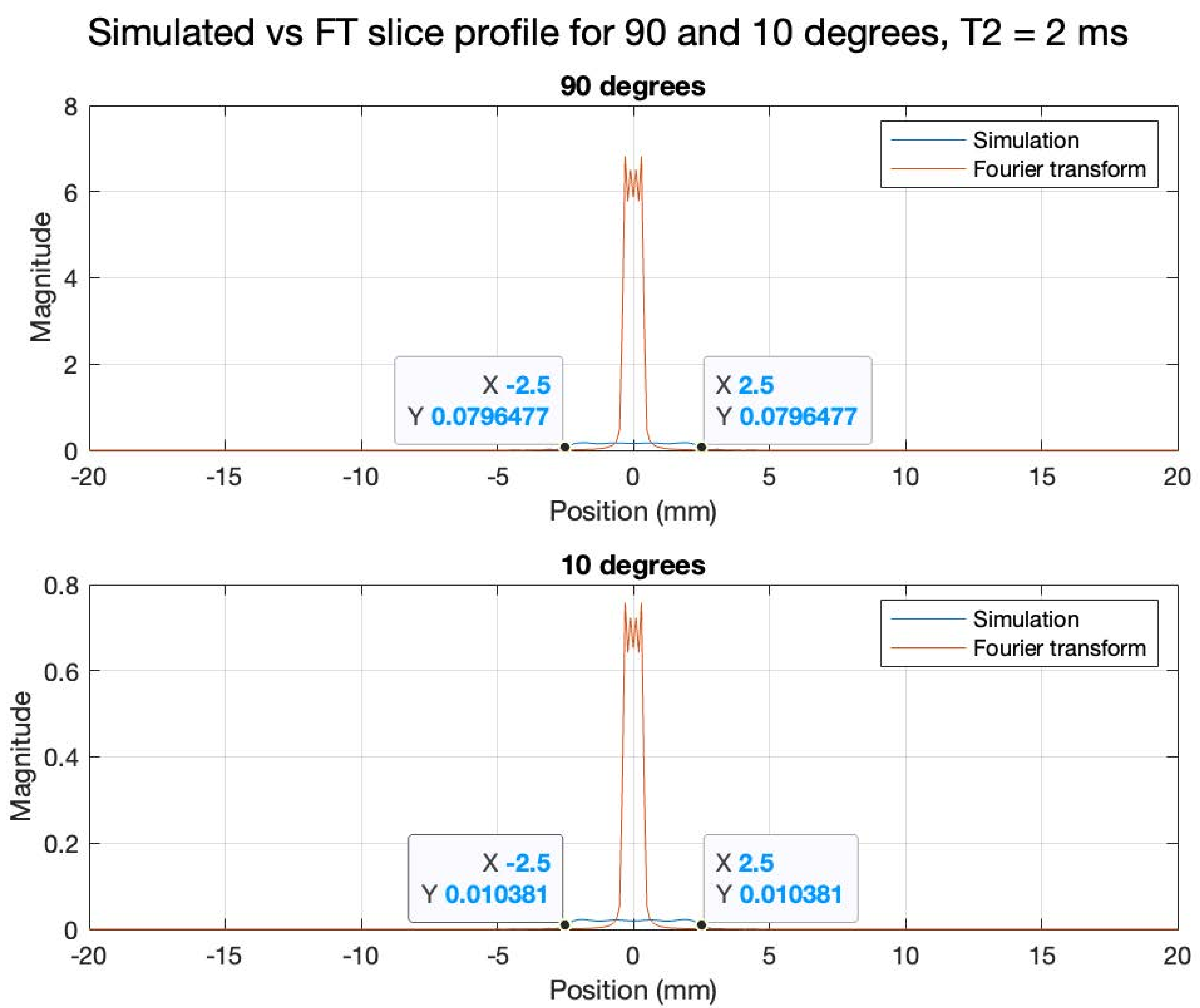
Transverse signal M_{xy}
Magnitude
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20
Position (mm)
Phase
2
1
0
-1
-2
-3
-20
-15
-10
-5
0
5
10
15
20
Position (mm)


Transverse signal M_{xy} at 200 Hz off-resonance
Magnitude
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20
Position (mm)
Phase
3
2
1
0
-1
-2
-3
-20
-15
-10
-5
0
5
10
15
20
Position (mm)

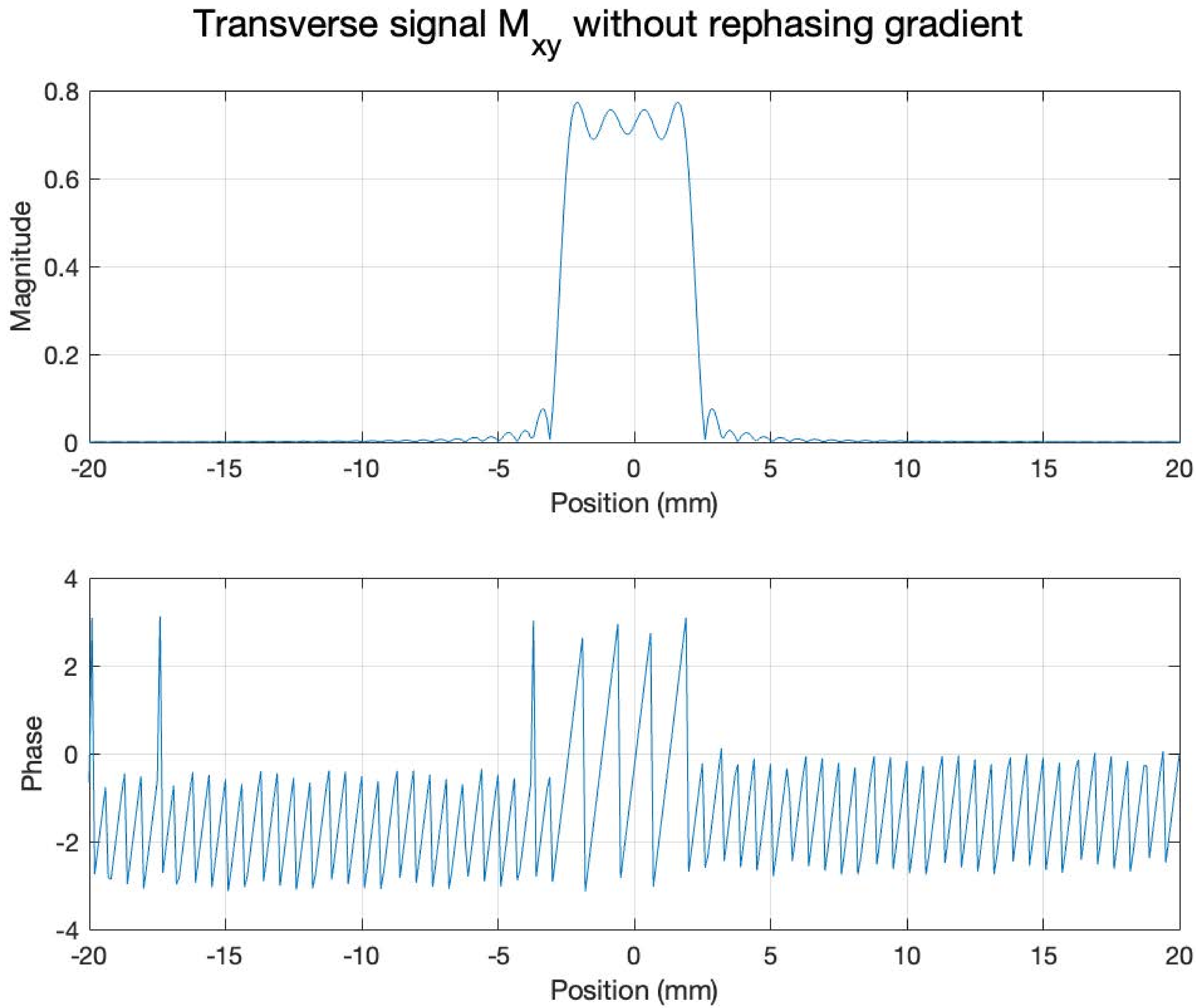


Transverse signal M_{xy} for different flip angles
Magnitude
Position (mm)
90 degrees
10 degrees
Phase
Position (mm)
90 degrees
10 degrees

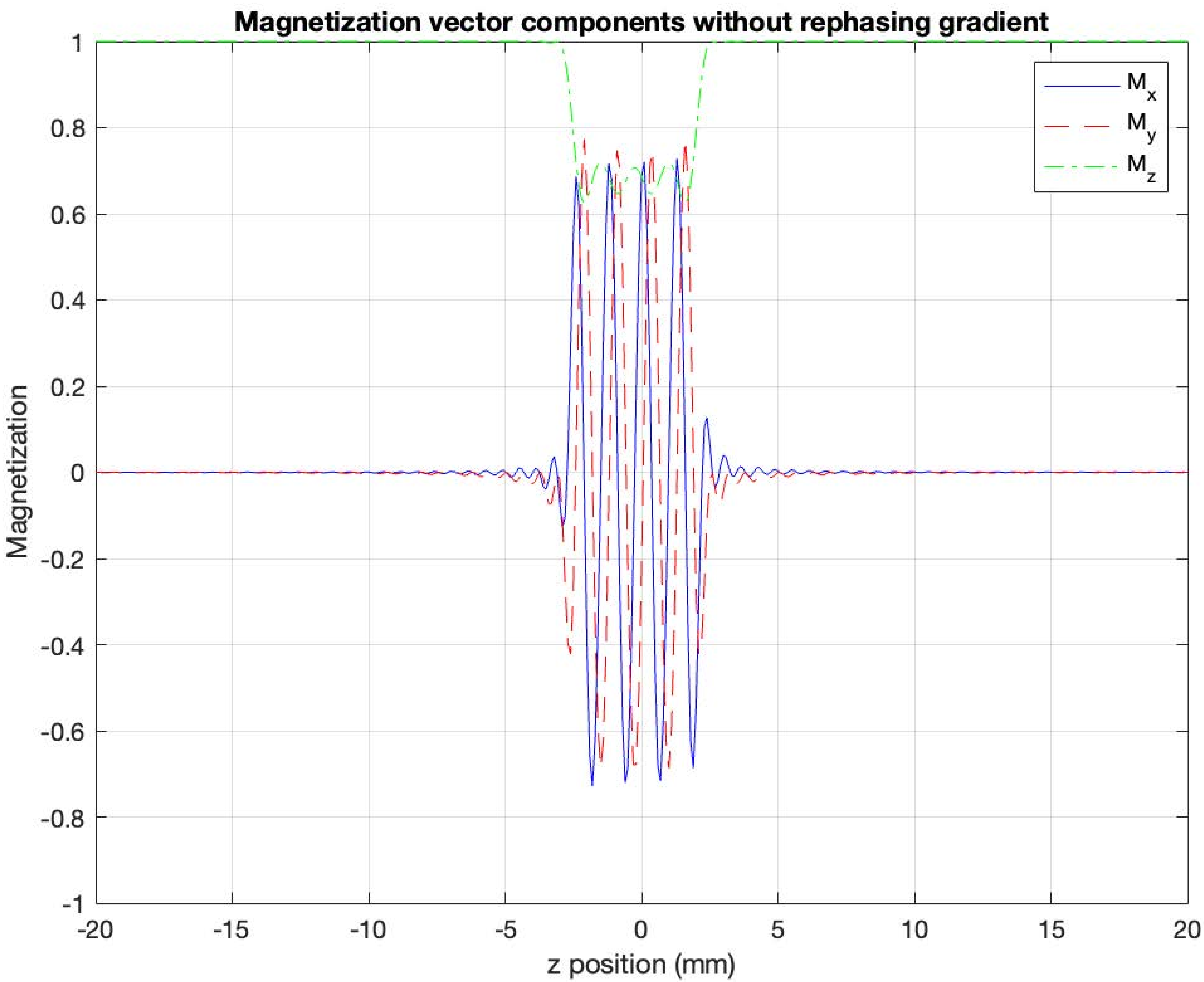

Simulated vs FT slice profile for 90 and 10 degrees, T2 = 100 ms
90 degrees
Simulation
Fourier transform
X -2.5
Y 0.334857
X 2.5
Y 0.334857
Magnitude
Position (mm)
10 degrees
90 degrees
10 degrees
X -2.5
Y 0.0431439
X 2.5
Y 0.0431439
Magnitude
Position (mm)


Simulated vs FT slice profile for 90 and 10 degrees, T2 = 2 ms
90 degrees
Simulation
Fourier transform
X -2.5
Y 0.0796477
X 2.5
Y 0.0796477
Magnitude
Position (mm)
10 degrees
Simulation
Fourier transform
X -2.5
Y 0.010381
X 2.5
Y 0.010381
Magnitude
Position (mm)

3) The slice profile obtained from Fourier Transforming the RF pulse results in a much narrower slice with much greater signal. The FT slice profile is also T2-independent, whereas the simulated slice profile basically decayed to zero when T2 decreased from 100 ms to 2 ms.

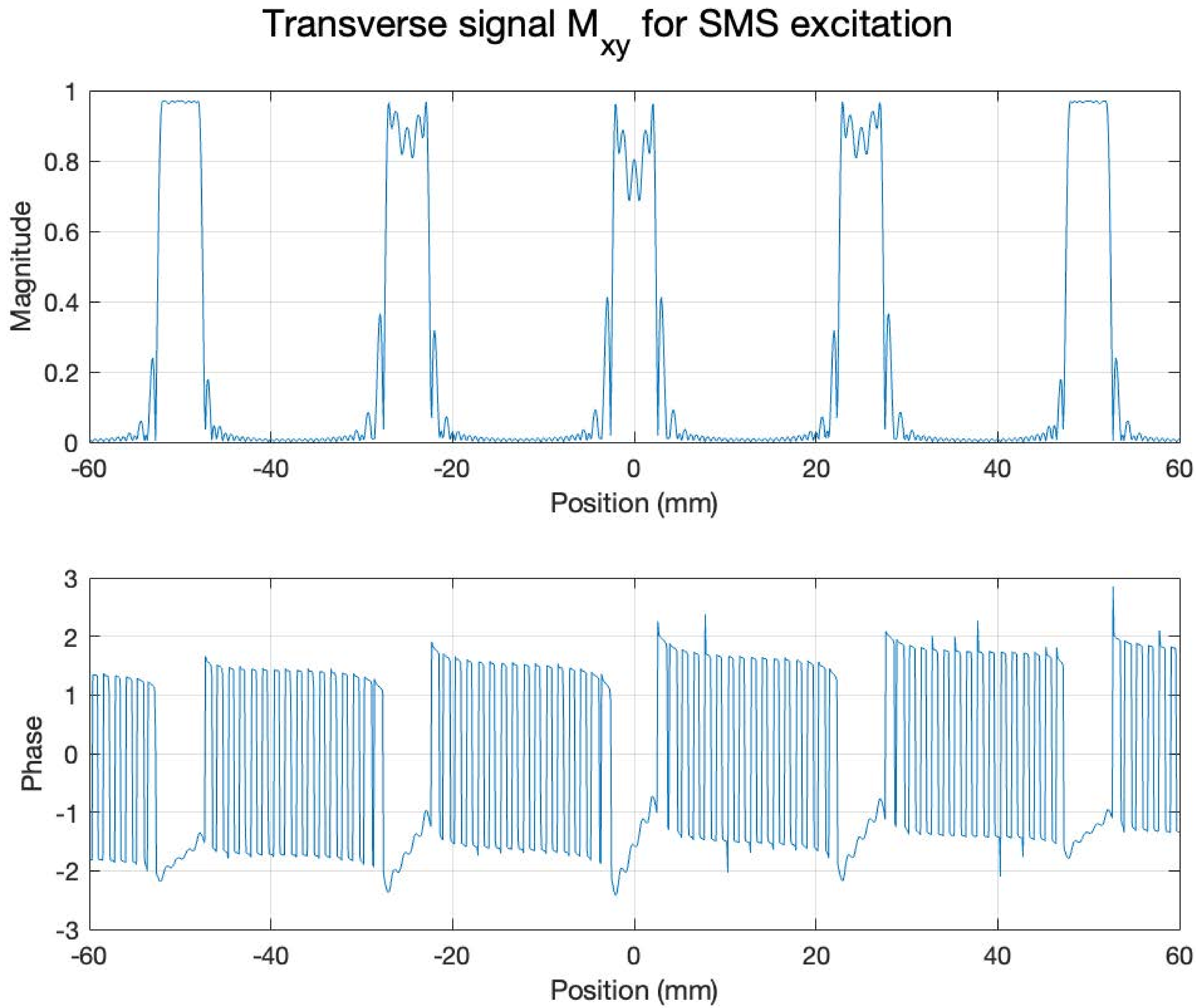


Transverse signal M xy without rephasing gradient
Magnitude
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20
Position (mm)
Phase
4
2
0
-2
-4
-20
-15
-10
-5
0
5
10
15
20
Position (mm)



Magnetization vector components without rephasing gradient
Magnetization
z position (mm)
M_x
M_y
M_z
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-20
-15
-10
-5
0
5
10
15
20

4) The slice rephasing gradient approximately cancels out the intraslice phase wrapping caure by the slice selection gradient.



Transverse signal M_xy for SMS excitation
Magnitude
Position (mm)
Phase
Position (mm)