3. (1.) $\Delta z = 5\,mm$ $\quad |G_{max}| = 25\ mT/m$

$\tau_{RF} = 2\,ms$ $\quad |Slew_{max}| = 180\ mT/m\cdot s$

$TBW = 8$ $\quad \boxed{\gamma = 42.58\times10^6\ \frac{Hz}{T}}$

$TBW = T \cdot BW = \tau_{RF}\cdot BW$

$BW = \frac{TBW}{T} = 8/2ms = 4\,kHz$

$TBW = \tau_{RF}\cdot BW_{RF,tx}$

$TBW = \tau_{RF}\Delta f_{RF,tx}$

$\Delta f_{RF,tx} = \gamma G \Delta z$

$TBW = \tau_{RF}\gamma G\Delta z$

$G = TBW/\tau_{RF}\gamma\Delta z$

$\tau_{ss}^{rise} = G_{ss}/Slew \quad \tau_{ss}^{top} = \tau_{RF}$

---

## Find $G_{ss}$ amplitude

$TBW = BW\cdot\tau_{RF} = \gamma G\Delta z$

$G_{ss} = \frac{BW}{\gamma\Delta z} = \frac{4\cdot10^3\,Hz}{42.58\times10^6\frac{Hz}{T}(5\times10^{-3}m)}$

$\boxed{G_{ss} = 18.788\ mT/m} = 1.8788\ G/cm$

## Find $G_{ss}$ duration ($\tau_{ss}^{rise}$, $\tau_{ss}^{top}$, $\tau_{ss}$ (total))

$\tau_{ss}^{rise} = \frac{G_{ss}}{Slew} = \frac{18.788\ mT/m}{180\ mT/m/ms}$

$\tau_{ss}^{rise} = 0.1044\,ms$

$\tau_{ss}^{top} = \tau_{RF} = 2\,ms$

$\tau_{ss} = \tau_{ss}^{rise}\cdot 2 + \tau_{ss}^{top}$

$= 2\,ms + 2(0.1044\,ms)$

$\boxed{\tau_{ss} = 2.208757\ ms}$

## Rephasing Slice-select Gradients:

Set rephasing gradient to the same duration as the $G_{ss}$ gradient but with half the amplitude & opposite polarity!

$G_{ss,rephasing} = \frac{G_{ss}}{2} = 9.394\ \frac{mT}{m}$

$\tau_{ss,rephasing} = \tau_{ss} = 2.208\,ms$ (w/ ramping)

## P3. (Part 1)

### Find RF waveform

- 1st zero crossing of Sinc $\equiv BW_{RF} = 4$ kHz
- # zero crossings of sinc $\equiv$ TBW $= 8$

### Find RF ($B_1$) amplitude:

$$\alpha = 2\pi \gamma \int^{T_{RF}} b_1(t)\,dt \qquad \bar{\gamma} = \frac{\gamma}{2\pi} = 42.58 \frac{\text{MHz}}{T} = 4258 \frac{\text{Hz}}{G}$$

$$\alpha = 2\pi \bar{\gamma} \int^{T_{RF}} b_{1,rf}(t)\,dt \qquad RF_{rot} = 2\pi \text{ rad}\left(4258 \tfrac{\text{Hz}}{G}\right) \cdot RF \cdot s$$

↳ Solve for $\int b_1(t)\,dt$ (or $\sum_i rf(i)$)

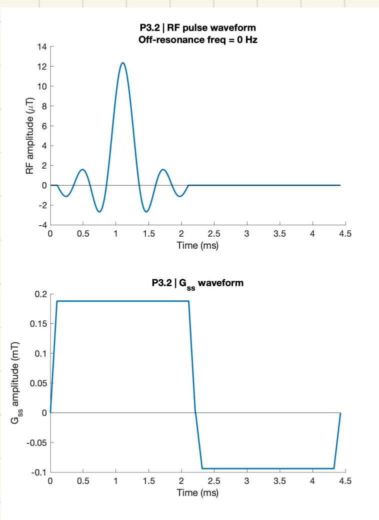
$$\int b_1(t)\,dt = \frac{\alpha}{2\pi\bar{\gamma}} = \frac{\pi/2}{2\pi(42.58\times10^6\ \text{Hz}/T)} = 5.8713e{-9}\ T\cdot s$$

$$= 5.8713\times10^{-3}\ \mu T\cdot s$$

Integral of RF waveform will be $5.8713\times10^{-3}\ \mu T\cdot s$

---

## 3. Part 2




Plot of designed waveforms for RF pulse (top; units = μT) and $G_{ss}$ w/ dephasing gradient (bottom; units mT) vs. time.

(The ramp times were not exact due to discretization, but were very close to calculated value.)

(See: p3part2.m)

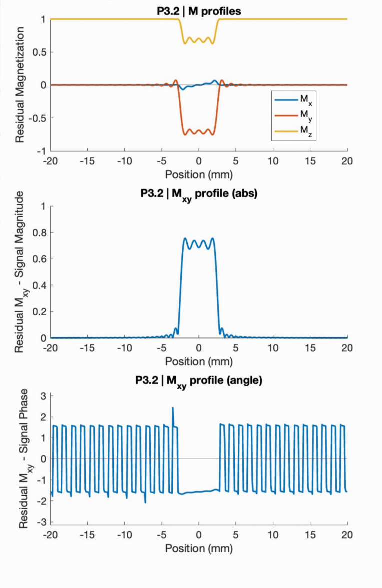
③ Part 2 [See matlab script: p3part2.m]

Flip α = 90°

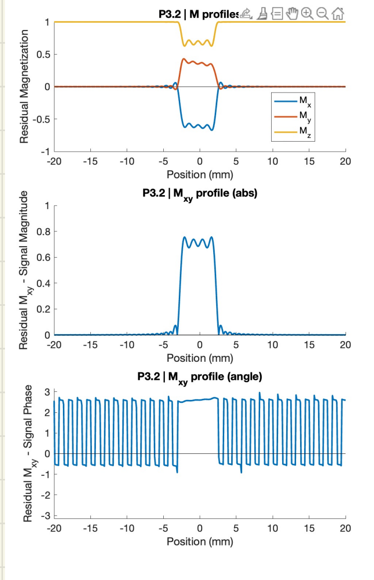
$T_1$ = 1000 ms

$T_2$ = 100 ms

off-resonance = 0 Hz




off-resonance = 200 Hz




Plots of Simulations w/ 0 Hz (left) and 200 Hz (right) off resonance, Showing:

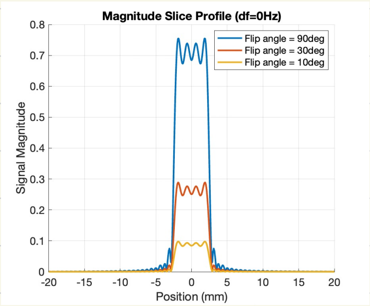
- $M_x, M_y, M_z$ vs. Spatial Position (top)
- $M_{xy}$ (transverse) signal magnitude (= Slice profile) vs. Spatial Position (middle)
- $M_{xy}$ (transverse) signal phase vs. spatial Position (bottom)

The 200 Hz off resonance shifts the Slice Profile ($M_{xy}$ signal magnitude) slightly towards negative positions; it also alters the $M_x$ and $M_y$ profiles, but does not change $M_z$ profile.

[See matlab script: p3part3.m]

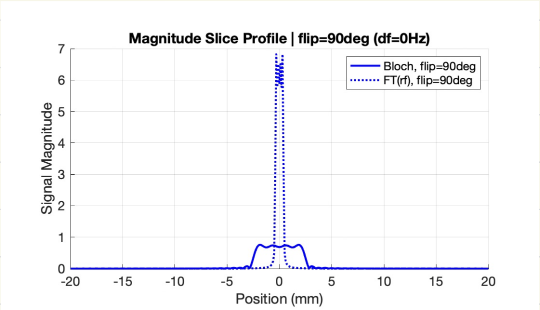
P3 Part 3

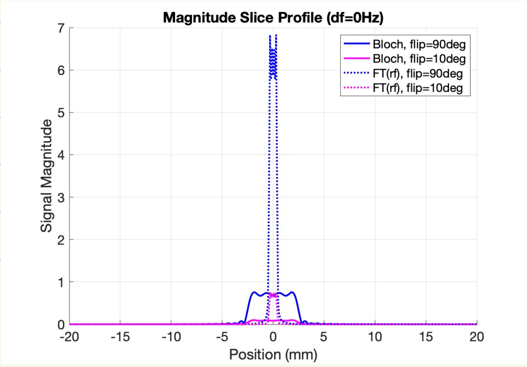
- Bullets 1-2: Decreasing flip angle results in a similar shaped slice profile, but reduces the overall (transverse, $M_{xy}$) signal magnitude, as flip decreases from $90° \rightarrow 10°$.

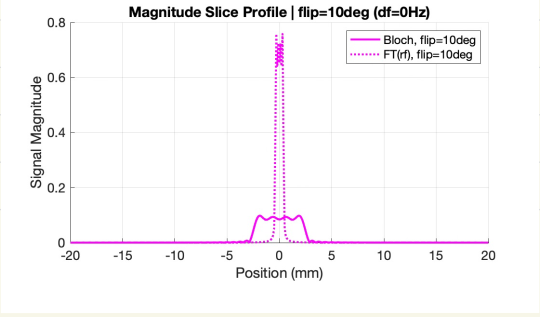



- Bullet 3: For both 10° and 90° RF pulses, the slice profile obtained by direct FT of the RF pulse had about the same spatial width, with slice profiles with FWHM ≈ 0.8 mm. These were narrower than the Bloch-sim slice profiles, which achieved FWHM ≈ 5 mm, as desired, for both the 90° and 10° RF pulses. The direct FT of RF pulses also had spiky artifacts atop their slice profiles. Thus, the FT of RF pulses seemed to lack the spatial selectivity to excite the desired spatial slice (without additional slice select gradient).

The slice profiles from Bloch sim w/ different flip angles are similar to each other, with same FWHM, just different signal magnitudes. The same is true for the slice profiles from FT of RF pulses w/ different flip angles.



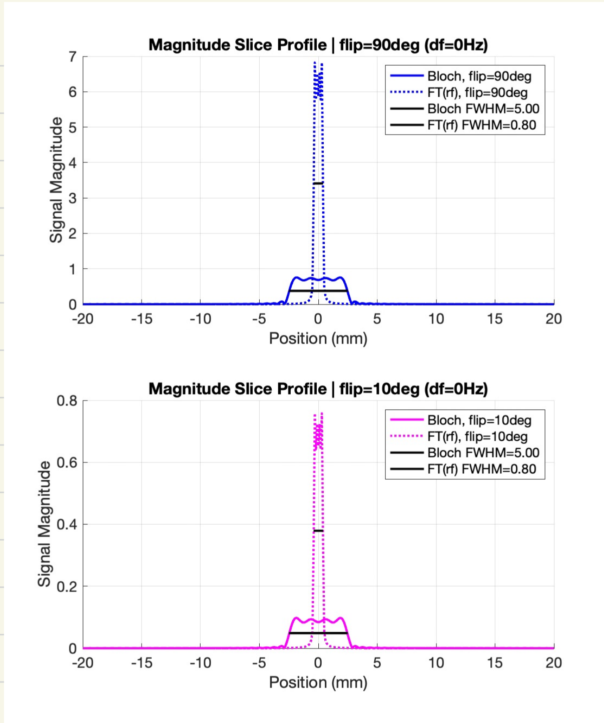



P3 Part 3

• Bullet 3 (continued)

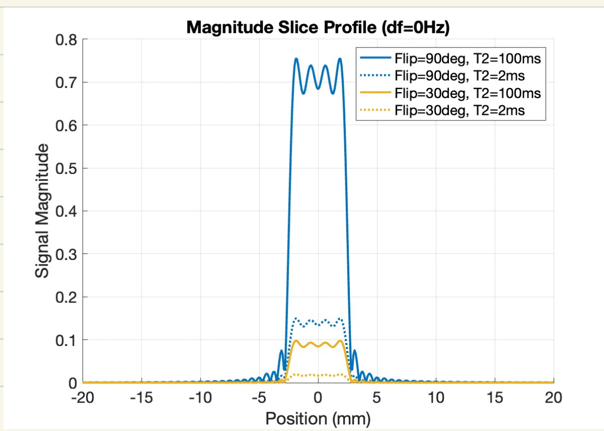
Same plot as on previous page, but with FWHM of slice profiles labeled, with FWHM values shown in legends.




P3. Part 3

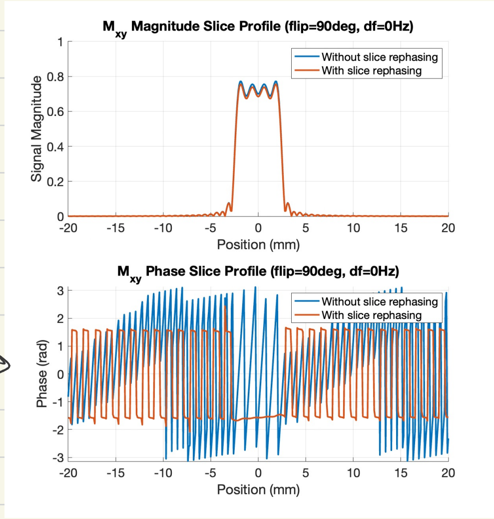
• Bullet 4:

Reducing T2 from 100ms to 2ms causes a reduction in signal magn. of slice profiles @ both 10° and 90° flip angles. The overall shape and spatial excitation coverage remains similar for both T2 values.



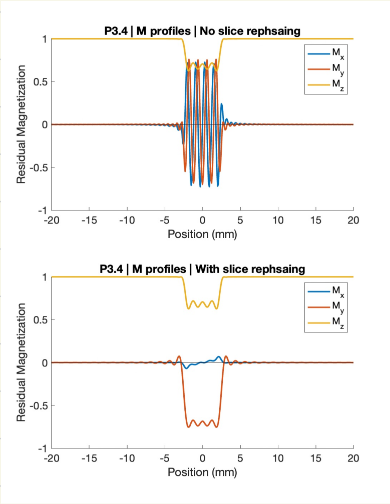
3. Part 4 [See matlab Script: p3part4.m]

Simulations for $M_x, M_y, M_z, M_{xy}$ at time b/w SS gradient + slice rephasing gradient.

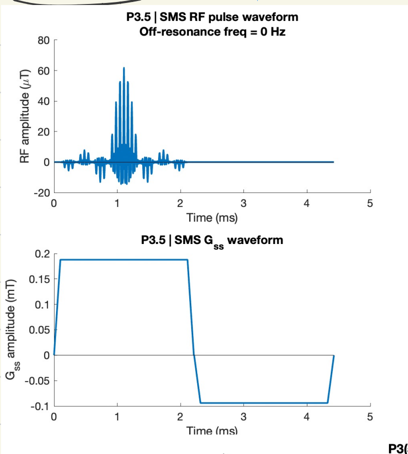


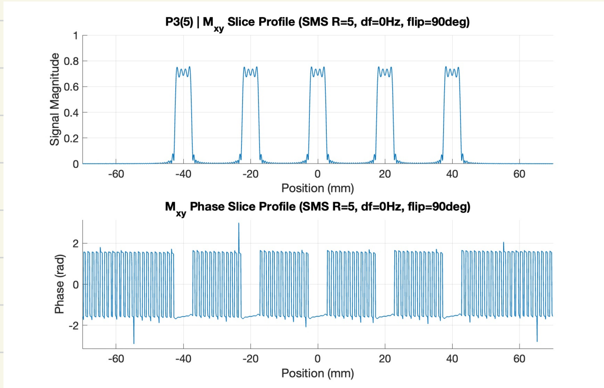

The slice profiles were very similar, but rephasing levels/evens out the phase variation across the excited slice positions and removes the linear phase wrapping cycles present w/out slice rephasing gradients.

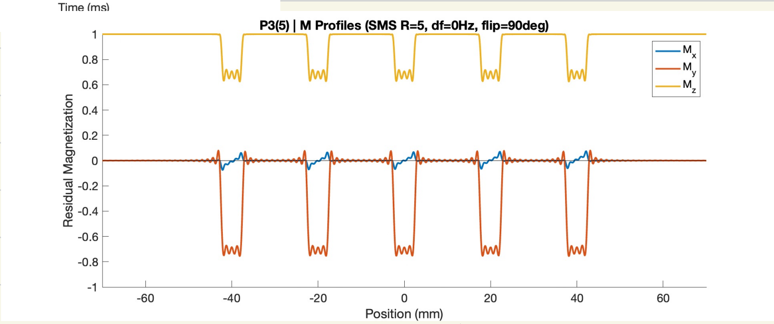
The effect of slice rephasing gradients on homogenizing phase across the slice is also shown in the $m_x, m_y$ profiles, which are much flatter when rephasing is applied.



3. Part 5 [See matlab script: p3part5.m]




See matlab code for generating SMS RF pulse by adding/modulating the RF waveform to include excitations at additional spatial frequencies.

```
%%% Add SMS excitations to RF pulse
% positions of additional slices to excite
sms_slice_pos = 20*[-2 1 1 2]; % mm
% frequencies of additional slices to excite
freq_pos = (gyro * 1e6) * (grad_amp * 1e-3) * (sms_slice_pos*1e-3);
%               Hz/T            T/m                    m
rf90sms = rf90;
for sms = 1:length(sms_slice_pos)
    rf90sms = rf90sms + rf90.*cos(2*pi*freq_pos(sms)*t);
end
```