# DEMO 1 model

Model in DEMO 1 deals with one set of decision variables *Xi,j* which define how much water should be allocated through links *i,j*. The optimal value for *Xi,j* depends upon parameters *ci,j* and *cci* where *cci* is the consumption coefficient at each demand node *i* while *ci,j* are the ‘costs’ assigned to links *i,j* in the network. The concept of ‘cost’ is directly connected to the one of priority: given that we are dealing with a minimization problem (see the objective function below), the higher the penalty *ci,j* the lower the priority to deliver water. The model objective function minimises the ‘cost’ of delivering water throughout the network so that the available water is allocated to all demand nodes based on priorities.

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|  | 1 |

The objective function above is subject to the mass balance equation at all nodes (set *I*), where *CON* is the set of links composing the network (network topology).

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|  | 2 |

In the equation above, term *inflowi* is the incoming flow (e.g. from a desalination plant or an in-stream from a river), while is a flow multiplier (e.g. equal 1 if there is no loss in a link).

Upper and lower bound constraints are also introduced to limit the links’ minimum and maximum extent of use.

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|  | 3 |

In the equation above *lqi,j* and *uqi,j* are respectively the lower and upper bounds capacities for link *i,j*, while *ai,j* is a loss coefficient.

The formulation above represents the classical example of the ‘network flow’ problem and literature on the subject can be found in Castillo (…) and Israel and Lund (1999).

# DEMO 2 model

Model in DEMO 2 differs from the one in DEMO 1 for the following reasons:

1. Since water manifests in networks over time, we have now introduced a time step *t* (set *T*).
2. Using a time step *t* allows introducing a new mass balance equation for reservoir nodes. In this way, storage levels can be tracked over the time horizon *T*, for each time-step *t*. Storage levels are defined through a new positively defined state variable *Sit*.

Under these considerations, the objective function to be minimised in each time-step *t* becomes:

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|  | 4 |

where is the extent of use variable introduced in the previous model now defined over the time set *T*.

The above objective function is subject to the mass balance equation 5 and to the upper and lower bound constraints (equation 6).

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|  | 5 |
|  | 6 |

Where *NS* represents the set of non-storage nodes (e.g. junction and demand nodes). For the storage nodes (set *ST*) we can apply a separate mass balance equation, as follows:

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| --- | --- |
| *T* | 7 |

Where is the storage at time-step *t* for storage node *i*. Upper and lower bound constraints on storage levels can also be set through the following constraints:

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| --- | --- |
| *TT* | 8 |

# DEMO 3 model

Model in DEMO 1 and DEMO 2 do not allow for any scarcity, i.e. if the available level of supply is lower than the required level of demand, both models become infeasible and no solution can be reached. We therefore modify the previous model formulation to allow for water shortage so that infeasibilities are not generated. The available water, even when lower than the target demand, will be distributed to demand nodes with higher priority (cost) first and then to those with lower priority.

To do so, a new decision variable *αit* is introduced and defined for each demand node (set *DEM*) that is equal to one if the demand is fully met and lower than one (with a minimum value of zero), if only a percentage of the target demand is met.

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|  | 10 |

The new objective function now becomes:

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|  | 11 |

The objective function maximises the level of demand satisfaction for each time-step *t* for all demand nodes based on the priorities *pit*. Since we are now dealing with a maximisation problem, the higher *pit* the higher is the priority for each demand node to receive water.

The objective function equation 11 is subject to the constraint equation 10 and to the mass balance equations 8 for storage nodes and to lower and upper bound constraints (equations 7 and 9) defined previously for model DEMO 2. With regards to the non-storage nodes, the mass balance equation becomes:

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|  | 5 |