ECE 550/650 QC
Two qubit gates

Robert Niffenegger

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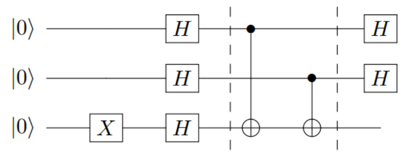
Outline of the course

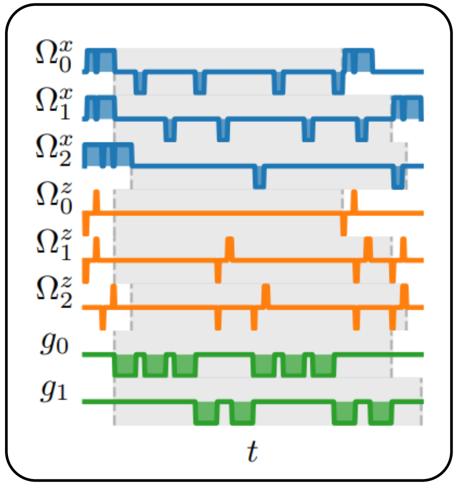
- Quantum Optics
  - What is interference (classical vs. single particle)
  - Superposition of states
  - Measurement and measurement basis
- Atomic physics
  - Spin states in magnetic fields and spin transitions
  - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
  - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
  - Error sources (dephasing, spontaneous decay)
  - Ramsey pulses and Spin echo pulse sequences
  - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
  - Two qubit interactions – gate speed vs. error rates
  - Entanglement – correlation at a distance
  - Bell states and the Bell basis
  - XX gates, Controlled Phase gates, Swap




```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```



- Quantum Hardware
  - Photonics – nonlinear phase shifts
  - Transmons – charge noise, SWAP gate
- Quantum Circuits
  - Single and two qubit gates
  - Hadamard gate , CNOT gate
- Quantum Algorithms
  - Amplitude amplification
  - Grover's Search
  - Oracle - Deutsch Jozsa
  - Bernstein Vazirani
  - Quantum Fourier Transform and period finding
  - Shor's algorithm

If time permits

- Error Correction
  - Repetition codes
  - Color Codes
  - Surface code

How do you build a quantum computer?

The Physical Implementation of Quantum Computation

David P. DiVincenzo

IBM T.J. Watson Research Center, Yorktown Heights, NY 10598 USA

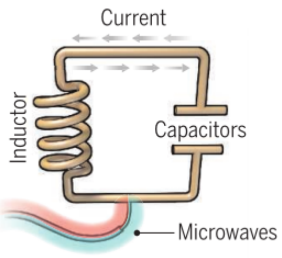
(February 1, 2008)

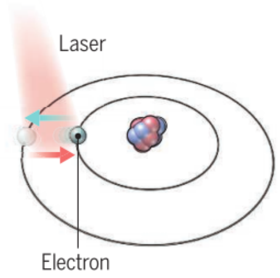
1. A scalable physical system with well characterized qubits
2. The ability to initialize the state of the qubits
3. Long relevant coherence times (vs. gate time)
4. A "universal" set of quantum gates
5. A qubit-specific measurement capability

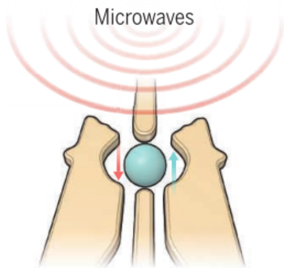
Many viable qubits but all have challenges scaling

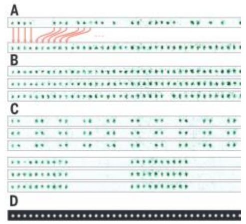
(Increasing qubit number without loss in performance)

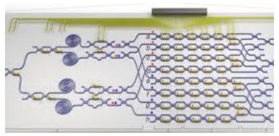
'New qubits'











	Superconducting loops	Trapped ions	Silicon quantum dots	Neutral atoms	Photonics
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Like ions but atoms are trapped with lasers and interact via Rydberg (nearly ionized) excitations	Single photons are guided along integrated pathways and interact via phase shifts during interference
Longevity (seconds)	0.00005	>1000	0.03	1	
Logic success rate	99.9%	99.99%	~99%	99%	97%* (heralded)
Number entangled	27	24	3	20	12
Company support	Google, IBM, Rigetti ...	IonQ, Honeywell, Oxionics, AlpineQ, UniversalQ	Intel, HRL/Boeing, SiliconQuantum	Quera , Atom Computing Cold Quanta	Xanadu, PsiQuantum
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	2D arrays becoming possible, faster gates	CMOS compatible photonics waveguide technology
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Gate fidelity	Cryogenic single photon sources and detectors

Science, Dec 2016 (modified)

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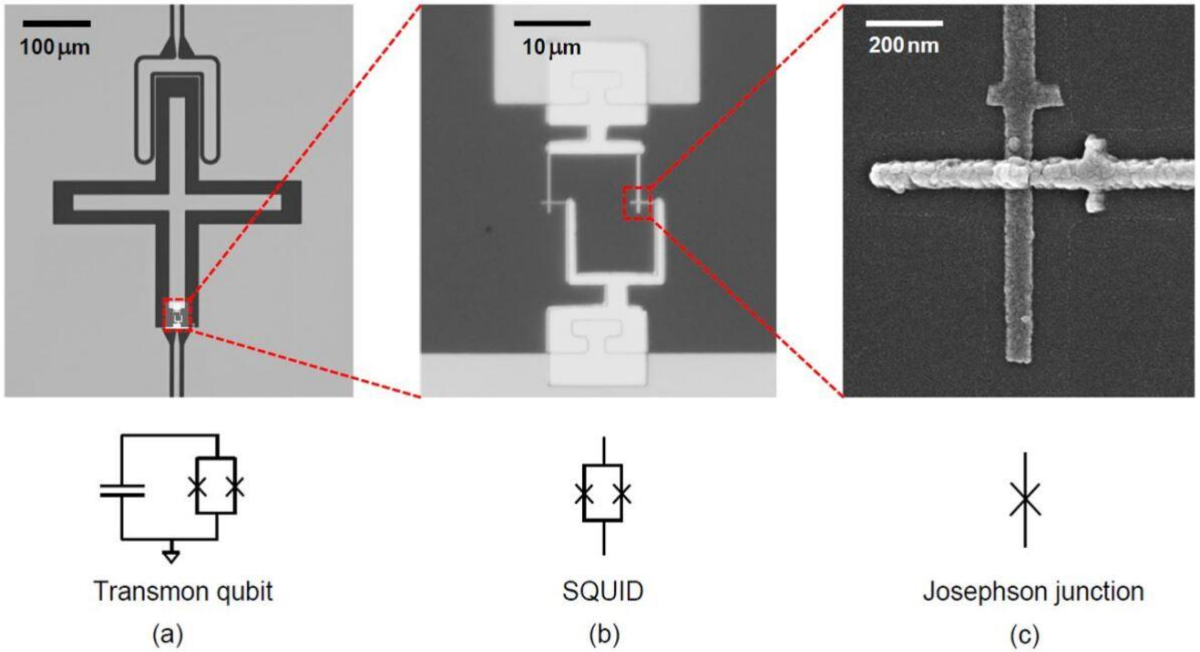
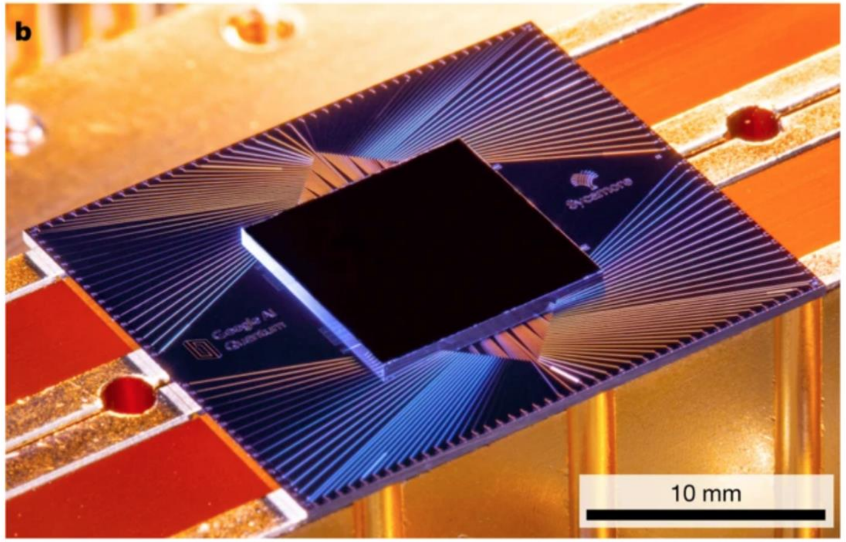
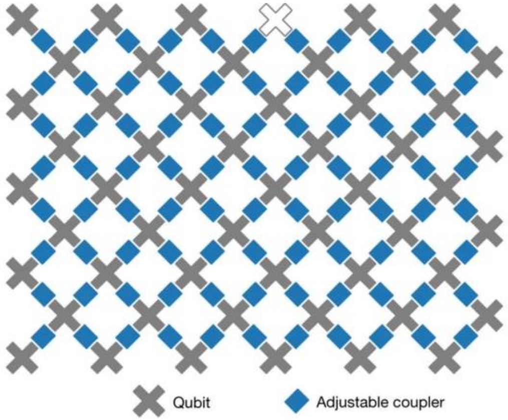
Transmons



100 μm
10 μm
200 nm
Transmon qubit
(a)
SQUID
(b)
Josephson junction
(c)



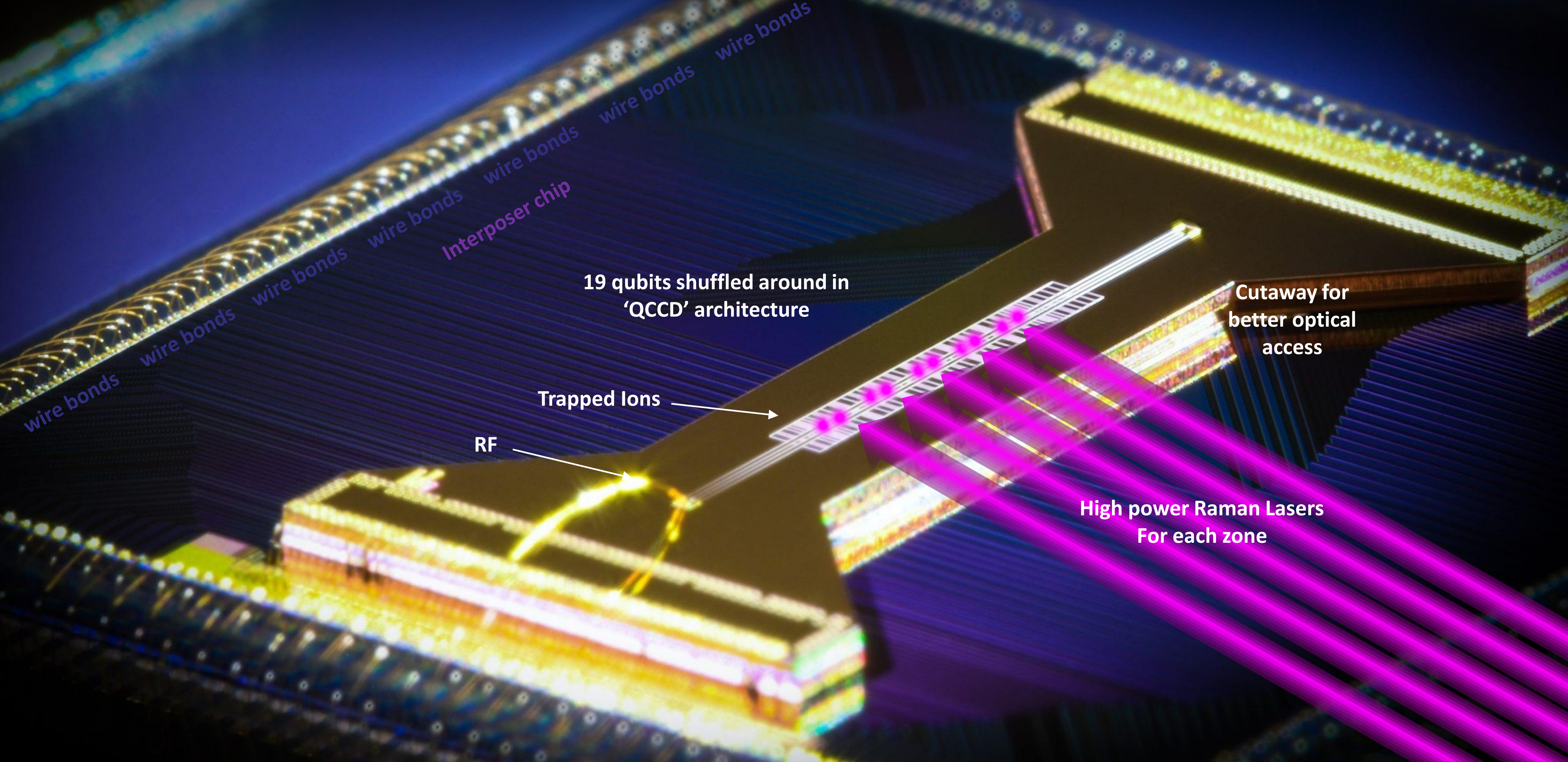
b
10 mm



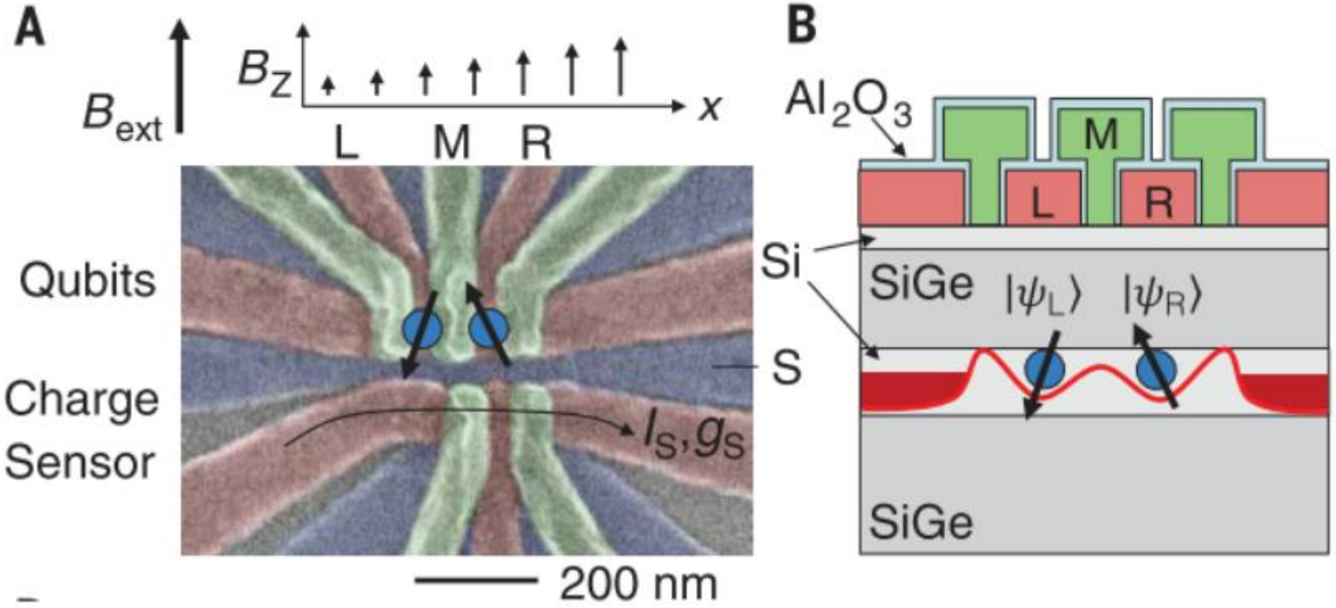
Qubit
Adjustable coupler

Trapped Ions

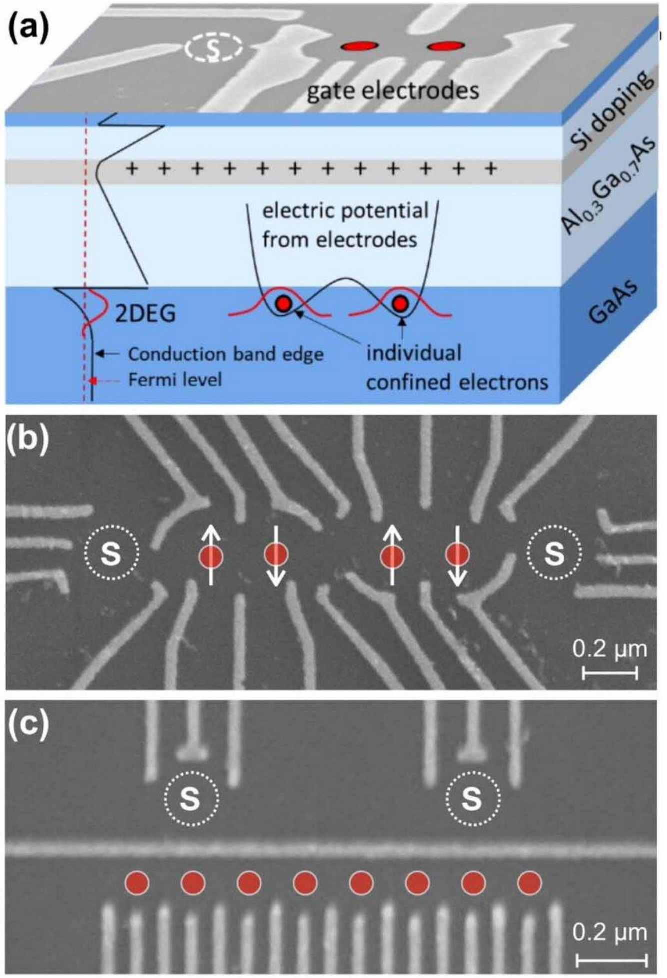
Honeywell (Quantinuum) H1 – linear trap



Spin Qubit

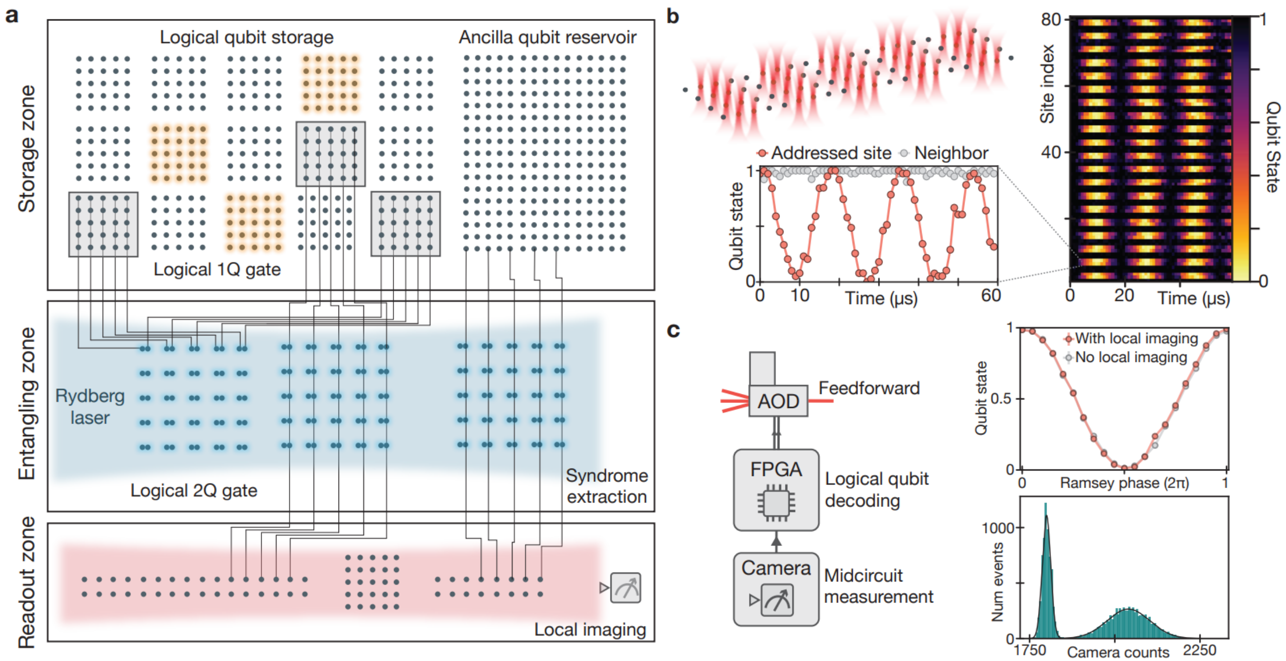


A
B_{ext}
B_Z
x
L M R
Qubits
Charge Sensor
S
I_S, g_S
200 nm
B
Al_2O_3
M
L
R
Si
SiGe
$|\psi_L\rangle$ $|\psi_R\rangle$
SiGe

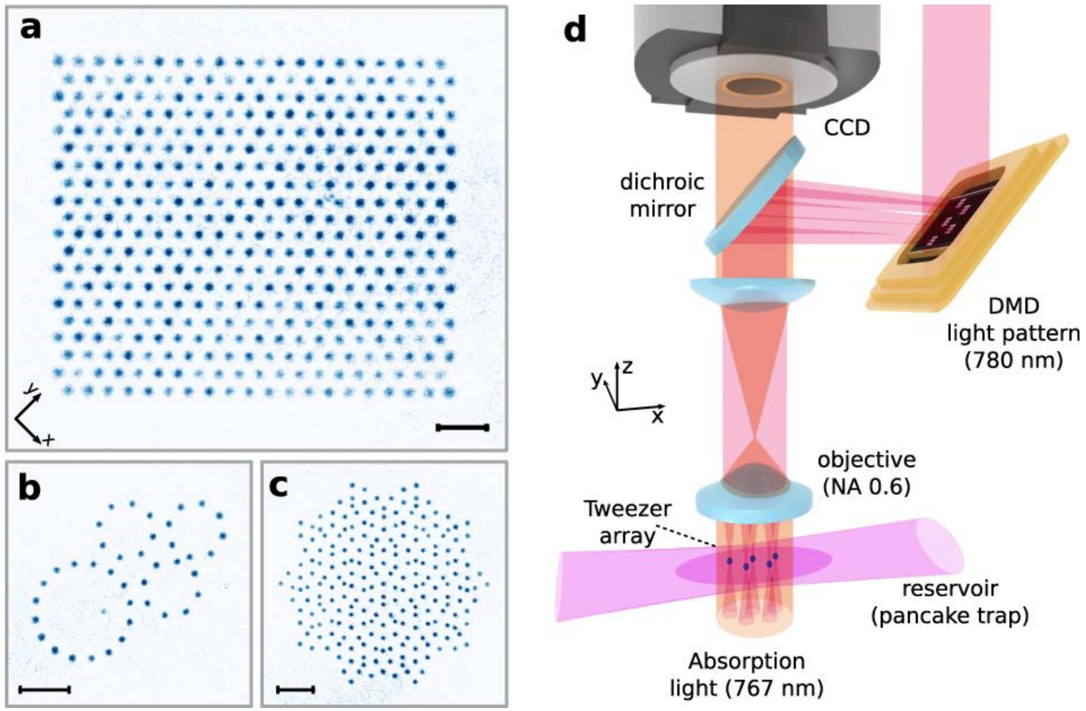


(a)
S
gate electrodes
Si doping
$Al_{0.3}Ga_{0.7}As$
GaAs
electric potential from electrodes
2DEG
Conduction band edge
Fermi level
individual confined electrons
(b)
S
S
0.2 μm
(c)
S
S
0.2 μm

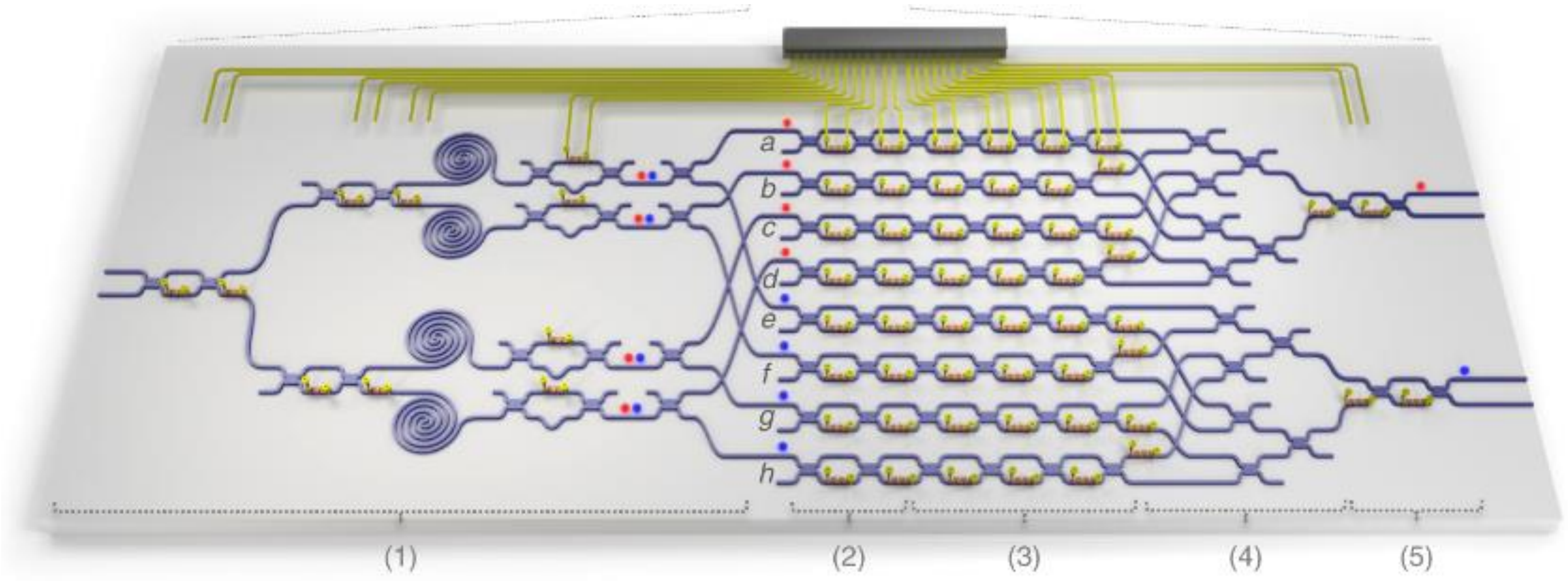
Neutral Atom Tweezers



Rydberg Qubits Lukin Harvard 2023

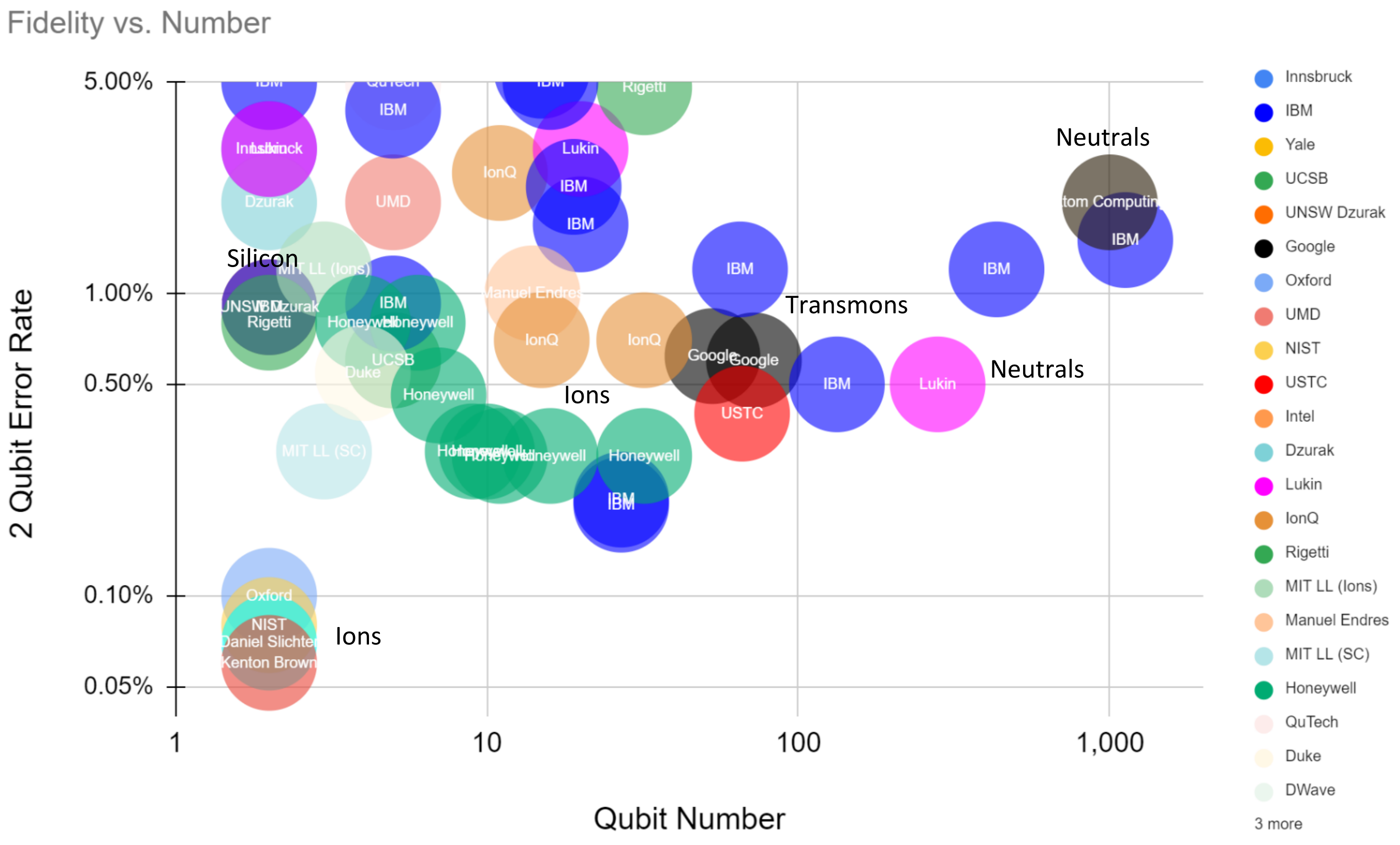


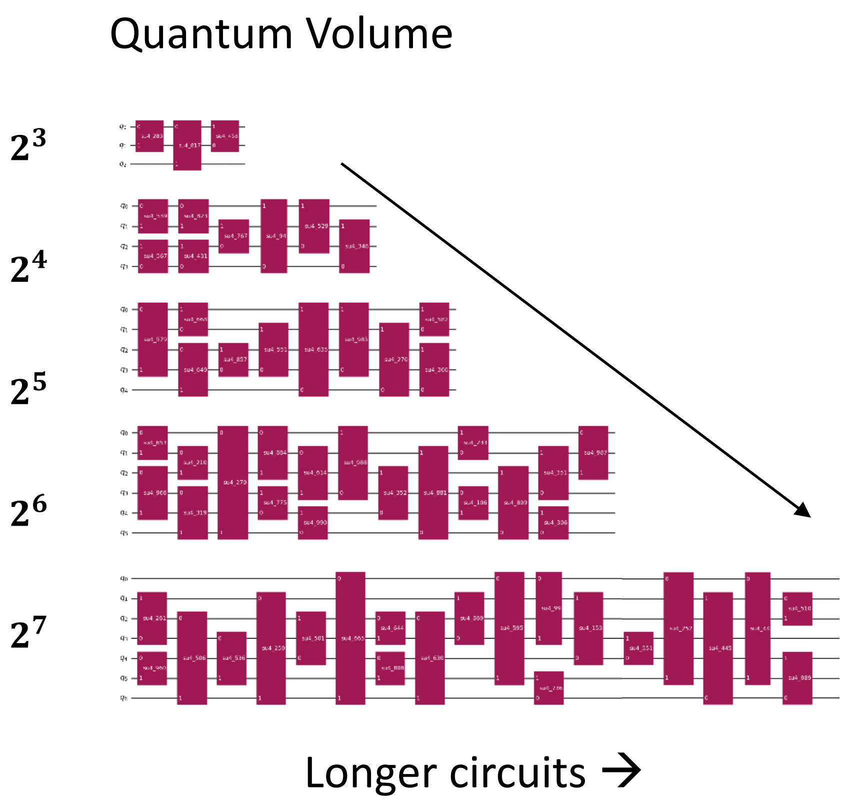
Photonics - MZIs



PsiQuantum 2020

Comparing Quantum Computers





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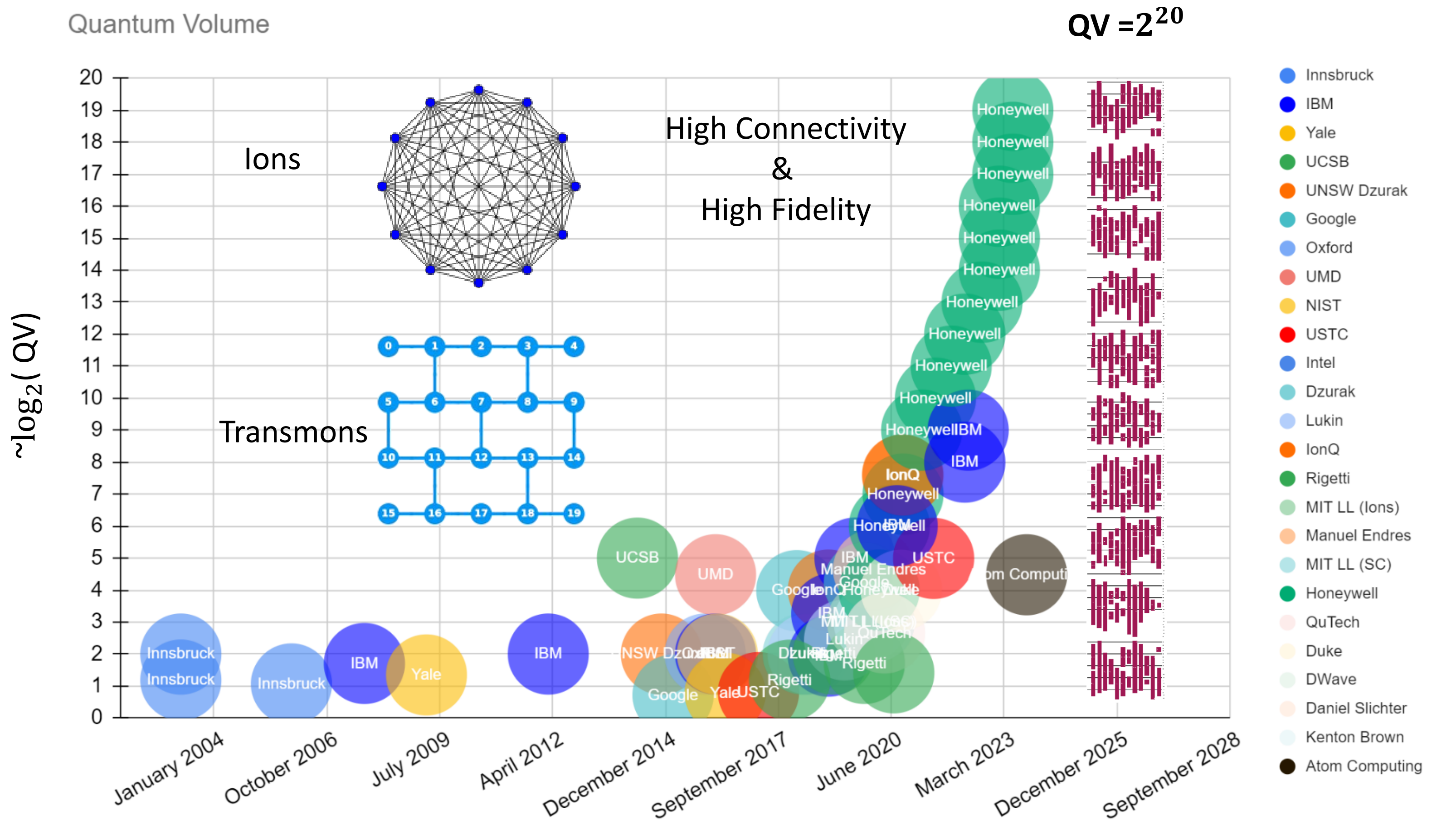
Quantum Volume



Trapped Ion Qubit Advantages

1. High fidelity operations

   2Q gates > 99.9%
   1Q gates > 99.9999%
   Readout > 99.99%
   State Preparation & Measurement > 99.97%

2. Long coherence time

   200ms → 600 seconds
   Ratio to gate time > 1,000,000

3. Atoms are identical (less tuning)
4. High connectivity
5. Low crosstalk (w/ focused lasers)

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2 Qubits

$$\Psi_1 = \alpha_1 |0\rangle + \beta_1 |1\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}$$

$$\Psi_2 = \alpha_2 |0\rangle + \beta_2 |1\rangle = \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}$$

$$|\Psi\rangle = \Psi_1 \otimes \Psi_2$$

Tensor Product

$$= (\alpha_1 |0\rangle_1 + \beta_1 |1\rangle_1)(\alpha_2 |0\rangle_2 + \beta_2 |1\rangle_2)$$

$$= \alpha_1\alpha_2 |00\rangle + \alpha_1\beta_2 |01\rangle + \beta_1\alpha_1 |10\rangle + \beta_1\beta_2 |11\rangle$$

Tensor Product

$$\Psi = \Psi_1 \otimes \Psi_2 = \begin{bmatrix} 1 \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & 0 \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\ 0 \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & 0 \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|\Psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} \otimes \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}$$

Tensor Product

$$= \begin{bmatrix} \alpha_1 \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix} \\ \beta_1 \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix}$$

Entanglement

$$\Psi = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Can we represent this state as a Tensor product?

Try:

$$\Psi_1 = \alpha_1 |0\rangle_1 + \beta_1 |1\rangle_1 \qquad \Psi_2 = \alpha_2 |0\rangle_2 + \beta_2 |1\rangle_2$$

$$\Rightarrow |\Psi_1 \Psi_2\rangle = \alpha_1\alpha_2 |00\rangle + \underbrace{\alpha_1\beta_1 |01\rangle}_{=0} + \underbrace{\beta_1\alpha_2 |10\rangle}_{=0} + \beta_1\beta_2 |11\rangle$$

check

$$\Rightarrow \Psi = |\Psi_1 \Psi_2\rangle \Rightarrow \alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \quad \& \quad \beta_1\beta_2 = \frac{1}{\sqrt{2}}$$

but then $\alpha_1\beta_2 \neq 0$, $\beta_1\alpha_2 \neq 0$

Entanglement

$$\Psi = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Can we represent this state as a Tensor product?

Try:

$\psi_1 = \alpha_1 |0\rangle_1 + \beta_1 |1\rangle_1 \qquad \psi_2 = \alpha_2 |0\rangle_2 + \beta_2 |1\rangle_2$

$$\Rightarrow |\psi_1 \psi_2\rangle = \alpha_1\alpha_2 |00\rangle + \underbrace{\alpha_1 \beta_1 |01\rangle}_{=0} + \underbrace{\beta_1 \alpha_2 |10\rangle}_{=0} + \beta_1\beta_2 |11\rangle$$

check

$$\Rightarrow \Psi = |\psi_1\psi_2\rangle \Rightarrow \alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \quad \& \quad \beta_1\beta_2 = \frac{1}{\sqrt{2}}$$

but then $\alpha_1\beta_2 \neq 0$, $\beta_1\alpha_2 \neq 0$

So either case could be true but not both

1.) Ψ is product state of ψ_1 & ψ_2

Then $\alpha_1, \alpha_2, \beta_1, \beta_2$ all defined

— or —

2) Ψ is not a product state

$\alpha_1, \alpha_2, \beta_1, \beta_2$ don't describe Ψ

Ψ is Entangled

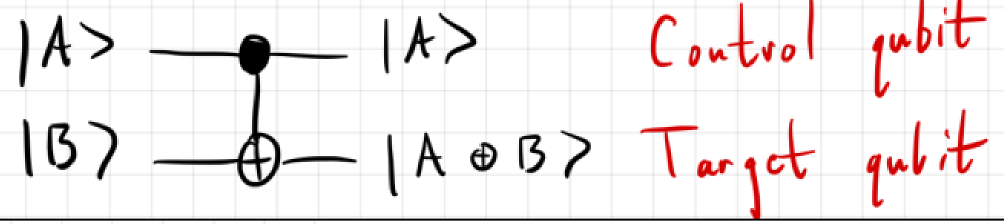
2 Qubit Gate

Controlled Not Gate

Truth table

$\|00\rangle$	→	$\|00\rangle$
$\|01\rangle$	→	$\|01\rangle$
$\|10\rangle$	→	$\|11\rangle$
$\|11\rangle$	→	$\|10\rangle$

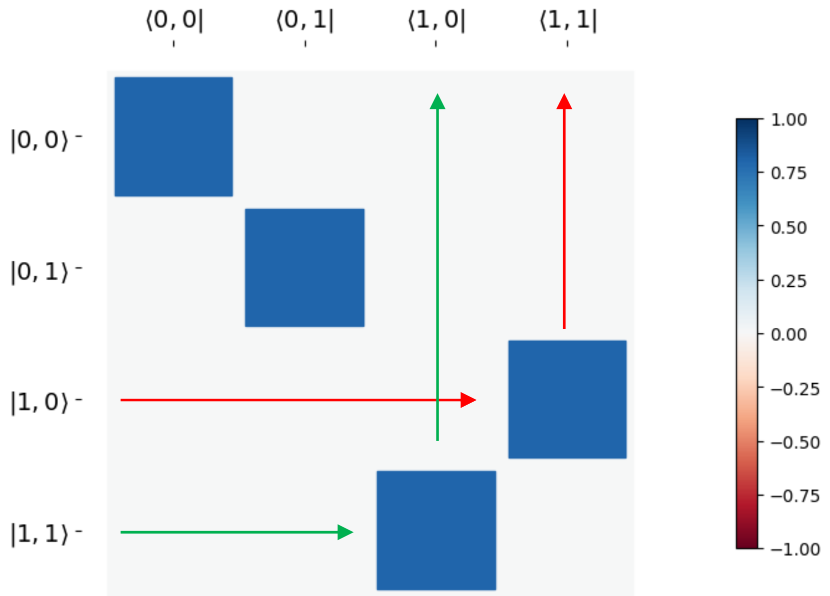
Circuit



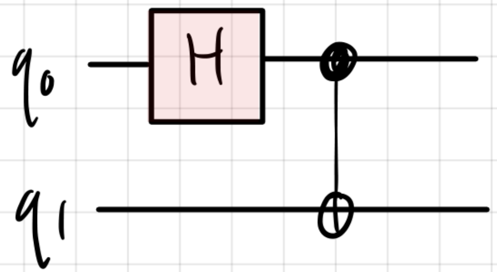
$|A, B\rangle \rightarrow |A, A \oplus B\rangle$

Addition mod 2
(flip 0 to 1, 1 to 0)

$$CX\, q_1, q_0 = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



Bell State Generation

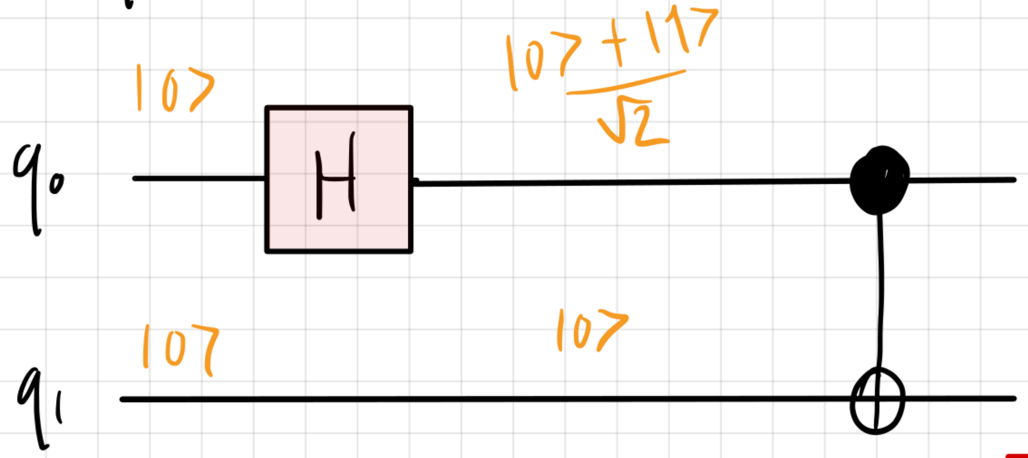


$\Psi_F = ?$

Superposition → CNOT → Entanglement!

1. CNOT $|00\rangle = |00\rangle$
2. CNOT $|10\rangle = |11\rangle$
3. CNOT $\frac{1}{\sqrt{2}}[|00\rangle + |10\rangle] = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$)

if $q_0 = |0\rangle$ & $q_1 = |0\rangle$



Ψ_0, Ψ_1 don't describe Ψ_F anymore...

Bell State

Ψ $\quad |0_0 0_1\rangle \quad \frac{|0_0 0_1\rangle + |1_0 0_1\rangle}{\sqrt{2}} \Rightarrow \boxed{\frac{|0_0 0_1\rangle + |1_0 1_1\rangle}{\sqrt{2}}}$

Bell basis states

Even
Bell states

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

Even Parity

$0+0=0$

$1+1=2 \mod 2 \Rightarrow = 0$

Odd
Bell states

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Odd Parity

$1+0=1$

$0+1=1$

Verifying Superposition?

Mixed States

Single qubit

How do you distinguish two possibilities

1) superposition state $prob(|+\rangle) = 100\%$

2) 'coin flip' $prob(|0\rangle) = 50\% + prob(|1\rangle) = 50\% = 100\%$

Density Matrix: $\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k|$

$$\rho = p_{|0\rangle}|0\rangle\langle 0| + p_{|1\rangle}|1\rangle\langle 1|$$

$$p_k(|0\rangle) = \langle 0|\, \rho_k |0\rangle$$

Density Matrix – Coins vs. qubits

Coin flip $\quad p(|0\rangle) = \frac{1}{2} \quad p(|1\rangle) = 1/2 \qquad$ probability $= 50\%$ Heads or Tails

$$\rho_{coin} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

Qubit $\quad p(|-\rangle) = 1 \quad$ then $\quad p\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = 1 \qquad \psi_- = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

"$|-\rangle$" $\quad \rho_- = |-\rangle\langle -| = \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\left(\frac{\langle 0| - \langle 1|}{\sqrt{2}}\right)$

$$= \frac{1}{2}|0\rangle\langle 0| \underbrace{- \frac{1}{2}|0\rangle\langle 1| - \frac{1}{2}|1\rangle\langle 0|} + \frac{1}{2}|1\rangle\langle 1|$$

$$\Rightarrow \rho = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

Off diagonal elements from coherence

Density Matrix - Coins vs. qubits

Coin flip $p(|0\rangle) = \frac{1}{2}$ $p(|1\rangle) = 1/2$ probability = 50% Heads or Tails

$\rho_{coin} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

$$Prob(|\psi\rangle) = \langle\psi|\,\rho|\psi\rangle$$

$$\rho_{coin}(|0\rangle) = \langle 0|\rho_{coin}|0\rangle = \frac{\langle 0|0\rangle\langle 0|0\rangle}{2} + \frac{\cancelto{0}{\langle 0|1\rangle}\cancelto{0}{\langle 1|0\rangle}}{2} = 1/2$$

$$\rho_{coin}(|1\rangle) = \langle 1|\rho_{coin}|1\rangle = \frac{\cancelto{0}{\langle 1|0\rangle}\cancelto{0}{\langle 0|1\rangle}}{2} + \frac{\langle 1|1\rangle\langle 1|1\rangle}{2} = 1/2$$

match!

$$\rho = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

Density Matrix – Coins vs. qubits

$$p_k(|\psi\rangle) = \langle\psi|\, \rho_k |\psi\rangle$$

$$\rho_-(|0\rangle) = \langle 0|\rho_-|0\rangle = \frac{\langle 0|0\rangle\langle 0|0\rangle}{2} - \frac{\langle 0|0\rangle\cancelto{0}{\langle 1|0\rangle}}{2} - \frac{\cancelto{0}{\langle 0|1\rangle}\langle 0|0\rangle}{2} + \frac{1}{2}\cancelto{0}{\langle 0|1\rangle}\cancelto{0}{\langle 1|0\rangle} = 1/2$$

$$\rho_-(|1\rangle) = \langle 1|\rho_-|1\rangle = \frac{\cancelto{0}{\langle 1|0\rangle}\cancelto{0}{\langle 0|1\rangle}}{2} - \frac{\cancelto{0}{\langle 1|0\rangle}\langle 1|1\rangle}{2} - \frac{\langle 1|1\rangle\cancelto{0}{\langle 0|1\rangle}}{2} + \frac{\langle 1|1\rangle\langle 1|1\rangle}{2} = 1/2$$

but in different basis!?

$$\rho_-(|-\rangle) = \langle -|-\rangle\langle -|-\rangle = 1 \quad \checkmark \qquad \rho_-(|+\rangle) = \cancelto{0}{\langle +|-\rangle}\cancelto{0}{\langle -|+\rangle} = 0 \quad \checkmark$$

$$\rho = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

Density Matrix – Coins vs. qubits

$$p_k(|\psi\rangle) = \langle\psi|\,\rho_k|\psi\rangle$$

$$\rho_-(|0\rangle) = \langle 0|\rho_-|0\rangle = \frac{\langle 0|0\rangle\langle 0|0\rangle}{2} - \frac{\langle 0|0\rangle\cancel{\langle 1|0\rangle}^{0}}{2} - \frac{\cancel{\langle 0|1\rangle}^{0}\langle 0|0\rangle}{2} + \frac{1}{2}\cancel{\langle 0|1\rangle}^{0}\cancel{\langle 1|0\rangle}^{0} = 1/2$$

$$\rho_-(|1\rangle) = \langle 1|\rho_-|1\rangle = \frac{\cancel{\langle 1|0\rangle}^{0}\cancel{\langle 0|1\rangle}^{0}}{2} - \frac{\cancel{\langle 1|0\rangle}^{0}\langle 1|1\rangle}{2} - \frac{\langle 1|1\rangle\cancel{\langle 0|1\rangle}^{0}}{2} + \frac{\langle 1|1\rangle\langle 1|1\rangle}{2} = 1/2$$

but in different basis!?

$$\rho_-(|-\rangle) = \langle -|-\rangle\langle -|-\rangle = 1 \checkmark \qquad \rho_-(|+\rangle) = \cancel{\langle +|-\rangle}^{0}\cancel{\langle -|+\rangle}^{0} = 0 \checkmark$$

$$\rho_{coin}(|-\rangle) = \langle -|\rho_{coin}|-\rangle = \frac{\overbrace{\langle -|0\rangle}^{1/\sqrt{2}}\overbrace{\langle 0|-\rangle}^{1/\sqrt{2}}}{2} + \frac{\overbrace{\langle -|1\rangle}^{1/\sqrt{2}}\overbrace{\langle 1|-\rangle}^{1/\sqrt{2}}}{2} = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\rho_{coin}(|+\rangle) = \langle +|\rho_{coin}|+\rangle = \frac{\langle +|0\rangle\langle 0|+\rangle}{2} + \frac{\langle +|1\rangle\langle 1|+\rangle}{2} = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

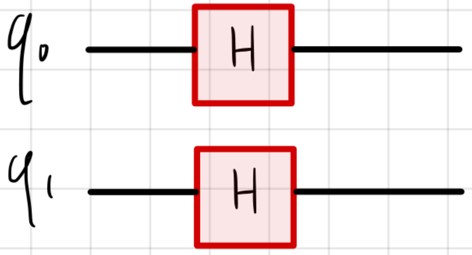
Coin is just a mix
not a coherent superposition

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix} = \frac{1}{2}$$

$$\rho_-(|-\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}}, & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}, & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{2}{2\sqrt{2}} \\ \frac{-2}{2\sqrt{2}} \end{pmatrix} = \frac{1}{2} - (-1/2) = 1 \checkmark$$

Verifying Entanglement

What if we can try this circuit



$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$

$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$

Entangled? No $\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$

$$= \frac{|00\rangle}{2} + \frac{|01\rangle}{2} + \frac{|10\rangle}{2} + \frac{|11\rangle}{2}$$

$\alpha_1 = \frac{1}{\sqrt{2}}$ $\alpha_2 = \frac{1}{\sqrt{2}}$ $\beta_1 = \frac{1}{\sqrt{2}}$ $\beta_2 = \frac{1}{\sqrt{2}}$

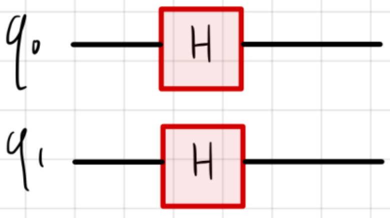
What if we measure q_0 and q_1 and keep getting correlations?

if $q_0 = |0\rangle$ then $q_1 = |0\rangle$ and if $q_0 = |1\rangle$ then $q_1 = |1\rangle$? Entangled?

One way to verify is to measure interference between the two states.

Verifying Entanglement

What if we can try this circuit



$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$

$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$

Entangled?

Z Basis $|0\rangle$

$\updownarrow \pi/2$

X Basis $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ $|+\rangle_x$

$\updownarrow \pi/2$

Z Basis $|1\rangle$

	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 0\rangle$	1	2
$\lvert 1\rangle$	2	3

Even Basis $|00\rangle$

$\pi/2$ $\pi/2$

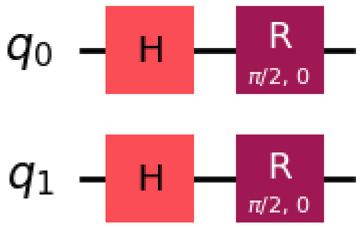
Odd Basis $|10\rangle$ $|01\rangle$ Intermediate states

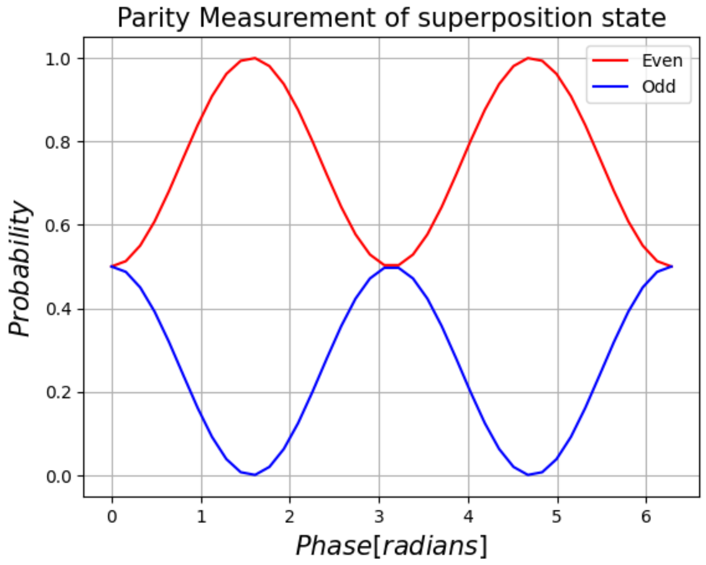
$\pi/2$ $\pi/2$

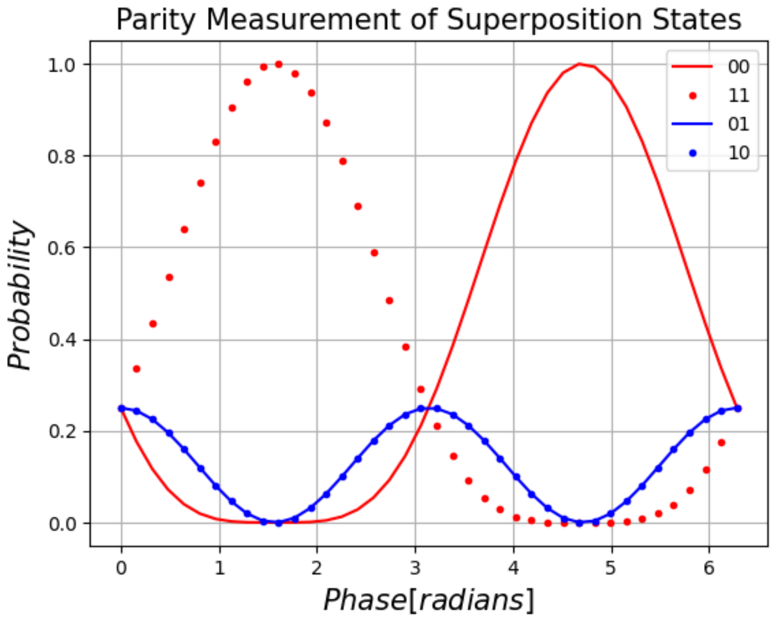
Even Basis $|11\rangle$

	00	01	10	11
00	1			
01		2		
10				
11				3

Two Superposition States







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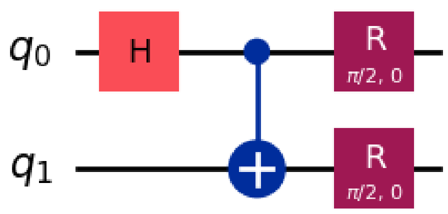
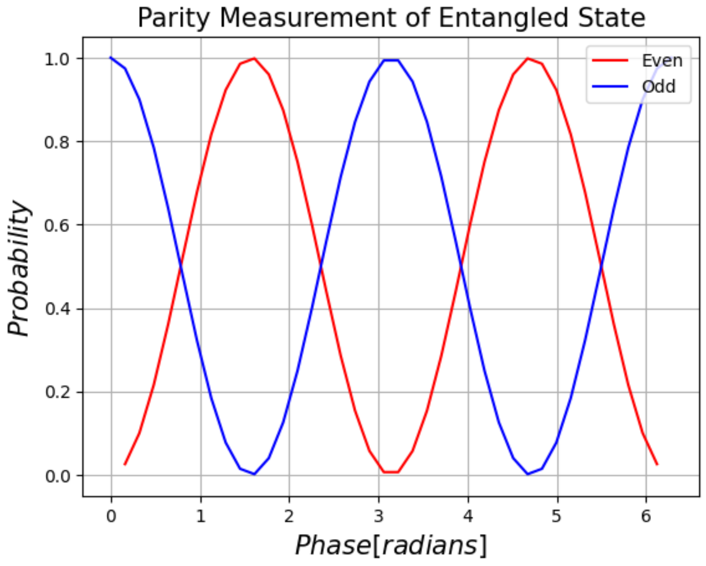
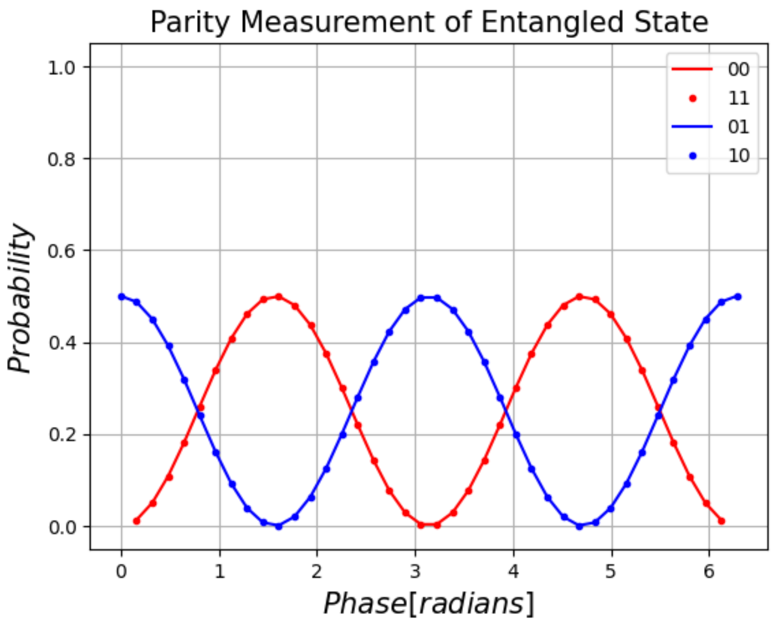
Entangled States







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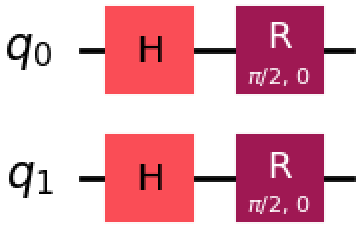
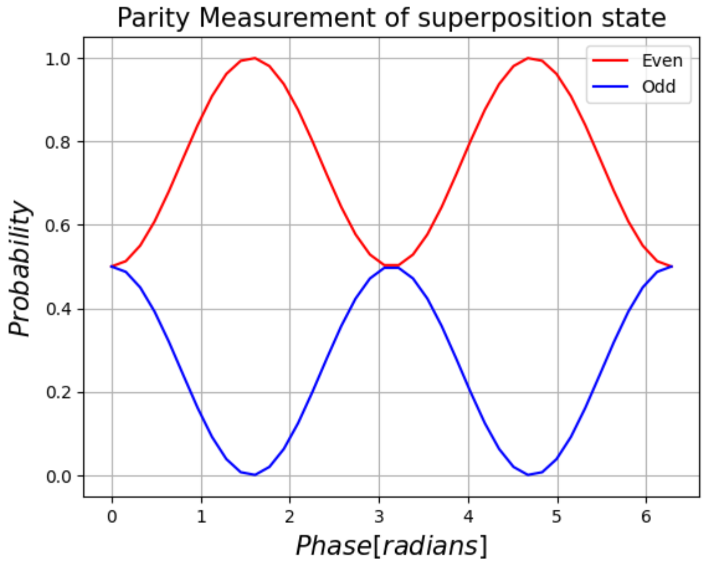
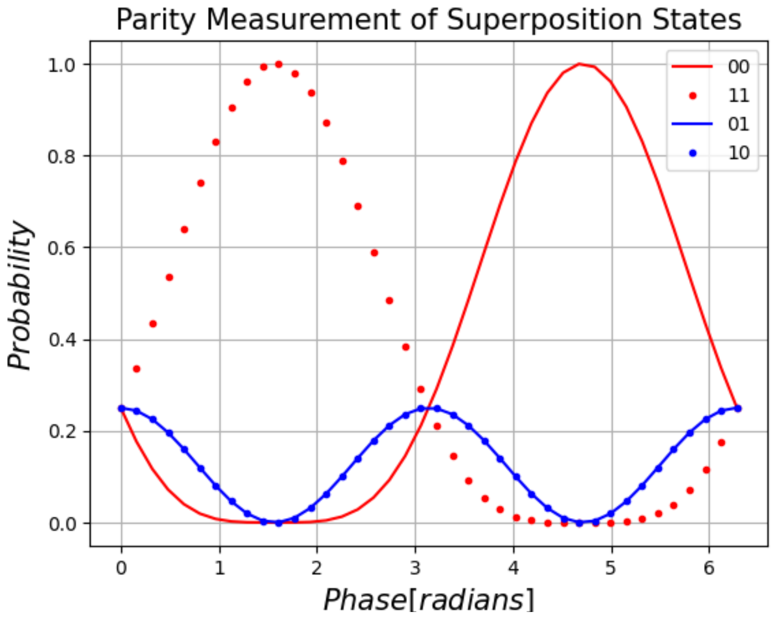
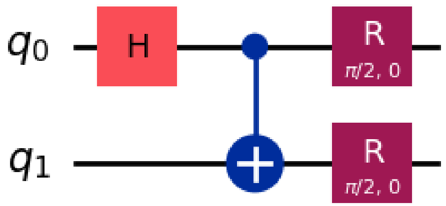
Two Superposition States

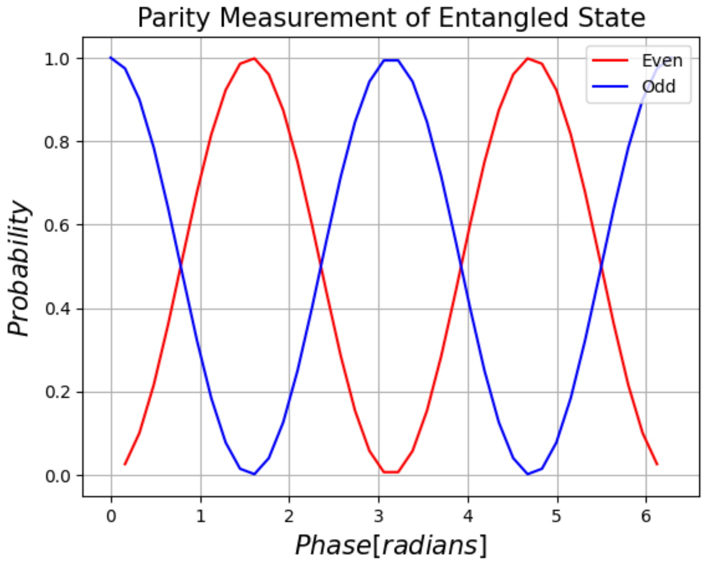


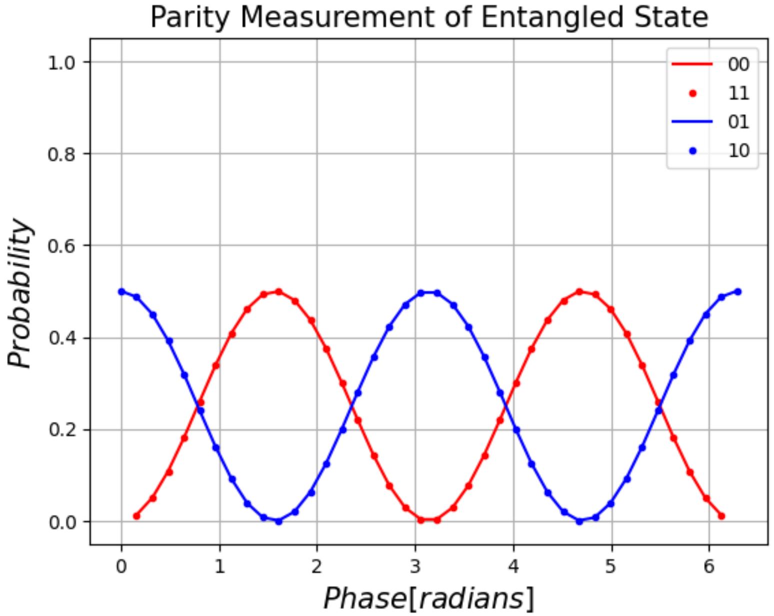




Entangled States







Two qubit gates with Trapped ions

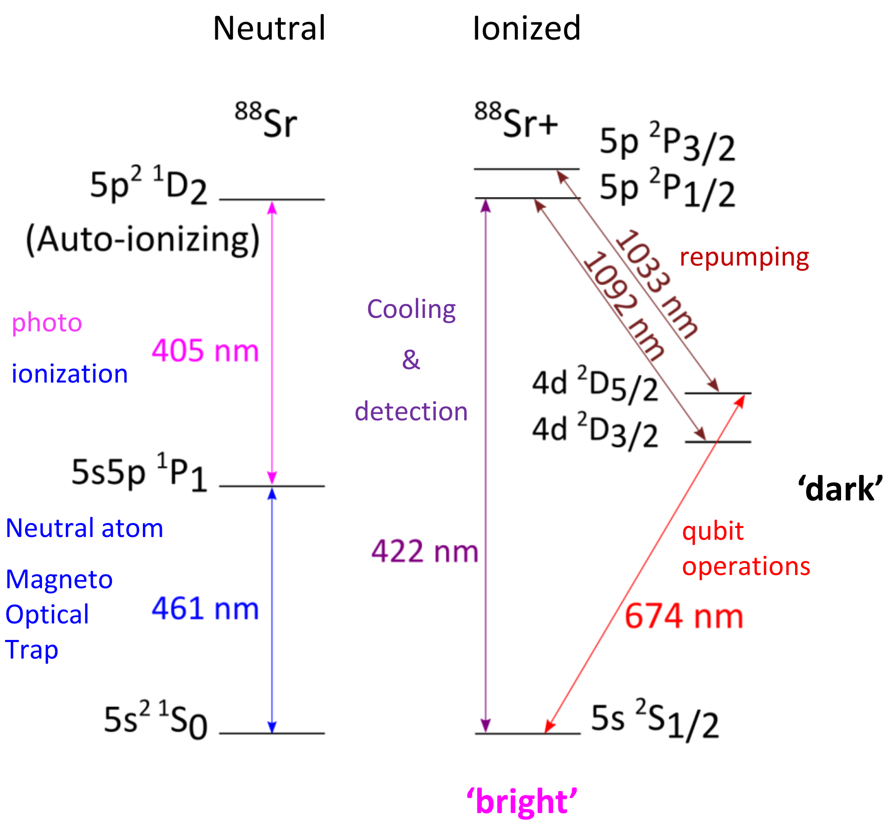
Ion trap operation

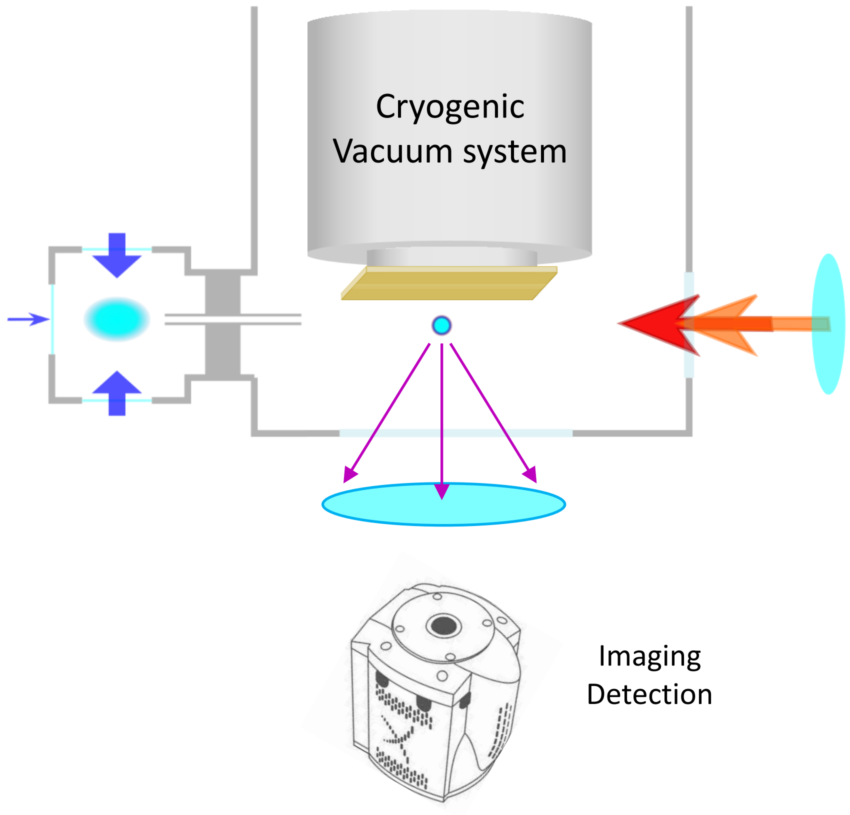
Ion trap loading

1. Laser cool neutral atom source
2. Push atoms towards ion trap
3. Photo-ionize atoms
4. Trap ion in RF Paul trap
5. Laser cool ion

Experiment:

1. Qubit initialization
   1. Doppler laser cool ion
   2. Prepare atomic state
   3. Sideband cool to ground state
2. Qubit operations
3. Ion state detection





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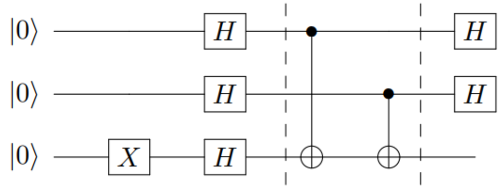
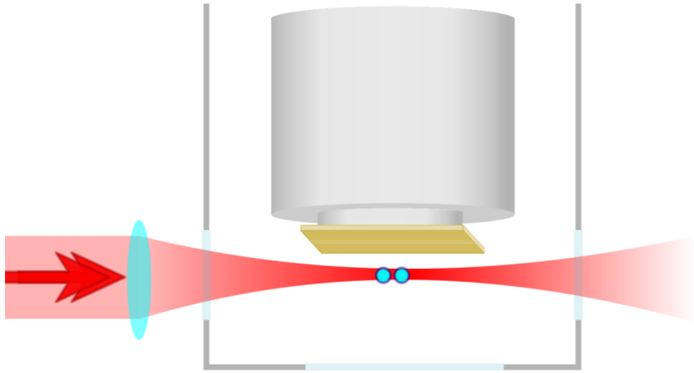
Trapped Ion Optical Qubit Gates

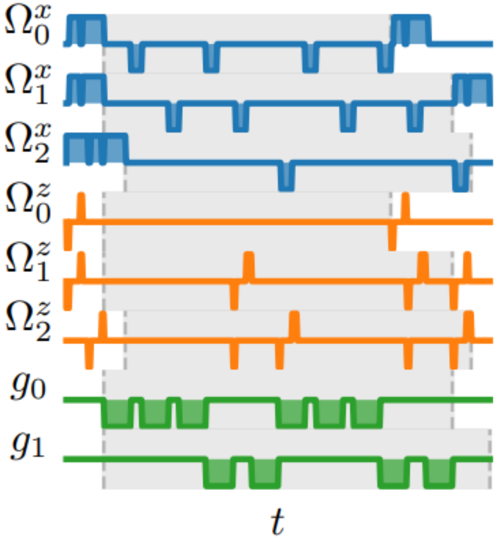


Gates = Laser beam pulses:

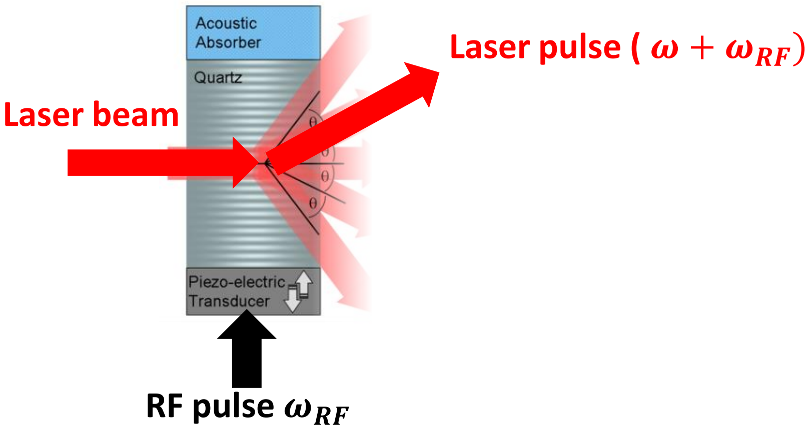
→ Electric field coupling electronic states
(RF pulse (and freq shift) of Terahertz optical carrier)
$\langle e|\,\overrightarrow{E_x}\,|\,g\rangle \propto \Omega$

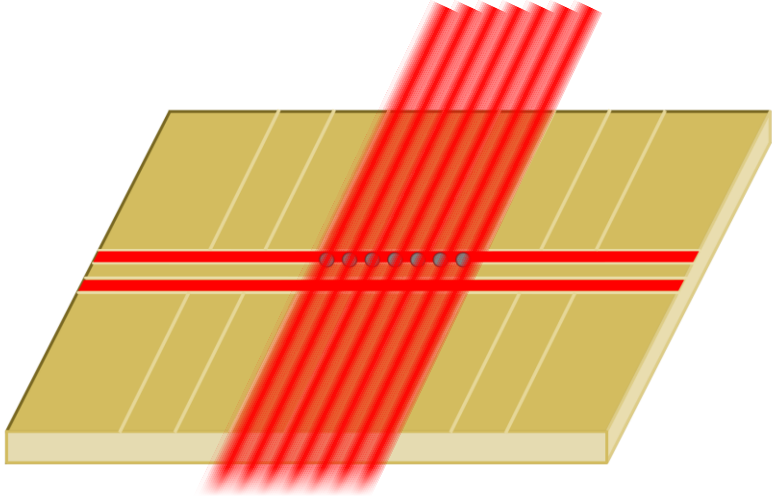


RF pulses



Acoustic Optical Modulator





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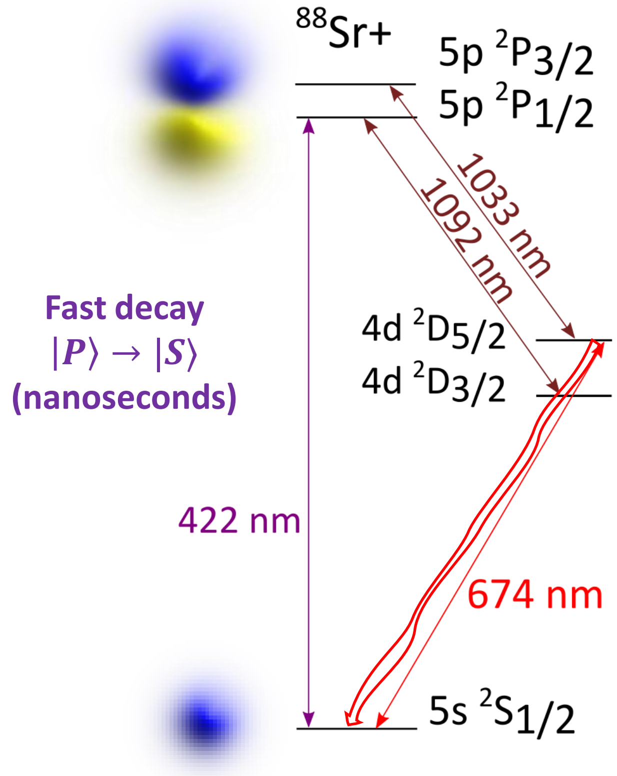
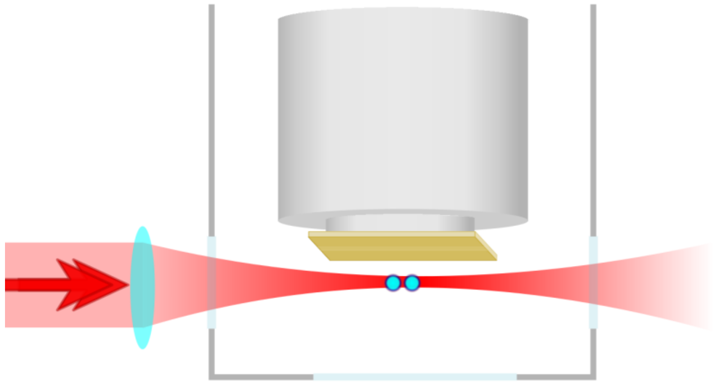
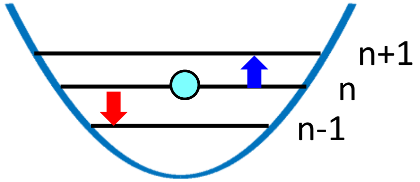
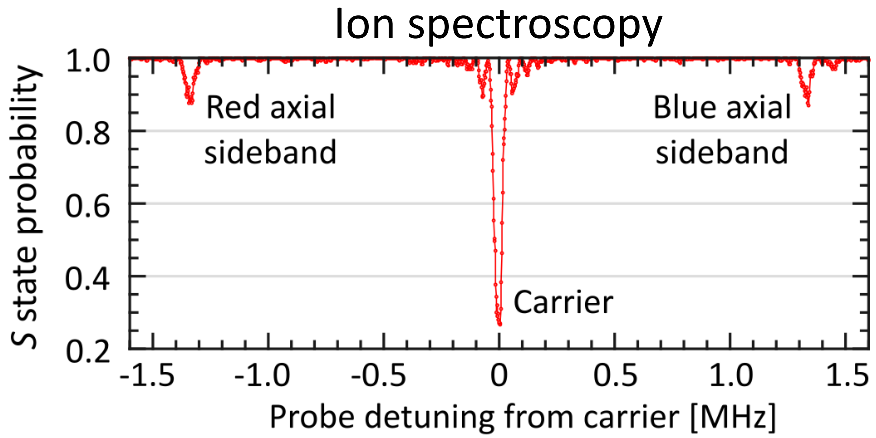
Trapped Ion Optical Qubit







Slow decay from excited D state to ground S state (350ms)
→ 'Meta-stable' Optical qubit



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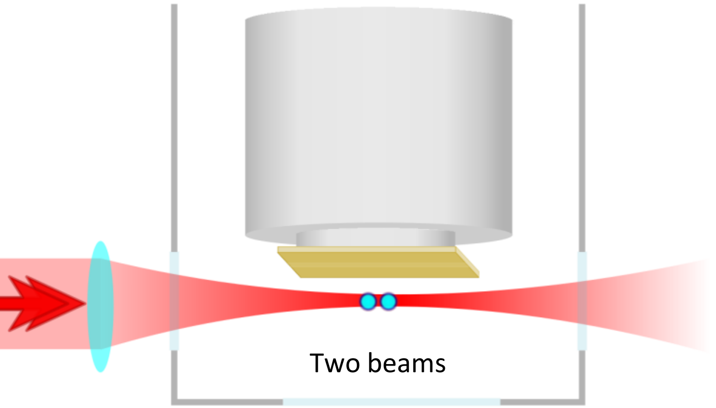
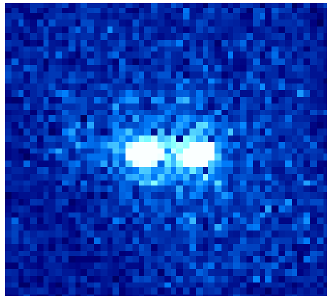
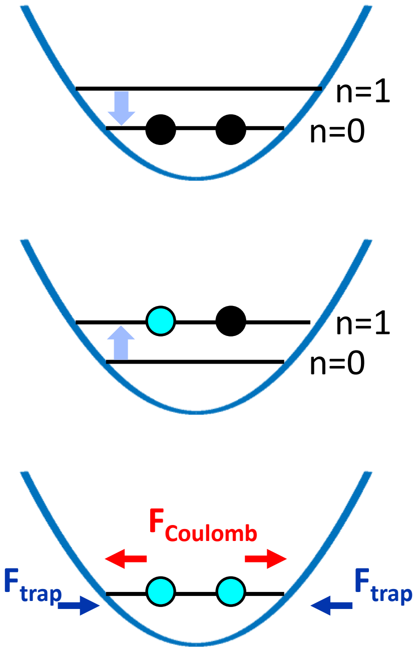
Two Qubit MS Gate





Both ions bright
|00> (|gg>)

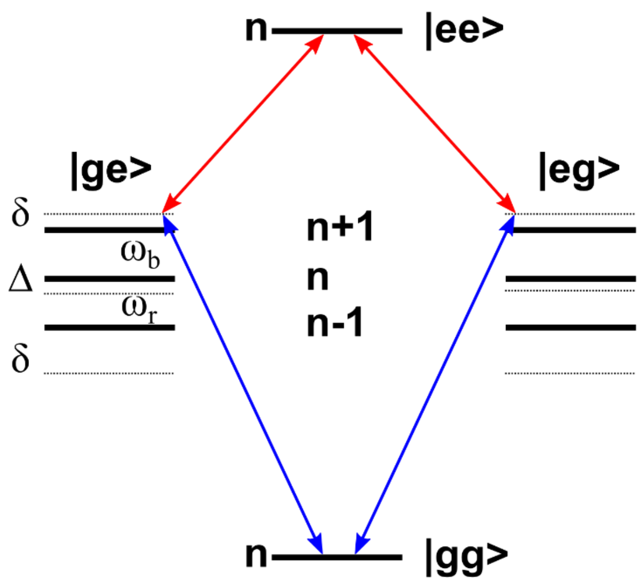


two photon
resonance
(no motional excitation)

detuned
intermediate state
(motional bus)

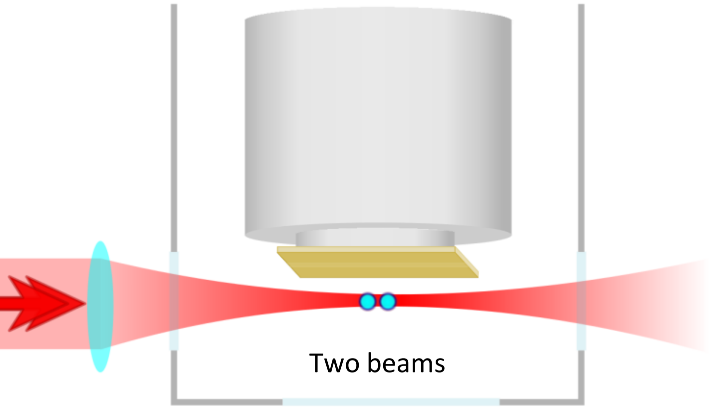
Ions
Interact via
Coulomb repulsion

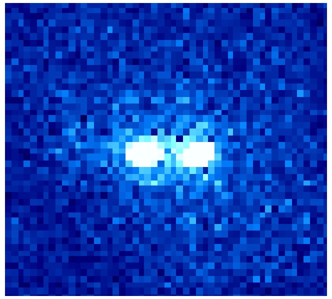
Mølmer-Sørensen Gate



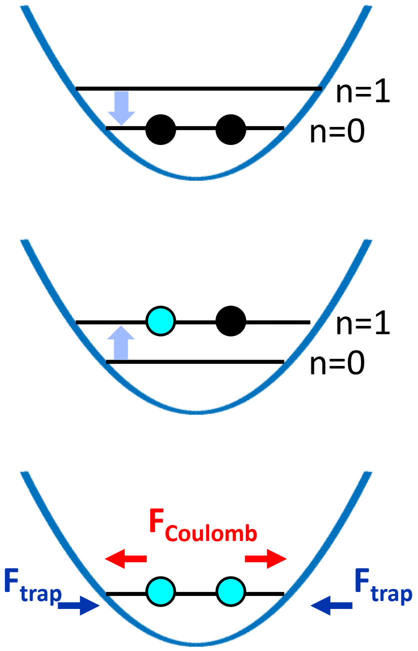
$$H_{\mathrm{int}} = \frac{\hbar\Omega}{2}\left(\mathrm{i}\eta \mathrm{e}^{-\mathrm{i}\Delta t}\left(a^{\dagger}\mathrm{e}^{-\mathrm{i}\delta t} + a\mathrm{e}^{\mathrm{i}\delta t}\right)\left(\sigma_{-}^{(1)} + \sigma_{-}^{(2)}\right) + \mathrm{h.c.}\right)$$

Two Qubit MS Gate





Both ions bright
|00> (|gg>)



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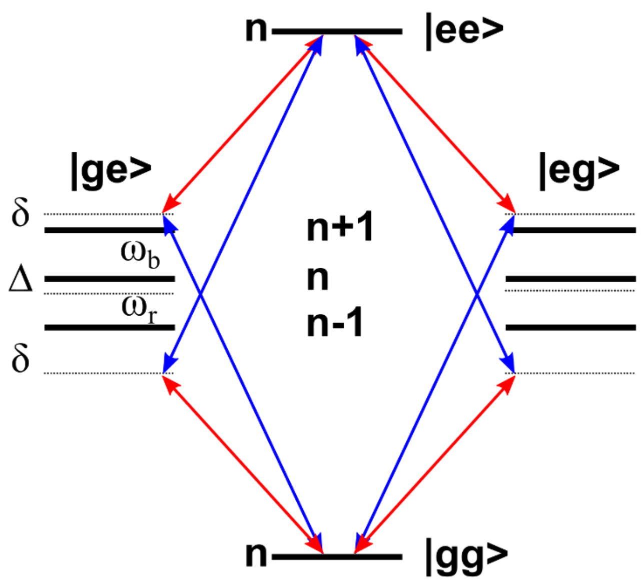


$$H_{\text{int}} = \frac{\hbar\Omega}{2}\left(i\eta e^{-i\Delta t}\left(a^{\dagger}e^{-i\delta t} + a e^{i\delta t}\right)\left(\sigma_-^{(1)} + \sigma_-^{(2)}\right) + \text{h.c.}\right)$$

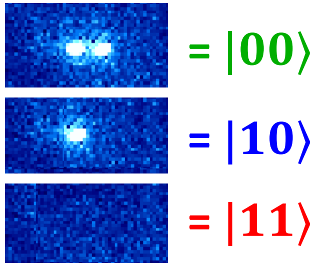
Motion excitation (Harmonic Oscillator)

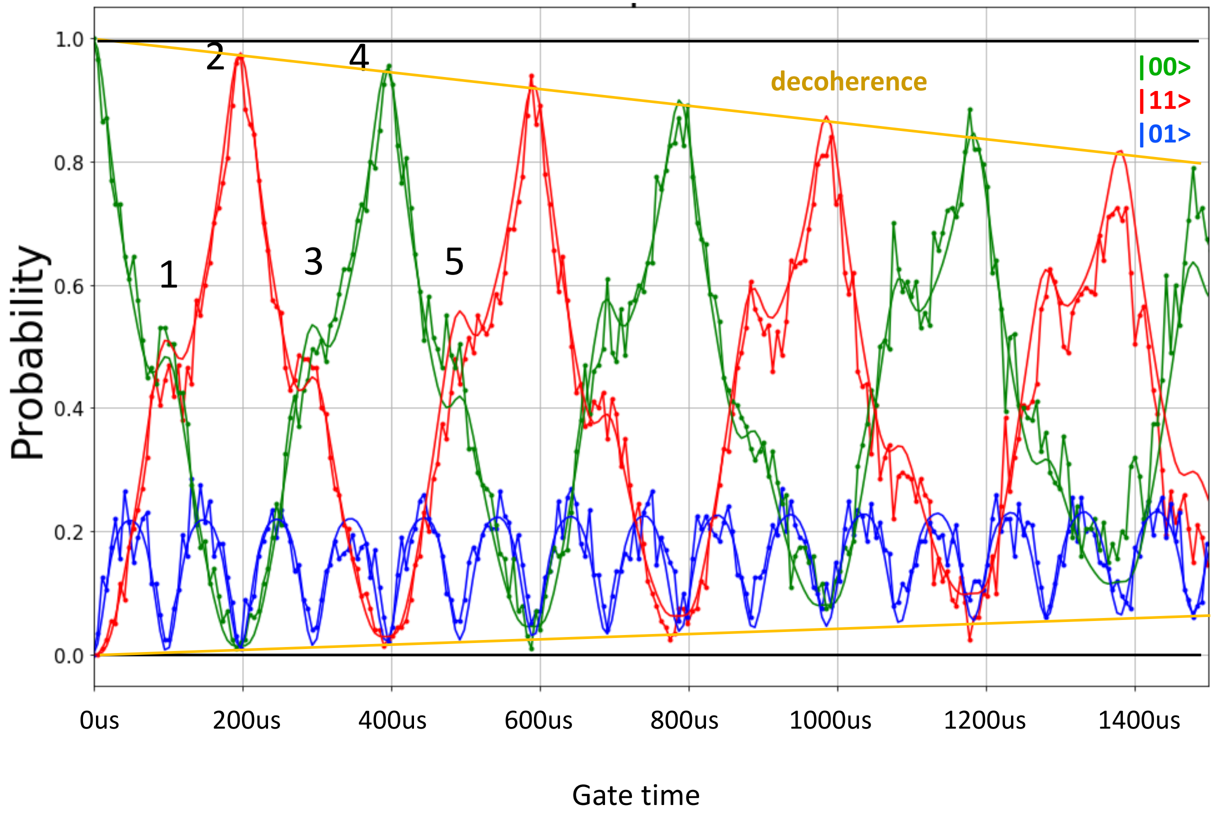
Electronic excitation (atomic orbitals)

Two Qubit Gate Fidelity

Two qubit gates

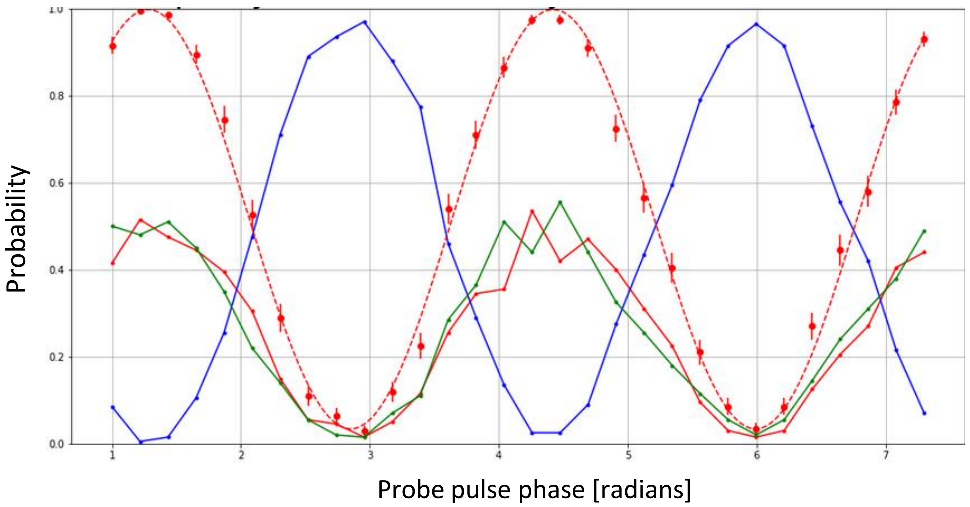
THz optical signal only 100 Hz off resonance

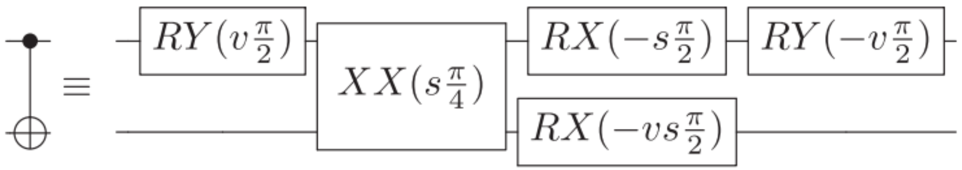




Parity measurement (probe $\pi/2$ pulse with varying phase)

Fidelity 98% (limited by vibrations)





Universal gate set with 1Q gates - Dmitri Maslov 2017 New J. Phys.

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Calculating MS 2Q gate time from 1Q rate

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\left(\mathrm{i}\eta \mathrm{e}^{-\mathrm{i}\Delta t}\left(a^{\dagger}\mathrm{e}^{-\mathrm{i}\delta t} + a\mathrm{e}^{\mathrm{i}\delta t}\right)\left(\sigma_{-}^{(1)} + \sigma_{-}^{(2)}\right) + \text{h.c.}\right)$$

Motion excitation (Harmonic Oscillator)

Electronic excitation (atomic orbitals)

Ω = **Single** qubit Rabi coupling rate of |e> to |g>

δ = Detuning of laser from optical resonance

η = 'Lamb-Dicke' parameter (motional coupling)

$\hat{a}$ = Harmonic Oscillator excitation operator (n=0→n=1)

ν = Harmonic Oscillator frequency detuning (Energy = ħν)

σ_{+} = Electronic excitation operator (g→e)

Ideal detuning $\delta = 2\Omega\eta$

Ideal time of 2Q gate , $t_{end} = \frac{2\pi}{\delta} = \frac{\pi}{\Omega\,\eta}$

```
#Parameters

Omega = 2*pi/(3*2*1e-6)      #Rabi freq from pi time

eta = 0.06                   #Lamb Dicke factor for trap 1.44MHz 674nm

'''

Calculate the ideal detuning

'''

delta = 2*Omega*eta   #assume ideal detuning

'''

Calculate the ideal 2Q gate time

'''

time_end = 2 pi /delta   #assume ideal gate time
```


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