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# PIC Method for Numerical Simulation

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# PIC Method for Numerical Simulation

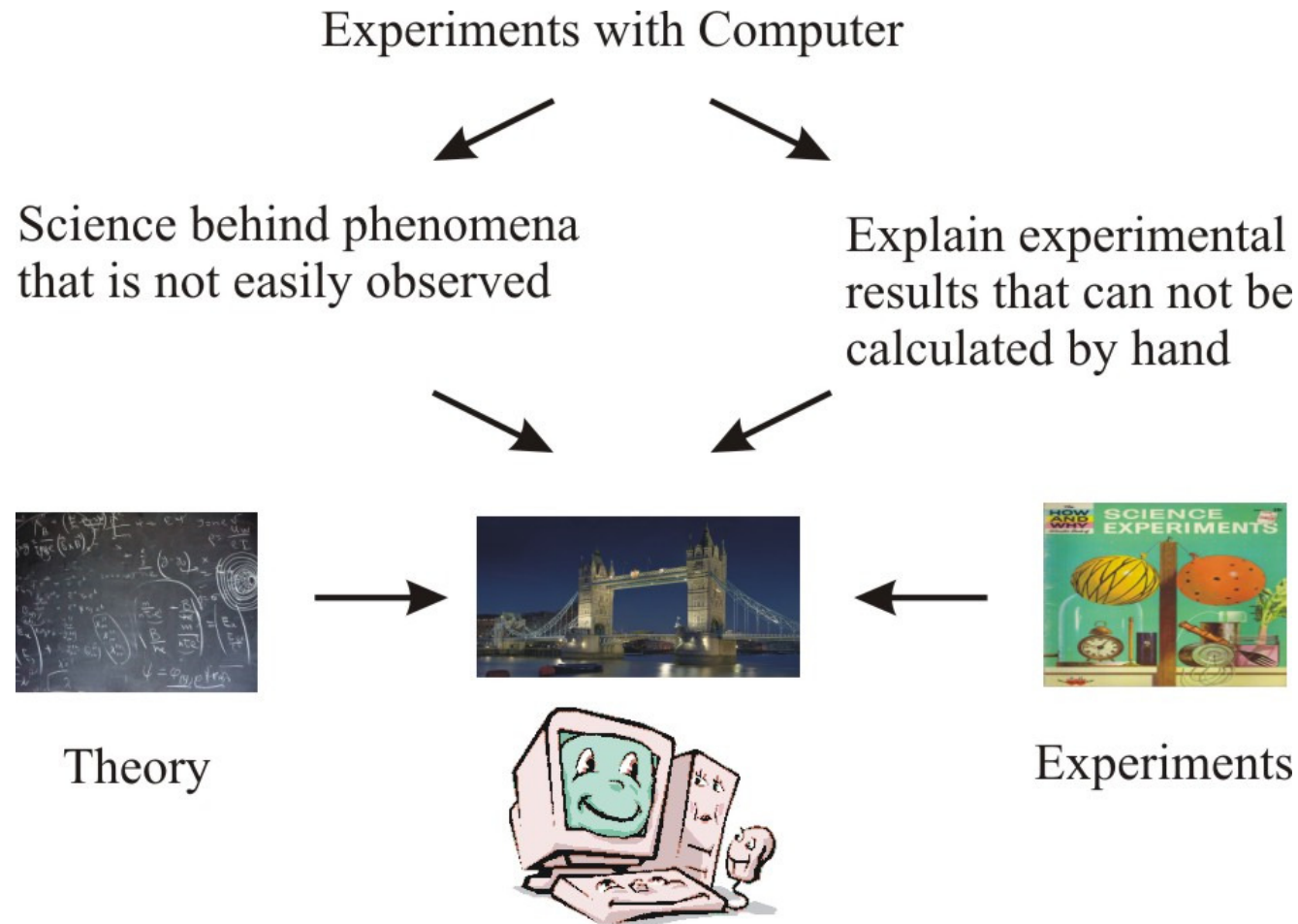
Ninad Joshi  
NNP – Group

# Contents

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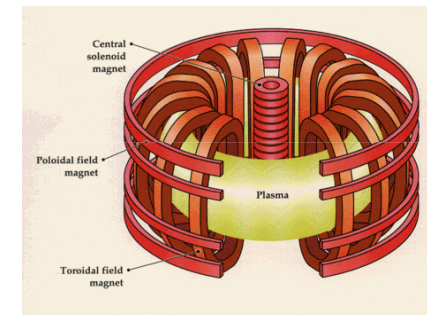
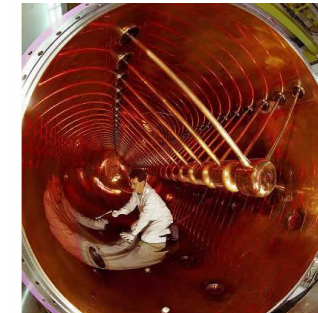
- Motivation
- Particle In Cell Method
- Projects
- Plasma and Ion Beam Simulations

# Motivation



## Ion beams and Plasmas

- Accelerators
  - Mostly single specie ions, different  $A/q$  ratio
  - Energy range from few  $keV$
  - Main momentum component in forward direction
- Stellarators and Tokamaks
  - Thermal / Maxwellian distribution of momentum
  - Energy range from few  $keV$  till  $MeV$
  - Magnetic confinement is the main issue
  - Neutral or non neutral
- Discharge plasmas
  - Pressure
  - Multi specie model: Ions, electrons, neutral atoms or molecules
  - Energy range upto few  $keV$

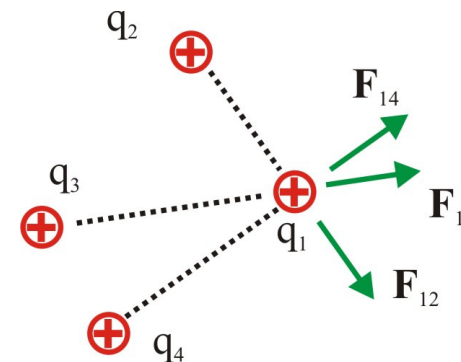


- Main Task
  - Calculate the force on charged particles through Lorentz Equation

$$\vec{F} = m\vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Simulate many particles

$$\vec{E}_i = \sum_{\substack{j=0 \\ j \neq i}}^n \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|^2} (\vec{r}_i - \vec{r}_j)$$



- Prof. Hartree and Phyllis Nicolson (1941-1944) used desk calculator  
30 electrons in Magnetron, 1D, Space charge included

- Computer with floating point operation  $1\mu\text{s}$

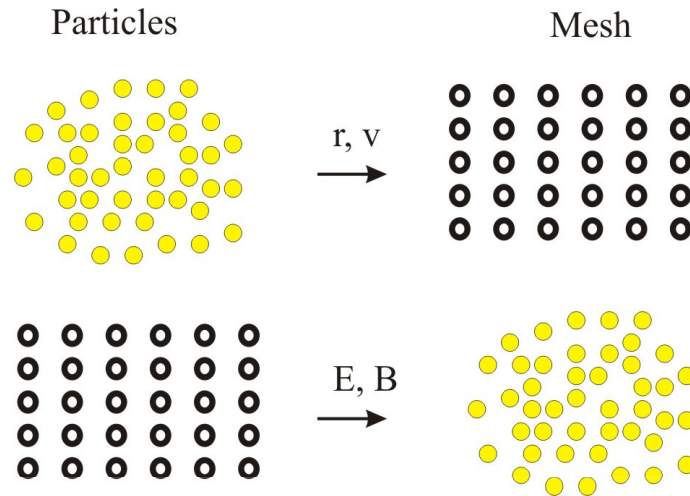
$$\text{Operation count} = \alpha N_p + \beta(N_g)$$

$$\alpha = 20 \quad \beta = 5N^3 \log_2 N^3 \quad N = 32 \quad N_p = 10^5$$

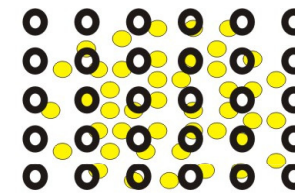
$$\text{CPU time} = 1 \text{ day for PP}$$

$$= 4.5 \text{ seconds for PM}$$

# PIC model

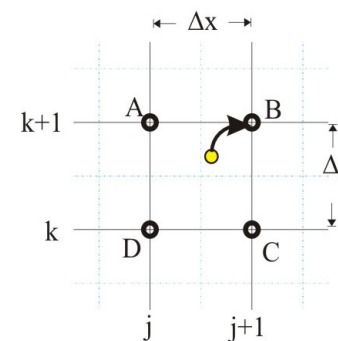


- Disordered data to the structured data form
- Periodicity, Symmetry

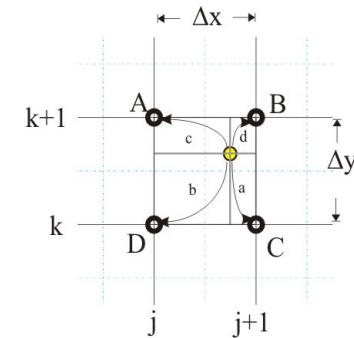


$$Q_i \longrightarrow \rho_j$$

- Charge distribution on grid points and back interpolation
- Weighing schemes according to necessity
  - Linear weighing
  - Functional form



Nearest Grid Point



Area weighing

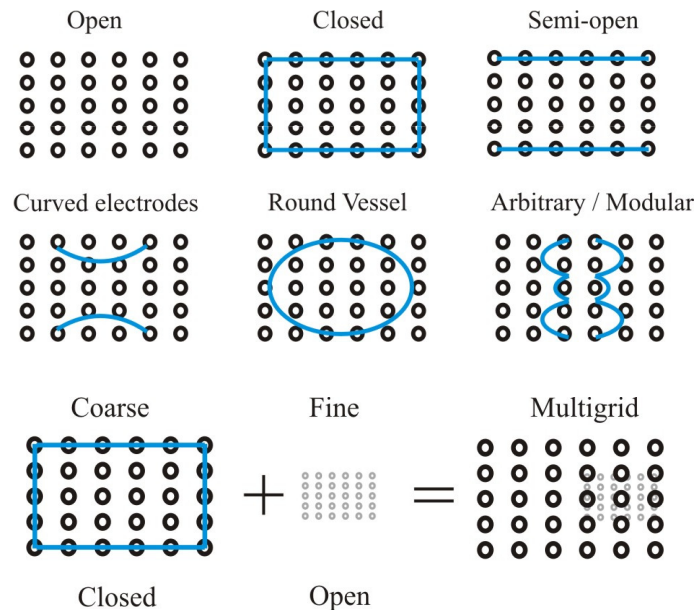
# Poisson Equation

$$\nabla^2 \phi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\epsilon_0}$$

$$\begin{aligned}\phi_{N_g} - 2\phi_1 + \phi_2 &= \rho_1 \\ \phi_1 - 2\phi_2 + \phi_3 &= \rho_2 \\ &\vdots \\ \phi_{N_g-2} - 2\phi_{N_g-1} + \phi_{N_g} &= \rho_{N_g-1} \\ \phi_{N_g-1} - 2\phi_{N_g} + \phi_1 &= \rho_{N_g}\end{aligned}$$

$$\mathbf{A} \cdot \phi = -\frac{\rho}{\epsilon_0}$$

## Boundary conditions



## 1. Fourier Transform

$$\rho(\mathbf{x}) \rightarrow \rho(\mathbf{k}) \rightarrow \phi(\mathbf{k}) \rightarrow \phi(\mathbf{x}) \rightarrow \mathbf{E}(\mathbf{x})$$

## 2. Iterative methods

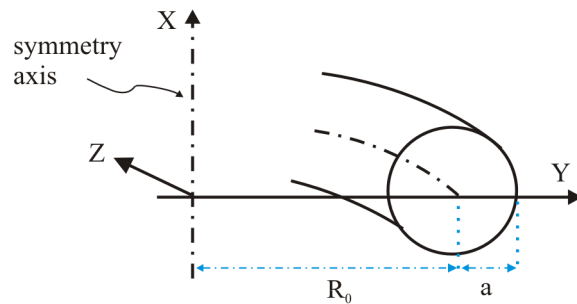
$$\rho(\mathbf{x}) \rightarrow \phi(\mathbf{x}) \rightarrow \mathbf{E}(\mathbf{x})$$



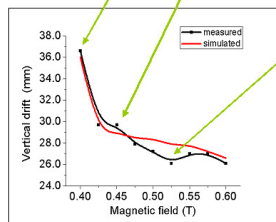
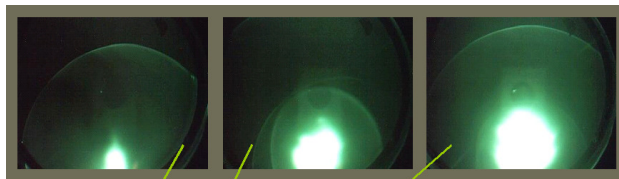
- Project MSR (**M**agnetostatic **S**torage **R**ing) at Frankfurt am Main
  - Ion Beam transport through toroidal segments
  - Confinement and guidance properties for ion storage
  - Injection scheme
- FRANZ facility
  - Chopper System in LEBT setion
- SPIE-Program: **S**imulation code for **P**lasma and **I**on **E**xtraction
  - Dynamics of plasmas in small volume type ion source
  - Ion extraction
  - Production mechanism different species

# Project MSR

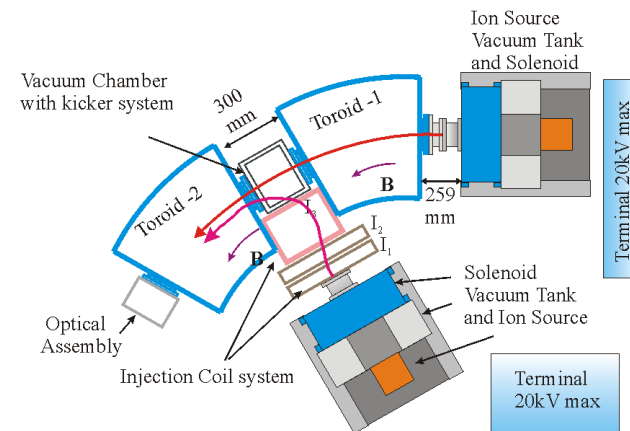
- High current beam storage in longitudinal magnetic field with closed magnetic field lines
  - Multi ampere proton beam with energy  $150\text{keV} \sim \text{few MeV}$



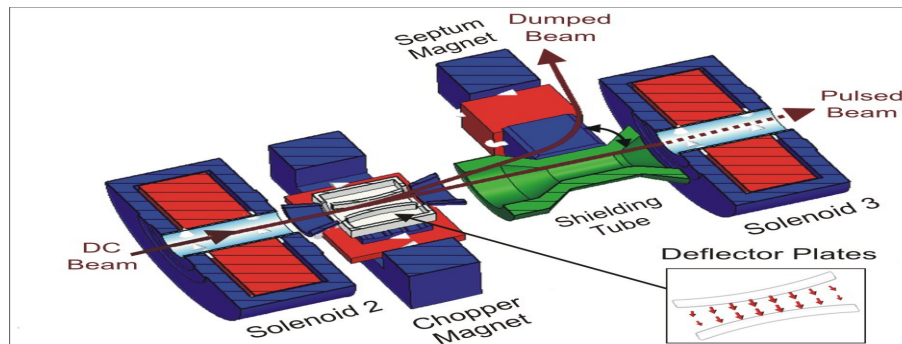
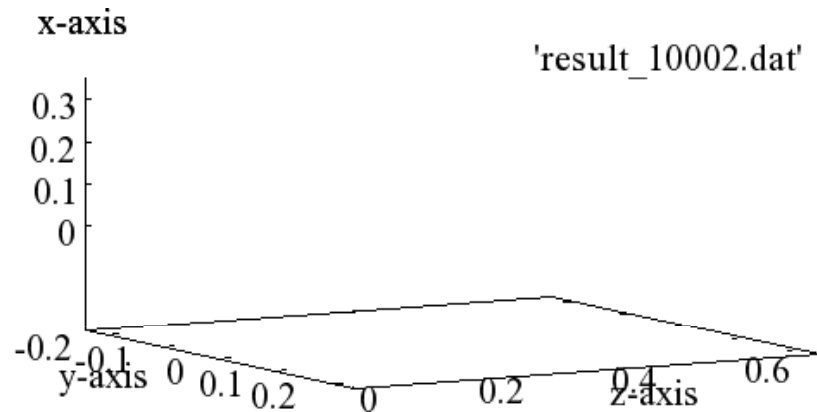
- PIC simulation in toroidal coordinates
- Ion beam transport with electrons
- Direct comparison with experiments



Experimental setup

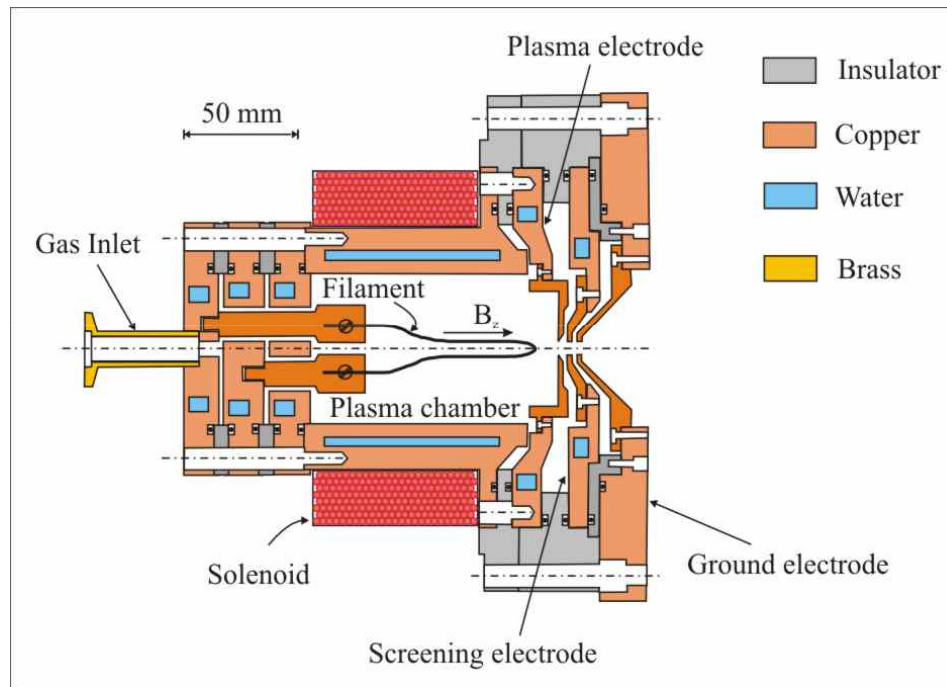


- Chopper in Low Energy Beam Transport (LEBT) section
  - Input proton beam with current  $200\text{ mA}$  at the energy of  $120\text{ keV}$



- Semi-open boundary conditions with definition of curved electrodes
- Continual generation of ions
- Space charge compensation due rest gas ionization and secondary electrons produced on wall
- Deflection due to electric field can be compared with experiments

## Simulation code for Plasma and Ion Extraction - Program



- Hot filament driven volume type ion source
- Originally constructed and developed by Peter Groß in 2000
- Triode extraction  $20\text{keV}$  max
- *He* ions as reference
- Proton beam for MSR experiments
- Ref: N. Joshi, doctoral thesis

# Ion species

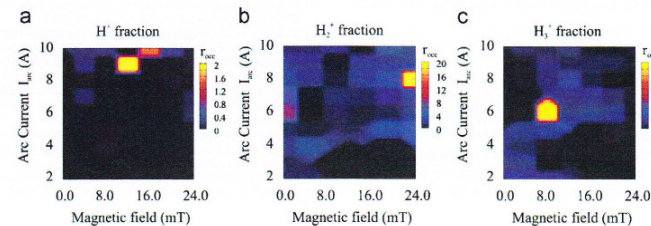
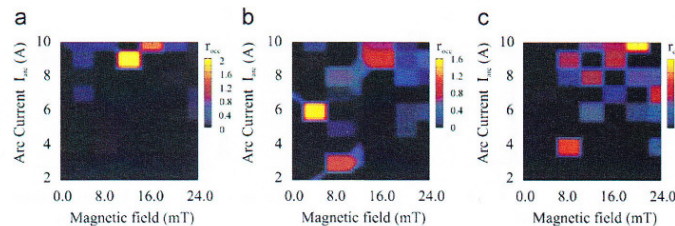
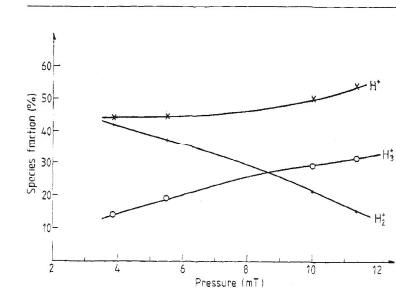
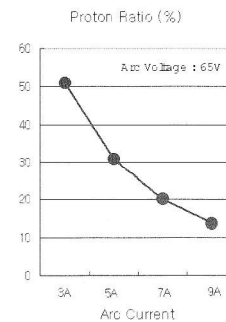
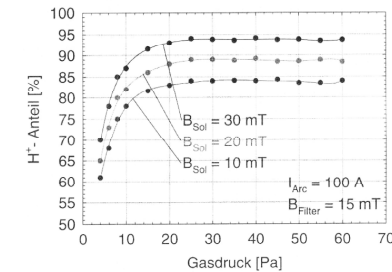
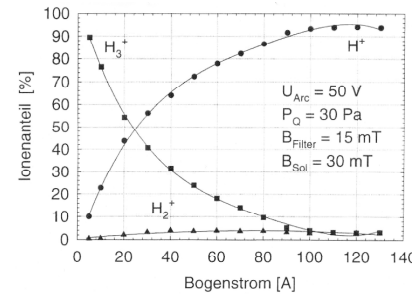
Ref: R. Hollinger, K. Volk, et.al., „Measurement of the beam emittance of the Frankfurt proton source“, RSI, 2002; and doctoral thesis

*200 mA, ~93% p, @ 55 keV*

Ref: A. J. T. Holmes, et.al., „A compact ion source with high brightness“, J. Phy. E, 1980

*40 mA, ~60% p, @ 50 keV*

Ref: N. Joshi, O. Meusel, et.al., NIM A, 2009, *5.0 mA, ~58% p, @ 10 keV*

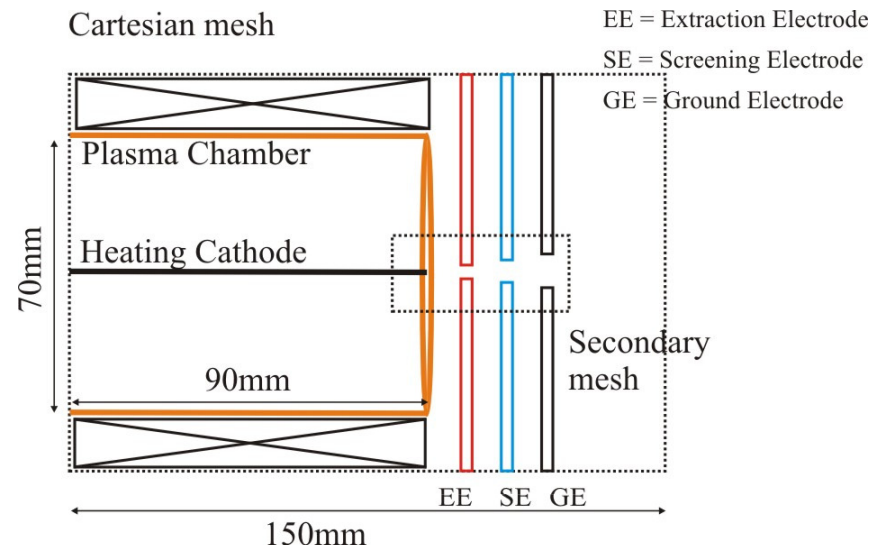


- Can we find theoretical limits of a given ion source by developing a simulation code ?
- How do the external components and fields influence the plasma properties and the production mechanism of different species in this type of source?
- Along with existing tools can we advance our code to investigate the hot filament driven ion source and find „science“ behind experimental results?

## Recent compititors

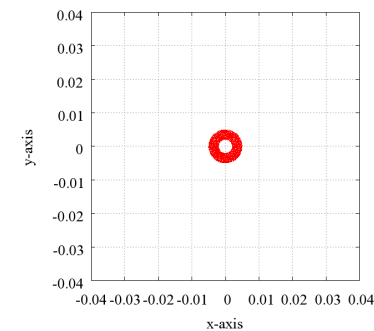
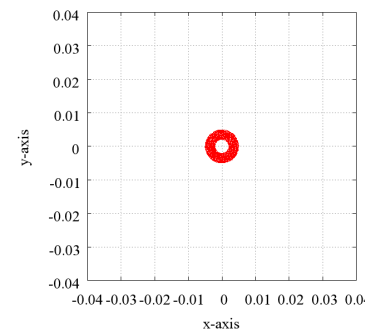
- Existing codes IGUN , KOBRA, OOPIC
- China: Simulation code for description in complete 3D, with 3D graphics
- France: Simulation code to describe H<sup>-</sup> production in negative ion source, ITER, (1D)

# Simulation parameter



- Multiple Grids
- Cartesian mesh with definition of circular plasma chamber and electrodes
- Magnetic field from coils
- Continual generation of electrons at cathode

Electron simulation in plasma chamber (vertical plane), filament at the center



Typical potentials: HV= extraction voltage

$U_{\text{cathode}} = 0 + HV$  ;  $U_{\text{arc}} = 100 + HV$

$U_{\text{extraction}} = 90 + HV$ ;

$U_{\text{screening}} = 10\% \text{ of } HV$

$U_{\text{ground}} = 0.0$  !!



# Hydrogen Plasma

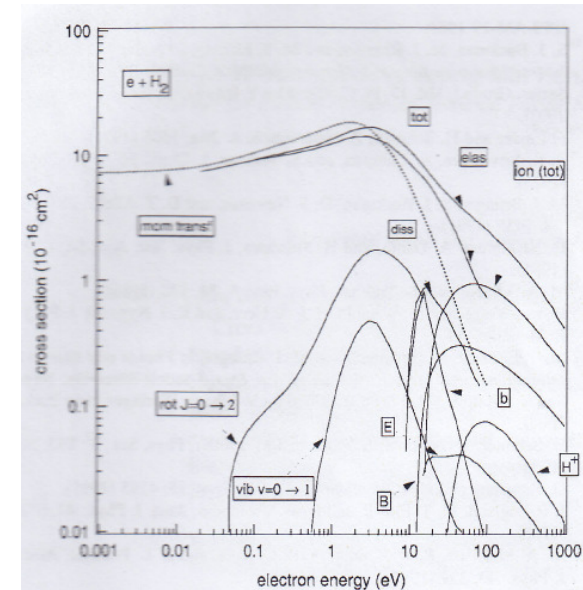
Reaction number	Process
1	$e + H \rightarrow e + H$ (elastic)
2	$e + H \rightarrow e + H^*$ (three energy levels)
3	$e + H \rightarrow 2e + H^+$
4	$e + H_2 \rightarrow e + H_2$
5	$e + H_2 \rightarrow e + H_2^*$ (17 energy levels)
6	$e + H_2 \rightarrow 2e + H_2^*$
7	$e + H_2 \rightarrow 2e + H^* + H$
8	$e + H_2(v > 3) \rightarrow H + H^-$
9	$e + H_2^+ \rightarrow H + H$
10	$e + H_2^+ \rightarrow e + H^* + H$
11	$e + H_3^+ \rightarrow H_2 + H$
12	$e + H_3^+ \rightarrow 3H$
13	$e + H_3^+ \rightarrow e + H^* + 2H$
14	$e + H_3^+ \rightarrow e + H^* + H_2$
15	$e + H^- \rightarrow 2e + H$

Reaction number	Process
16	$H^+ + H \rightarrow H + H^+$ (charge exchange)
17	$H^+ + H \rightarrow H^+ + H$ (elastic)
18	$H^+ + H_2 \rightarrow H^+ + H_2$ (elastic)
19	$H^+ + H_2 \rightarrow H_2^+ + H$ (charge exchange)
20	$H_2^+ + H_2 \rightarrow H_2 + H_2^+$ (charge exchange)
21	$H_2^+ + H_2 \rightarrow H_3^+ + H$
22	$H_3^+ + H_2 \rightarrow H_3^+ + H_2$ (elastic)
23	$H^- + H \rightarrow e + 2H$
24	$H^- + H \rightarrow e + H_2^*$
25	$H^- + H \rightarrow H + H^-$ (charge exchange)
26	$H^- + H_2 \rightarrow H^- + H_2$ (elastic)
27	$H^- + H_2 \rightarrow e + H + H_2$
28	$H^+ + H^- \rightarrow H + H^*$
29	$H^+ + H^- \rightarrow e + H_2^*$

Species =>  $H_2$   $H_2^*$   $H$   $H^*$   $H^+$   $H_2^+$   $H_3^+$   $H^-$   $e^-$

=> Total = 9

	Creation	Annihilation
$H^+$	5	3
$H_2^+$	3	3
$H_3^+$	1	4
$H^-$	1	6



Ref: „Cross Sections for Electron Collisions with Hydrogen Molecules“, Jung-Sik Yoon et.al., J. Phys Chem.

„Electron Collisions with Atoms and Molecules“, Atomic data and Nuclear data tables



# PIC with Monte Carlo Collision

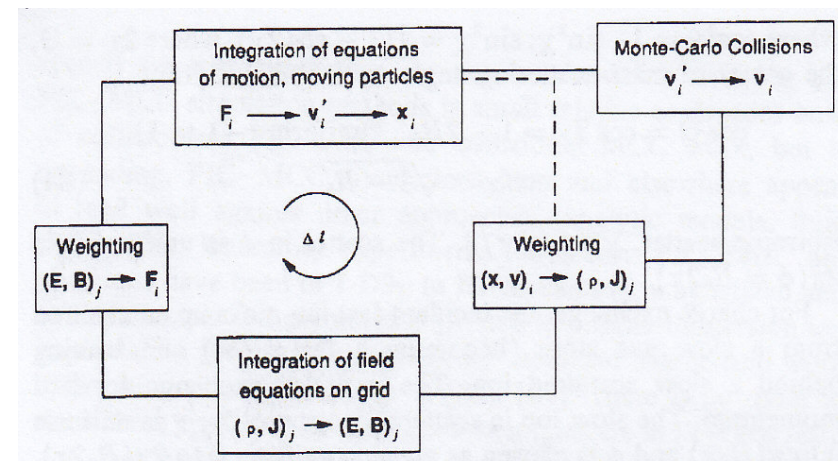
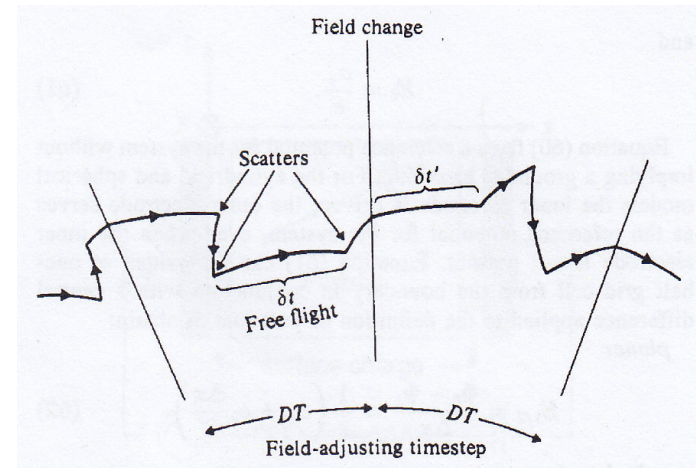
## PIC

- Deterministic classical mechanics
- Moving particles in small time step
- Collective effect through self and applied fields

## MC

- Probabilistic
- Collisional effects in relatively weak electric fields

$$\sigma(E) \quad n(x) \quad \Delta s \rightarrow \nu_{coll} \quad P$$



Kinetic Energy for  $i$  th particle of specie  $s$

$$E_i = \frac{1}{2} m_s v_i^2$$

Total collisional cross section is sum of  
where  $j$  is type of cross section

$$\sigma_T(E_i) = \sum \sigma_j(E_i)$$

Collisional probability for  $i$  th particle

$$P_i = 1 - \exp(-\Delta t v_i \sigma_T(E_i) n_t(\mathbf{x}_i))$$

Random numbers  $R_1, R_2 \Rightarrow [0,1]$

$$R_1 < P_i \Rightarrow \text{collision}$$

$$R_2 \Rightarrow \text{type of collision}$$

Random numbers  $R_3, R_4 \Rightarrow [0,1]$

$$R_3, R_4 \Rightarrow \text{Scattering}$$

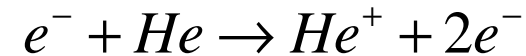
Error missed collision in  $\Delta t$

$$r < 0.01 \Rightarrow P_i < 0.095$$

$$r \sim \sum_{k=2}^{\infty} P_i^k = \frac{P_i^2}{1 - P_i}$$

# Example He- ions

Single specie plasma from Helium



Plasma chamber  
length  $0.1m$   
radius  $r=0.035m$

Filament  $90mm$

Green : Electrons

Red : He - ions

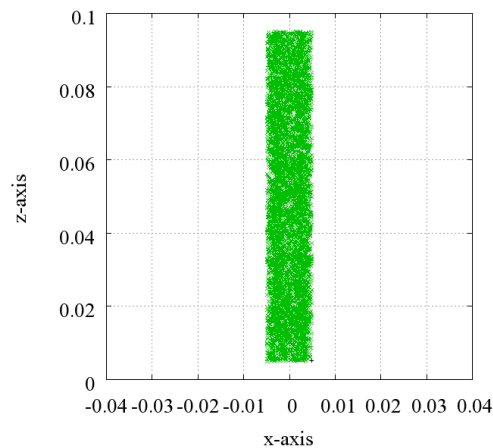
Simulation time

Each step  $\sim ns$

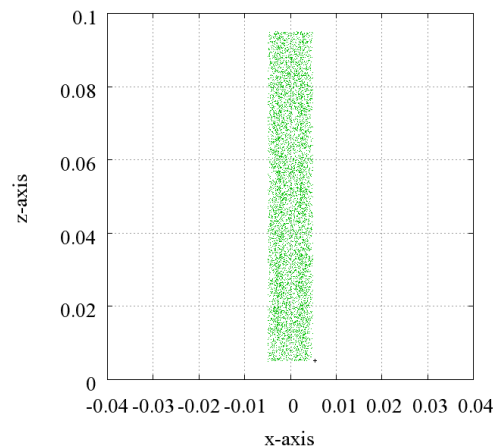
Total time max  $0.5 \mu s$

Horizontal plane

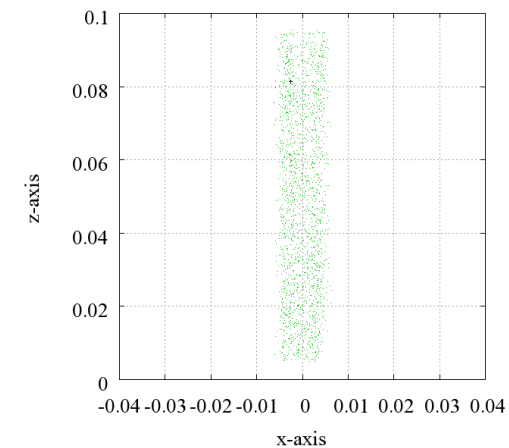
$B \sim 0$  Gauss



$B \sim \text{few Gauss}$

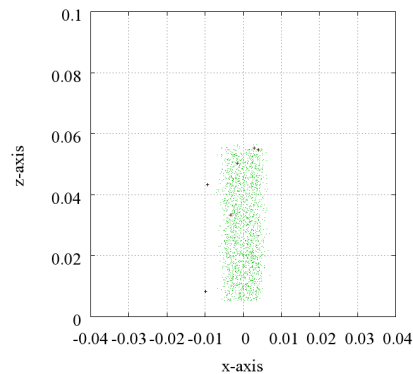


Potential Plasma  
electrode changed

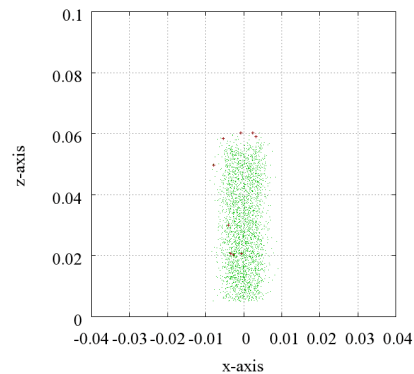


# Influence on plasma

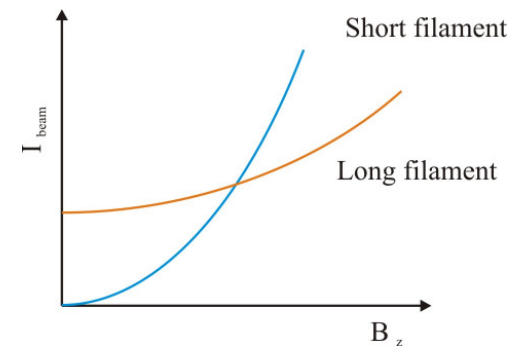
## Shorter filament length 55 mm



w/o  $B$



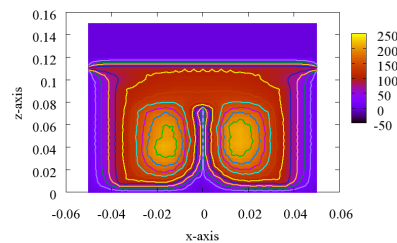
with  $B$



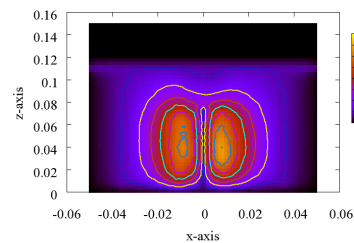
Estimated beam current behaviour

## Potential distribution (Horizontal plane)

Long filament

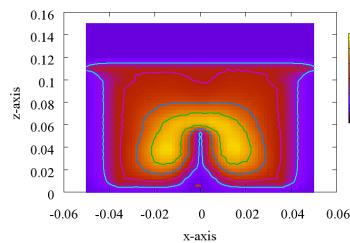


w/o  $B$

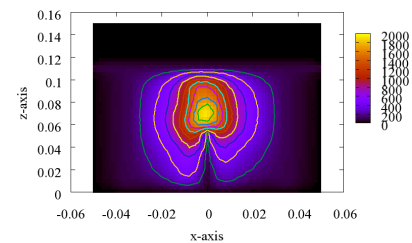


with  $B$

Short filament

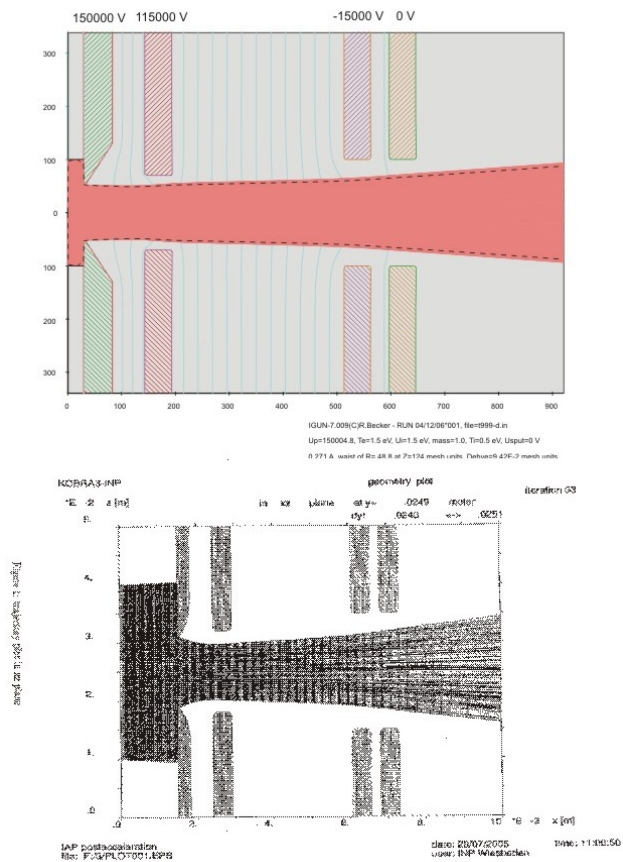


w/o  $B$

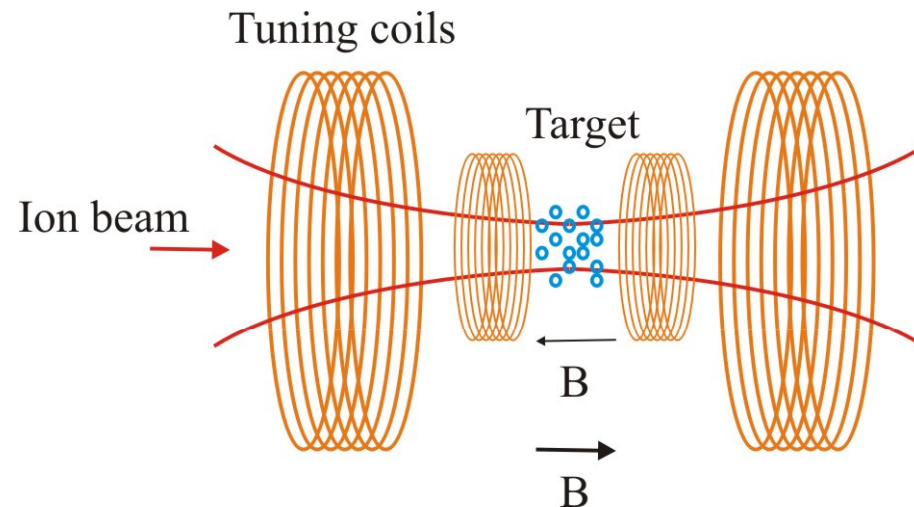


with  $B$

# Comparison



- Designing the target chamber for collision experiments



- Different Larmor gyration for ions and electrons in strong magnetic fields
- Configuration of tuning coil can be investigated for optimized particle density and scattering cross sections

# References

- Computer simulation using particles
  - R. W. Hockney and J. W. Eastwood
- Plasma-based ion beam sources
  - Loeb, Plasma Phys. Control. fusion, (47), 2005
- Charged particle beams
  - S. Humphries, Jr.
- Particle-in-Cell Charged-Particle Simulations, Plus Monte Carlo Collisions With Neutral Atoms, PIC-MCC
  - C. K. Birdsall, IEEE Transactions on Plasma science, vol. 19 (2), 1991
- A Monte Carlo collision model for the particle-in-cell method: applications to argon and oxygen discharge
  - V. Vahedi, et.al., Comp. Phys. Commun., 87,1995
- Particle-in-cell with Monte Carlo collision modelling of the electron and negative hydrogen ion transport across a localized transverse magnetic field
  - St. Kolev, et.al., Phys. Plasmas, (16), 2009
- For more informaton see:  
<http://www.uni-frankfurt.de/~joshi>

# Conclusions and Outlook

- Particle-in-cell model has been investigated and successfully applied for different problems
- The code is being upgraded Monte Carlo subroutine for simulation of collision
- The results can be directly compared with experiments
- New approaches required to study the target designing in magnetic field

## Acknowledgements

- Prof. U. Ratzinger
- NNP – Group

Thank you ...!!