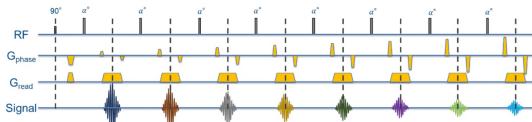


(1.)

PROBLEM 1: Extended phase graphs. Shown below is a sequence diagram of a fast spin echo (FSE) with its first 8 refocusing pulses within one repetition time (TR). Each RF pulse is designated as a delta function, so you do not need worry about the slice profile. Refocusing pulses are spaced 5ms apart (echo spacing).



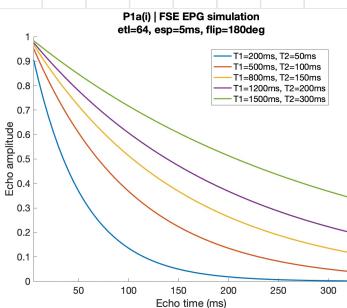
a. Please write a function using EPG to simulate spin echo train echo amplitudes for a sequence with 90° excitation, followed by refocusing pulses of $[\alpha(90 - \alpha/2)]_y$, $\alpha_y, \alpha_y, \dots$ for 64 echoes for $T1 = [200:100:1500]$ ms, and $T2 = [50:30:300]$ ms

- Simulate echo amplitudes with $\alpha = 180^\circ$, and plot amplitudes with five different $T1$ and $T2$ combinations
 - Simulate echo amplitudes with $\alpha = 120^\circ$, and plot amplitudes with five different $T1$ and $T2$ combinations
 - Simulate echo amplitudes with $\alpha = 60^\circ$, and plot amplitudes with five different $T1$ and $T2$ combinations
- b. Plot 4 contour plots signals vs $T1$ and $T2$ for the 6th, 16th, 32nd, and 48th echoes using various α
- c. Discuss the contrast you get with different flip angles.

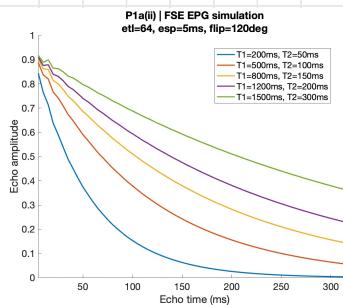
a.) 5 different $T1, T2$ combinations were tested:

$$[T1, T2] = [200, 50] \text{ ms}, [500, 100] \text{ ms}, [800, 150] \text{ ms}, [1200, 200] \text{ ms}, [1500, 300] \text{ ms}$$

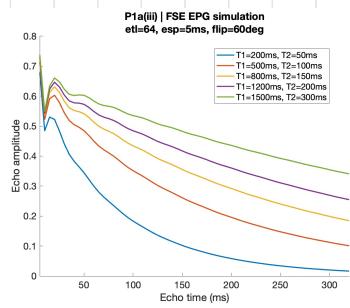
(i.) $\alpha = 180^\circ$



(ii.) $\alpha = 120^\circ$



(iii.) $\alpha = 60^\circ$



(1.) b.)

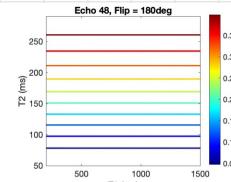
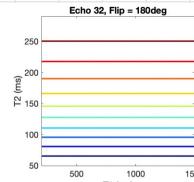
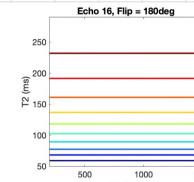
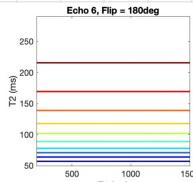
Echo 6

Echo 16

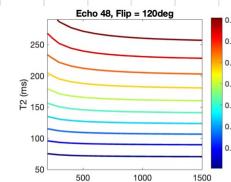
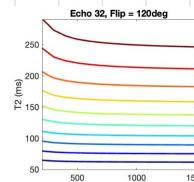
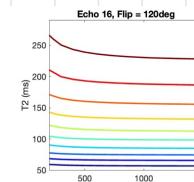
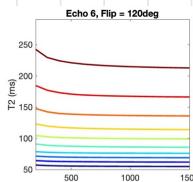
Echo 32

Echo 64

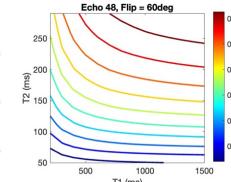
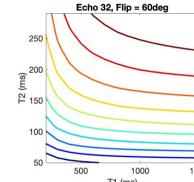
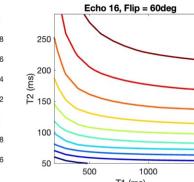
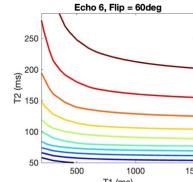
$$\alpha = 180^\circ$$



$$\alpha = 120^\circ$$



$$\alpha = 60^\circ$$



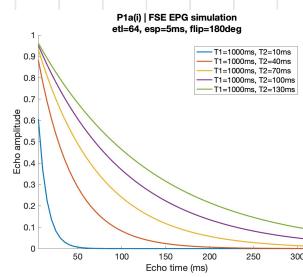
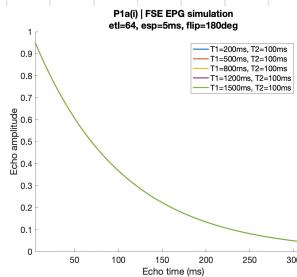
In the contour plots above,

- x-axes = T_1 (ms), y-axes = T_2 (ms),
- Colorbars depict echo (transverse magn. $|M_{xy}|$) signal $\in [0, 1]$ corresponding to colors of contours.

(1.) c.) Using a flip angle of $\alpha=180^\circ$, the echo signals are constant for fixed T_1 values, regardless of the T_2 value.

This suggests that large (inversion) flip angles of $\alpha=180^\circ$ would be used to generate $T_2\text{-w}$ contrast:

Differences in signal intensity arise solely due to differences in T_2 ; tissues with identical T_2 will have identical signal, regardless of their T_1 time.



This can also be seen by generating plots similar to those in part 1(a.), (shown above) but holding either T_2 (left) or T_1 (right) constant. This shows when $\alpha=180^\circ$, contrast is generated by differences in T_2 times, and not by T_1 times.

When flip angle decreases, ($\alpha=120^\circ$, $\alpha=60^\circ$), the contour plots show more signal variation with respect to both varying T_1 and T_2 times. This suggests smaller flip angles ($<180^\circ$) could be used to generate more $T_1\text{-w}$ or mixed ($T_1/T_2\text{-w}$) contrasts (than using $\alpha=180^\circ$).

(2) (a.) For the T_{1w} contrast, we want a short TE and short TR;

a short TE decreases T₂-weighting and a short TR increases T₁-weighting.
Thus, we chose TR/TE values of:

$$\text{TR/TE} = 500 / 15 \text{ ms for T}_{1w} \text{ contrast}$$

For T_{2w} contrast, we want to choose a long TE and long TR;

a long TE increases T_{2w} contrast and a long TR decreases T_{1w} contrast.

$$\text{TR/TE} = 6000 / 100 \text{ ms for T}_{2w} \text{ contrast}$$

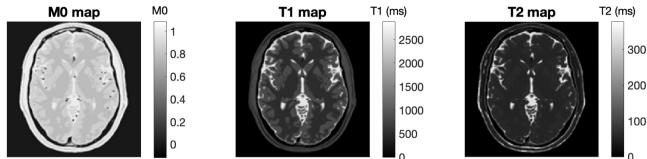
For PD weighted contrast, we want to choose a short TE and long TR;

a short TE decreases T_{2w} contrast and a long TR decreases T_{1w} contrast,
giving us a PD weighted contrast that is neither T₁ or T₂ weighted.

$$\text{TR/TE} = 6000 / 15 \text{ ms for PD weighted contrast}$$

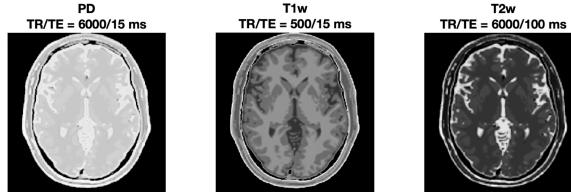
Given

Brain parameter maps



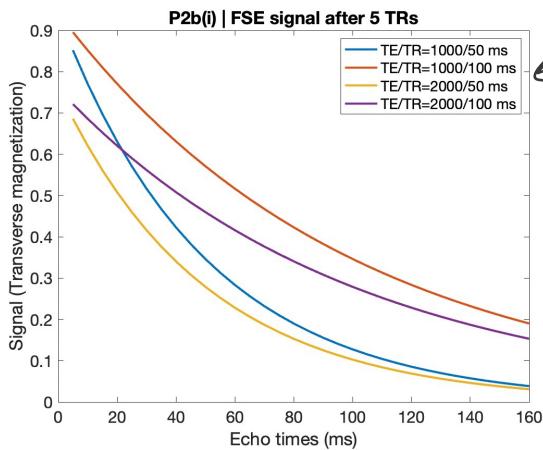
Simulated

SE contrasts



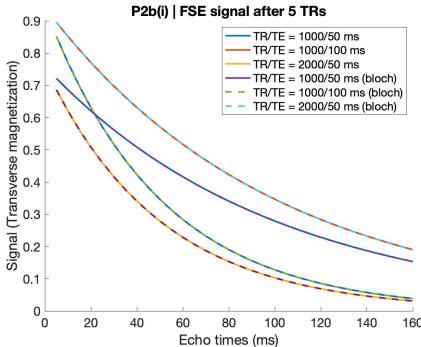
(2) b.) (i.) FSE: $\underline{TR = 3s}$, $ESP = 5ms$, $ETL = 32$, $\alpha_{exc.} = 90^\circ$, $\alpha_{refocus} = 180^\circ$

With a TR of 3s, we can assume the residual magnetization has decayed/relaxed completely to equilibrium before the start of the next TR; thus the steady state signal can be used.



Line colors depict different (T_1, T_2) values;
see legend

Comparing the EPG multi-TR simulation to the steady state FSE Bloch simulation, we see the 2 methods yield identical signal profiles after the 5th (final) TR.

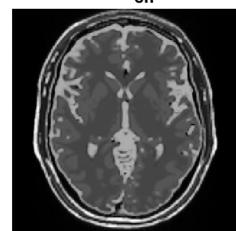


(2.)

(ii.) We chose the k-space filling order so that the center PE line in k-space would be filled by the echo at time $TE = 80\text{ms}$. With $ESP = 5\text{ms}$, this was echo #16.

The simulated image is shown on the right. We filled k-space from bottom to top, with $TE_{\text{eff}} = 80\text{ms}$, which created a T₂W contrast.

FSE sim | $TE_{\text{eff}} = 80\text{ ms}$



For $ETL = 32$, $ESP = 5\text{ms}$, we filled kspace as shown on the right:

- Each color shows the lines filled by each echo from the 32-echo train.
- Each echo filled 8 different lines, which correspond to 8 different TRs.
- The first echo filled the bottom 8 lines of kspace.

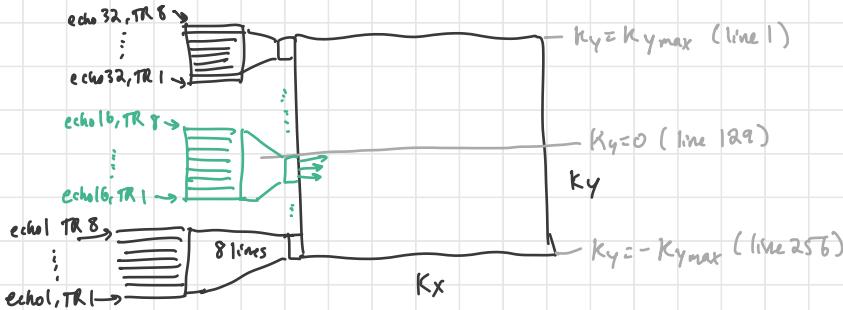
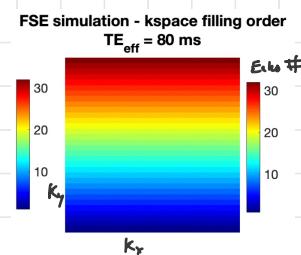
\hookrightarrow Echo 1, TR 1 = line 256 (bottom line)

Echo 1, TR 2 = line 255 (second from bottom)

;

Echo 1, TR 8 = line 249 (8th from bottom)

- The center kspace line (line 128) was filled by echo 16, with its echo time being 80ms in order to make $TE_{\text{eff}} = 80\text{ms}$.



(2.)

(ii.) We chose the k-space filling order so that the center PE line in k-space would be filled by the echo at time $TE = 80\text{ms}$. With $ESE = 5\text{ms}$, this was echo #16.

With an $ETL = 32$, we can fill 32 lines of k-space in each TR. Thus, to fill 256 (PE) lines, we need $256/32 = 8$ TR's.

The total acquisition time would be:

$$T_{acq} = \frac{TR \cdot Ny}{ETL} = \frac{3s \cdot 256}{32} = 24s = T_{acq, ESE, ETL=32}$$

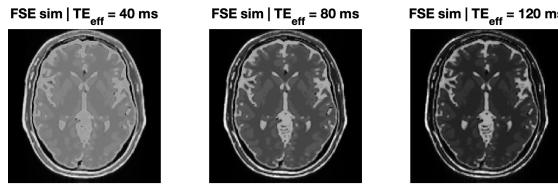
For a single SE scan with equivalent contrast, we would fill one line of k-space (PE line) per TR; or our ETL would be 1. Thus :

$$T_{acq} = TR \cdot Ny = (3s)(256) = 768\text{ sec} = T_{acq, SE, ETL=1}$$

The FSE sequence results in a reduction in total acquisition time directly proportional to ETL.

(2.) (iii.) We used a similar approach as (ii.) to simulate

T_E eff = 40 ms and T_E eff = 120 ms images, shown below.



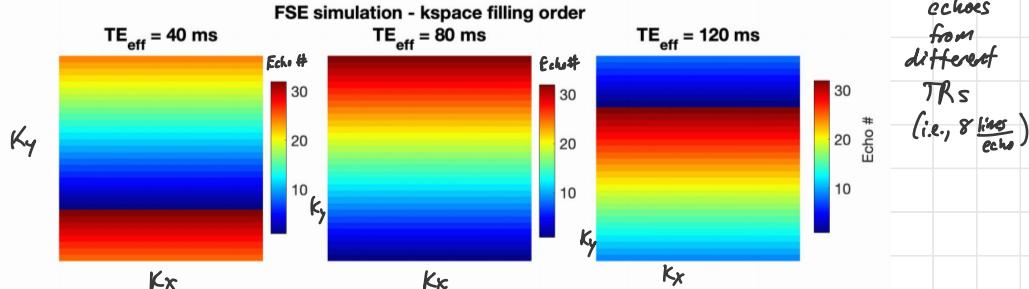
To fill k-space, we ensured the echo filling the central line ($k_y=0$) of k-space came from the echo occurring at the desired TEeff.

- k-center-echo # = TE_{eff}/ESP
 - For $TE_{eff} = 40ms$, we used the 8th to fill the kspace center
 $(40ms / 5ms/echo = 8^{th} echo)$
 - For $TE_{eff} = 120ms$, we used the 24th echo $(120ms / 5ms/echo = 24^{th} echo)$

For simplicity, we used a similar "bottom-to-top" approach to fill k-space, but simply shifted the starting line to be filled by the first echo to ensure the center line was filled to make T_{Epp} the desired time.

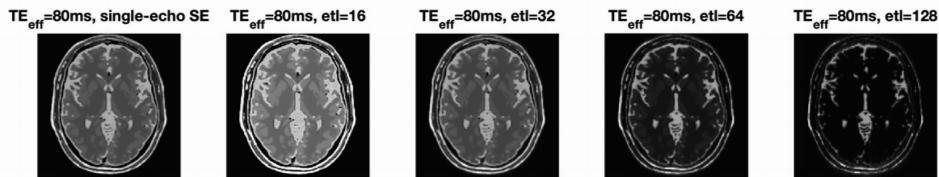
For $T_{\text{Eeff}} = 40\text{ms}$, we started filling k-space with echo 1 at line 192 (64th line from bottom), and for $T_{\text{Eeff}} = 120\text{ms}$, we started filling k-space at line 64 (192nd from bottom); when the top of kspace was reached, we wrapped around to the bottom line and continued.

(color = echo # used to fill line of kspace; Note each color echo contains 8 different



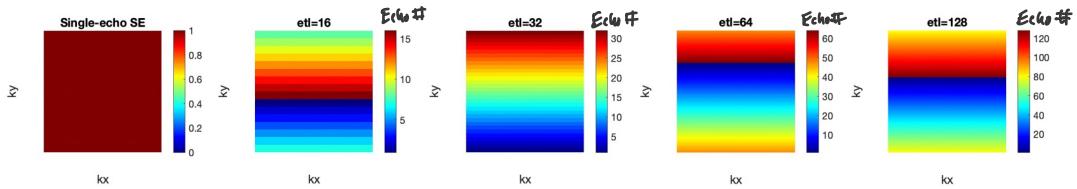
(2.) b.) (iii.)

(iv.) As the ETL becomes longer, the image quality degrades. This is due to the later echoes in long echo trains having less overall signal, as they are occurring a long time after the initial excitation. This causes the k-space to contain some lines with very small/low signals, resulting in a loss of SNR and GNR in the reconstructed images.



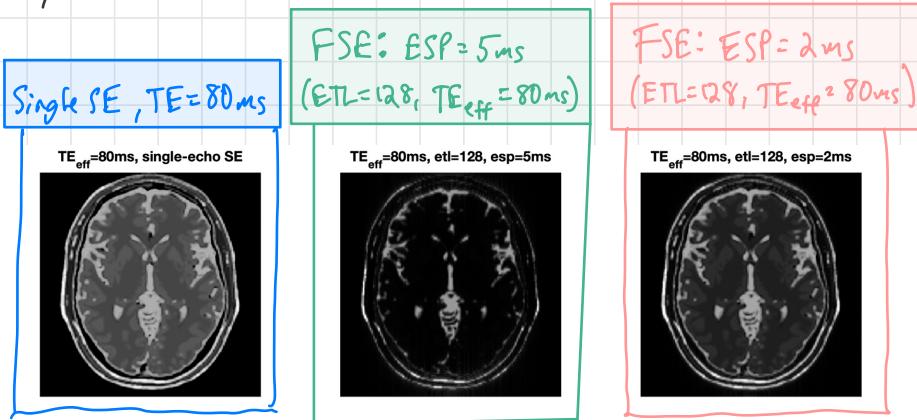
Increasing ETL
(Fixed TE_{eff}, Fixed ESP)

Corresponding k-space filling order plots are below.
(color = echo # used to fill that k_y line)



(2.) C.) One way to improve image quality for very long echo trains could be to decrease the echo spacing. This would leave more signal left at later echoes which could help improve image quality, SNR and CiVR.

I tested using an FSE sequence with exactly the same parameters, but using an echo spacing of 2ms instead of 5ms. This appeared to improve the image quality.



The k-space filling orders are shown below. Again, the color of each k_y line indicates the echo used to fill that k-space line.

