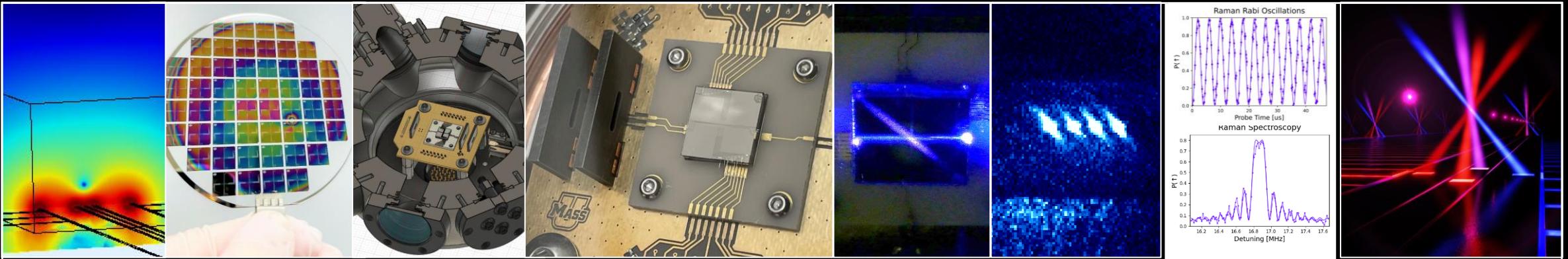

ECE 550/650 – Intro to Quantum Computing

WW06 Lecture 02

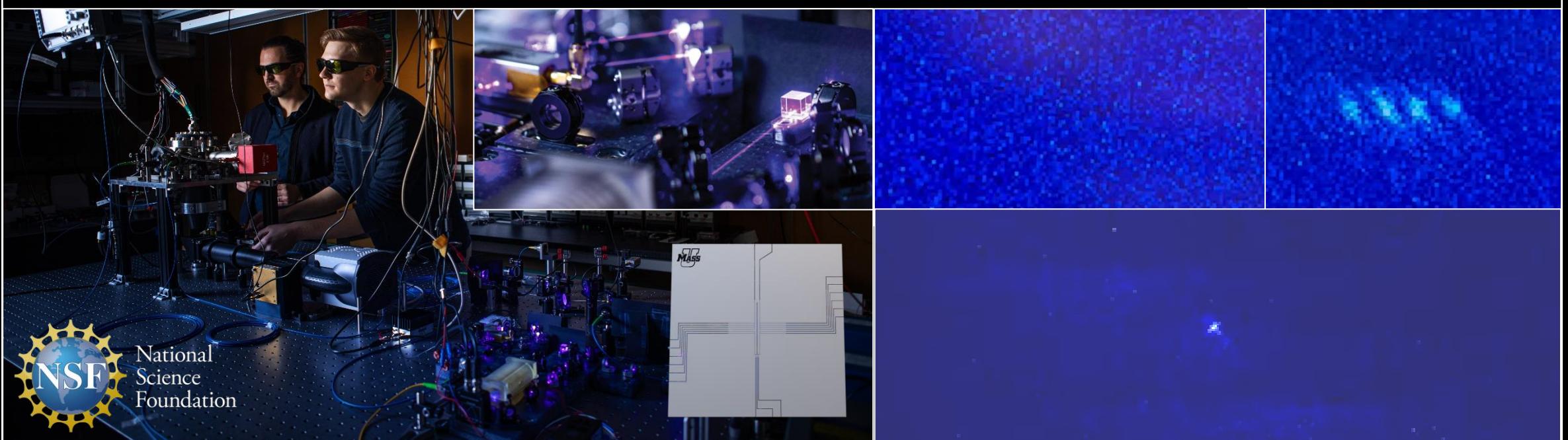
Robert Niffenegger



Niffenegger Lab - Developing Integrated Technologies for Trapped Ion Qubits and Optical Clocks



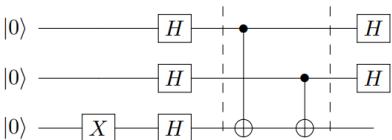
Design → Layout → Fabrication → Packaging → Ultra High Vacuum → Optics → Lasers → Trapped Ions → Zeeman Qubits → Integrated Photonics



National
Science
Foundation

Outline of the course

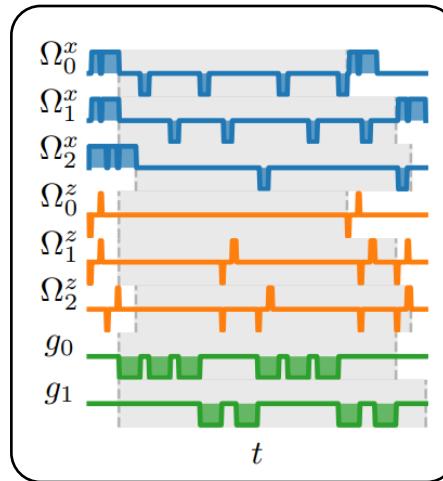
- Quantum Optics
 - What is interference (classical vs. single particle)
 - Superposition of states
 - Measurement and measurement basis
- Atomic physics
 - Spin states in magnetic fields and spin transitions
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 - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
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 - XX gates, Controlled Phase gates, Swap



```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
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qc.add_gate("SNOT", targets=2)

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qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```

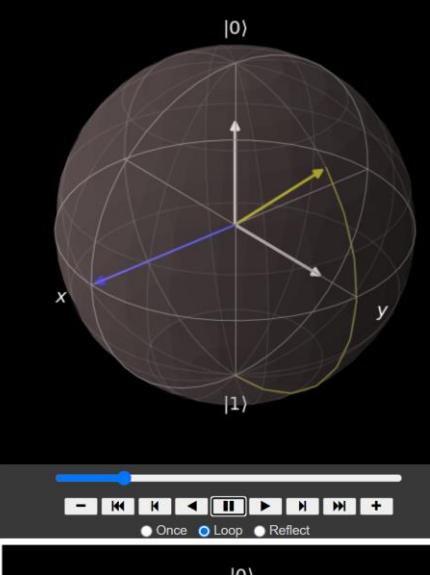


- Quantum Hardware
 - Photonics – nonlinear phase shifts
 - Transmons – charge noise, SWAP gate
- Quantum Circuits
 - Single and two qubit gates
 - Hadamard gate , CNOT gate
- Quantum Algorithms
 - Amplitude amplification
 - Grover's Search
 - Oracle - Deutsch Jozsa
 - Bernstein Vazirani
 - Quantum Fourier Transform and period finding
 - Shor's algorithm
- If time permits
 - Error Correction
 - Repetition codes
 - Color Codes
 - Surface code

Labs – Colab notebooks QuTiP then QISKit

Table of contents

- ECE 397 QC
- How to use this Notebook
- QuTiP
- Qiskit
- Initialize
- REVIEW FROM LECTURE:
- Lab 2
- Time Dependent Schrödinger equation
 - Exercise 1 :
 - Unitary
 - Rotation operators about the Bloch basis
 - Plotting rotation about the Bloch Sphere
 - Rotation about Z:
 - Rotation about X:
 - Numerical integration of the Schrödinger Equation
- Animate the rotation on the Bloch sphere
 - σ_z is the 'state' basis
 - Exercise 2:
 - Plotting all projections at the same time
 - Halfway point of Lab 2
 - State Measurement
 - Projection = measurement
 - Many Trials:
 - Measurement Uncertainty
 - What does 'real' quantum data look like?
 - We're only getting up or down for each trial of each time step.
- Exercises
 - Exercise 1 :



Lab 10 - Shor

File Edit View Insert Runtime Tools Help Last edited on Apr 29, 2022

Table of contents

- ECE 397 QC
- How to use this Notebook
- Qiskit
- Initialize
- Part II
- Shor's Algorithm
 - Repeated Squaring
 - POW()
 - Classical FFT
 - Eigenstate of U
 - However, there are other eigenvectors of U.
 - The Magic
 - Back to the circuit
 - U function
 - Inverse QFT
 - Control Swap
 - Phase estimation of a Control Swap
 - Ripple Swap
 - U and U repeated squares
 - Estimate r from phase
 - Shor from scratch
 - More Shor (bits)
 - But how does the phase give the factors of N?
 - Conclusion
 - Exercises
 - Section

+ Code + Text

```
n_phe_qubits = 1
n_U_qubits = 3
n_qubits = n_phe_qubits+n_U_qubits
qc = QuantumCircuit(n_qubits,n_phe_qubits)

for i in range(n_phe_qubits):
    qc.h(i)

qc.x(n_qubits-1)

qc.barrier()

qc.toffoli([0],[n_phe_qubits+1],[n_phe_qubits])
qc.toffoli([0],[n_phe_qubits+1],[n_phe_qubits+1])
qc.toffoli([0],[n_phe_qubits+2],[n_phe_qubits+1])

qc.toffoli([0],[n_phe_qubits+2],[n_phe_qubits+2])
qc.toffoli([0],[n_phe_qubits+1],[n_phe_qubits+2])
qc.toffoli([0],[n_phe_qubits+2],[n_phe_qubits+1])

qc.barrier()

qc.append(qft_dagger(n_phe_qubits), range(n_phe_qubits))

qc.barrier()

qc.measure( range(n_phe_qubits), range(n_phe_qubits))

qc.save_statevector()

qc.draw('mpl')
```

<https://github.com/UMassIonTrappers/quantum-computing-labs>

Quick intro

Brief overview of quantum computing



Why compute with qubits?

qubit
'Each photon then interferes only with itself'

- Dirac

A qubit can experience multiple 'paths' simultaneously (superposition, double slit)

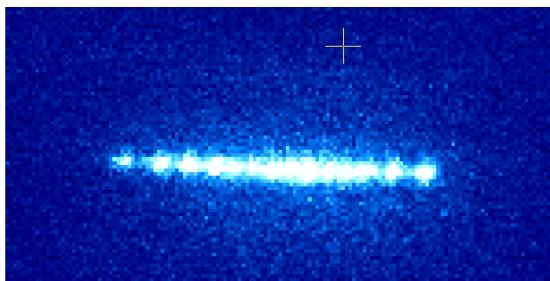
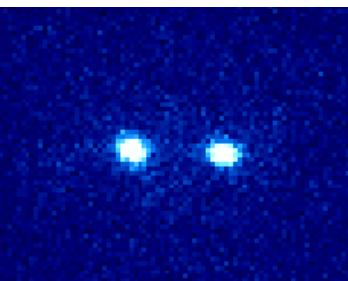
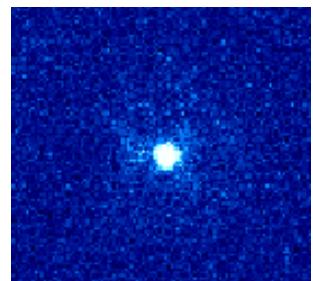
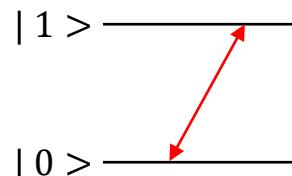
Until you entangle them

Each additional entangled two-level qubit adds those two states to the permutation

→ Qubits explore twice as many paths for each additional qubit

2^n possible states (for n qubits with two levels)

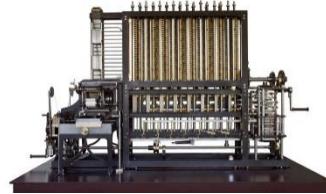
→ Higher dimensional interference is the 'power' of quantum computation



Trapped ion

Classical

32 bits in SRAM store just one number at a time



0000000 00000001 = 1
0000000 00000010 = 2
0000000 00000011 = 3
0000000 00000100 = 4

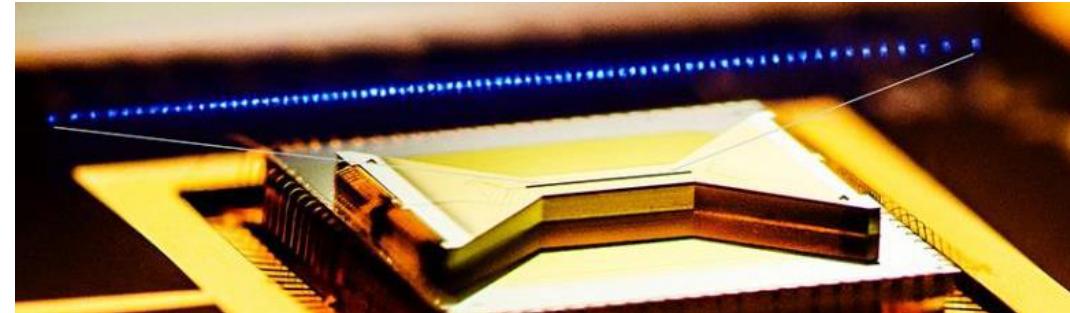
'01' or '10'

Quantum

32 qubits can represent all permutations

$2^{32} > 4$ Billion (~Quantum volume)

'01' and '10'

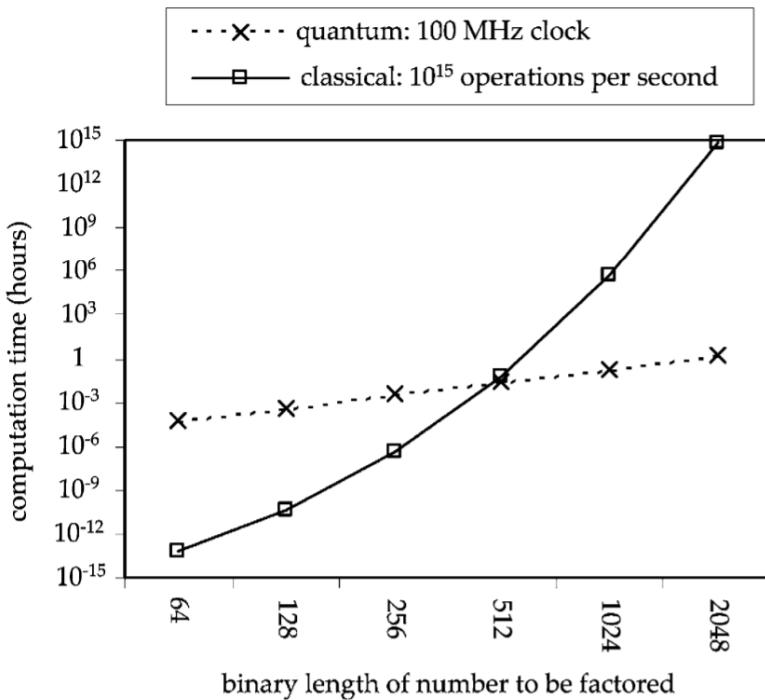


Quantum speedups?

Shor's Algorithm
(Quantum Fourier Transform → period finding → factoring)

Quantum Computing With Superconductors

KARL K. BERGGREN, MEMBER, IEEE



Andrew Childs
University of Maryland

Overview

0. Introduction

1. Quantum query complexity

2. Algebraic problems

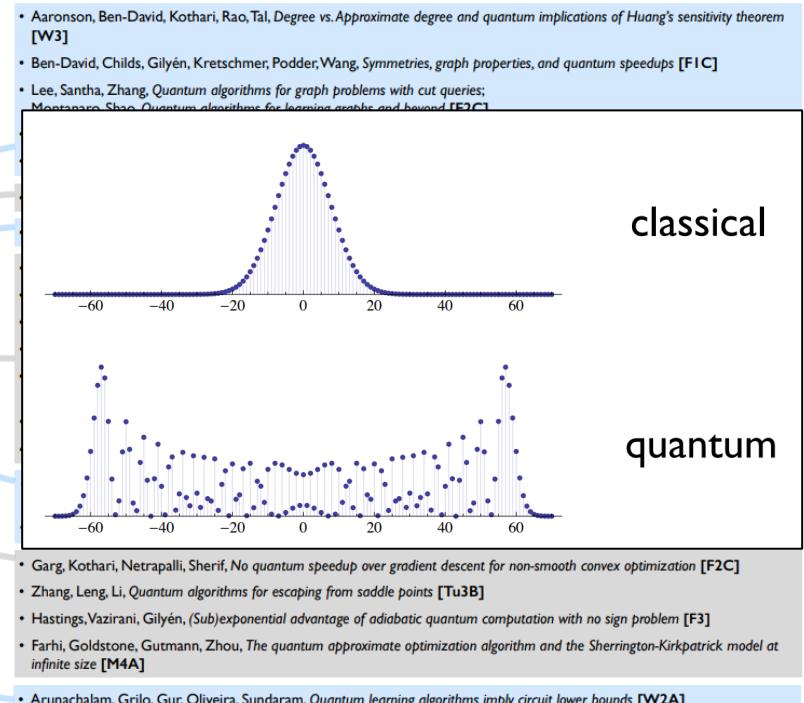
3. Quantum walk

4. Hamiltonian simulation

5. Quantum linear algebra

6. Optimization

7. Machine learning



How do you build a quantum computer?

The Physical Implementation of Quantum Computation

David P. DiVincenzo

IBM T.J. Watson Research Center, Yorktown Heights, NY 10598 USA
(February 1, 2008)

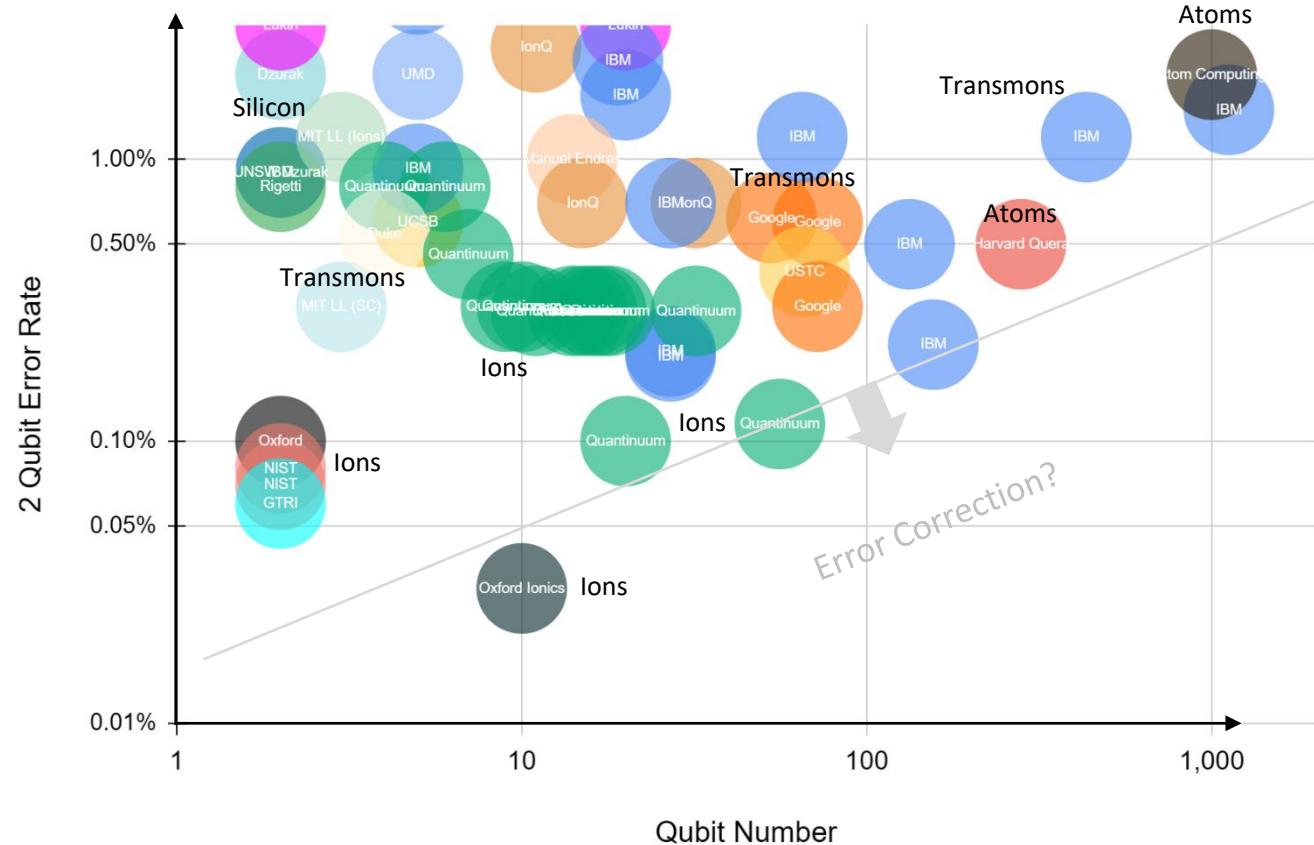
1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits
3. Long relevant coherence times (vs. gate time)
4. A “universal” set of quantum gates
5. A qubit-specific measurement capability

Many viable qubits but all have challenges scaling
(Increasing qubit number without loss in performance)

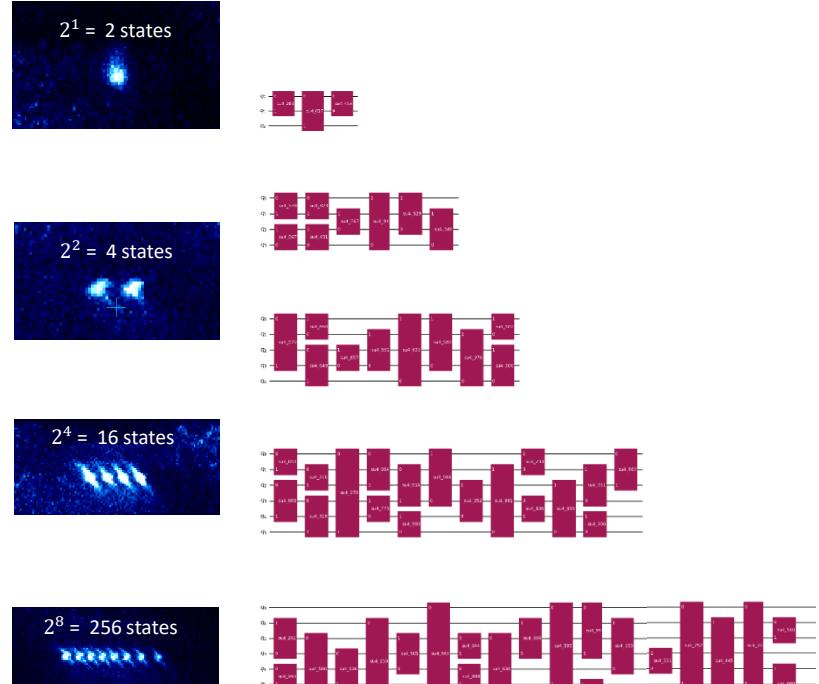
		‘New qubits’	
		A	B
Superconducting loops	Trapped ions	Silicon quantum dots	Neutral atoms
A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into superposition states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These “artificial atoms” are made by adding an electron to a small piece of pure silicon. Microwaves control the electron’s quantum state.	Like ions but atoms are trapped with lasers and interact via Rydberg (nearly ionized) excitations
Longevity (seconds) 0.00005	>1000	0.03	1
Logic success rate 99.4%	99.9%	~99%	97% 97%* (heralded)
Number entangled 27	24	3	20 12
Company support Google, IBM, Rigetti ...	IonQ, Honeywell, AlpineQ, UniversalIQ	Intel, HRL/Boeing, SiliconQuantum	Quera , Atom Computing Cold Quanta Xanadu, PsiQuantum
Pros Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	2D arrays becoming possible, faster gates CMOS compatible photonics waveguide technology
Cons Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Gate fidelity Cryogenic single photon sources and detectors
Science, Dec 2016 (modified)			

Comparing Quantum Computers

Fidelity vs. Number

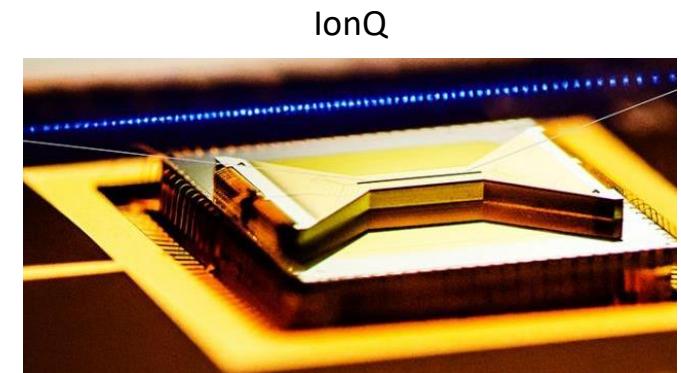
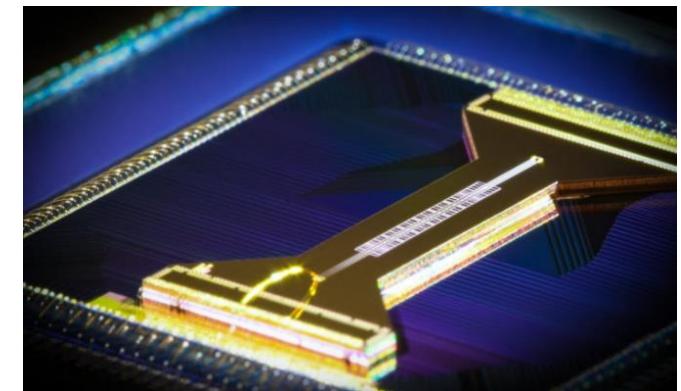
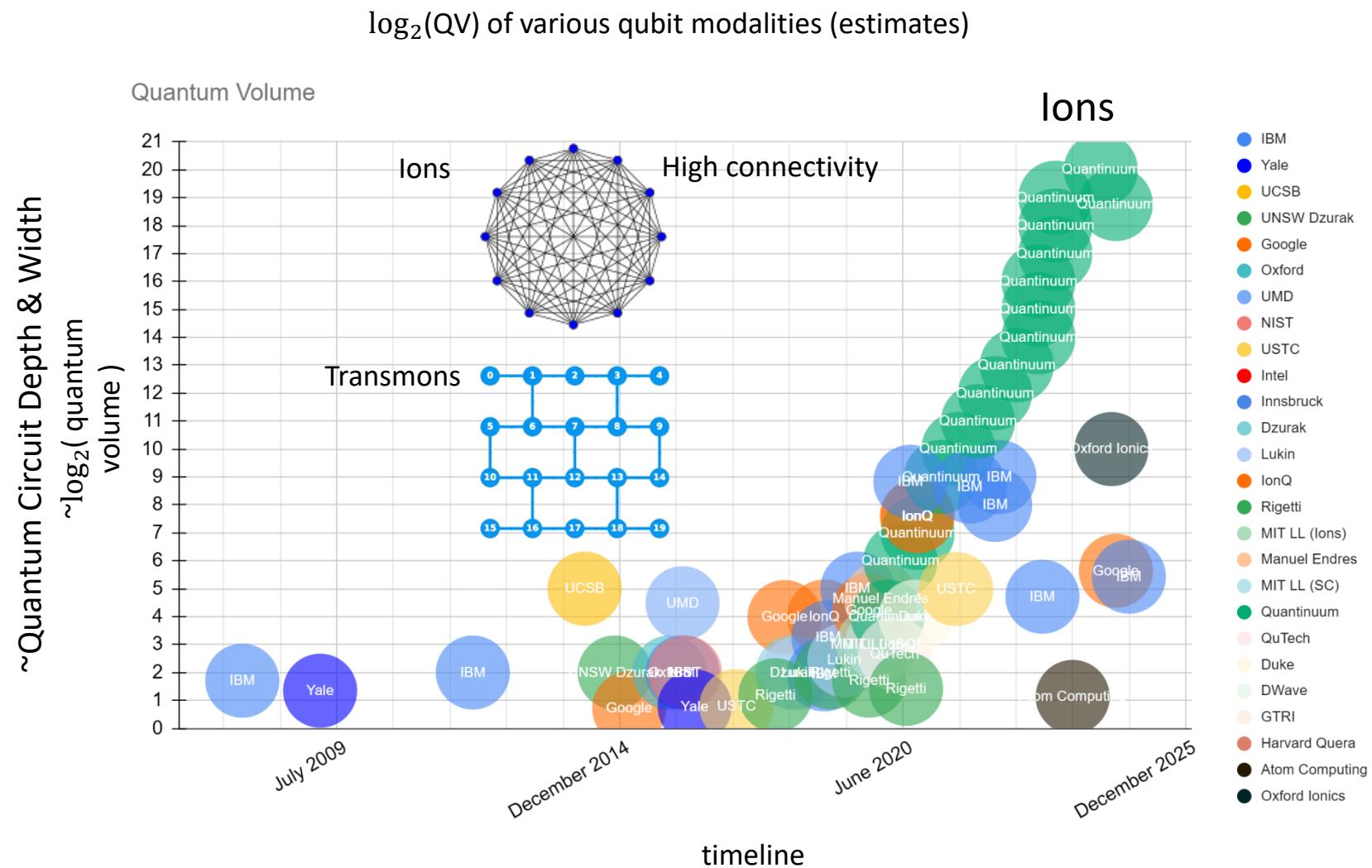


Quantum Volume

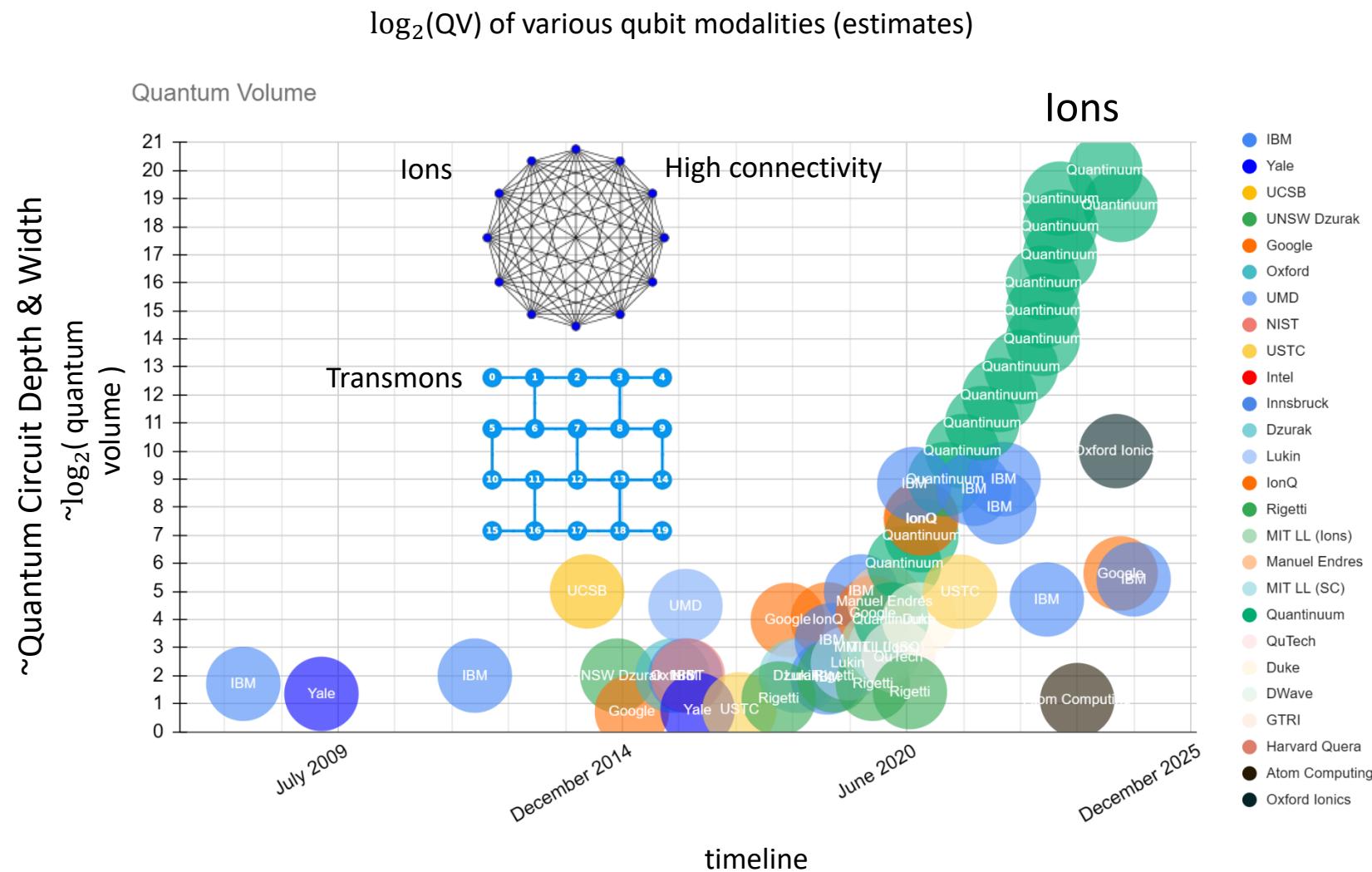


Qubit number &
Circuit depth
Both increasing

Quantum Volume



Quantum Volume – Trapped ions in the lead



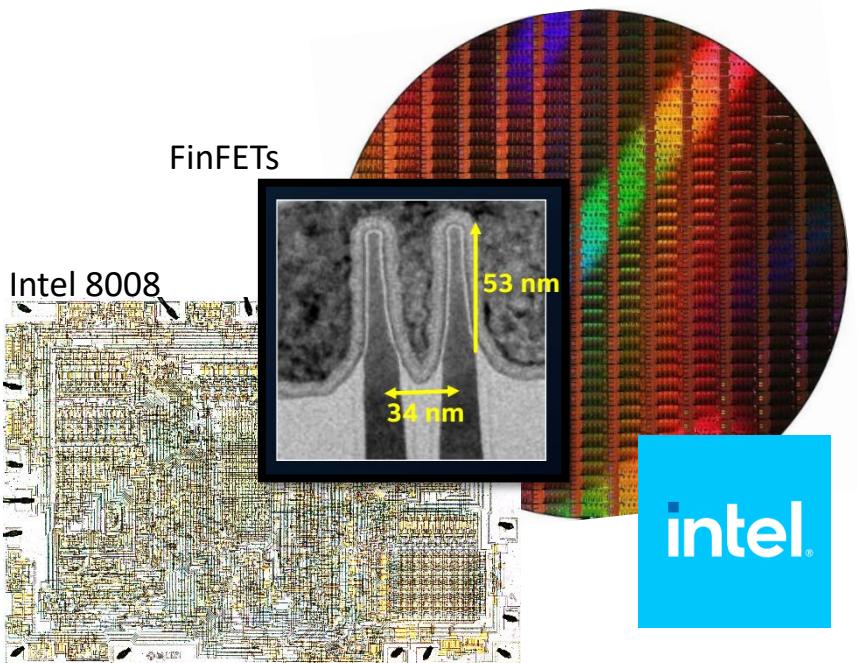
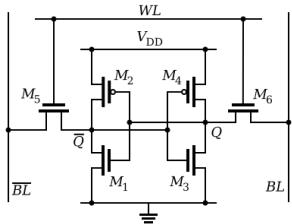
Trapped Ion Qubit Advantages

1. High fidelity operations
 - 2Q gates > 99.97%
 - 1Q gates > 99.9999%
 - Readout > 99.99%
 - State Preparation & Measurement > 99.97%
 2. Long coherence time
 - 200ms → 600 seconds
 - Ratio to gate time > 1,000,000
 3. Atoms are identical (less tuning)
 4. High connectivity
 5. Low crosstalk (w/ focused lasers)

6T SRAM bit flip errors $< 10^{-18}$
(i.e. >99.9999999999999%)

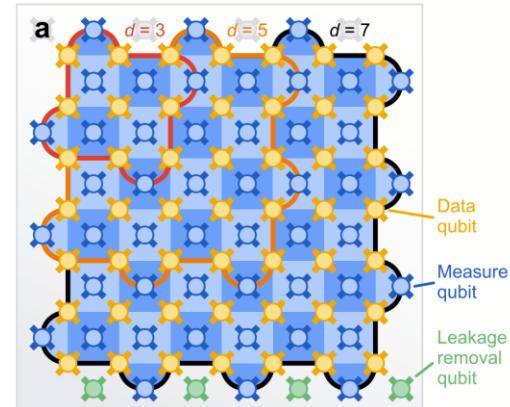
Scaling towards Quantum Error Correction

1 T – Transistor
6 T - SRAM (error correction)
100 T - Logic Chains
3,100 T – Intel 8008 (1971)
285,000 T – Intel 386 (1985)
1,000,000,000 T – Modern CPU
1,000,000,000,000 T - Full wafer

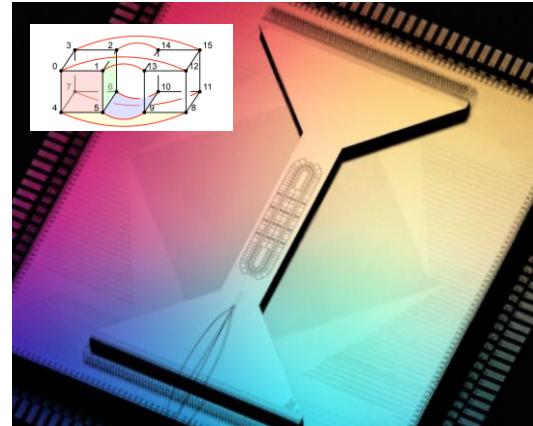


1 Q – Qubit
7-101 Q = 1 Logical Qubit (LQ)
56 Q – 12 Logical Qubits (12 LGHZ)
280 Q - 40 Logical Qubits (4 LGHZ)
1,000 Q – 100 Logic qubits (?)
10,000 Q – 100 Fault tolerant LQ
100,000 Q - 1000 FT LQ
1,000,000 Q – Shor's Algorithm?

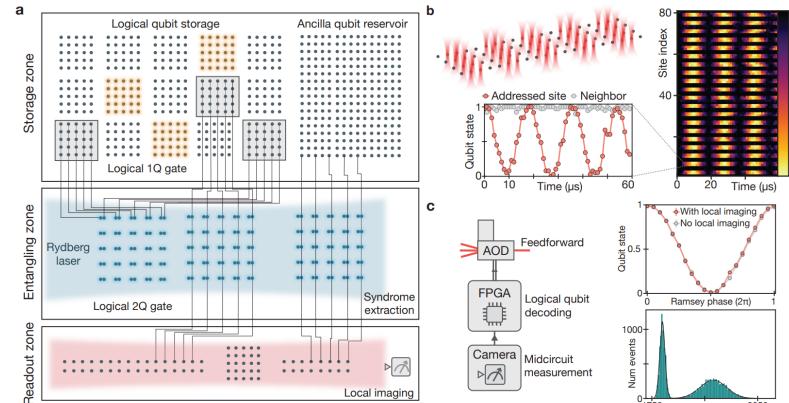
Google - arXiv:2408.13687



Quantinuum - arXiv:2409.04628



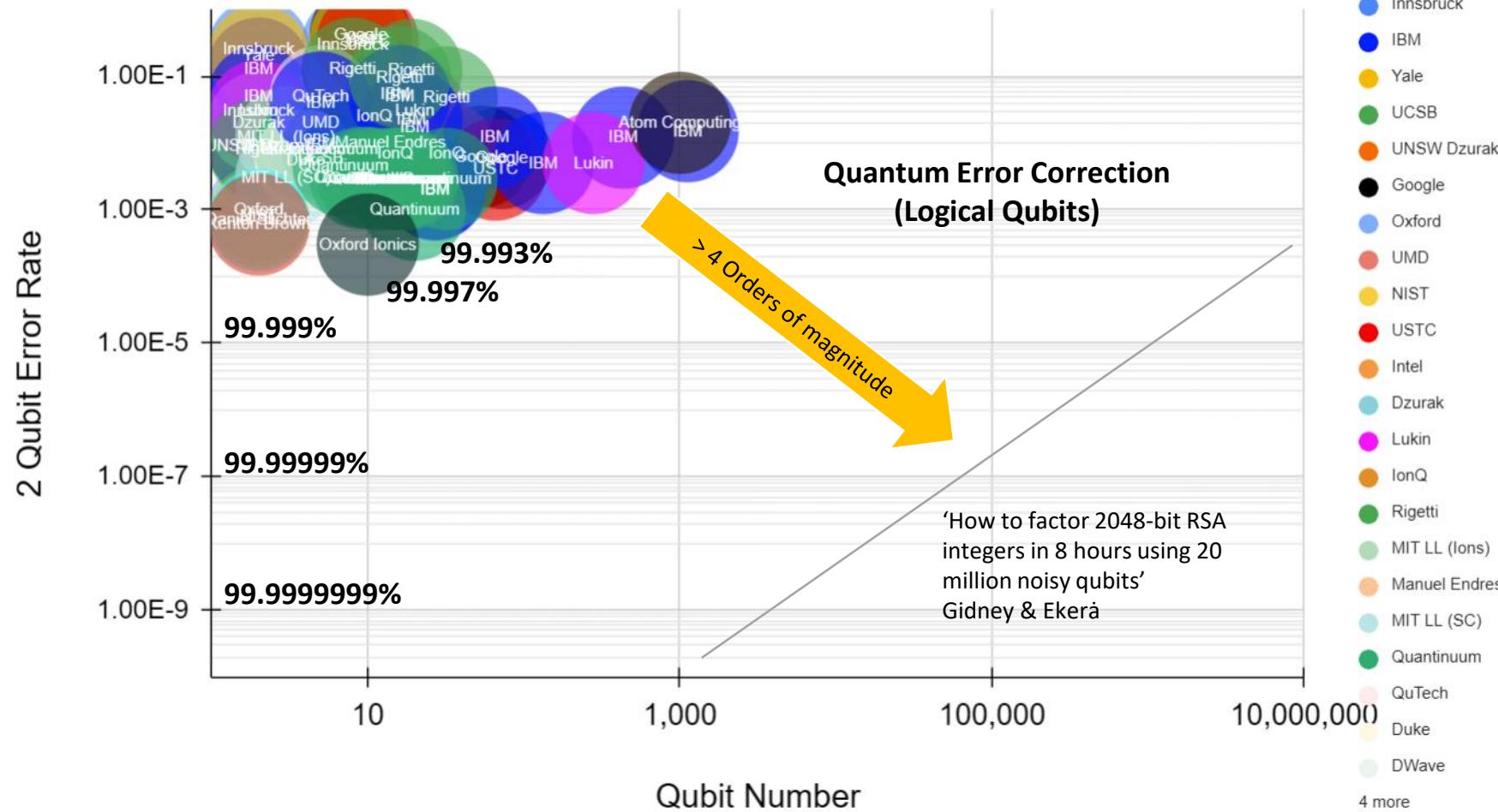
Rydberg atoms - Lukin Harvard 2023



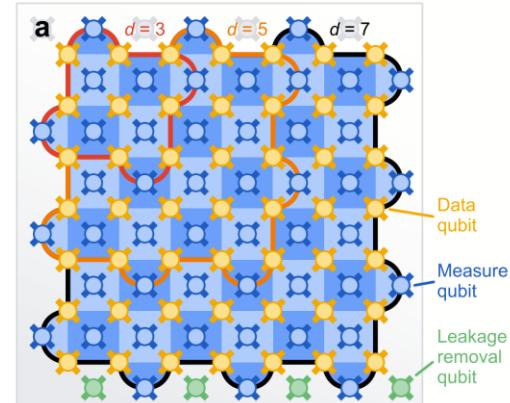
6T SRAM bit flip errors $< 10^{-18}$
(i.e. $>99.9999999999999\%$)

Scaling towards Error Correction

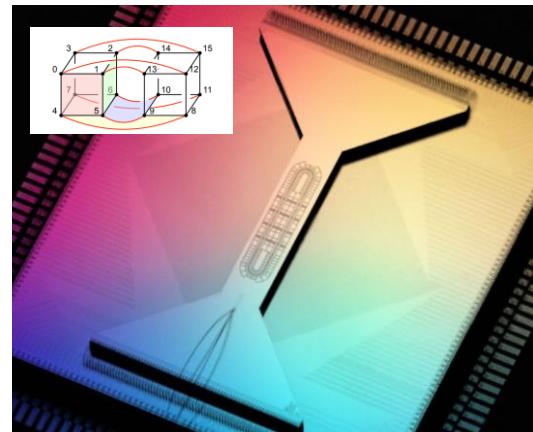
Fidelity vs. Number



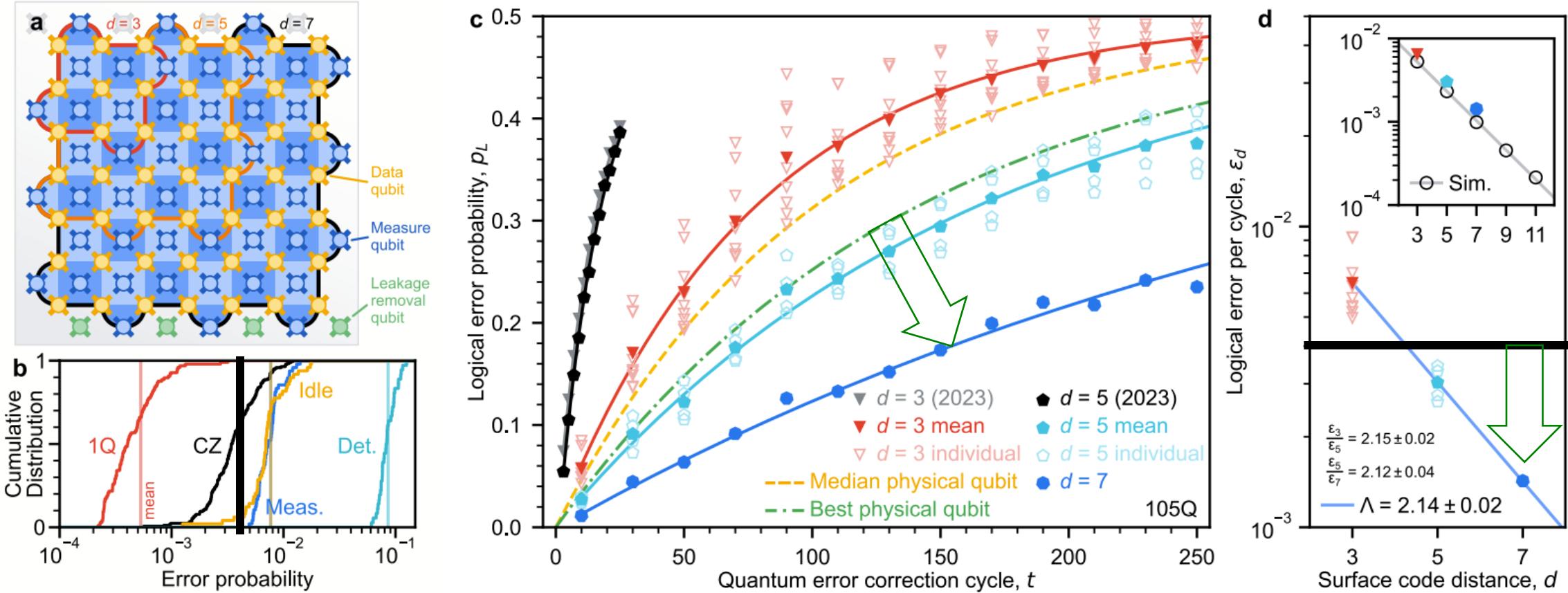
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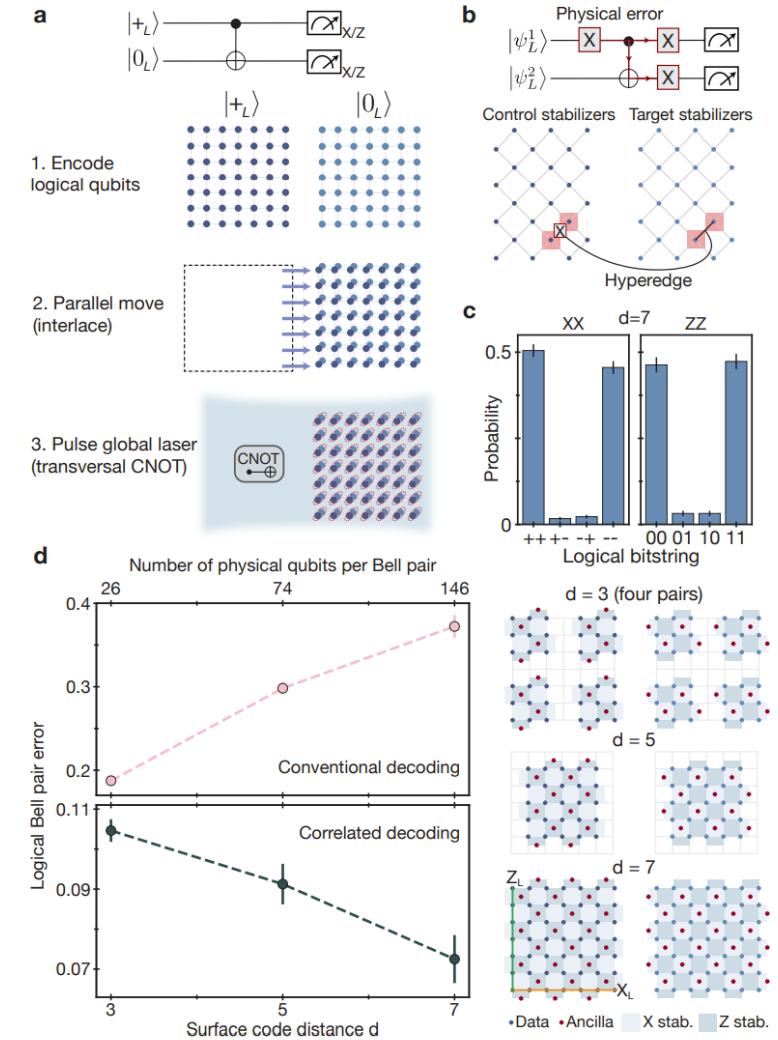
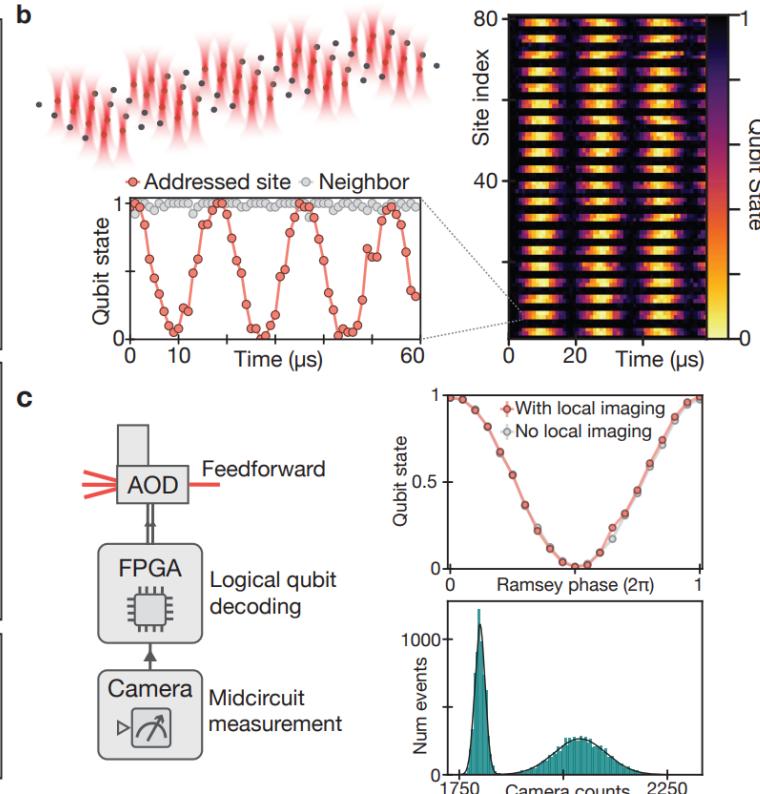
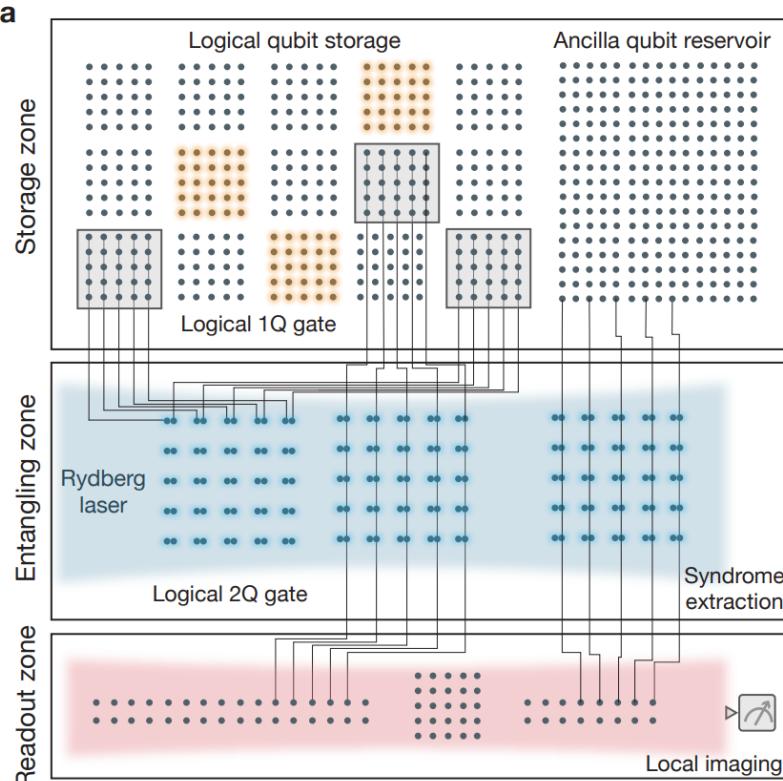


Google – Surface Code Threshold



A single, large, logical qubit (surface code) – with fidelity that scales

Rydberg Atoms – Entangling of multiple Logical Qubits



Trapped Ions – Error correction & entangling of Logical Qubits

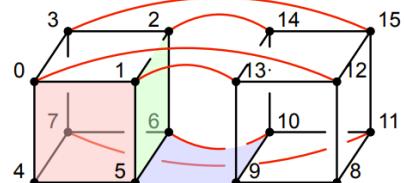
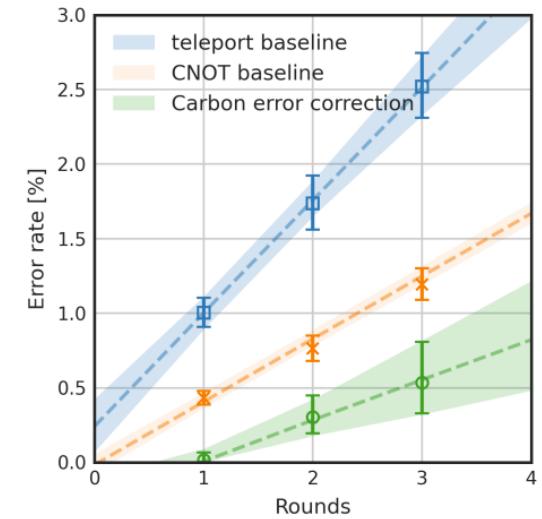
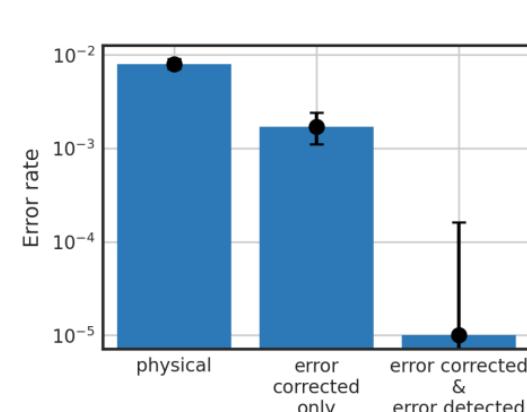


FIG. 1. The $[[16, 6, 4]]$ color code on the 4D hypercube, or tesseract. Each of the 16 vertices is a qubit. Cubes are X and Z stabilizers, and squares are logical operators, e.g., 0145.



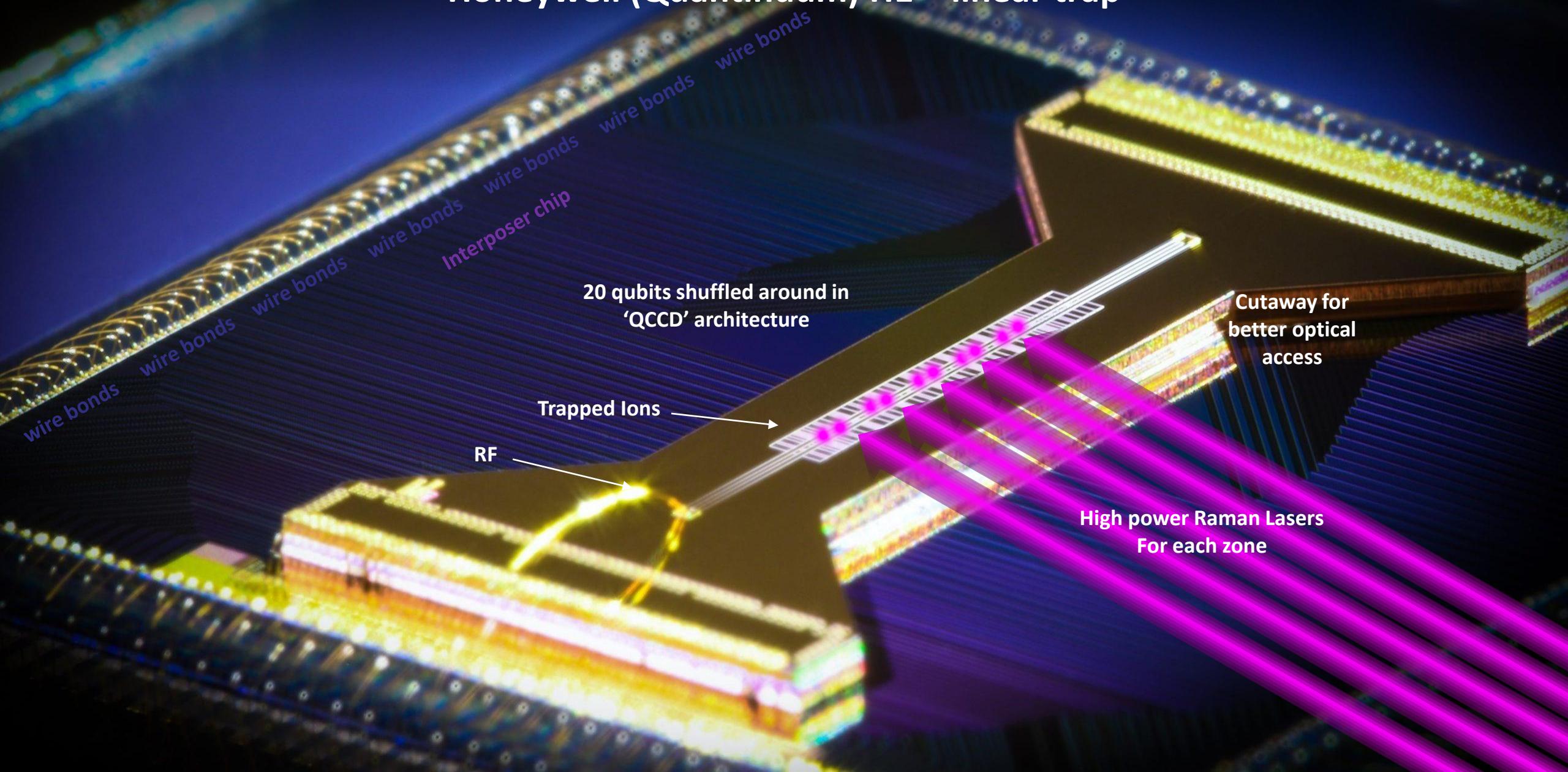
Experiment	Qubits	Baseline error rate	Encoded error rate	Gain
Path-4	4	1.5(2)%	$0.10^{+0.11}_{-0.06}\%$	15×
Cube-8	8	2.3(3)%	$0.2^{+0.2}_{-0.1}\%$	11×
$ 0^{12}\rangle + 1^{12}\rangle$ Cat-12	12	2.4(3)%	$0.11^{+0.16}_{-0.08}\%$	22×
Error correction 5×	4	2.7(4)%	$0.11^{+0.21}_{-0.09}\%$	24×
	8	5.6(6)%	$0.7^{+0.7}_{-0.4}\%$	8×

TABLE II. Experiments preparing encoded cat states.

Reference	Logical qubits	Fidelity
[HDHL24]	4 in $[[25, 4, 3]]$ code	$99.5^{+0.2}_{-0.4}\%$ to $99.7^{+0.2}_{-0.3}\%$
[BEG ⁺ 24]	4 in $[[7, 1, 3]]$ code	$\begin{cases} 72(2)\% \text{ error correction} \\ 99.85^{+0.1}_{-1.0}\% \text{ error detection} \end{cases}$
This work	12 in $[[16, 4, 4]]$ code	$99.82^{+0.12}_{-0.4}\%$ to $99.90^{+0.1}_{-0.3}\%$

Surface Electrode Trap

Honeywell (Quantinuum) H1 – linear trap



Quantinuum H2 – Racetrack Ion Trap

50 qubits shuffled around in
'QCCD' architecture

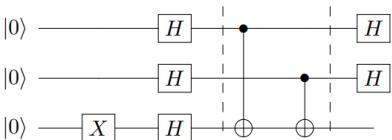
4 zones for 2Q gates

Cutaway for
better optical
access

RF

Outline of the course

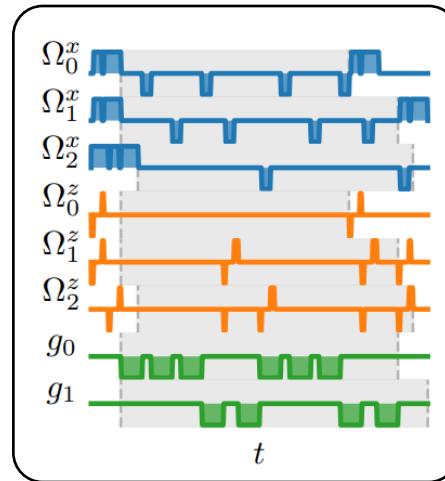
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```



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 - Color Codes
 - Surface code

Benchmarking Quantum Hardware with Algorithms

"Application-Oriented Performance Benchmarks for Quantum Computing."

Lubinski, Thomas, et al. *arXiv preprint arXiv:2110.03137* (2021).

QED-C Technical Advisory Committee on Standards and Performance Benchmarks Chairman

Honeywell

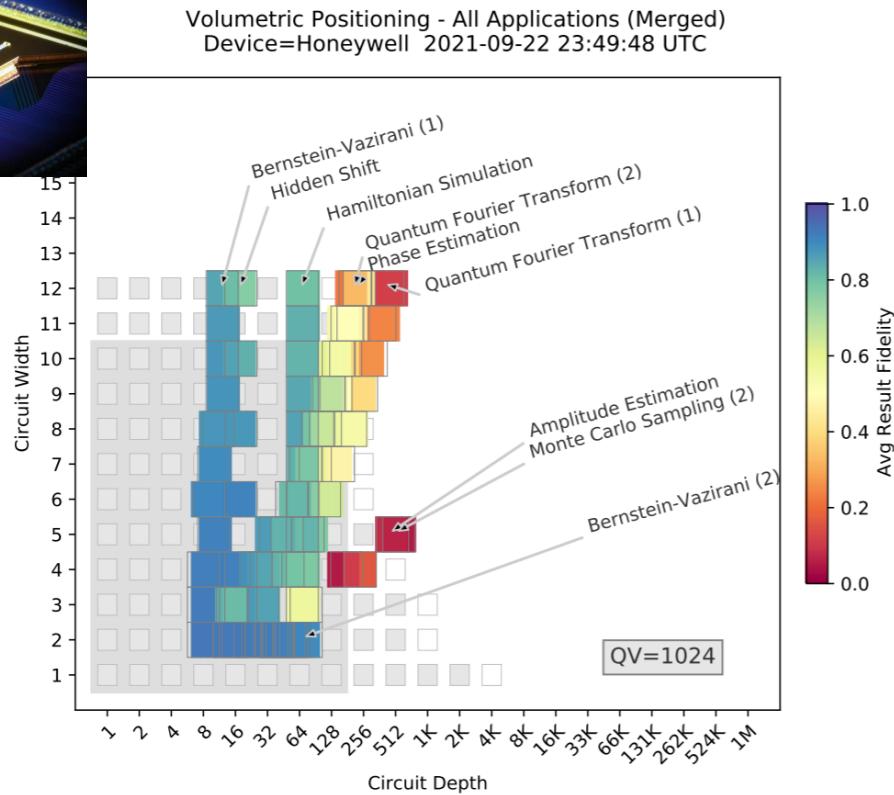
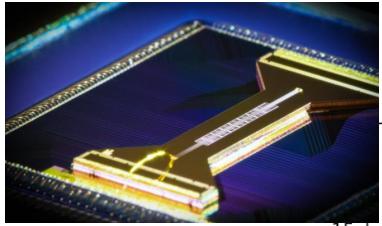


FIG. 12. Benchmark results on Honeywell System Model H1. The

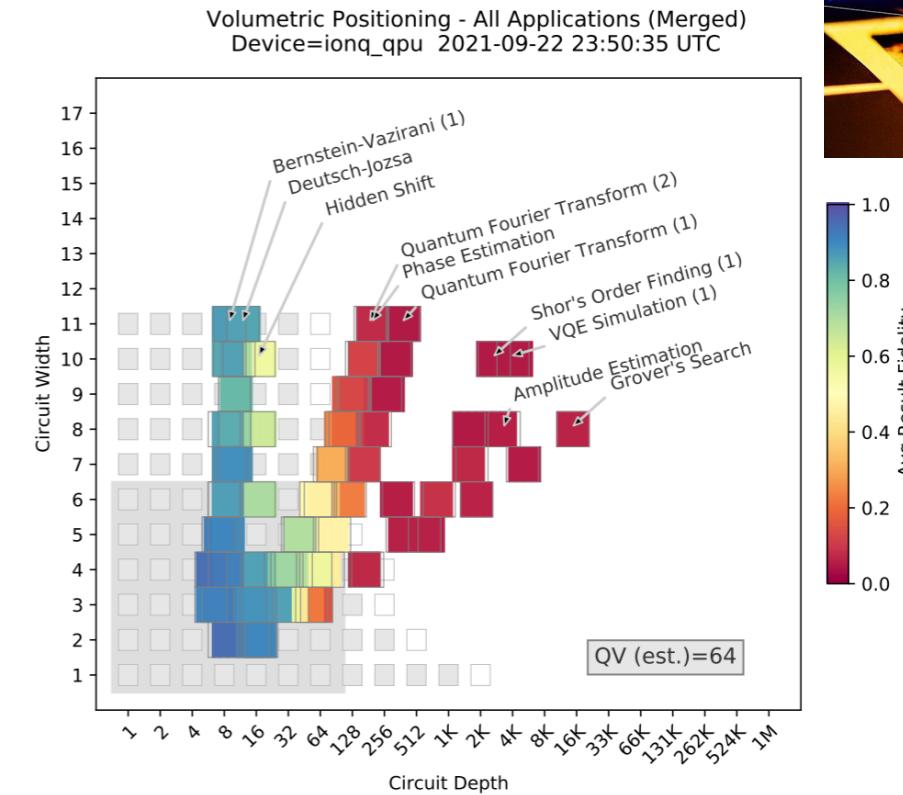
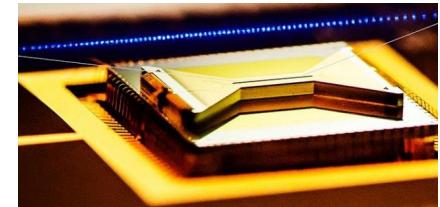


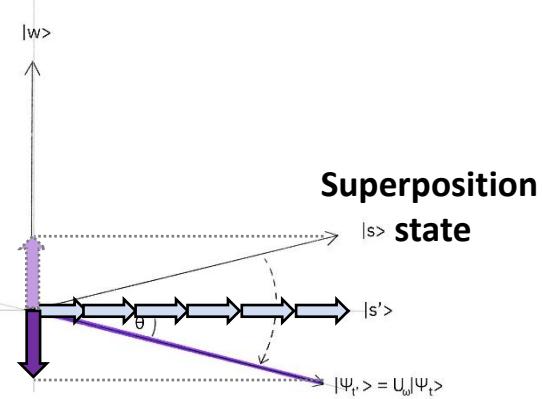
FIG. 13. Benchmark results on IonQ's cloud-accessible system.

IonQ

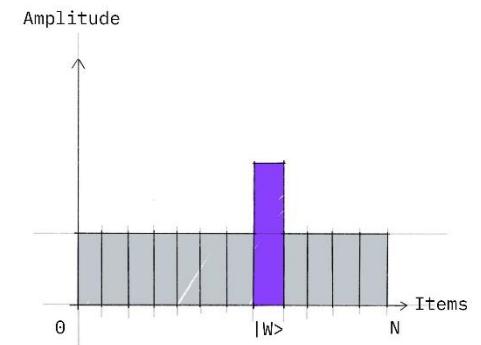
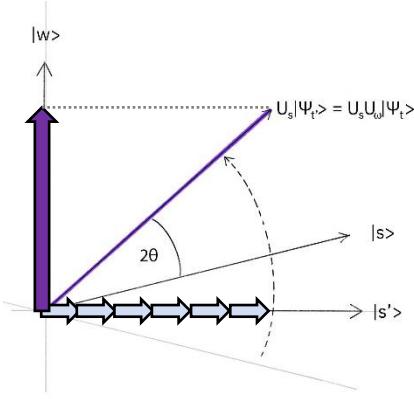
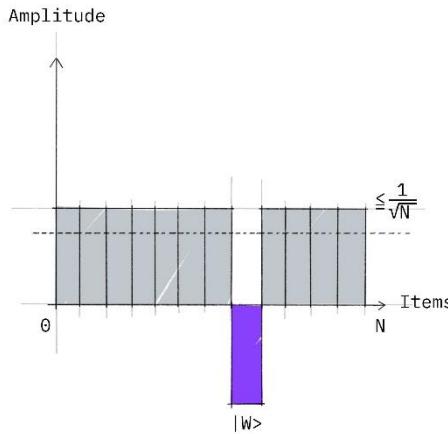


Grover Search (amplitude amplification)

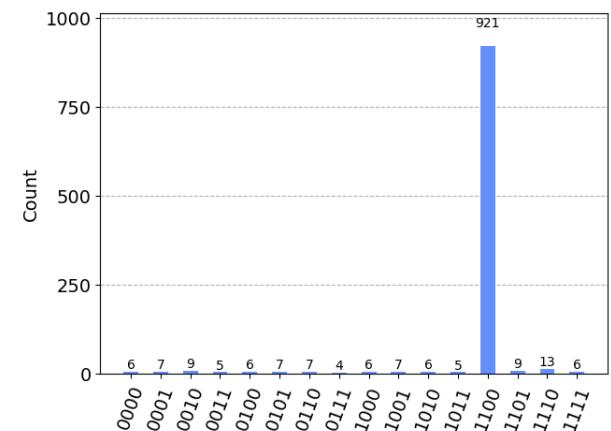
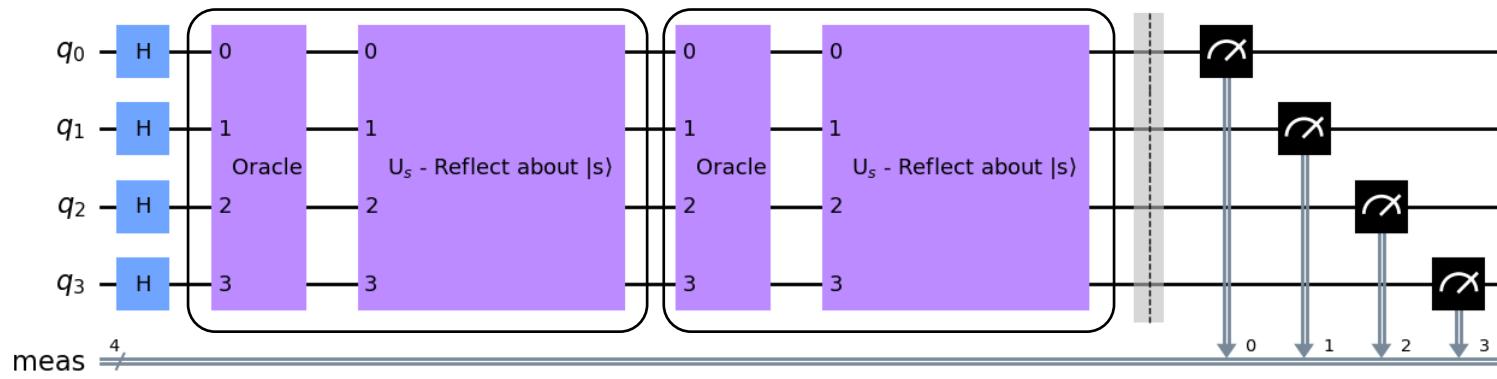
Oracle → Flips sign of ‘answer’



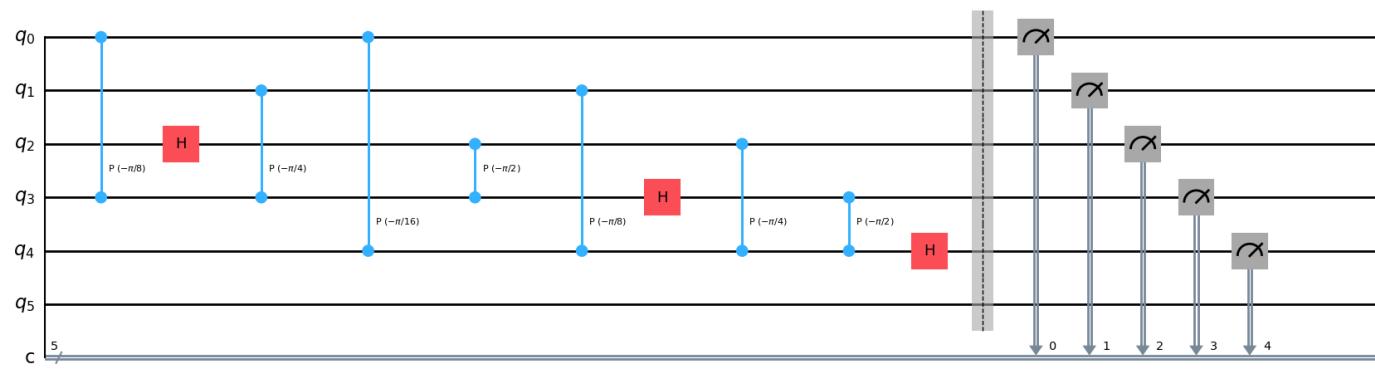
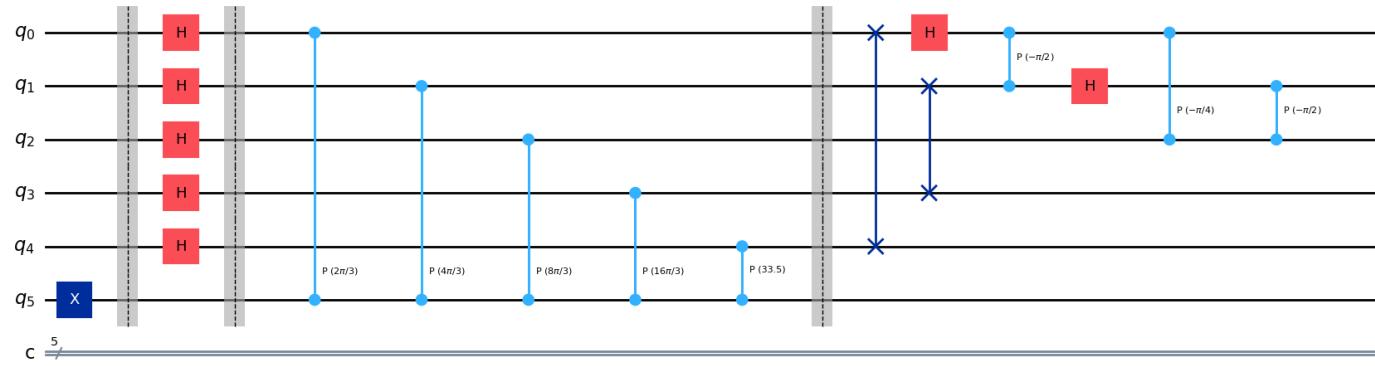
Amplitude Amplification → Answer more probable



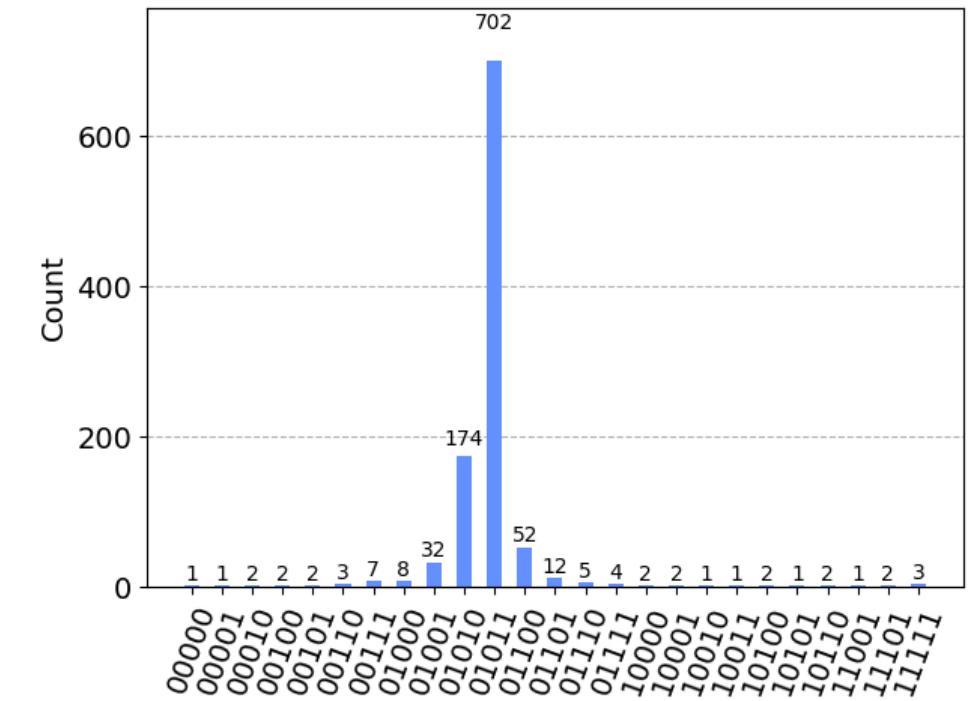
Repeat \sqrt{N} times (twice for 4 bits)



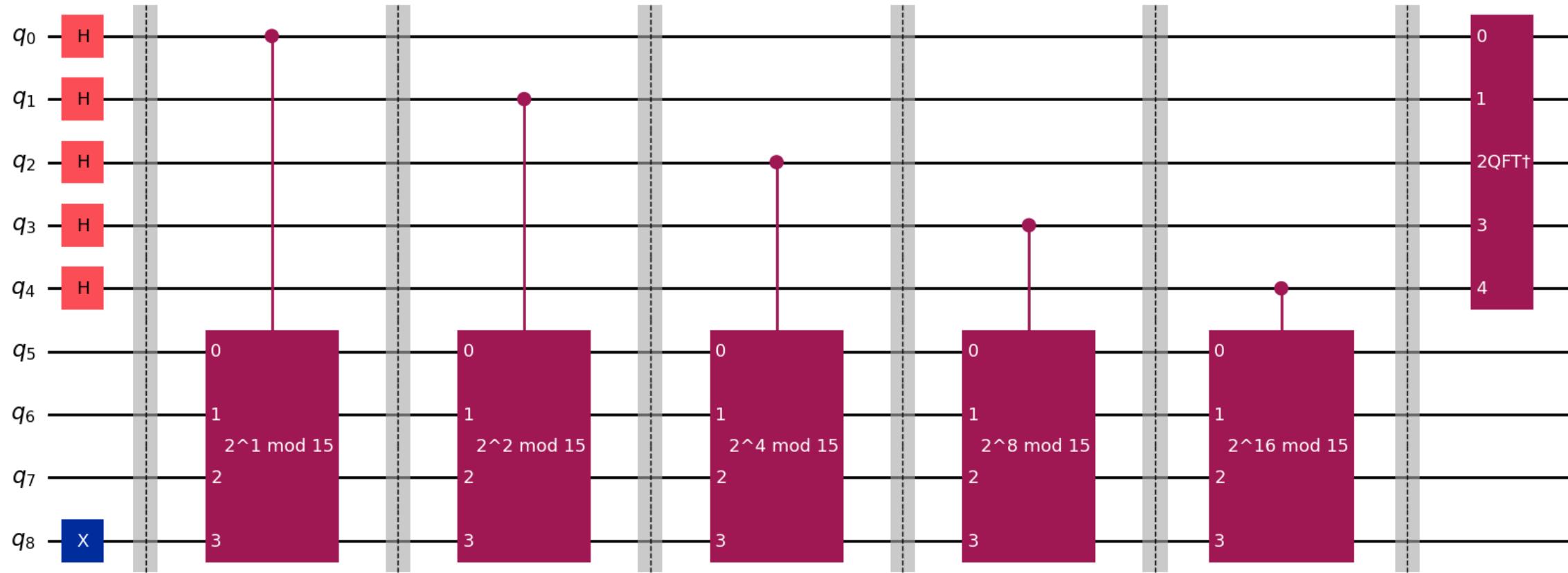
Quantum Phase Estimation



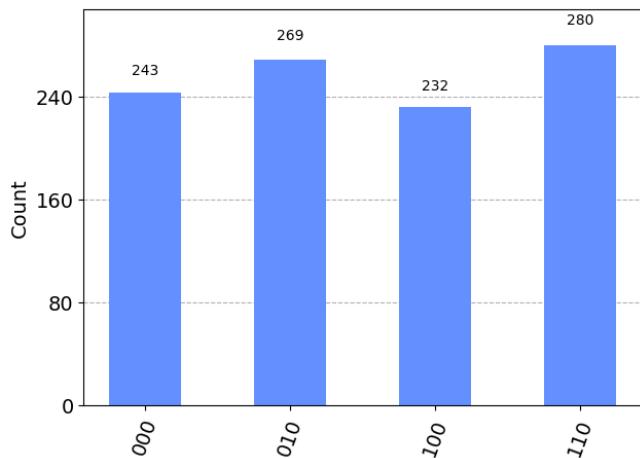
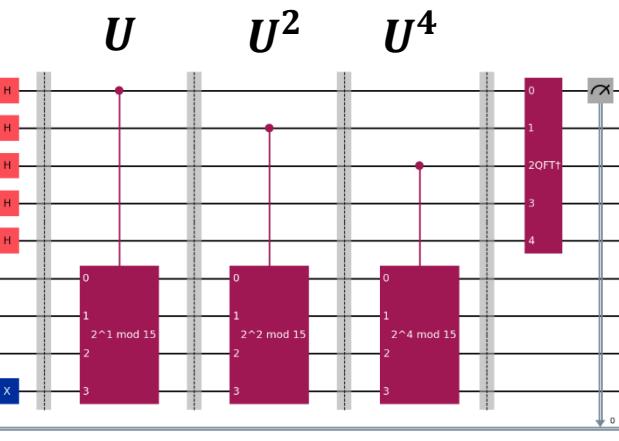
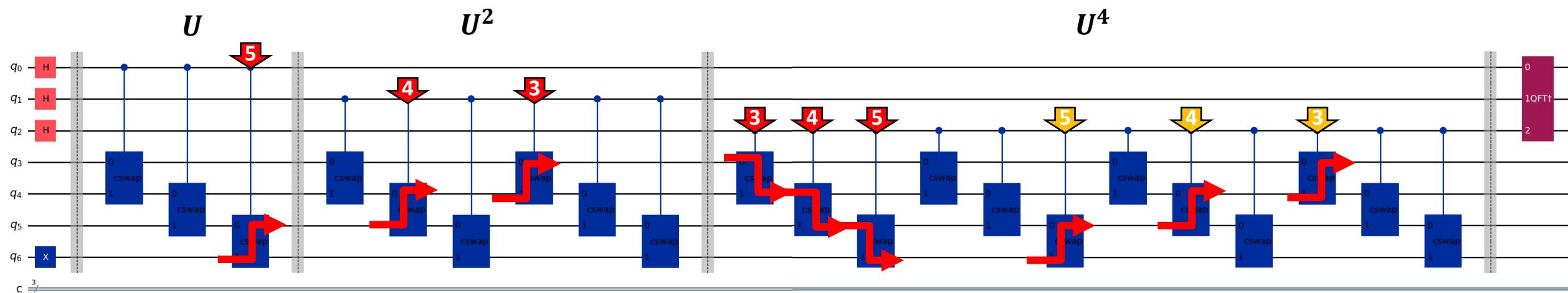
$$01011 = 11, \quad 11 \cdot \frac{1}{2^5} = \frac{11}{32} = 0.34375$$



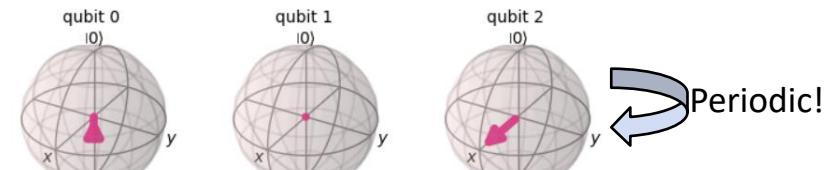
Shor's Algorithm



Shor's Algorithm - Repeated Squares!



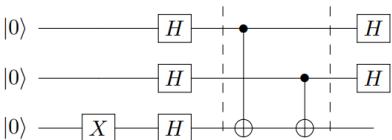
Phase kickbacks before InvQFT



Outputs	Phase	r=
110(bin) = 6(dec)	$6/8 = 3/4$	4
000(bin) = 0(dec)	$0/8 = 0$	0
010(bin) = 2(dec)	$2/8 = 1/4$	4
100(bin) = 4(dec)	$4/8 = 2/4$	2

Outline of the course

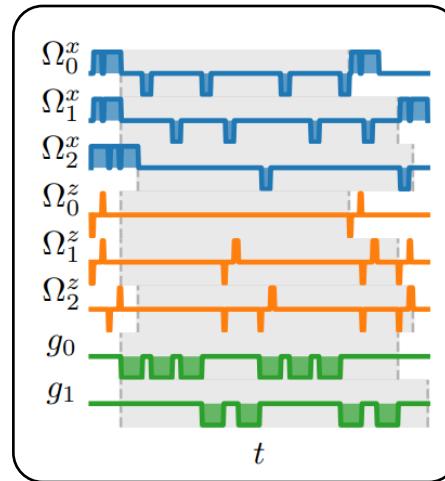
- Quantum Optics
 - What is interference (classical vs. single particle)
 - Superposition of states
 - Measurement and measurement basis
- Atomic physics
 - Spin states in magnetic fields and spin transitions
 - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
 - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
 - Error sources (dephasing, spontaneous decay)
 - Ramsey pulses and Spin echo pulse sequences
 - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
 - Two qubit interactions – gate speed vs. error rates
 - Entanglement – correlation at a distance
 - Bell states and the Bell basis
 - XX gates, Controlled Phase gates, Swap



```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

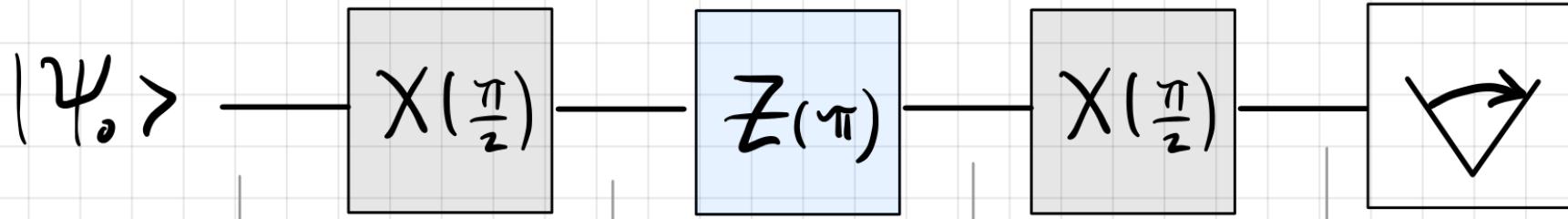
# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```



- Quantum Hardware
 - Photonics – nonlinear phase shifts
 - Transmons – charge noise, SWAP gate
- Quantum Circuits
 - Single and two qubit gates
 - Hadamard gate , CNOT gate
- Quantum Algorithms
 - Amplitude amplification
 - Grover's Search
 - Oracle - Deutsch Jozsa
 - Bernstein Vazirani
 - Quantum Fourier Transform and period finding
 - Shor's algorithm
- If time permits
 - Error Correction
 - Repetition codes
 - Color Codes
 - Surface code

Quantum Computing

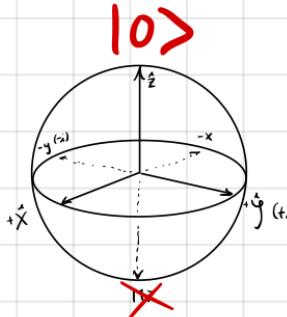
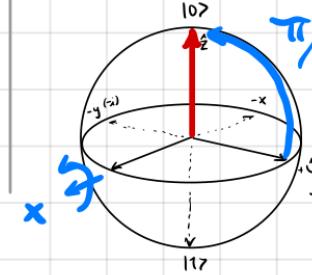
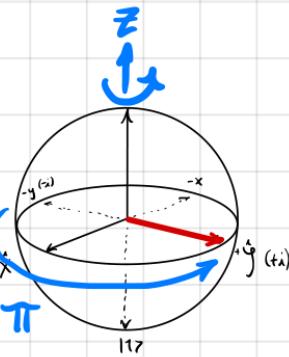
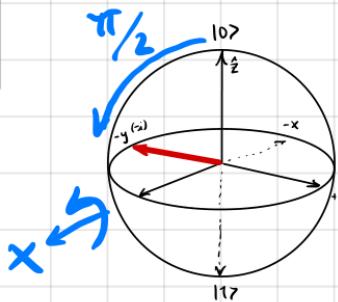
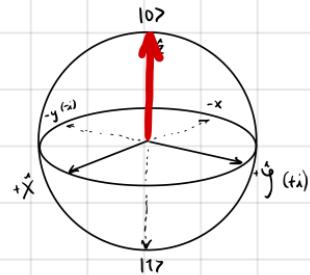


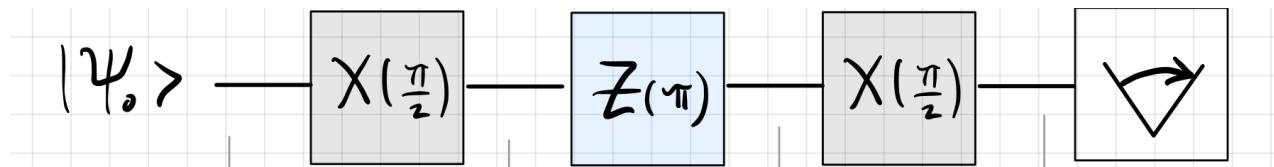
Superposition

Phase gate

Interference
of
superposition

Measurement



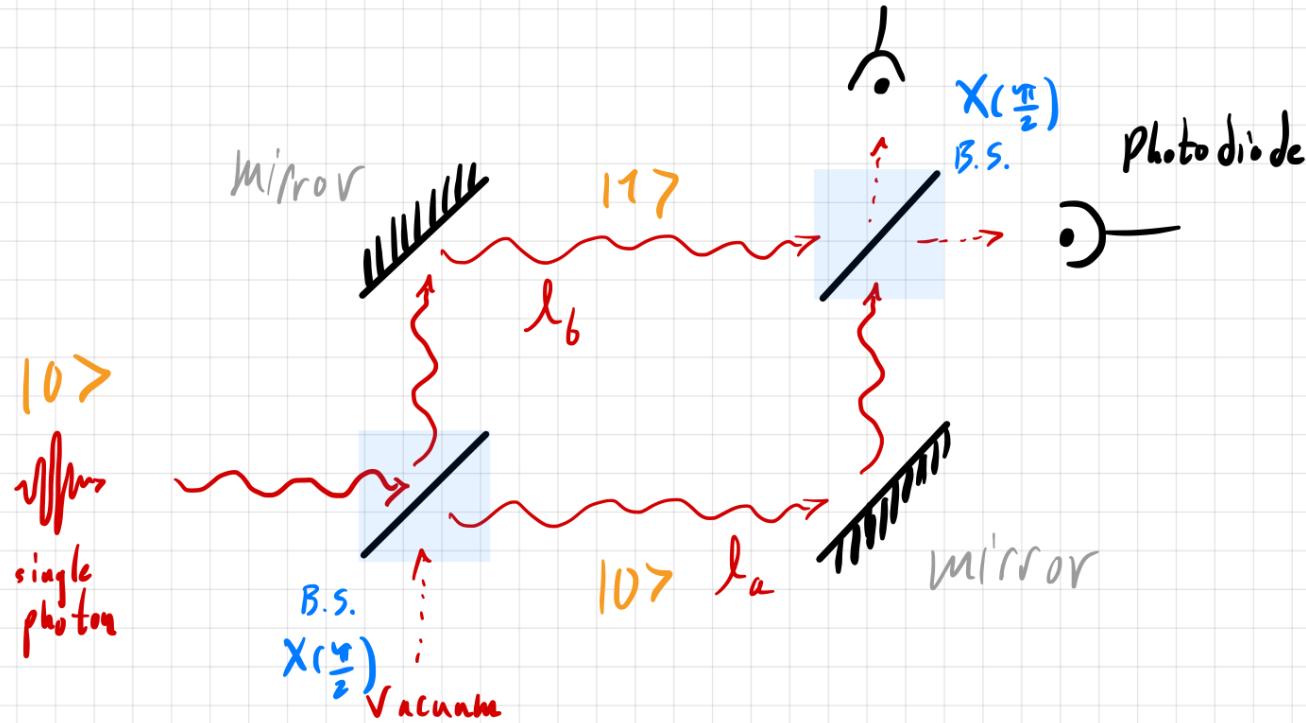
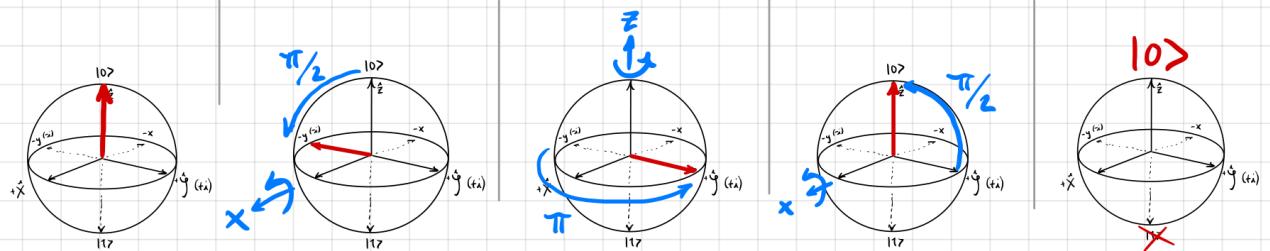


Superposition

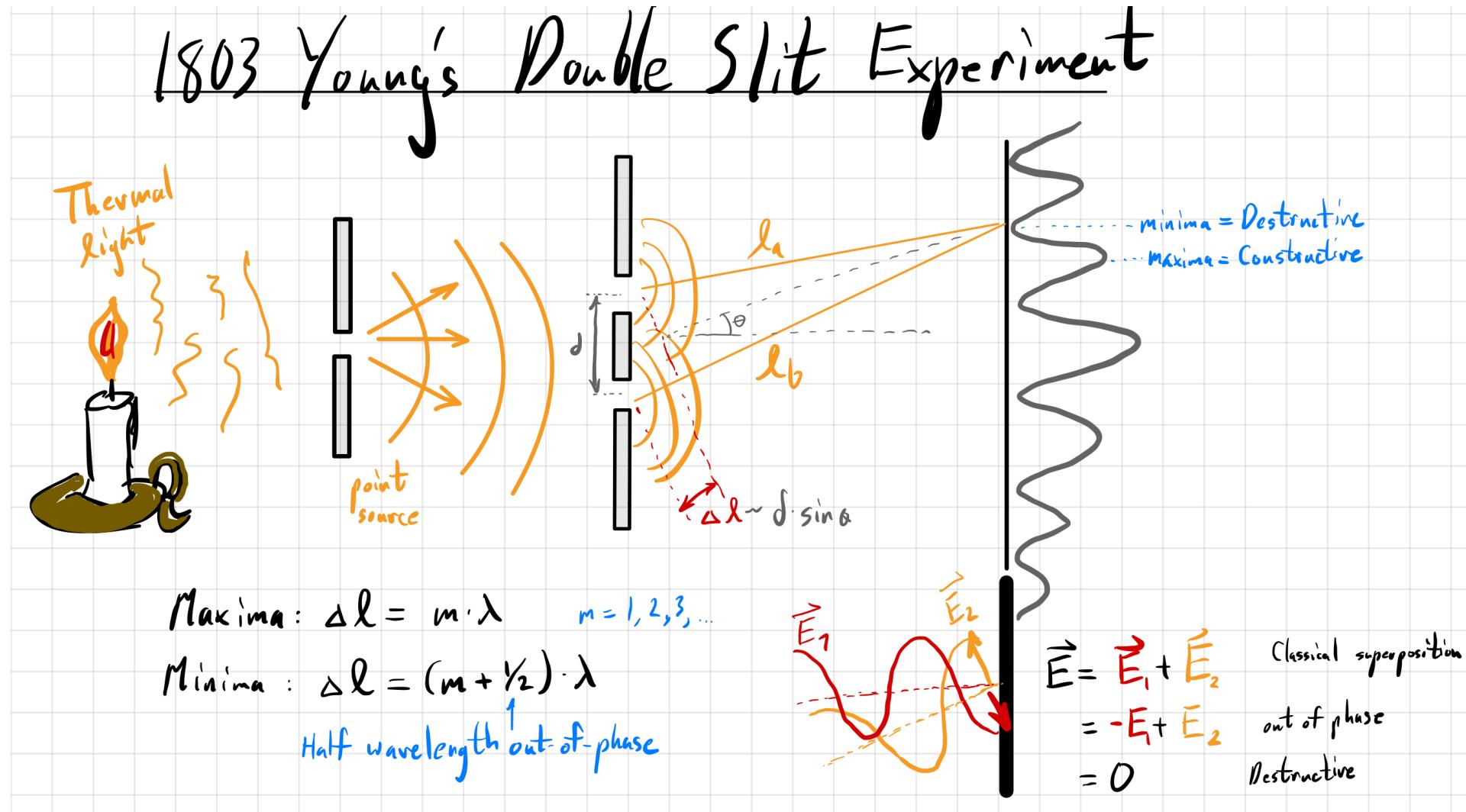
Phase gate

Interference
of
superposition

Measurement



Interference – Young's Double Slit 1803



Interference with feeble light

114 Mr Taylor, Interference fringes with feeble light.

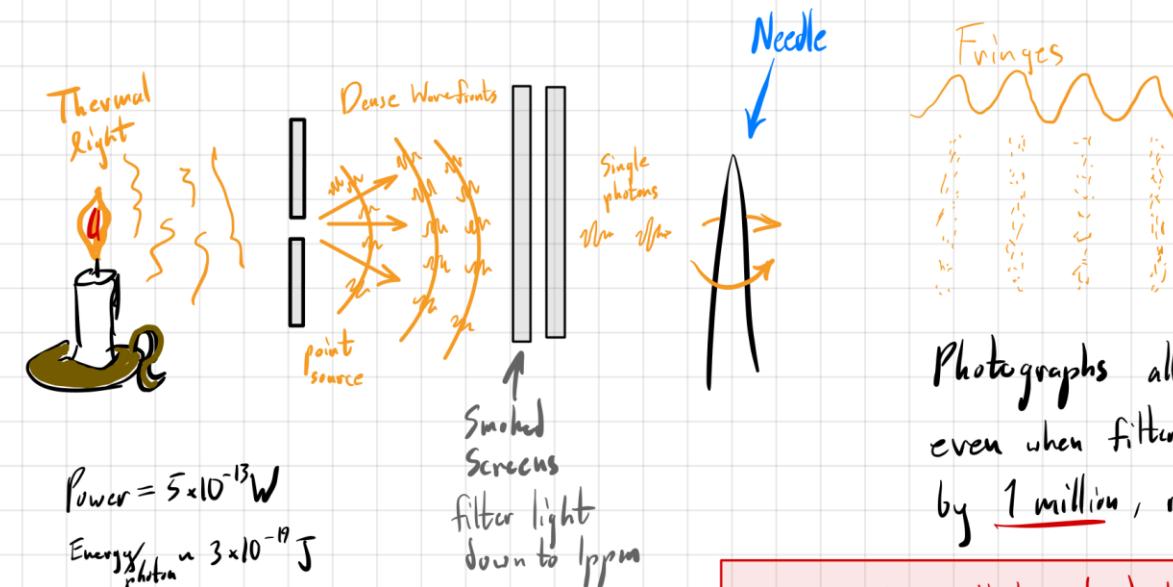
Interference fringes with feeble light. By G. I. TAYLOR, B.A.,
Trinity College. (Communicated by Professor Sir J. J. Thomson,
F.R.S.)

[Read 25 January 1909.]

The phenomena of ionisation by light and by Röntgen rays have led to a theory according to which energy is distributed unevenly over the wave-front (J. J. Thomson, *Froc. Camb. Phil. Soc.* XIV. p. 417, 1907). There are regions of maximum energy widely separated by large undisturbed areas. When the intensity of light is reduced these regions become more widely separated, but the amount of energy in any one of them does not change; that is, they are indivisible units.

So far all the evidence brought forward in support of the theory has been of an indirect nature; for all ordinary optical phenomena are average effects, and are therefore incapable of differentiating between the usual electromagnetic theory and the modification of it that we are considering. Sir J. J. Thomson however suggested that if the intensity of light in a diffraction pattern were so greatly reduced that only a few of these indivisible units of energy should occur on a Huygens zone at once the ordinary phenomena of diffraction would be modified. Photographs were taken of the shadow of a needle, the source of light being a narrow slit placed in front of a gas flame. The intensity of the light was reduced by means of smoked glass screens.

1909 Thompson-Taylor Interference Fringes with Feeble Light



$$\text{Power} = 5 \times 10^{-13} \text{ W}$$

$$\text{Energy}_\text{photon} = 3 \times 10^{-19} \text{ J}$$

$$\approx 10^6 \text{ photons/s/cm}^2$$

Photographs all showed interference even when filters had reduced light by 1 million, requiring 3 months exposure.

1st evidence that individual particles can experience multiple paths simultaneously!

Interference with feeble light

Before making any exposures it was necessary to find out what proportion of the light was cut off by these screens. A plate was exposed to direct gas light for a certain time. The gas flame was then shaded by the various screens that were to be used, and other plates of the same kind were exposed till they came out as black as the first plate on being completely developed. The times of exposure necessary to produce this result were taken as inversely proportional to the intensities. Experiments made to test the truth of this assumption shewed it to be true if the light was not very feeble.

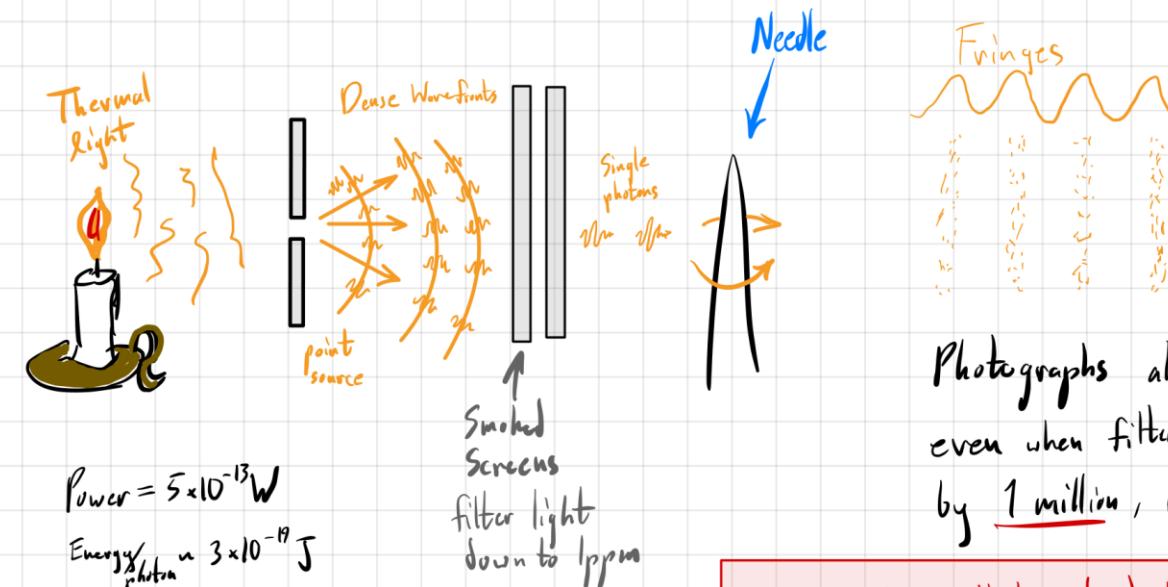
Five diffraction photographs were then taken, the first with direct light and the others with the various screens inserted between the gas flame and the slit. The time of exposure for the first photograph was obtained by trial, a certain standard of blackness being attained by the plate when fully developed. The

remaining times of exposure were taken from the first in the inverse ratio of the corresponding intensities. The longest time was 2000 hours or about 3 months. In no case was there any diminution in the sharpness of the pattern although the plates did not all reach the standard blackness of the first photograph.

In order to get some idea of the energy of the light falling on the plates in these experiments a plate of the same kind was exposed at a distance of two metres from a standard candle till complete development brought it up to the standard of blackness. Ten seconds sufficed for this. A simple calculation will shew that the amount of energy falling on the plate during the longest exposure was the same as that due to a standard candle burning at a distance slightly exceeding a mile. Taking the value given by Drude for the energy in the visible part of the spectrum of a standard candle, the amount of energy falling on 1 square centimetre of the plate is 5×10^{-8} ergs per sec. and the amount of energy per cubic centimetre of this radiation is 1.6×10^{-16} ergs.

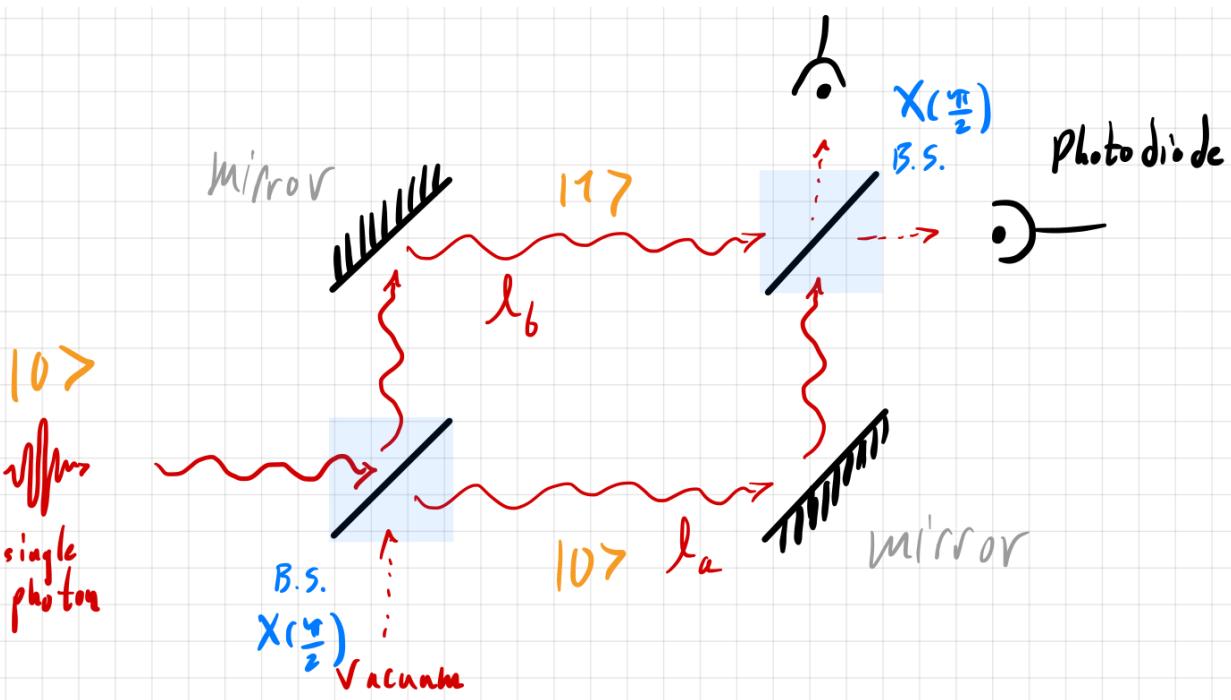
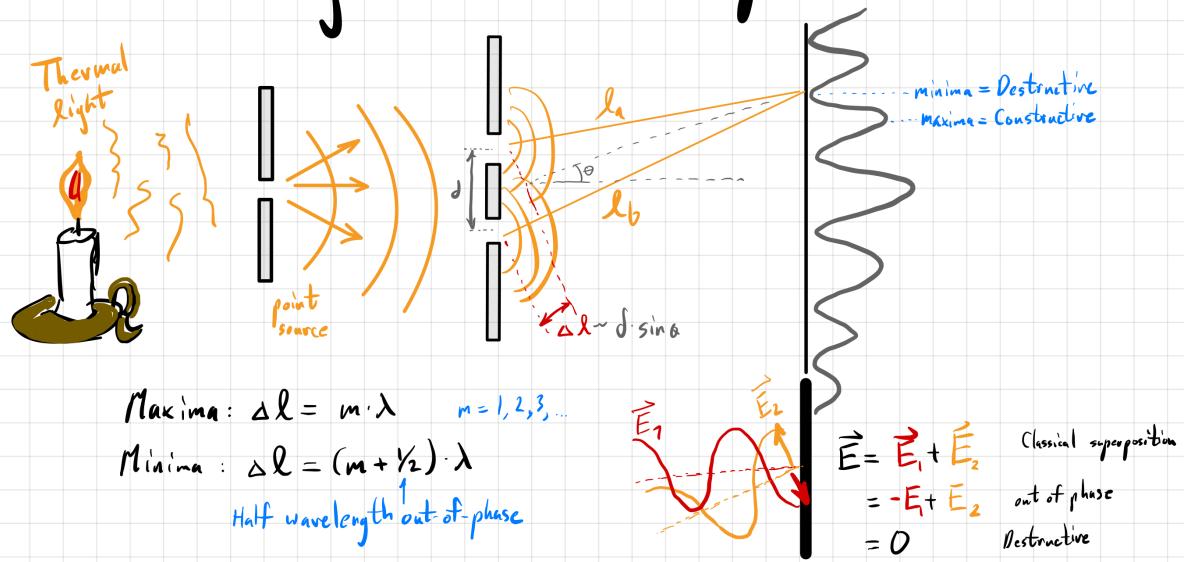
According to Sir J. J. Thomson this value sets an upper limit to the amount of energy contained in one of the indivisible units mentioned above.

1909 Thompson-Taylor Interference Fringes with Feeble Light



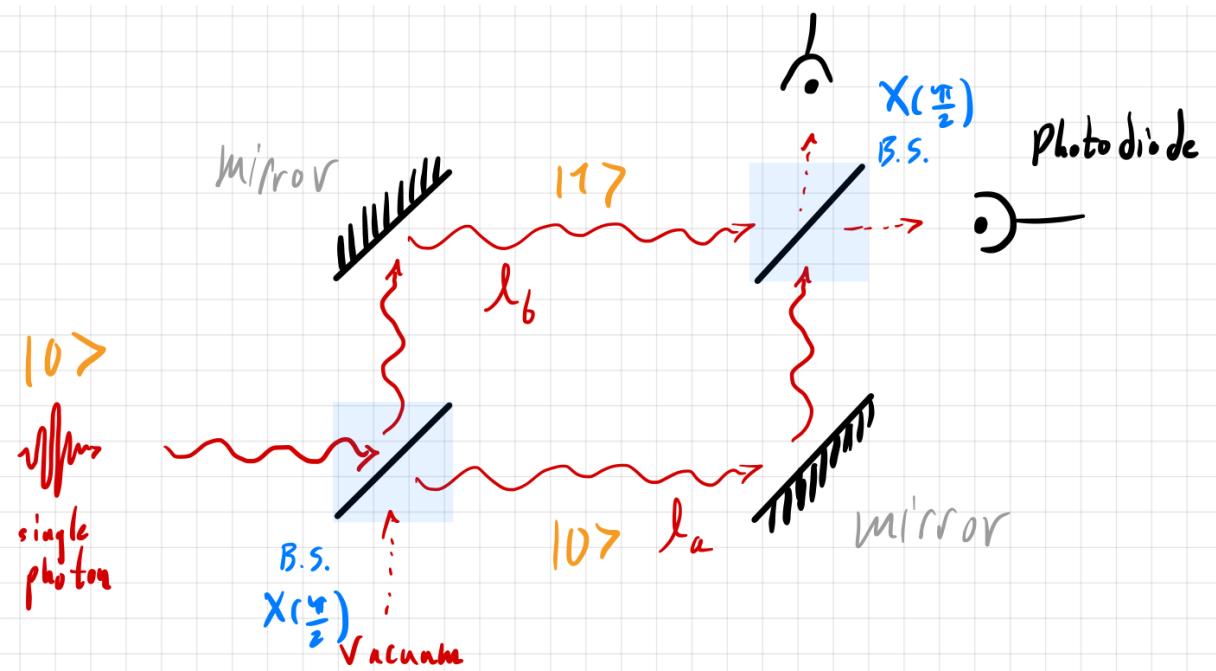
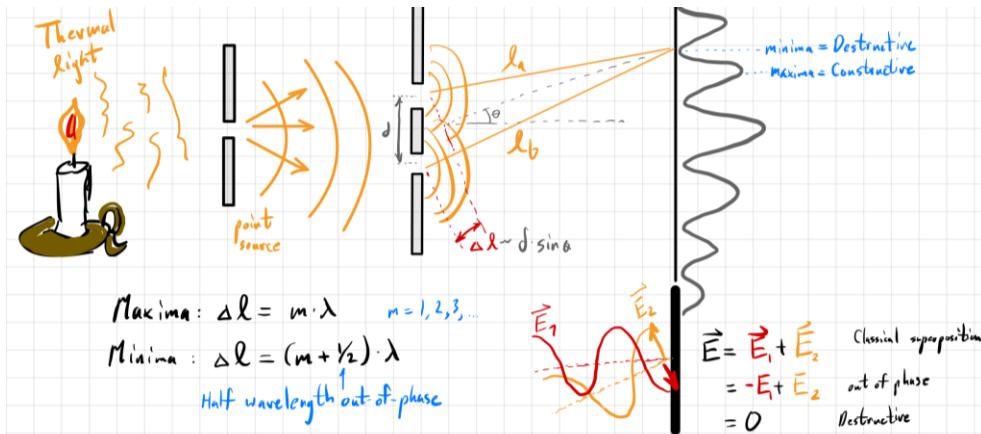
Mach Zehnder Interferometer

1803 Young's Double Slit Experiment



Mach Zehnder Interferometer

1803 Classical Interference



1909 Quantum* Interference (single photons)

