

## Problem Setup

3	4	2	5
1	3	2	5
2	3	4	4

Selection bias  
 $p_T(u, i) \neq p_D(u, i)$

3	4		5
	3		5
	3	4	4

**Missing-At-Random (MAR)** means the probability of missing data depends only on observed variables, not on the true unobserved values—so the missingness can be explained by available information rather than hidden preferences. In recommender systems, observed ratings are often Missing-Not-At-Random (MNAR) because users selectively rate items. This causes *selection bias*, making the observed ratings unrepresentative of the full user–item space.

- $\mathcal{U} = \{u_1, u_2, \dots, u_m\}$  is the set of  $m$  users.  $\mathcal{I} = \{i_1, i_2, \dots, i_n\}$  is the set of  $n$  items.
- $\mathcal{D} = \mathcal{U} \times \mathcal{I}$  is the set of all user-item pairs.  $x_{u,i}$  is the feature of user  $u$  and item  $i$ .
- $\mathbf{R}^* \in \mathbb{R}^{m \times n}$  is the true rating matrix,  $\hat{\mathbf{R}} \in \mathbb{R}^{m \times n}$  is the rating prediction matrix for  $\mathbf{R}^*$ .  $r_{u,i}^*$  is the true rating of user  $u$  on item  $i$ , while  $r_{u,i}$  is the observed rating.
- $o_{u,i}$  is an indicator.  $o_{u,i} = 1$  means the rating of user  $u$  for item  $i$  is observed.

With full observability, the model would minimize the ideal risk:

$$\mathcal{L}_{\text{ideal}}(\theta) = \frac{1}{|\mathcal{D}|} \sum_{(u,i) \in \mathcal{D}} \ell(\hat{y}_{u,i}, y_{u,i}),$$

where  $\ell(\cdot, \cdot)$  is a standard pointwise loss, like cross entropy (CE), mean absolute error (MAE) or mean squared error (MSE).

but only a biased subset  $\mathcal{O} \subset \mathcal{D}$  is observed, yielding the naive loss:

$$\mathcal{L}_{\text{naive}} = \frac{1}{|\mathcal{O}|} \sum_{(u,i) \in \mathcal{O}} \ell(\hat{y}_{u,i}, y_{u,i}).$$

- If the missingness is missing at random, the naive estimator is unbiased to the ideal loss, but MAR-style evaluation slices are small and unevenly distributed across users. Following common experimental protocols and prior analyses, we assume the observed set is a mixture  $\mathcal{O} = \mathcal{O}_{\text{MNAR}} \cup \mathcal{O}_{\text{MAR}}$  with  $|\mathcal{O}_{\text{MNAR}}| \gg |\mathcal{O}_{\text{MAR}}|$ : a large MNAR portion collected from natural use, plus a small MAR-like portion (e.g., randomized unbiased slices) that is costly and uneven across users. Users with MAR-like behavior—called pivot users—can serve as references for debiasing others. *SemiSL-RDC* leverages these pivots to calibrate rating distributions across users in a semi-supervised way.

## Motivation

### Limitations of existing methods

- Imputation / Pseudo-labeling → sensitive to unobserved factors, unreliable under sparsity.
- Propensity-based reweighting → fragile when exposure probabilities are misspecified.
- Doubly-Robust & balancing methods → still depend on model correctness, unstable with few unbiased labels.
- RDC (Rating Distribution Calibration) → assumes every user has enough unbiased feedback, unrealistic in real data.

### Observations

- In real datasets (e.g., Yahoo! R3, Coat), unbiased (MAR) supervision is small and unevenly distributed.
- User-level bias varies: some users behave close to MAR and can be used as reliable anchors (“pivot users”).

### Motivation

To overcome scarce and heterogeneous unbiased data, we propose a *Semi-Supervised Rating Distribution Calibration (SemiSL-RDC)* framework that automatically discovers MAR-like pivot users, and calibrates other users’ predicted rating distributions toward these pivots, thereby mitigating selection bias without explicit imputation or propensity modeling.

## Method

*SemiSL-RDC* mitigates selection bias by using pivot users—those whose feedback patterns are closer to MAR—as anchors to calibrate others’ predicted rating distributions. It integrates semi-supervised pivot discovery, distribution feature construction, and pivot-aware calibration into one training loop.

- Mean Absolute Error (MAE) for labeled ratings:

$$\text{MAE}_u = \frac{1}{|\mathcal{O}_u|} \sum_{i \in \mathcal{O}_u} |y_{u,i} - f_\theta(x_{u,i})| \in [0, 1].$$

- Normalized rating distribution entropy. Let  $n_{u,y}$  be the count of observed labels  $y \in \{0, 1\}$  for user  $u$ , and  $q_u(y) = \frac{n_{u,y}}{n_{u,0} + n_{u,1}}$ . Define

$$\tilde{H}_u = - \sum_{y=0}^1 q_u(y) \log q_u(y), \quad H_u = \frac{\tilde{H}_u}{\log 2} \in [0, 1].$$

- To capture adequacy of  $\text{su\_x0002\_pervision}$  and mitigate the variance of label-only signals, we use

$$C_u = \frac{|\mathcal{O}_u|}{M} \in [0, 1],$$

We form a convex score that decreases with “pivot-likeness”:

$$s_u = -\alpha_1 \cdot \text{MAE}_u + \alpha_2 \cdot H_u + \alpha_3 \cdot C_u, \quad \alpha_j \geq 0, \sum_{j=1}^3 \alpha_j = 1,$$

so  $s_u \in [-1, 0]$  and we map it to a probability by normalizing the sigmoid:

$$\pi_u = \frac{\sigma(s_u)}{\sigma(0) - \sigma(-1)} \in [0, 1],$$

- Rating Distribution Feature Extraction. For each user  $u$ , collect predicted probabilities over all items (observed and unobserved):

$$t_k = \frac{k}{K}, \quad \mathbf{g}_u = [f^{(1)}(\hat{\mathbf{y}}_u), \dots, f^{(K)}(\hat{\mathbf{y}}_u)],$$

$$f^{(k)}(\hat{\mathbf{y}}_u) = \frac{1}{M} \sum_{i=1}^M \sigma(\tau(t_k - \hat{y}_{u,i})), \quad k = 1, \dots, K.$$

- Semi-Supervised Calibration Loss. For a non-pivot user  $u \in \mathcal{P}_-$ , measure distributional similarity to each pivot  $v \in \mathcal{P}_+$  with a distance  $\delta(\mathbf{g}_u, \mathbf{g}_v)$  (where we use squared  $\ell_2$ -norm by default, though other metrics are possible). Define a soft neighborhood over pivots:

$$w_{u \rightarrow v} = \frac{\exp(1/\delta(\mathbf{g}_u, \mathbf{g}_v))}{\sum_{v' \in \mathcal{P}_+} \exp(1/\delta(\mathbf{g}_u, \mathbf{g}_{v'}))}, \quad \mathcal{L}_{\text{semi}}^u = \frac{1}{|\mathcal{P}_+|} \sum_{v \in \mathcal{P}_+} w_{u \rightarrow v} \delta(\mathbf{g}_u, \mathbf{g}_v),$$

$$\mathcal{L}_{\text{semi-cal}}(\theta) = \frac{1}{|\mathcal{P}_-|} \sum_{u \in \mathcal{P}_-} \mathcal{L}_{\text{semi}}^u.$$

Overall Objective:

$$\mathcal{L}_{\text{SemiSL-RDC}} = \mathcal{L}_{\text{base}} + \mu \mathcal{L}_{\text{semi-cal}},$$

where  $L_{\text{base}}$  is cross-entropy on observed ratings and  $L_{\text{semi}}$  enforces distribution calibration toward MAR-like pivots. This end-to-end optimization simultaneously learns prediction and debiasing.

## Learning Algorithm of SemiSL-RDC

### Algorithm 1 SemiSL-RDC Optimization Algorithm

- 1: **Input:** Set of user-item features  $X$ ; All user-item pairs matrix  $\mathcal{D}$ ; Observed user-item pairs matrix  $\mathcal{O}$ ; Hyperparameters  $\alpha_1, \alpha_2, \alpha_3, \lambda, \mu$ .
- 2: **Output:** A rating prediction model  $f_\theta$ .
- 3: Initialize the prediction model  $f_\theta$  by pre-training it on the observed set  $\mathcal{O}$ .
- 4: **while** not converge **do**
- 5:   **for** number of training iterations **do**
- 6:     Sample a mini-batch  $\{(u_j, i_j)\}_{j=1}^J$  from  $\mathcal{D}$ ;
- 7:     Compute 3 per-user criteria,  $\text{MAE}_u$  from  $\mathcal{O}$ , and  $H_u, C_u$  from  $\mathcal{D}$ ;
- 8:     For each user, calculate the pivot-user probability  $\pi_u$  and classify to set  $\mathcal{P}_+$  or  $\mathcal{P}_-$ ;
- 9:     For each non-pivot user  $u \in \mathcal{P}_-$ , calculate its distribution difference between each pivot user  $v \in \mathcal{P}_+$ ,  $\delta(\mathbf{g}_u, \mathbf{g}_v)$ ;
- 10:    Calculate calibration loss  $\mathcal{L}_{\text{semi-cal}}$ , and overall loss  $\mathcal{L}_{\text{SemiSL-RDC}}$ ;
- 11:    Update  $f_\theta$  by minimizing  $\mathcal{L}_{\text{SemiSL-RDC}}$ ;
- 12:   **end for**
- 13: **end while**

## Experimental Results

TABLE I  
PERFORMANCE IN TERMS OF AUC, NDCG@5, AND RECALL@5 ON THE UNBIASED DATASET OF COAT AND YAHOO! R3. THE BEST RESULTS ARE BOLD, AND THE BEST BASELINE RESULTS ARE UNDERLINED.

Method	COAT			YAHOO! R3		
	AUC	NDCG@5	Recall@5	AUC	NDCG@5	Recall@5
MF (Bias)	0.747	0.500	0.546	0.721	0.553	0.716
MF (Uniform)	0.580	0.363	0.386	0.574	0.455	0.611
MF (Combine)	0.751	0.504	0.546	0.724	0.558	0.717
CausE	0.763	0.512	0.575	0.730	0.555	0.736
ESMM	0.745	0.506	0.525	0.708	0.545	0.693
KD-Label	0.760	0.509	0.562	0.726	0.583	0.752
AutoDebias	0.762	0.540	0.580	0.735	0.632	0.785
KD-Feature	0.766	0.522	0.584	0.717	0.557	0.736
IPS	0.761	0.513	0.566	0.722	0.555	0.733
Multi-IPS	0.758	0.514	0.531	0.719	0.546	0.710
ESCM2-IPS	0.757	0.514	0.558	0.729	0.559	0.714
RD-IPS	0.764	0.514	0.566	0.730	0.571	0.735
BRD-IPS	0.763	0.511	0.564	0.735	0.582	0.743
DR	0.766	0.525	0.552	0.725	0.553	0.727
Multi-DR	0.759	0.527	0.565	0.719	0.553	0.712
ESCM2-DR	0.760	0.553	0.568	0.715	0.566	0.722
RD-DR	0.768	0.539	0.571	0.732	0.569	0.738
BRD-DR	0.770	0.546	0.577	0.735	0.576	0.737
RDC	0.761	0.521	0.549	0.729	0.559	0.719
<b>SemiSL-RDC</b>	<b>0.771</b>	<b>0.557</b>	<b>0.589</b>	<b>0.751</b>	<b>0.640</b>	<b>0.796</b>