P1.

Problem 1: Spatial Encoding

Suppose you want to collect a 2D bSSFP image at a spatial resolution of 1.2 x 1.2 x 3.0 mm^3 with 256 x 256 matrix size. The receiver bandwidth is 750 Hz / pixel. You are imaging on a 1.5T scanner with maximum gradient amplitude 25 mT/m (for one axis) and maximum slew rate of 180 T/m/s. In the following problems, you will design gradients for slice-selection, phase encoding, and frequency encoding and calculate the shortest possible TR and TE.

$TBW = \tau_{RF} \cdot BW = \tau_{RF} \cdot \Delta f_{RF}$ 8z

$\Delta f = \gamma G \Delta z$

$TBW = 8 = \tau_{RF} \gamma G \Delta z$

$\Delta f = \gamma G \Delta z = 25 \frac{mT}{m} \cdot 8mm$

$\boxed{\tau_{RF} = 8/\Delta f \approx 1ms = 0.94ms}$

Parameters:

1.5T	25 mT/m $\lvert G_{max} \rvert$	1.2×1.2×3 mm^3 voxel size		
180 T/m/s Slew$_{max}$	256×256 mtx	750 Hz/px BW$_{rec}$		

$N = 256$

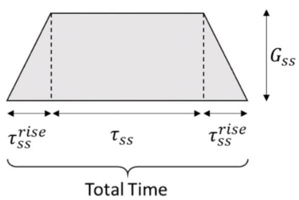
$FOV = 300\ mm$

$rBW_{pix} = 200\ Hz/px$

$\text{Total } rBW = \gamma G\, FOV = rBW_{pix} \cdot N$

$G = \frac{rBW_{pix} \cdot N}{\gamma\, FOV} \approx 4 mT/m$

1a. You decide to use a sinc-shaped RF pulse with time bandwidth product ~~4~~ 2 and a duration of 1 ms. Calculate the amplitude and total duration of the slice-select gradient. Note that the total duration includes both the flat-top time (τ_{ss}) and the time needed to ramp-up and ramp-down the gradient (τ_{ss}^{rise}), as shown in the figure below.



[Hz] [mT/m] [mm]

$$\Delta f = \gamma \cdot G \cdot \Delta z$$

RF transmit bandwidth — Gradient Amplitude — Slice Thickness

(a.) Calculate Slice Select Gradient Amplitude

$TBW = \tau_{RF} \cdot BW_{RF,tx} = \tau_{RF} \Delta f_{RF,tx}$

$\Delta f_{RF,tx} = \gamma G \Delta z$

$TBW = \tau_{RF} \gamma G \Delta z$

$G = \frac{TBW}{\tau_{RF} \gamma \Delta z} = \frac{2}{(1\times10^{-3} s)(42.58\times10^{6} \frac{Hz}{T})(3\times10^{-3} m)}$

$\approx 0.01566\ T/m$

$\boxed{G_{ss} = 15.6568\ mT/m} < \lvert G_{max} \rvert$

TBW = time bandwidth product

Δf_{RF} = RF transmit bandwidth = BW

$\gamma = 42.577 \frac{MHz}{T}$

TBW = 2

$\tau_{RF} = 1ms$

$\Delta z = 3mm$

$\gamma = 42.58\ MHz$

Calculate Slice Select Gradient duration:

The rise time of the G_{ss} is:

$\tau_{ss}^{rise} = \frac{G_{ss}}{\text{Slew Rate}} \quad \frac{0.01566\ T/m}{180\ T/m/s} = 8.7\times10^{-5} s$

$\tau_{ss}^{rise} = 0.087 ms$

We assume the flat-top duration equals the RF pulse duration:

$\tau_{ss}^{flat} = \tau_{RF} = 1 ms$

Then the total duration of the slice select gradient is:

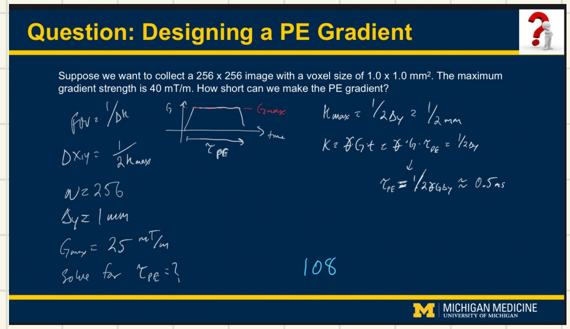
$\tau_{ss}^{total} = \tau_{ss}^{flat} + 2\tau_{ss}^{rise} = 1ms + 2(0.087ms) = \boxed{1.174ms = \tau_{ss}^{total}}$

1.

1b. Calculate the shortest possible duration of the phase encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

1c. Calculate the amplitude and total duration of the frequency encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

1d. Based on your results in parts a-d, calculate the shortest possible TE and TR. Please draw a sequence diagram and label the timings and amplitudes of all RF and gradient events. It is okay to submit a handwritten diagram.



(b.) Calculate the shortest possible duration of the PE gradient:

$$\text{Matrix size} = 256 \times 256 \qquad G_{max} = 25\ mT/m$$

$$\text{Voxel size} = 1.2 \times 1.2\ mm \qquad \text{Slew} = 180\ T/m/s$$

$$\Delta y = 1.2\ mm \rightarrow k_{y,max} = 1/2\Delta y = \frac{1}{2.4}\ mm^{-1} = \frac{1000}{2.4}\ m^{-1}$$

$$G_{PE} = G_{max} = 25\ mT/m$$

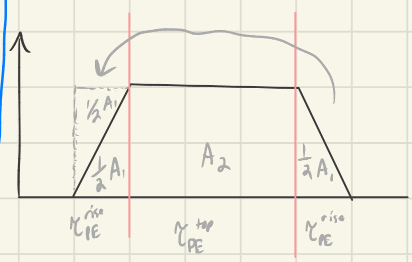
$$\tau_{PE}^{rise} = \frac{G_{PE}}{\text{Slew}} = \frac{0.025\ T/m}{180\ T/m/s}$$

$$= 1.389 \times 10^{-4}\ s$$

$$= 0.1389\ ms$$

$$k_y = \gamma G_{PE}\left(\tau_{PE}^{flat} + \tau_{PE}^{rise}\right)$$

$$k_y = \gamma G_{PE}\tau_{PE}$$



$\hookrightarrow k_y \equiv$ total area under G_{PE}

$$\parallel$$

$$2\left(\tfrac{1}{2}\tau_{PE}^{rise} G_{PE}\right) + \tau_{PE}^{top} G_{PE}$$

$$\parallel$$

$$\tau_{PE}^{rise} G_{PE} + \tau_{PE}^{top} G_{PE} = G_{PE}\left(\tau_{PE}^{rise} + \tau_{PE}^{top}\right)$$

$$\tau_{PE}^{flat} = \frac{k_y}{\gamma G_{PE}} - \tau_{PE}^{rise}$$

$$= \frac{(1000/2.4\ m^{-1})}{(42.58\ \frac{MHz}{T})(0.025\frac{T}{m})} - 1.389 \times 10^{-4}\ s$$

$$k_y = \gamma G_{PE}\left(\tfrac{1}{2}\tau_{PE}^{rise} + \tau_{PE}^{top} + \tfrac{1}{2}\tau_{PE}^{rise}\right)$$

$$= 2.5252 \times 10^{-4}\ s$$

$$\tau_{PE}^{flat} = 0.2525\ ms$$

$$\tau_{PE,Total} = \tau_{PE}^{flat} + 2\tau_{PE}^{rise} = 0.2525\ ms + 2(0.1389\ ms)$$

$$\boxed{\tau_{PE,Total} = 0.5303\ ms}$$

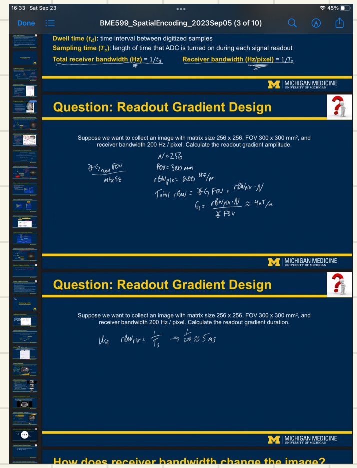
The shortest possible duration of the phase encoding gradient is: $\tau_{PE} = 0.5303\ ms$.

1.

1b. Calculate the shortest possible duration of the phase encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

1c. Calculate the amplitude and total duration of the frequency encoding gradient. As in part a, consider both the flat-top time and rise time when calculating the total duration.

1d. Based on your results in parts a-d, calculate the shortest possible TE and TR. Please draw a sequence diagram and label the timings and amplitudes of all RF and gradient events. It is okay to submit a handwritten diagram.



1c.) Calculate the amplitude and duration of the frequency encoding gradient.

$$FOV_x = \Delta x \cdot N_x = 1.2\,mm \cdot 256 = 307.2\,mm$$

$$rBW_{pix} = 750\ Hz/px$$

$$rBW_{pix} = \frac{1}{T_s}$$

$$\text{total } rBW = \gamma G_{read} FOV = rBW_{pix} \cdot MtxSize$$

Amplitude of freq. encoding gradient:

$$G_{read} = \frac{rBW_{pix} \cdot N}{\gamma FOV}$$

$$= \frac{750 \frac{Hz}{px} \cdot 256\,px}{(42.58 \frac{MHz}{T})(0.307\,m)} \qquad \frac{\cancel{\frac{1}{px \cdot s}} \cdot \cancel{px}}{\frac{1}{\cancel{s} \cdot T} \cdot m} = \frac{1}{\frac{m}{T}} = \frac{T}{m}$$

$$= 0.01469\ \frac{T}{m}$$

$$\boxed{G_{read} = G_{FE} = 14.69\ \frac{mT}{m}}$$

Grad. Duration:

$$\tau_{read} = \tau_{FE} = T_s = 1/rBW_{pix} = 1/750\ Hz/px$$

$$\boxed{\tau_{read} = \tau_{FE} = 1.33\,ms}$$

The amplitude and duration of the frequency encoding gradient are $14.69 \frac{mT}{m}$ and $1.333\,ms$, respectively.

1.

1d. Based on your results in parts a-d, calculate the shortest possible TE and TR. Please draw a sequence diagram and label the timings and amplitudes of all RF and gradient events. It is okay to submit a handwritten diagram.

- HINT #1: You can assume that the duration of the slice-rephasing gradient equals half the duration of the slice-selection gradient, and likewise that the duration of the read pre-phasing gradient is half that of the readout gradient.
- HINT #2: You can let certain gradients overlap to minimize the TE and TR.

(d.)

$\boxed{G_{SS} = 15.6568 \text{ mT/m}}$

$\boxed{G_{PE} = G_{max} = 25 \text{ mT/m}}$

$\boxed{G_{read} = G_{FE} = 14.69 \frac{\text{mT}}{\text{m}}}$

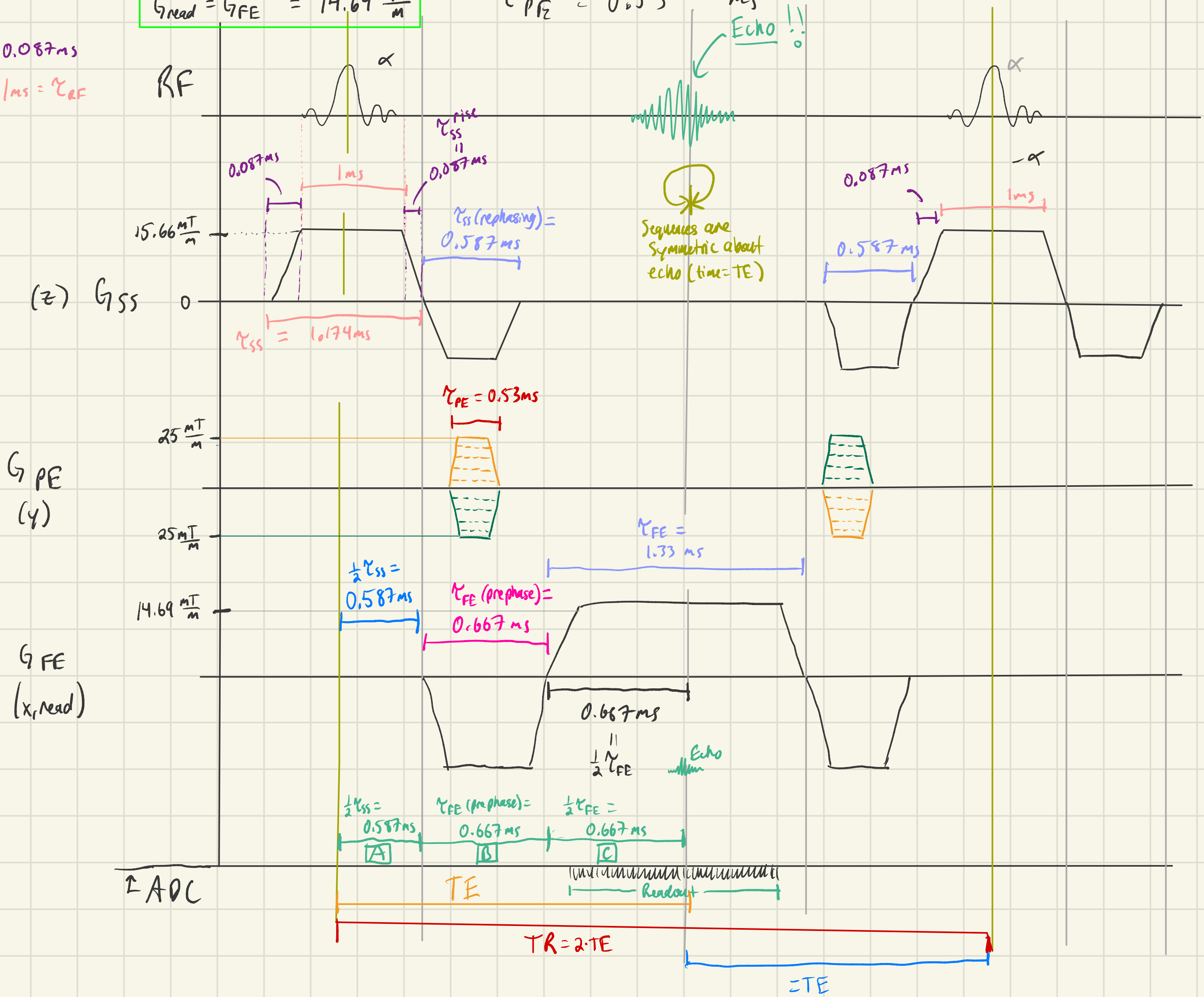
$\tau_{SS} = 1.174 \text{ ms} \rightarrow /2 = 0.587 \text{ ms}$

$\tau_{read} = 1.33 \text{ ms} \rightarrow /2 = 0.667 \text{ ms}$

$\tau_{PE} = 0.53 \text{ ms}$

$\tau_{SS}^{rise} = 0.087 \text{ ms}$

$\tau_{SS}^{top} = 1 \text{ ms} = \tau_{RF}$



$$TE = \frac{1}{2}\tau_{SS} + \tau_{FE}^{Prephasing} + \frac{1}{2}\tau_{FE} = \frac{1}{2}\tau_{SS} + \frac{1}{2}\tau_{FE} + \frac{1}{2}\tau_{FE} = \frac{1}{2}\tau_{SS} + \tau_{FE}$$

$$= \underbrace{0.587 \text{ ms}}_{A} + \underbrace{0.667 \text{ ms}}_{B} + \underbrace{0.667 \text{ ms}}_{C} = \boxed{1.9203 \text{ ms} = TE}$$

$$TR = \frac{1}{2}\tau_{SS} + \tau_{FE}^{Prephasing} + \tau_{FE} + \tau_{FE}^{Rephasing} + \frac{1}{2}\tau_{SS} = 2 \cdot TE$$

$$= 0.587 \text{ ms} + 0.667 \text{ ms} + 1.333 \text{ ms} + 0.667 \text{ ms} + 0.587 \text{ ms} = \boxed{3.841 \text{ ms} = TR}$$

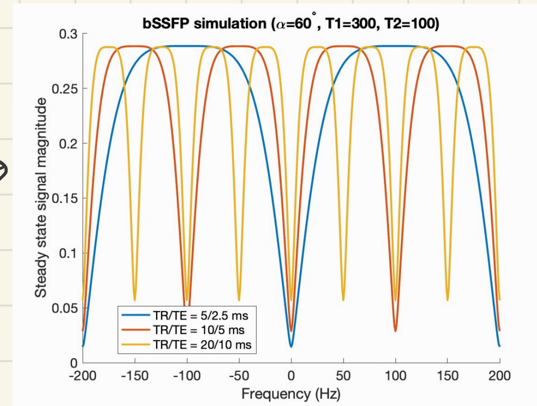
(2.)

Problem 2: Balanced and Spoiled Steady-State Sequences

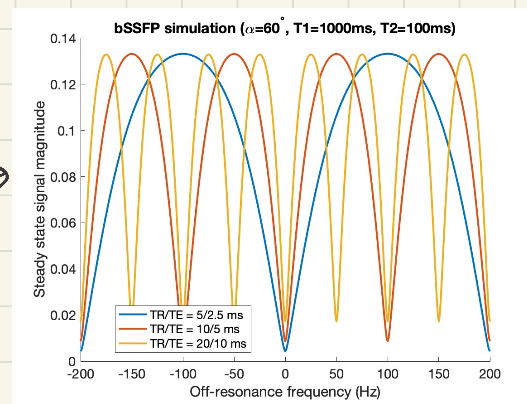
2a. Simulate the steady-state frequency response of a bSSFP sequence with a flip angle of 60° using Bloch equation simulations. Show results for (1) TR=5ms and TE=2.5ms, (2) TR=10ms and TE=5ms, and (3) TR=20ms and TE=10ms.

(a.) steady state freq. response of bSSFP sequence
w/ flip angle $\alpha = 60°$ for:
(1) TR/TE = 5/2.5 ms
(2) TR/TE = 10/5 ms
(3) TR/TE = 20/10 ms

[I set T1 = 300 ms and T2 = 100 ms, and simulated the steady state signal over an off-resonance frequency range from −200 Hz to 200 Hz.] →



[I set T1 = 1000 ms and T2 = 100 ms, and simulated the steady state signal over an off-resonance frequency range from −200 Hz to 200 Hz.] →



2. b.)

i. Assuming the spoiler gradient as a "perfect spoiler" that completely eliminates transverse magnetization components, please calculate the steady state signal at T1 = 1000 ms, T2 = 100 ms and the flip angle = 10° using a Bloch equation simulation

Modify bSSFP sequence to get FLASH [add grad. spoilers along slice select direction]

- Use TR/TE = 10/5 ms

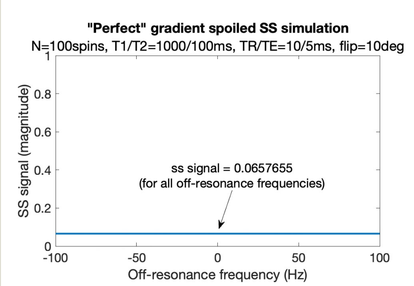
Part (i.) [See matlab script: p2b1.m; See Also: srsignal.m]

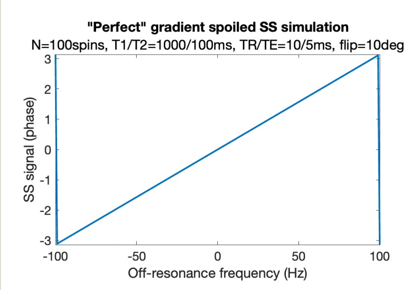
- Assume "perfect spoiler", and use T1/T2 = 1000/100 ms and flip angle $\alpha = 10°$.
  ↳ To assume "perfect spoiling", we will simply force the transverse magnetization to zero prior to excitation. This is done in the Matlab function "srsignal.m", via
  Atr = [0 0 0; 0 0 0; 0 0 1] * Atr

The magnitude of the steady state signal at T1 = 1000 ms, T2 = 100 ms, $\alpha = 10°$ was calculated to be:

$$\begin{cases} |M_x + iM_y| = \text{Signal}_{ss,\ \text{magnitude}} = 0.0657655 \\ \text{Residual Magnetization} = [M_x\ M_y\ M_z] = [0.065766\ \ 0.0\ \ 0.395] \end{cases}$$

The signal was consistent across a wide range of off-resonance frequencies ($< 5 \times 10^{-17}$ difference in signal magnitudes across off-resonance frequencies from -100 Hz to 100 Hz).



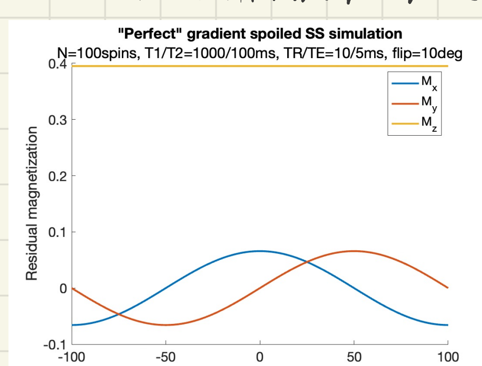


Results from P2b(i) for "perfect" spoiling where transverse magnetization is forced to zero before excitation.

Top: Magnitude of SS signal (y axis) across different off-resonant frequencies (x axis)

Bottom: Phase of SS signal (y axis) across different off-resonant frequencies (x axis)

Residual Magnetization (M_x, M_y, M_z components) across different off-resonant frequencies (x axis)



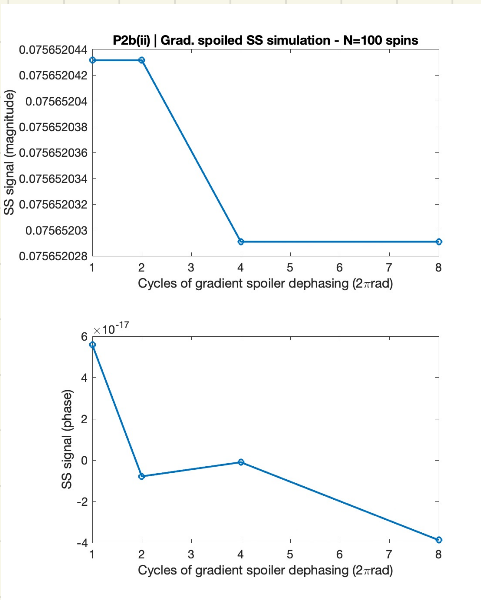
② b.)

ii. Rather than assuming ideal spoiling, now let us examine the effect of a gradient spoiler using a Bloch equation simulation. Simulate the steady state signal with the gradient spoiler achieving 2π, 4π, 8π, and 16π dephasing (1 cycle, 2 cycles, 4 cycles, and 8 cycles) per voxel (each with the same TR). Plot the signal as a function of dephasing moment.

Part (ii.)

[See matlab script: p2b2.m ; See Also: gssignal.m, gresignal.m]

Note: An off resonance frequency of 0 Hz was used in simulations here.



Simulated Grad. Spoiled SS signal

[top = Magnitude ≡ abs($M_x + i \cdot M_y$)]

[Bottom: Phase ≡ angle($M_x + i \cdot M_y$)]

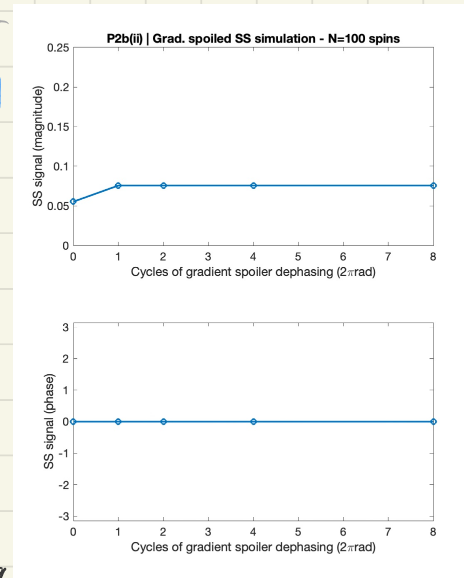
Left:

Signal magn (top) and phase (bottom) as a function of # of Grad. Spoiling Dephasing Cycles.

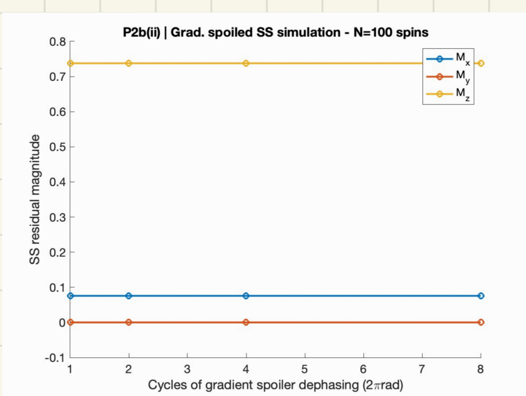
NOTE: the y-axis limits/labels – the differences in signal over # dephasing cycles is very small!

Right:

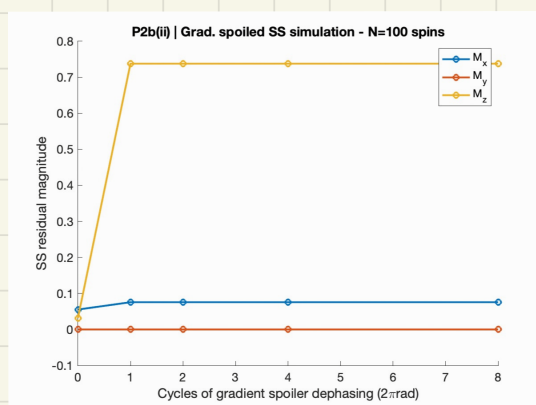
Same plot as left, but including the signal from using 0π dephasing (i.e., no grad. spoiling), to illustrate the similarity in signal obtained when at least ≥2π (≥1 cycle) dephasing of grad. spoilers.



Overall, these results indicate using at least 2π (1 cycle) for dephasing gradient spoiling achieves the same desired effect.



Similar to plots above, but plotting the residual magnetization (M_x, M_y, M_z) across grad. spoiling dephasing cycles.



iii. Now simulate the effects of *both* the gradient spoiler and RF spoiling. Simulate the steady-state signal by sweeping RF phase from 0°-180° with a reasonable step. Plot the signal as a function of RF phase. Please list your choice of RF phase that best eliminates transverse magnetization.

② b.)

$\boxed{\text{Part (iii.)}}$ [See matlab script: p2b3.m ; See Also: spgrsignal.m]

The effects of both gradient & RF spoiling were simulated by

① simulating gradient spoiling with 1 cycle (2π) dephasing, with $N=100$ spins in the voxel experiencing 100 different frequencies evenly spaced b/w $-\pi$ and π.

② Also adding RF spoiling by incrementing RF pulse phase following each excitation. 181 values of RF phase increments were tested, every 1° b/w 0° and 180°.

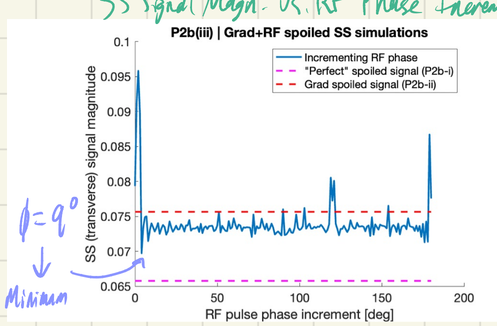
③ The average signal in the simulated voxel after the final (100^{th}) excitation was recorded.

□ Based on my results, it appears that either 117° or 9° were the best choices for RF phase (increment).

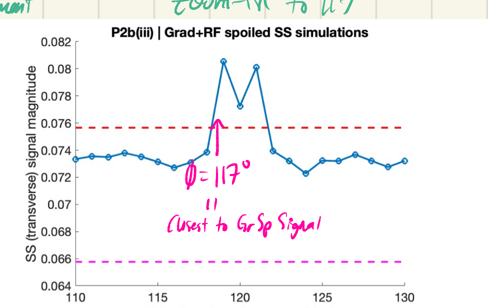
↳ At 117°, the RF spoiled Signal is close to the Grad spoiled Signal; based on Dr. Hargreaves' RF Spoiling Slides (Next Page), this 117° is the ideal RF phase increment, and the left + center plots below replicate the ☆ plot on the Slides on the next page

↳ At 9° RF phase, the SS (transverse) signal magnitude is minimized, so it may also be a good choice. However, based on Dr. Hargreaves' video cited on next slide, it seems 117° is the commonly adopted RF phase increment.

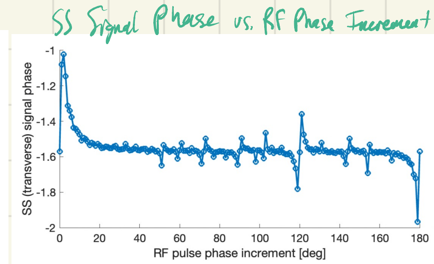
SS Signal Magn. vs. RF Phase Increment



Zoom-in to 117°

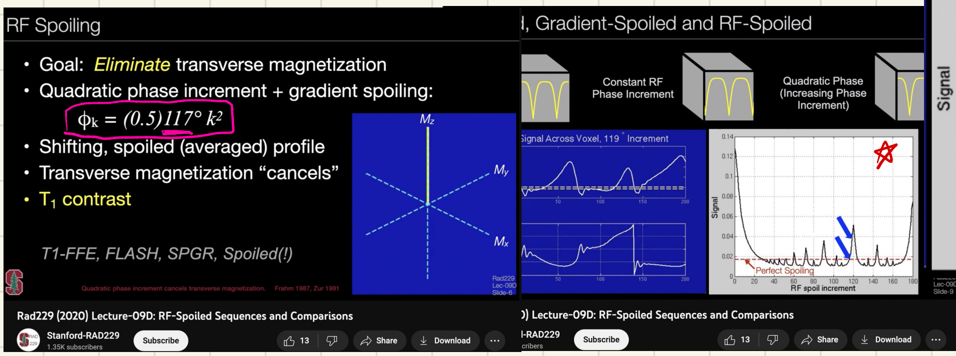


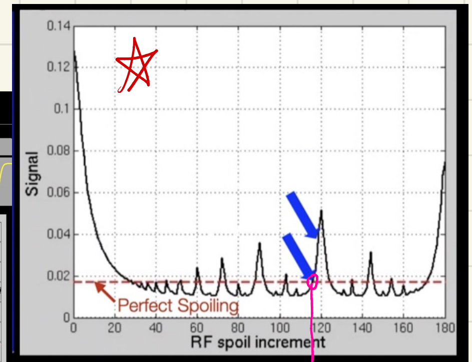
SS Signal Phase vs. RF Phase Increment



(2.) b.) Part (iii.) (continued)

On Brian Hargreaves' YouTube Lecture Slides on RF spoiling, as well as in his Matlab code, 117° is given as the ideal RF phase increment for RF spoiling,





117°

Signal using 117° RF spoil increment intersects the "perfect spoiling" Signal

[Screenshots from: (Brian Hargreaves lecture Slides https://www.youtube.com/watch?v=cD5IbFyZK6w]