ECE 550/650 – Intro to Quantum Computing

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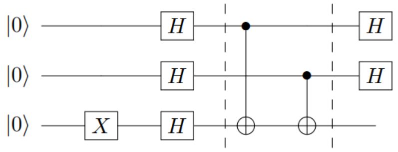
Outline of the course

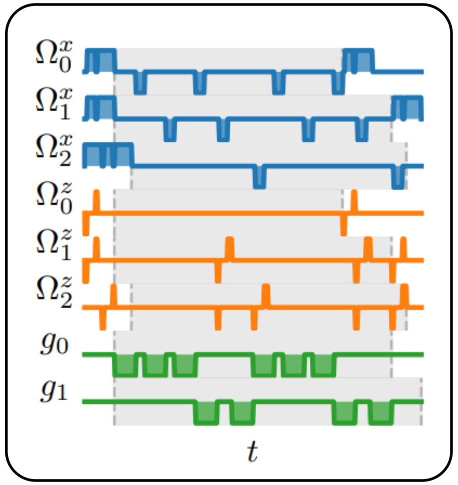
- Quantum Optics
  - What is interference (classical vs. single particle)
  - Superposition of states
  - Measurement and measurement basis
- Atomic physics
  - Spin states in magnetic fields and spin transitions
  - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
  - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
  - Error sources (dephasing, spontaneous decay)
  - Ramsey pulses and Spin echo pulse sequences
  - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
  - Two qubit interactions – gate speed vs. error rates
  - Entanglement – correlation at a distance
  - Bell states and the Bell basis
  - XX gates, Controlled Phase gates, Swap




```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```



- Quantum Hardware
  - Photonics – nonlinear phase shifts
  - Transmons – charge noise, SWAP gate
- Quantum Circuits
  - Single and two qubit gates
  - Hadamard gate , CNOT gate
- Quantum Algorithms
  - Amplitude amplification
  - Grover's Search
  - Oracle - Deutsch Jozsa
  - Bernstein Vazirani
  - Quantum Fourier Transform and period finding
  - Shor's algorithm

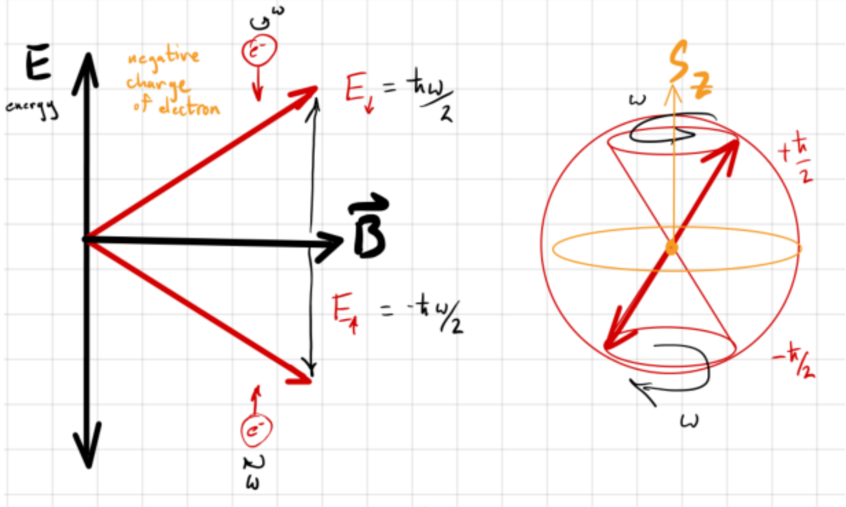
If time permits

- Error Correction
  - Repetition codes
  - Color Codes
  - Surface code

General Solution:

$$|\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\text{Rotation!!!}} \cdot \underbrace{|\psi(t=0)\rangle}_{\text{Initial state}}$$

$$\hat{H} = \sum E_{ij} |E_i\rangle\langle E_j|$$



Energy of the 'wave'!

$$\boldsymbol{E = \hbar\omega_z/2}$$

Phase of the wave!

$$\boldsymbol{\theta = \omega_z \cdot t}$$

Time Dep. Schrodinger Eqn.

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Energy of waves (rotation about Z!)

$$H_{00} = \frac{\hbar\omega_z}{2}|0\rangle\langle 0|$$

$$H_{11} = \frac{-\hbar\omega_z}{2}|1\rangle\langle 1|$$

$$\hat{H} = \begin{bmatrix} \frac{\hbar\omega_z}{2} & 0 \\ 0 & -\frac{\hbar\omega_z}{2} \end{bmatrix}$$

$$\hat{H} = \frac{\hbar\omega_z}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega_z}{2}\cdot\widehat{\sigma_z}$$

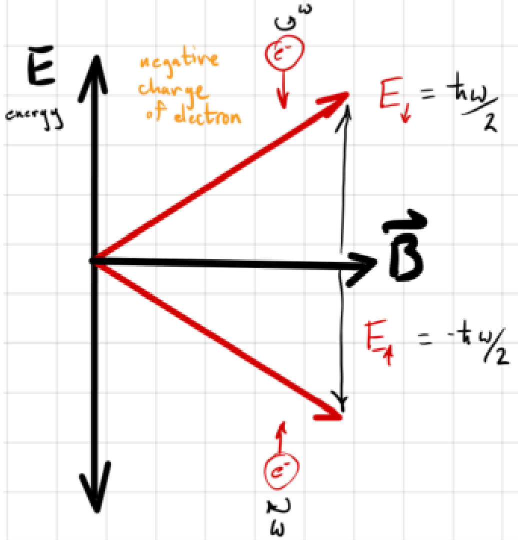
$$\hat{H} \equiv \sum E_{ij} \, |E_i\rangle\langle E_j|$$

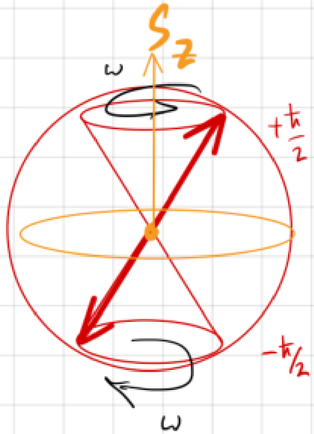
$$\hat{H} = \frac{\hbar\omega}{2} |\uparrow\rangle\langle\uparrow| - \frac{\hbar\omega}{2} |\downarrow\rangle\langle\downarrow|$$

$$= \frac{\hbar\omega}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega}{2} \cdot \hat{\sigma}_Z$$

Time Dep. Schrodinger Eqn.

$$i\,\hbar \, \frac{\partial \hat{\Psi}}{\partial t} = \hat{H} \cdot \hat{\Psi}$$





Energy of waves (rotation about Z!)

$$|\psi(t{=}0)\rangle = |0\rangle \Rightarrow \psi(t) = e^{i\omega t/2} |0\rangle$$

or

$$|\psi(t{=}0)\rangle = |1\rangle \Rightarrow \psi(t) = e^{-i\omega t/2} |1\rangle$$

or

$$|\psi(t{=}0)\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \psi(t) = \frac{1}{\sqrt{2}} \left[e^{i\omega t/2} |0\rangle + e^{-i\omega t/2} |1\rangle \right]$$

$$H_{00} = \frac{\hbar\omega_z}{2} |0\rangle\langle 0|$$

$$H_{11} = \frac{-\hbar\omega_z}{2} |1\rangle\langle 1|$$

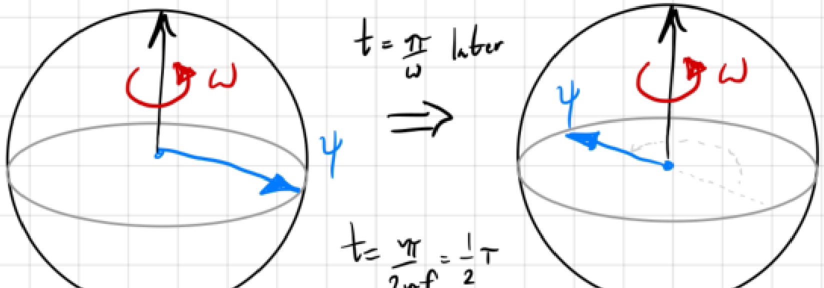
$$\hat{H} = \begin{bmatrix} \frac{\hbar\omega_z}{2} & 0 \\ 0 & -\frac{\hbar\omega_z}{2} \end{bmatrix}$$

σ_z requires 1/2

$$R_Z(\theta) = e^{i\theta \hat{\sigma}_Z/2} = \underbrace{\cos(\theta/2)\,\hat{1} - i \sin(\theta/2)\,\hat{\sigma}_Z}_{\text{factor of } 1/2}$$

no factor of 1/2

$$= \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \qquad \theta = \omega t \implies \psi \text{ rotating about } Z \text{ at } \omega \qquad \omega = 2\pi f$$



Phase of the wave!

$$\boldsymbol{\theta = \omega_z \cdot t}$$

$$\hat{H} = \frac{\hbar\omega_z}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega_z}{2} \cdot \hat{\sigma_z}$$

T.D.S.E. & Rabi Oscillations

$$|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\Psi(t=0)\rangle$$

Let $\hat{H}$ couple two states $\Rightarrow \hat{H} = \frac{\hbar\Omega}{2}|0\rangle\langle 1| + \frac{\hbar\Omega}{2}|1\rangle\langle 0|$

$$\hat{H} = \frac{\hbar\Omega}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar\Omega}{2}\hat{\sigma}_x$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$R_x(\theta) = e^{i\theta\cdot\hat{\sigma}_x/2} \Rightarrow R_x(\Omega\cdot t) = e^{i\Omega\cdot t\,\hat{\sigma}_x/2}$$

Time Dep. Schrodinger Eqn.

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Rabi Coupling (coupling states)

$$H_{01} = \frac{\hbar\Omega_{Rabi}}{2}|0\rangle\langle 1|$$

$$H_{10} = \frac{\hbar\Omega_{Rabi}}{2}|1\rangle\langle 0|$$

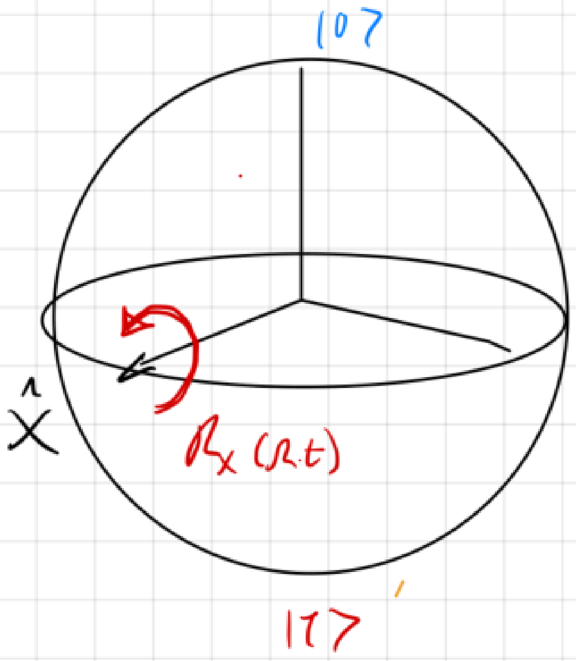
$$\hat{H} = \begin{bmatrix} 0 & \frac{\hbar\Omega_{Rabi}}{2} \\ \frac{\hbar\Omega_{Rabi}}{2} & 0 \end{bmatrix}$$

$$\hat{H} = \frac{\hbar\Omega_{Rabi}}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{\hbar\Omega_{Rabi}}{2}\cdot\widehat{\sigma_x}$$

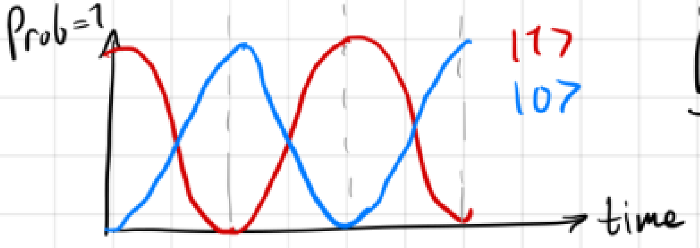
$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(t=0)\rangle$$

$$= e^{-i\cancel{\hbar}\frac{\Omega}{2}\cdot\hat{\sigma}_x\cdot t/\cancel{\hbar}}|\psi(t=0)\rangle$$

$$= \underbrace{e^{-i\Omega t\cdot\hat{\sigma}_x/2}}_{R_x(\Omega\cdot t)\,!!!}|\psi(t=0)\rangle$$



Rotation about Bloch Sphere



Rabi Oscillations

Time Dep. Schrodinger Eqn.

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

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Time Dep. Schrodinger Eqn.

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Time Dep. wavefunction

$$\hat{\Psi} = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

Rabi Coupling (coupling states)

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \begin{bmatrix}0 & \Omega_{Rabi}\\ \Omega_{Rabi} & 0\end{bmatrix}\cdot\hat{\Psi}$$

$$i\,\hbar\,\frac{\partial}{\partial t}\begin{bmatrix}\alpha(t)\\ \beta(t)\end{bmatrix} = \begin{bmatrix}0 & \Omega_{Rabi}\\ \Omega_{Rabi} & 0\end{bmatrix}\cdot\hat{\Psi}$$

case 1: $\delta = 0$ (Resonance)

$\frac{d\alpha}{dt} = \dot{\alpha}$ $\quad \frac{d^2\alpha}{dt^2} = \ddot{\alpha}$

$i\hbar\,\dot{\alpha}(t) = \frac{\hbar\Omega}{2}\beta(t)$

$i\hbar\,\dot{\beta}(t) = \frac{\hbar\Omega}{2}\alpha(t)$

or

$i\hbar\frac{\partial\Psi}{\partial t} = \frac{\hbar\Omega}{2}\sigma_x\cdot\Psi$

$\frac{\partial}{\partial t}\left(i\hbar\,\dot{\alpha}(t)\right) = \frac{\partial}{\partial t}\left(\frac{\hbar\Omega}{2}\cdot\beta(t)\right)$

$i\hbar\,\ddot{\alpha}(t) = \frac{\hbar\Omega}{2}\dot{\beta}(t)$

$\dot{\beta} = \frac{i\cancel{\hbar}\,\ddot{\alpha}}{\cancel{\hbar}\Omega/2} = \frac{i\ddot{\alpha}\cdot 2}{\Omega}$

$i\hbar\dot{\beta} = \frac{\hbar\Omega}{2}\alpha$

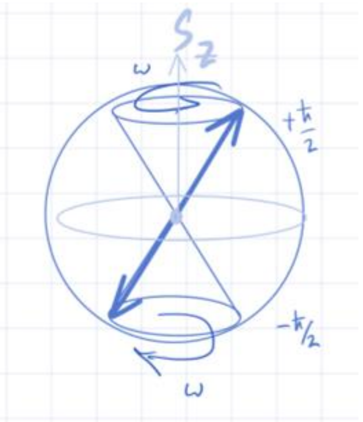
$\dot{\beta} = \frac{\cancel{\hbar}\Omega}{i\cancel{\hbar}2}\alpha = \frac{-i\Omega\alpha}{2}$

equal $\Rightarrow \frac{i\ddot{\alpha}\cdot 2}{\Omega} = \frac{-i\Omega\alpha}{2}$

$\boxed{\ddot{\alpha} = -\frac{\Omega^2}{4}\alpha}$

Energy of states (waves) ≡ rotation about Z!

$$\hat{H} = \begin{bmatrix} \frac{\hbar\omega_z}{2} & 0 \\ 0 & -\frac{\hbar\omega_z}{2} \end{bmatrix}$$



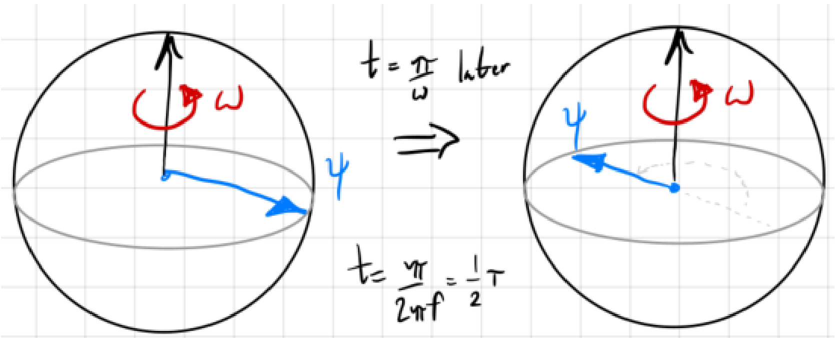
Energy of the 'wave'!

$$E = \hbar\omega_z/2$$

Phase of the wave!

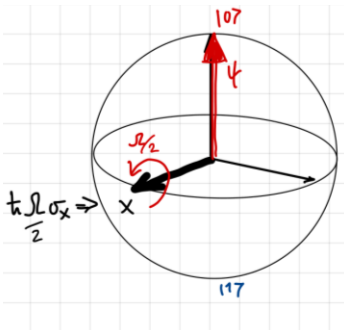
$$\theta = \omega_z \cdot t$$

$$\hat{H} = \frac{\hbar\omega_z}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega_z}{2} \cdot \widehat{\sigma_z}$$

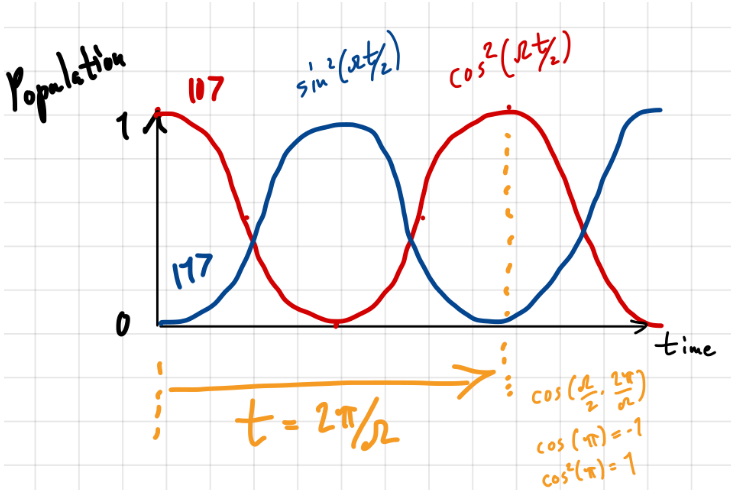


Rabi Coupling (coupling states) ≡ rotation about X (or Y)

$$\hat{H} = \begin{bmatrix} 0 & \frac{\hbar\Omega_{Rabi}}{2} \\ \frac{\hbar\Omega_{Rabi}}{2} & 0 \end{bmatrix}$$



$$\hat{H} = \frac{\hbar\Omega_{Rabi}}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{\hbar\Omega_{Rabi}}{2} \cdot \widehat{\sigma_x}$$



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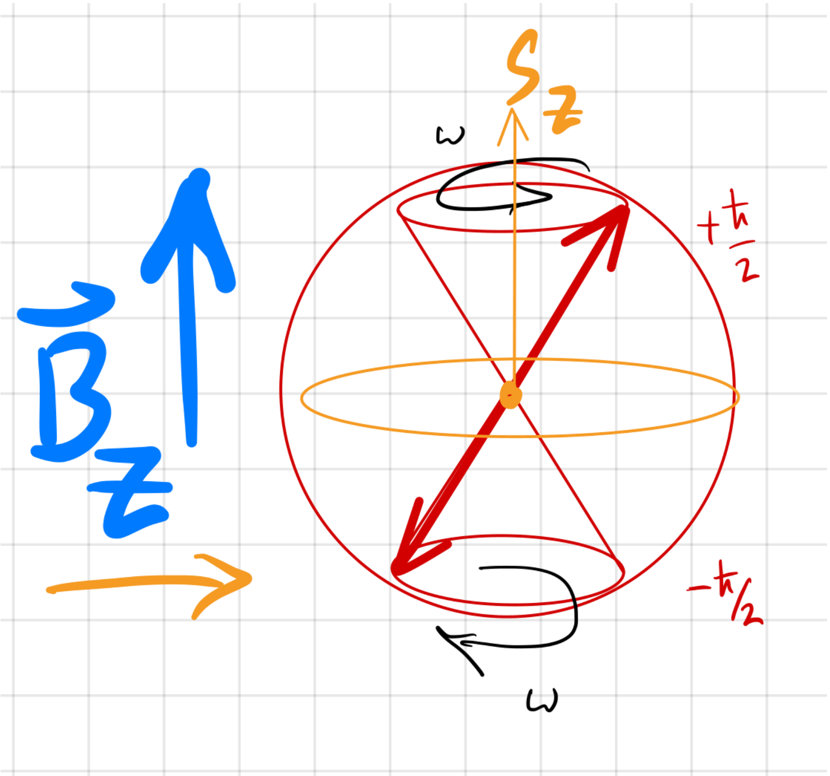
Lecture 2/26

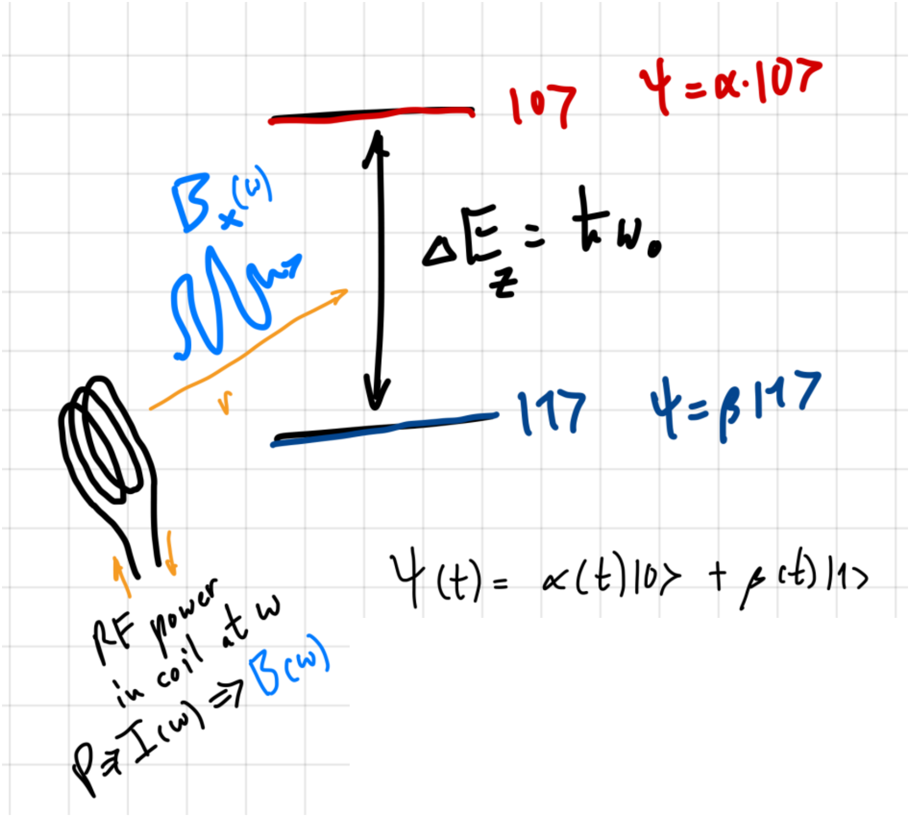
- **The Rotating Wave Approximation**

Applying Radiation (B_x) to Control Qubit State

Strong applied Field ($\overrightarrow{B_z}$) = Defines Basis



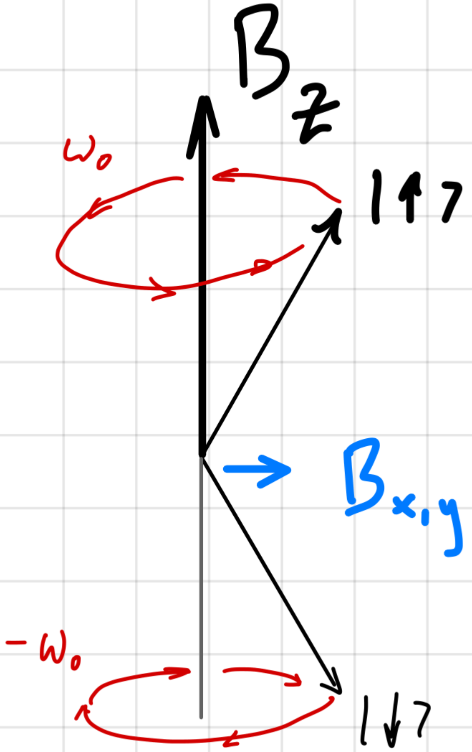
Smaller perturbation($\overrightarrow{B_X}(t)$)
→ Rotates state in stronger fields basis



Perturbation

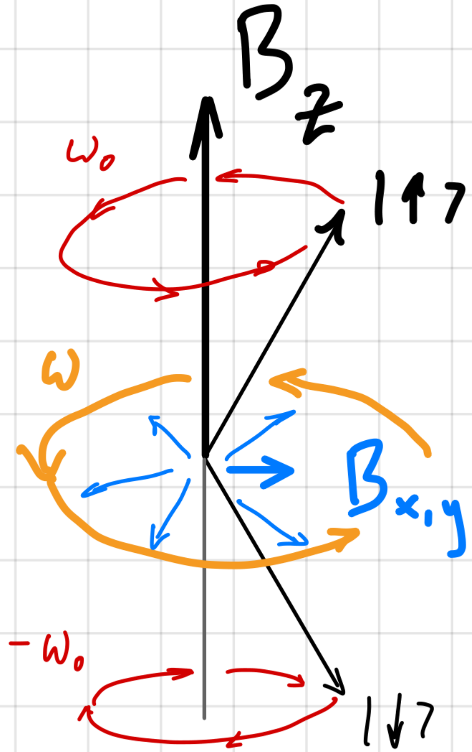
To Flip spin, apply an orthogonal field

Qubit rotating



Rotate $B_{x,y}$ about z with qubit! $\Rightarrow$

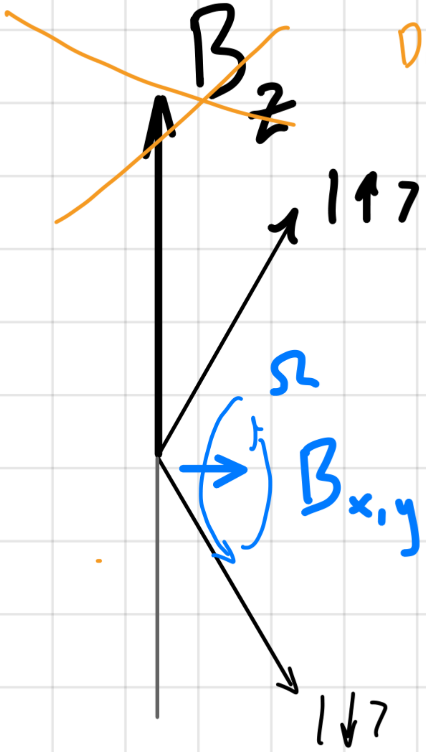
Field rotating with Qubit



Rotating Frame

Qubit rotates about new field

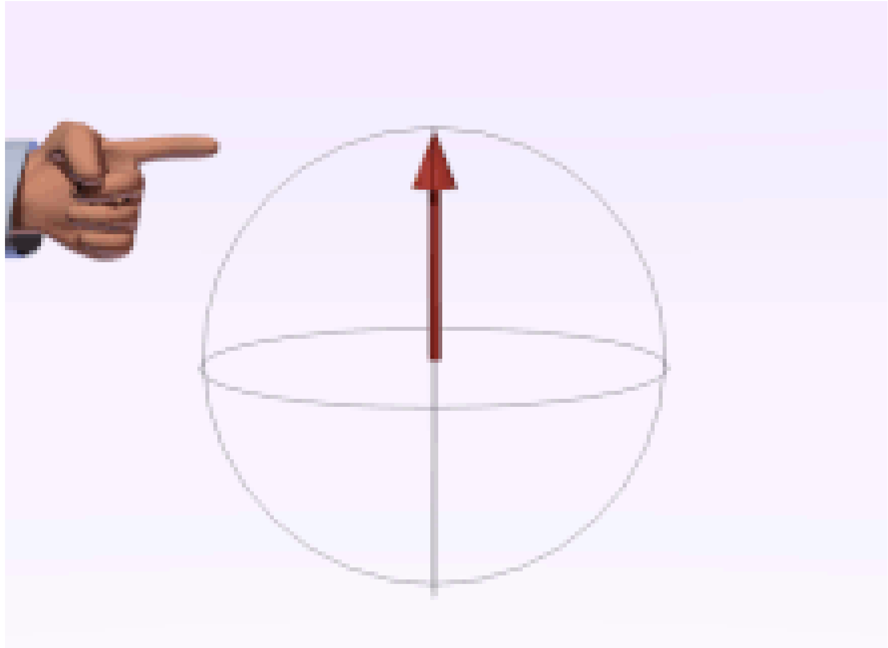
Doesn't "see" B_z in rotating frame



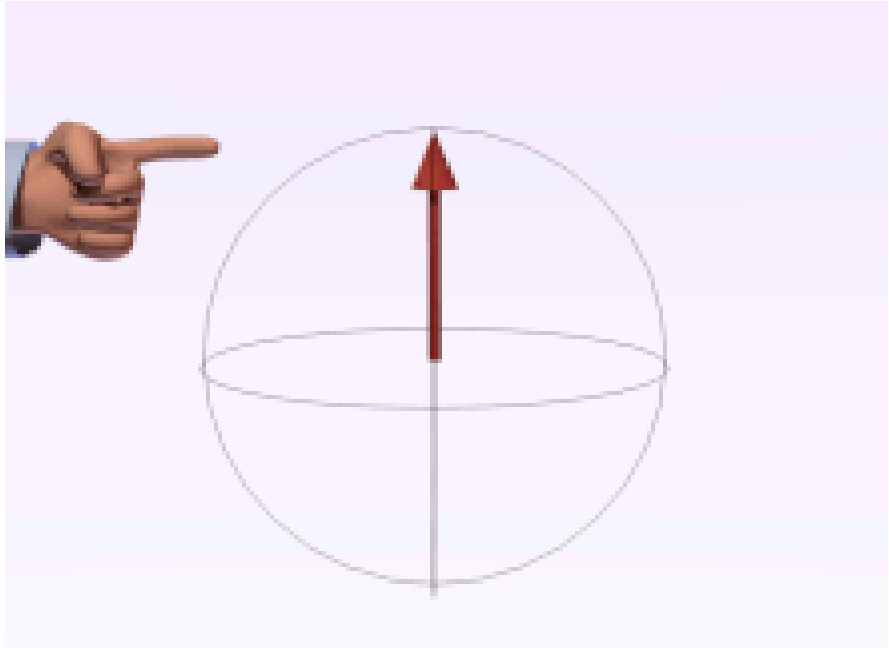
Qubit will now rotate about small perturbing field at rate Ω

Rotating Frame (with rotating Perturbation)

Lab Frame

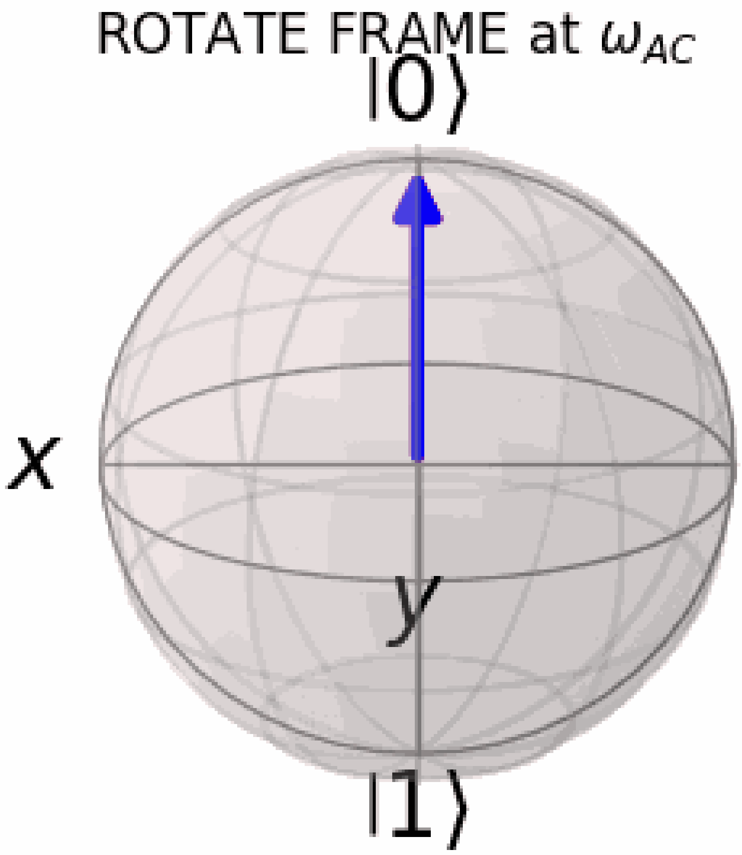


Rotating Frame



https://en.wikipedia.org/wiki/Unitary_transformation_(quantum_mechanics)#Rotating_frame

Rotating Frame



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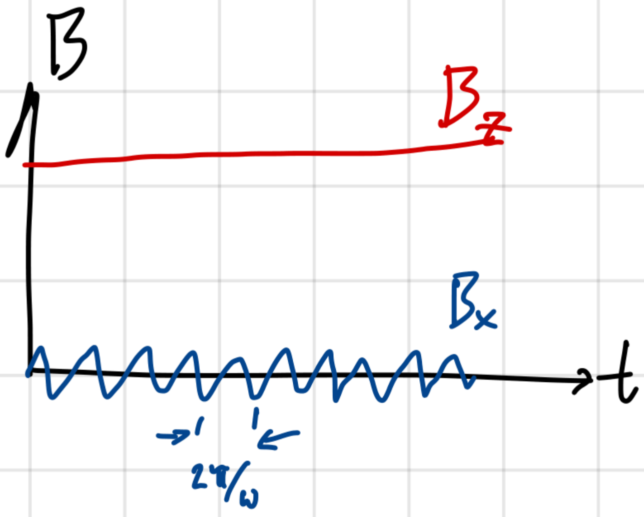
Rotating Frame

about z at rate ω of $\vec{B}_x(t)$

$$\text{Rotation} = \hat{R}_z(\omega\cdot t) = e^{-i\,\omega t/2\cdot\hat{\sigma}_z}$$



Rotating State — Rotation Operator — stationary state

$$|\psi_R(t)\rangle = \underbrace{\hat{R}_z^{\dagger}\cdot|\psi_s(t)\rangle}$$

$$i\hbar\frac{\partial\hat{\psi}}{\partial t} = \hat{H}\hat{\psi} = E\cdot\psi$$

$$i\hbar\frac{\partial}{\partial t}|\psi_R(t)\rangle = i\hbar\frac{\partial}{\partial t}\left[\overbrace{\hat{R}_z^{\dagger}\cdot|\psi_s(t)\rangle}\right]$$

Product rule

$$= \overbrace{i\hbar\,\hat{R}_z^{\dagger}\Big[\underbrace{\frac{\partial}{\partial t}|\psi_s(t)\rangle}_{H_s\cdot|\psi_s(t)\rangle}\Big]} + \overbrace{i\hbar\frac{\partial\hat{R}_z^{\dagger}}{\partial t}\Big[|\psi_s(t)\rangle\Big]}$$

$$= \left[\hat{R}_z^{\dagger}\,\hat{H}_s + i\hbar\frac{\partial\hat{R}_z^{\dagger}}{\partial t}\right]|\psi_s\rangle$$

$$|\psi_R\rangle = \hat{R}_z\cdot|\psi_s\rangle$$

$$|\psi_s\rangle = \underbrace{\hat{R}_z^{\dagger}\cdot|\psi_R\rangle}$$

Product rule

$$= i\hbar\, \hat{R}_z^{+}\left[\frac{\partial}{\partial t}|\psi_s(t)\rangle\right] + i\hbar\,\frac{\partial \hat{R}_z^{+}}{\partial t}\left[|\psi_s(t)\rangle\right]$$

$H_s \cdot |\psi_s(t)\rangle$

$$= \left[\hat{R}_z^{+}\hat{H}_s + i\hbar\,\frac{\partial \hat{R}_z^{+}}{\partial t}\right]|\psi_s\rangle$$

$$|\psi_R\rangle = \hat{R}_z \cdot |\psi_s\rangle$$

$$|\psi_s\rangle = \hat{R}_z^{+} \cdot |\psi_R\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\psi_R(t)\rangle = \underbrace{\hat{R}_z^{+}\cdot\hat{H}_s\cdot\hat{R}_z + i\hbar\,\frac{\partial R_z^{+}}{\partial t}\cdot R_z}_{=H_{Rotating}}\cdot|\psi_R\rangle$$

$$= \hat{H}_R \cdot |\psi_R(t)\rangle$$

Rotation – is - Energy

$$i\hbar\frac{\partial}{\partial t}\left(e^{+i\omega t/2\cdot\hat{\sigma}_z}\right)\cdot e^{-i\omega t/2\,\hat{\sigma}_z}$$

cancel

$$= i\hbar\,(+i)\cdot\frac{\omega}{2}\cdot\hat{\sigma}_z = -\hbar\frac{\omega}{2}\hat{\sigma}_z$$

Rotation of qubit was energy so now rotating w/ qubit cancels Energy

$$\Rightarrow \boxed{\hat{H}_{rotating} = \hat{R}_z^{+}\cdot\hat{H}_s\cdot\hat{R}_z - i\hbar\,\frac{\partial \hat{R}_z}{\partial t}\cdot R_z}$$

In terms of stationary Hamiltonian and Rotation

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$$\Rightarrow \hat{H}_{rotating} = \underbrace{\hat{R}_z^\dagger \cdot \hat{H}_s \cdot \hat{R}_z}_{\text{In terms of stationary Hamiltonian and Rotation}} - i\hbar \frac{\partial \hat{R}_z}{\partial t} \cdot R_z$$

$$\boxed{\hat{H}_{stationary} = \frac{\hbar\omega_0}{2} \cdot \hat{\sigma}_z + \underbrace{\hbar\Omega \cdot \hat{\sigma}_x \cdot \cos(\omega t)}_{\text{Perturbing field coupling spins}}}$$

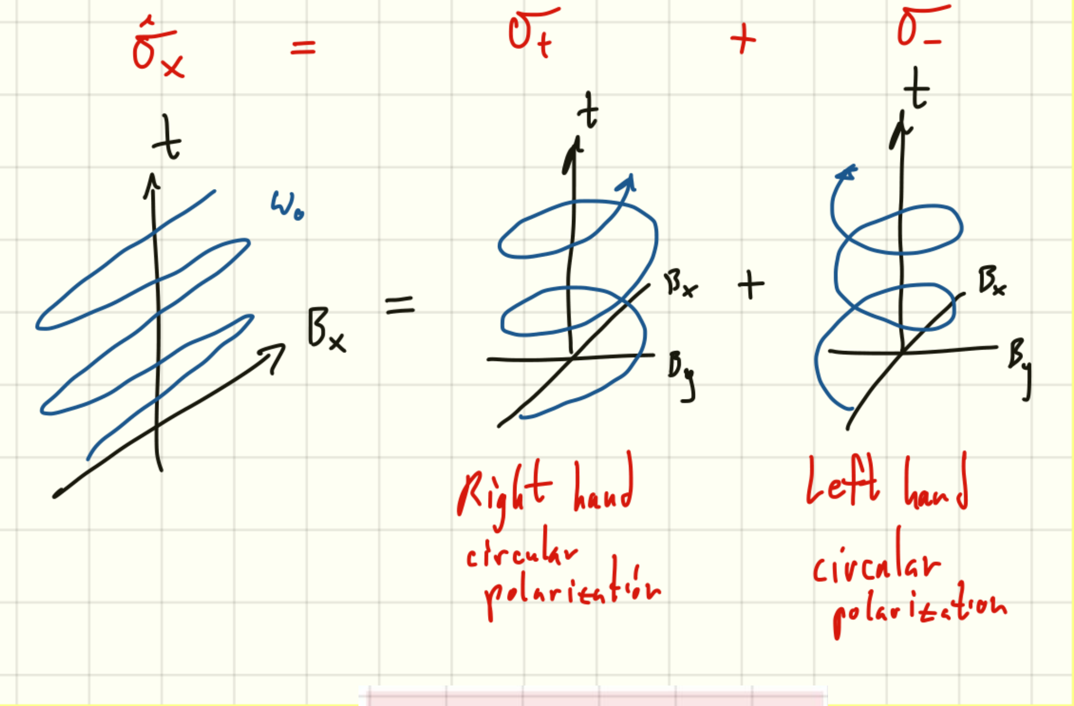
$$\hat{R}_z^\dagger \hat{H}_s \hat{R}_z = \hat{R}_z^\dagger \left[\frac{\hbar\omega_0}{2} \cdot \hat{\sigma}_z + \hbar \cdot \Omega \, \hat{\sigma}_x \cdot \cos(\omega t) \right] \hat{R}_z$$

$$= \hat{R}_z^\dagger \left(\frac{\hbar\omega_0}{2} \cdot \hat{\sigma}_z \right) \hat{R}_z + \hat{R}_z^\dagger \left(\hbar \cdot \Omega \, \hat{\sigma}_x \cdot \cos(\omega t) \right) \hat{R}_z$$

$$= \frac{\hbar\omega_0}{2} \cdot \hat{\sigma}_z + \hat{R}_z^\dagger \left(\hbar\Omega \cdot \hat{\sigma}_x \cdot \cos(\omega t) \right) \hat{R}_z$$

Like Stern-Gerlach projection of basis

X basis = superposition of other bases



$$\hat{\sigma}_+ + \hat{\sigma}_- = \hat{\sigma}_x$$

$$\hat{\sigma}_+ - \hat{\sigma}_- = \hat{\sigma}_y$$

$$= \frac{\hbar\omega_0}{2}\cdot\hat{\sigma}_z + \underbrace{\hat{R}_z^{\dagger}\left(\hbar\Omega\cdot\hat{\sigma}_x\cdot\cos(\omega t)\right)\hat{R}_z}_{=?}$$

Short cut mathematically is to add and subtract $\hat{\sigma}_y$

Similar to how $|+\rangle + |-\rangle = |0\rangle$, $|+\rangle - |-\rangle = |1\rangle$ identities work out

add but also subtract σ_y

$$\hat{\sigma}_x = \frac{\hat{\sigma}_x + \hat{\sigma}_y}{2} + \frac{\sigma_x - \sigma_y}{2} = \hat{\sigma}_+ + \hat{\sigma}_- \Rightarrow \hat{\sigma}_x = \hat{\sigma}_+ + \hat{\sigma}_-$$

$$\hbar\cdot\Omega\cdot\hat{\sigma}_x\cos(\omega t) \Rightarrow \hbar\cdot\Omega\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$$

$$\boxed{\hat{R}_z^{\dagger}\cdot\hat{\sigma}_{\pm}\cdot\hat{R}_z = e^{\pm i\omega t}\hat{\sigma}_{\pm}}$$ rotates faster (or slower) about z

$$\hat{R}_z^{\dagger}\cdot\hat{\sigma}_z\cdot\hat{R}_z = \hat{\sigma}_z$$ stays rotating about z ...

$$\hat{\sigma}_+ + \hat{\sigma}_- = \hat{\sigma}_x$$

$$\hat{\sigma}_+ - \hat{\sigma}_- = \hat{\sigma}_y$$

$$\hat{R}_z^{\dagger} \cdot H_s \hat{R}_z = \hat{R}_z^{\dagger} \cdot \left[\hbar \cdot \Omega \left(\hat{\sigma}_+ + \hat{\sigma}_- \right) \cdot \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right) \right] \cdot \hat{R}_z + \frac{\hbar \omega_0}{2} \hat{\sigma}_z$$

$$= \frac{\hbar \Omega}{2} \cdot \hat{R}_z^{\dagger} \left[\hat{\sigma}_+ e^{i\omega t} + \hat{\sigma}_- e^{i\omega t} + \hat{\sigma}_+ e^{-i\omega t} + \hat{\sigma}_- e^{-i\omega t} \right] \cdot \hat{R}_z + \frac{\hbar \omega_0}{2} \hat{\sigma}_z$$

"Stationary" in Rotating Frame

$$= \frac{\hbar \Omega}{2} \cdot \left[\cancel{\hat{\sigma}_+ e^{2i\omega t}} + \overbrace{\sigma_- \cdot \underbrace{e^{i\omega t - i\omega t}}_{=1} + \hat{\sigma}_+ \underbrace{e^{-i\omega t + i\omega t}}_{=1}} + \cancel{\sigma_- e^{-2i\omega t}} \right] + \frac{\hbar \omega_0}{2} \hat{\sigma}_z$$

Rotating twice as fast $(2 \cdot \omega)$ ⇒ set to zero

Rotating Wave Approximation = Fast dynamics $(2 \cdot \omega)$ average, only "stationary" dynamics important

5. RWA - CCW = stationary , but CW too fast...

$$= \frac{\hbar\Omega}{2}\cdot\Big[\underbrace{\cancel{\hat\sigma_+ e^{2i\omega t}}} + \overbrace{\hat\sigma_-\cdot \underbrace{e^{i\omega t - i\omega t}}_{=1} + \hat\sigma_+ \underbrace{e^{-i\omega t + i\omega t}}_{=1}}^{\text{"Stationary" in Rotating Frame}} + \underbrace{\cancel{\hat\sigma_- e^{-2i\omega t}}}\Big] + \frac{\hbar\omega_0}{2}\hat\sigma_z$$

Rotating twice as fast $(2\cdot\omega)$ ⇒ set to zero

Rotating Wave Approximation ≡ Fast dynamics $(2\cdot\omega)$ average, only "stationary" dynamics important

$$= \frac{\hbar\cdot\Omega}{2}\cdot\left(\hat\sigma_- + \hat\sigma_+\right) + \frac{\hbar\omega_0}{2}\hat\sigma_z$$

$$\boxed{\hat R_z^{+}\cdot\hat H_s\,\hat R_z = \underbrace{\frac{\hbar\Omega}{2}\hat\sigma_x + \frac{\hbar\omega_0}{2}\hat\sigma_z}}\ \blacksquare$$

Stationary!!! No more time dependent terms

Because field is co-rotating with qubit

Now

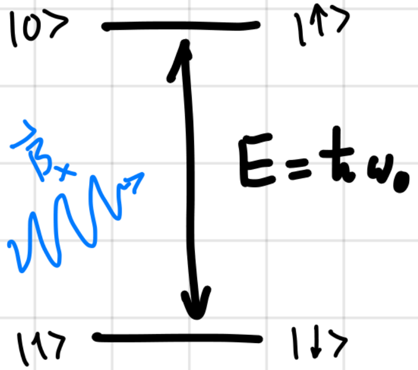
$$\hat{H}_{Rotating} = \hat{R}_z^{\dagger} \cdot \hat{H}_s \cdot \hat{R}_z - i\hbar \frac{\partial R_z}{\partial t} \cdot R_z = \frac{\hbar \omega_0}{2} \cdot \hat{\sigma}_z + \frac{\hbar \Omega}{2} \hat{\sigma}_x - \frac{\hbar \omega}{2} \sigma_z$$

both about z

$$= \frac{\hbar}{2} \underbrace{(\omega_0 - \omega)}_{= \text{detuning "delta" } \delta} \sigma_z + \frac{\hbar \Omega}{2} \hat{\sigma}_x$$

Detuning from resonance $\delta \equiv \omega_0 - \omega$

$$\boxed{\hat{H}_R = \frac{\hbar \delta}{2} \hat{\sigma}_z + \frac{\hbar \Omega}{2} \hat{\sigma}_x = \frac{\hbar}{2} \begin{bmatrix} \delta & \Omega \\ \Omega & -\delta \end{bmatrix}}$$

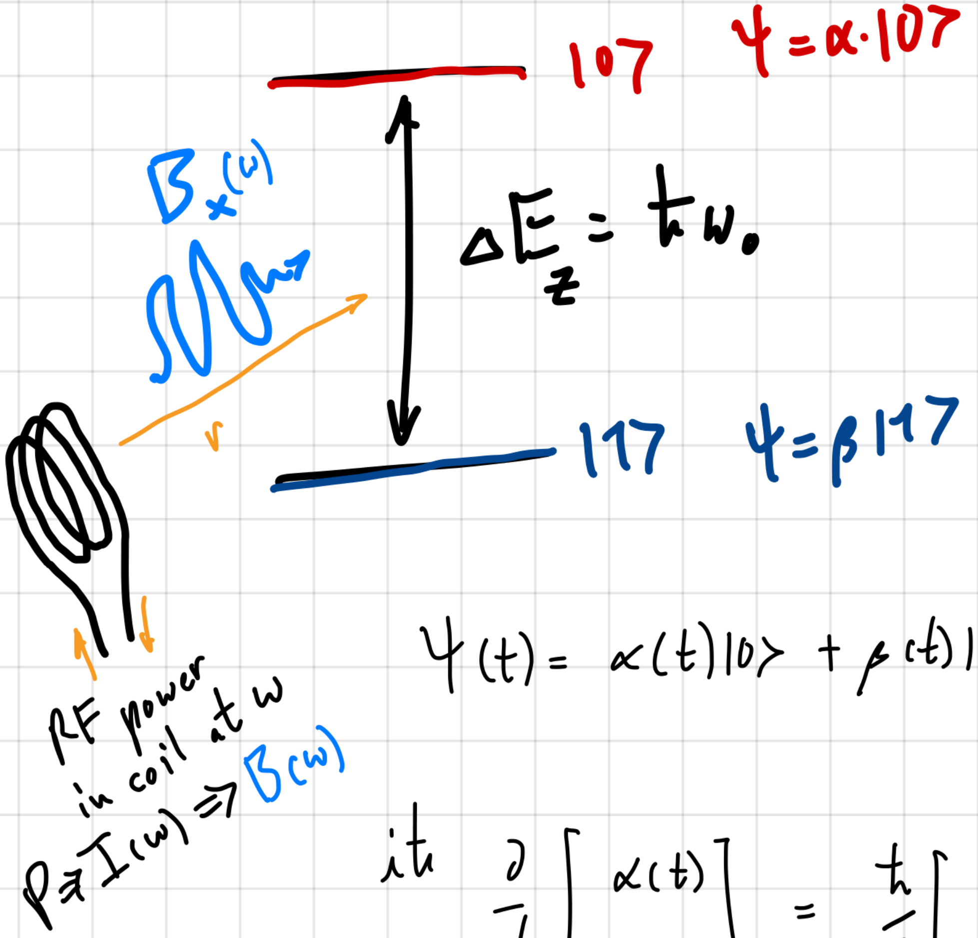


$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \cdot \hat{\psi} = \frac{\hbar}{2} \begin{bmatrix} \delta & \Omega \\ \Omega & -\delta \end{bmatrix} \cdot \psi$$

Lecture 2/28

Rabi Oscillations

Apply fluctuating $\underline{B_x}$ at $\omega = \omega_0$ to flip spin



$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi = \frac{\hbar}{2}\begin{bmatrix} \delta & \Omega \\ \Omega & -\delta \end{bmatrix}\psi$$

$$\omega_0 = \mu_B \frac{g_s}{\hbar} B_0$$

$$\Omega = -\mu_B \frac{g_s}{\hbar} B_x$$

$$\boxed{B_x = 2\sqrt{\frac{\mu_0}{c}\cdot\frac{\text{Power}}{4\pi r^2}}}$$

$$\psi(t) = \alpha(t)|0\rangle + \beta(t)|1\rangle$$

$$i\hbar \frac{\partial}{\partial t}\begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} \delta & \Omega \\ \Omega & -\delta \end{bmatrix}\begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix} =$$

$$i\hbar \frac{\partial \alpha(t)}{\partial t} = \hbar\, \delta/_2 \cdot \alpha(t) + \hbar \frac{\Omega}{2}\cdot \beta(t)$$

$$i\hbar \frac{\partial \beta(t)}{\partial t} = \hbar \frac{\Omega}{2}\cdot \alpha(t) + \hbar\, \delta/_2 \cdot \beta(t)$$

case 1: $\delta = 0$ (Resonance)

$$i\hbar \, \dot{\alpha}(t) = \hbar \frac{\Omega}{2} \beta(t)$$

$$i\hbar \, \dot{\beta}(t) = \hbar \frac{\Omega}{2} \alpha(t)$$

or

$$i\hbar \frac{\partial \psi}{\partial t} = \hbar \frac{\Omega}{2} \sigma_x \cdot \psi$$

$$\frac{d\alpha}{dt} = \dot{\alpha} \qquad \frac{d^2\alpha}{dt^2} = \ddot{\alpha}$$

$$\frac{\partial}{\partial t}\left(i\hbar \, \dot{\alpha}(t)\right) = \frac{\partial}{\partial t}\left(\hbar \frac{\Omega}{2} \cdot \beta(t)\right)$$

$$i\hbar \, \ddot{\alpha}(t) = \hbar \frac{\Omega}{2} \dot{\beta}(t)$$

$$\dot{\beta} = \frac{\cancel{i\hbar}\,\ddot{\alpha}}{\cancel{\hbar}\,\Omega/2} = \frac{i\ddot{\alpha} \cdot 2}{\Omega}$$

$$i\hbar \dot{\beta} = \frac{\hbar\Omega}{2} \alpha$$

$$\dot{\beta} = \frac{\cancel{\hbar}\,\Omega}{\cancel{i\hbar}\,2} \alpha = -i\frac{\Omega}{2}\alpha$$

equal $\Rightarrow$ $\frac{i\ddot{\alpha} \cdot 2}{\Omega} = -i\frac{\Omega}{2}\alpha$

$$\boxed{\ddot{\alpha} = -\frac{\Omega^2}{4}\alpha}$$

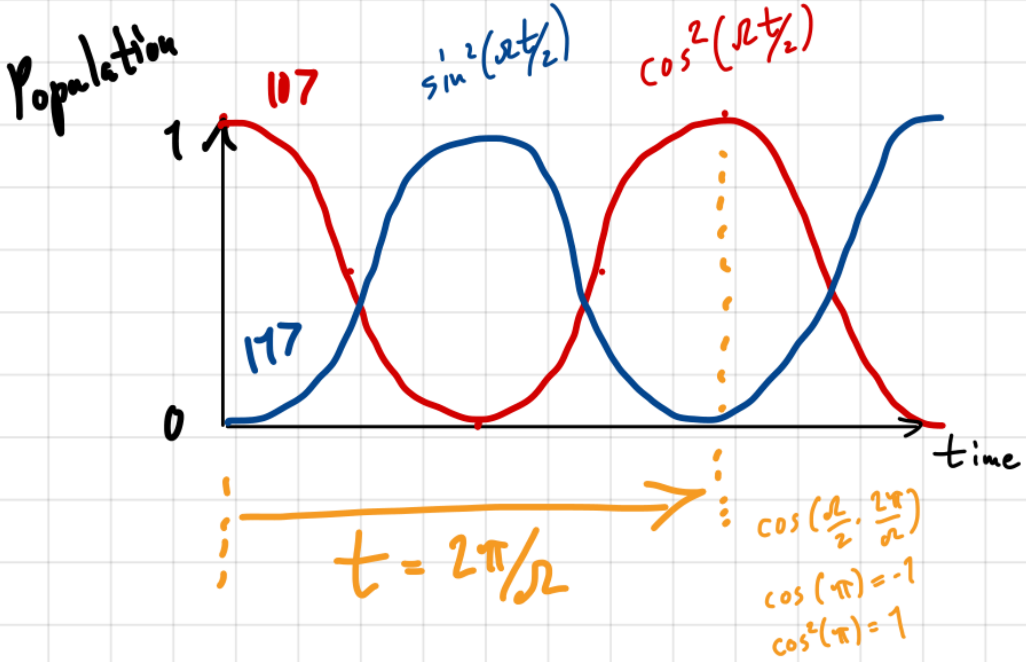
if $\alpha(t=0) = 1$ and $\beta(t=0) = 0$ then $\alpha(t) = \cos\left(\frac{\Omega t}{2}\right)$ $\beta = \sin\left(\frac{\Omega t}{2}\right)$

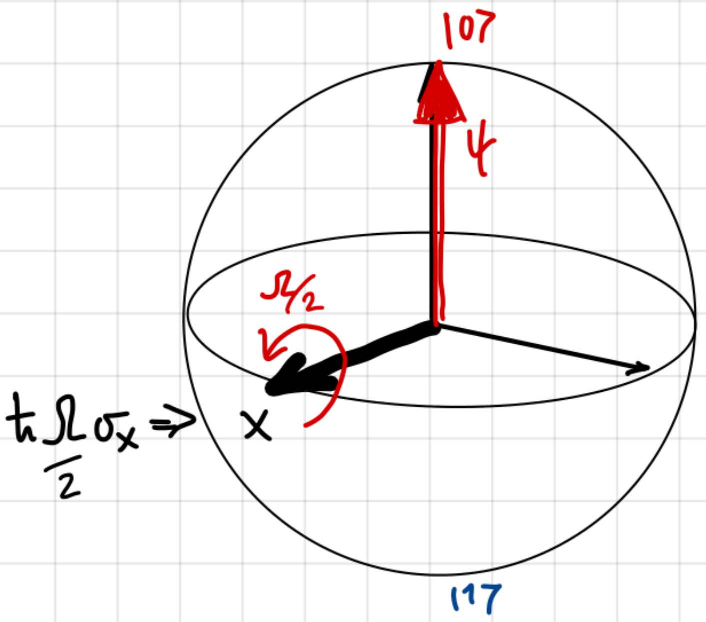
$$\langle \psi | \psi \rangle = \cos^2\left(\frac{\Omega t}{2}\right)|0\rangle + \sin^2\left(\frac{\Omega t}{2}\right)|1\rangle$$

Rabi Oscillations ($\delta = 0$)

if $\alpha(t=0) = 1$ and $\beta(t=0) = 0$ then $\alpha(t) = \cos\left(\frac{\Omega t}{2}\right)$ $\beta = \sin\left(\frac{\Omega t}{2}\right)$

$\langle\psi|\psi\rangle = \cos^2\left(\frac{\Omega t}{2}\right)|0\rangle + \sin^2\left(\frac{\Omega t}{2}\right)|1\rangle$





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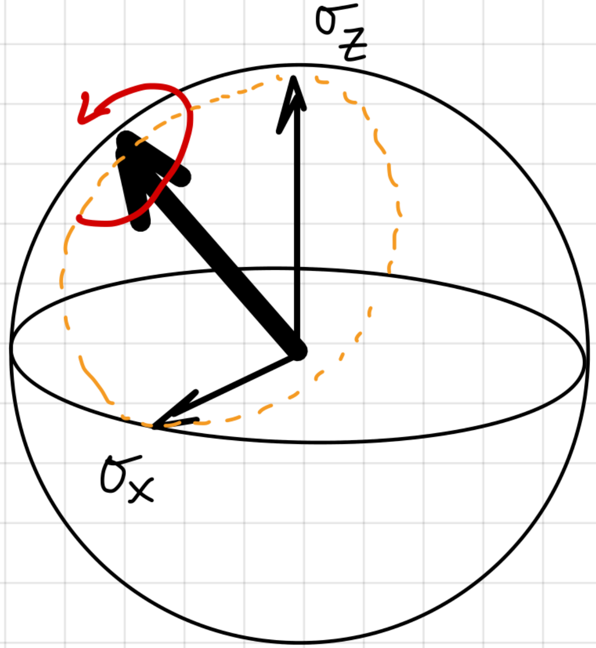
Hadamard Gate

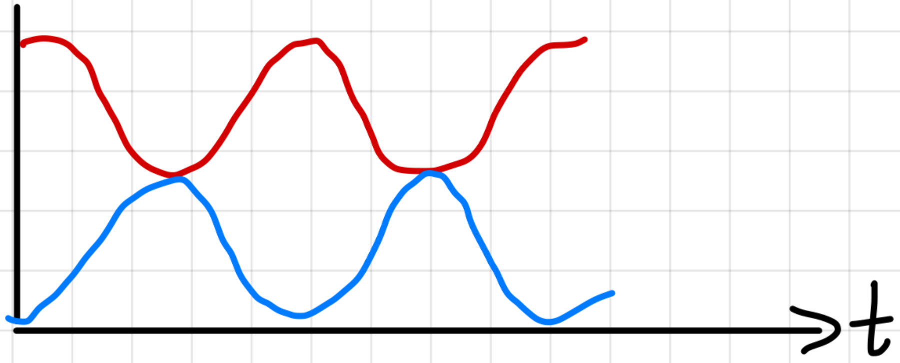
case 2: $\delta = \Omega$

$$\hat{H} = \frac{\hbar\delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega}{2}\hat{\sigma}_x$$

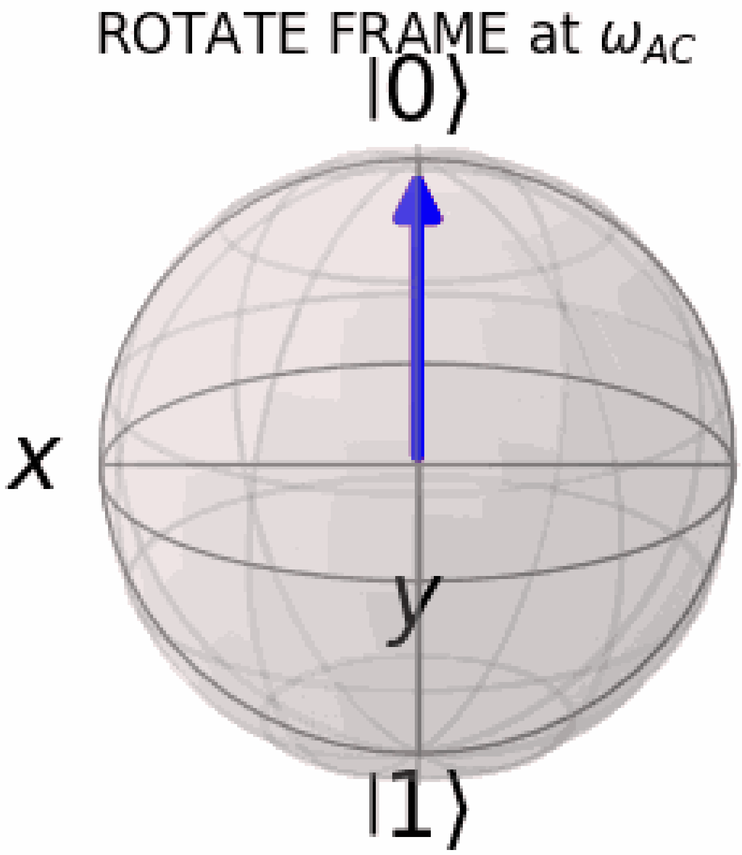
$$= \frac{\hbar}{2}\Omega\,\underbrace{(\hat{\sigma}_z + \hat{\sigma}_x)} \quad \text{or} \quad \frac{\hbar}{2}\delta\,\underbrace{(\hat{\sigma}_z + \hat{\sigma}_x)}$$

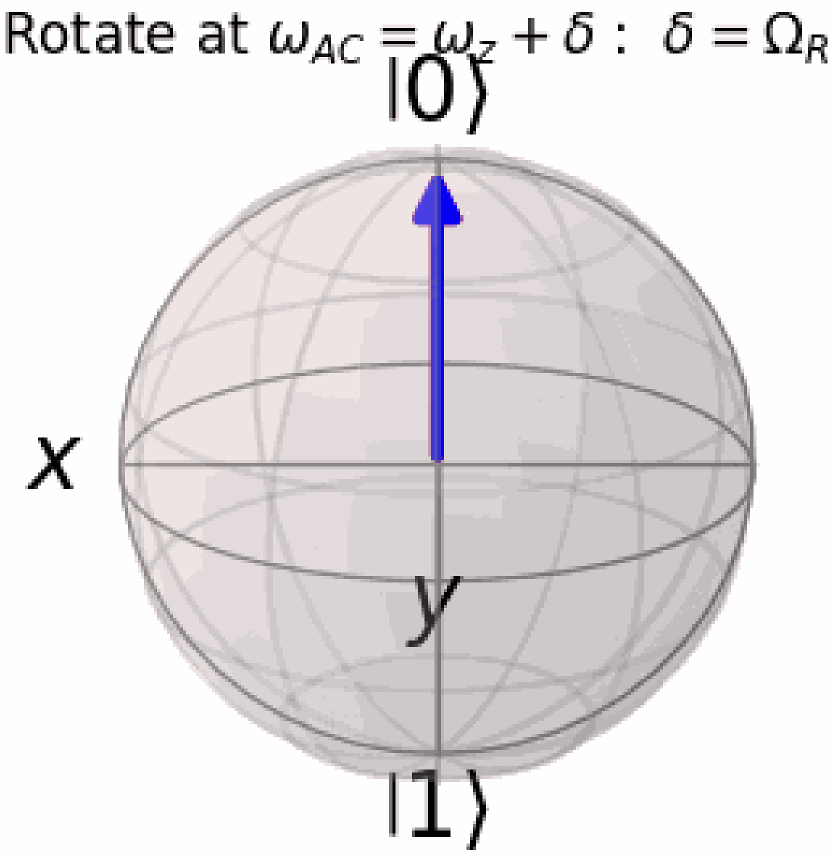
Equal rotation about BOTH $\hat{x}$ and $\hat{z}$ axes



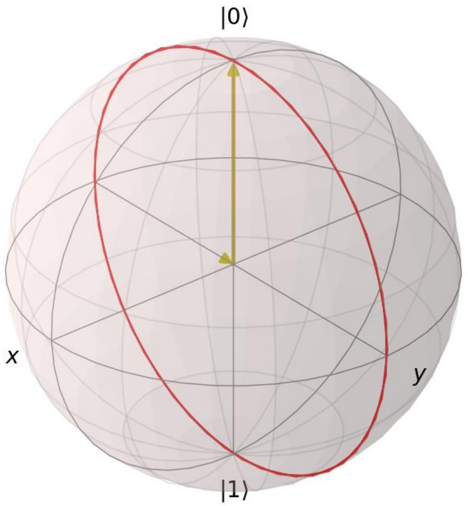


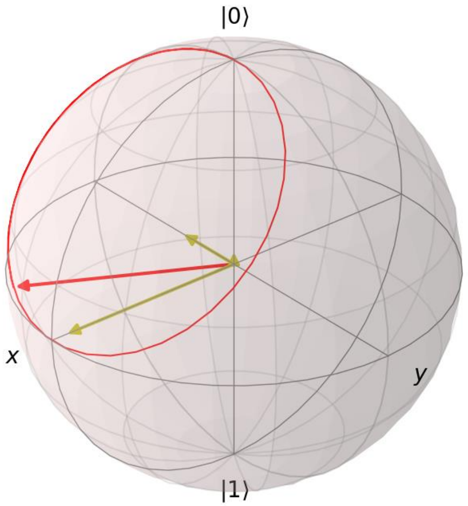
Rotating Frame - with residual relative rotation (***detuning***!)

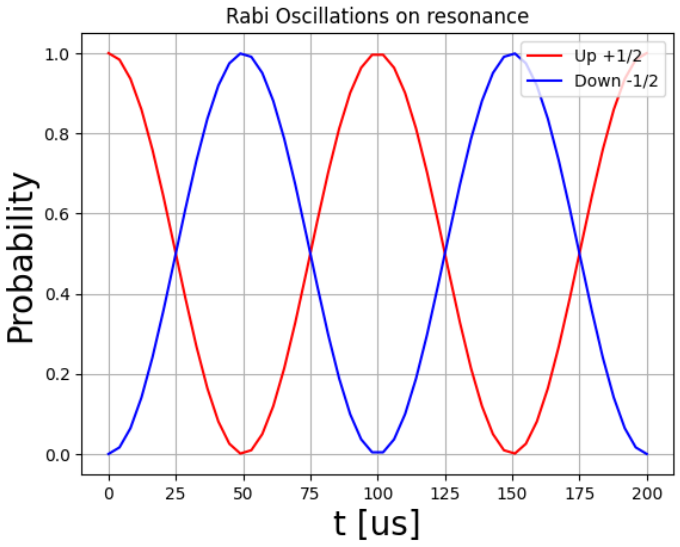


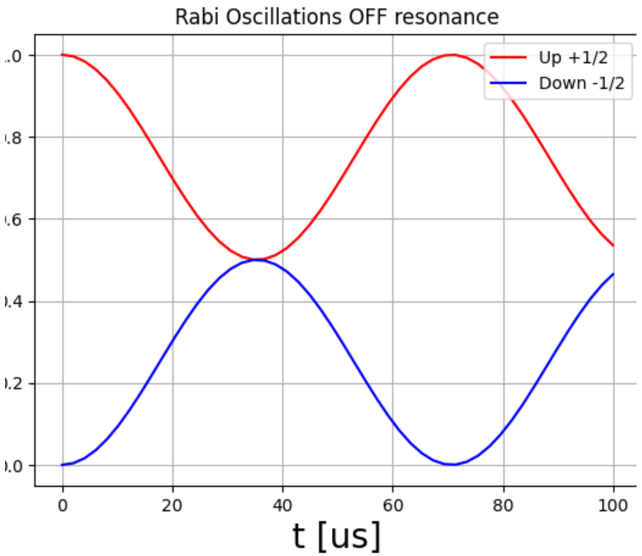


Rabi Oscillations (on resonance)









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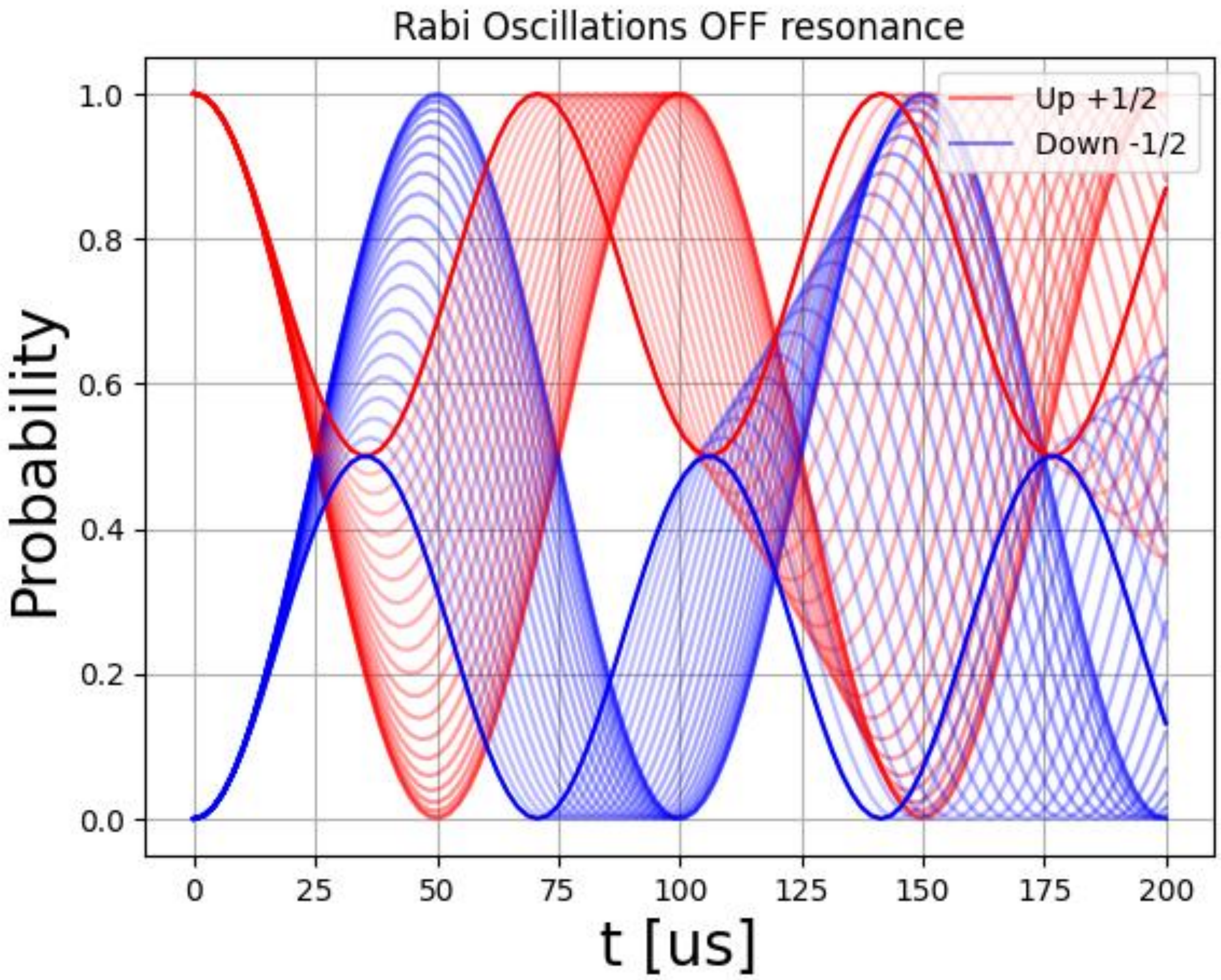

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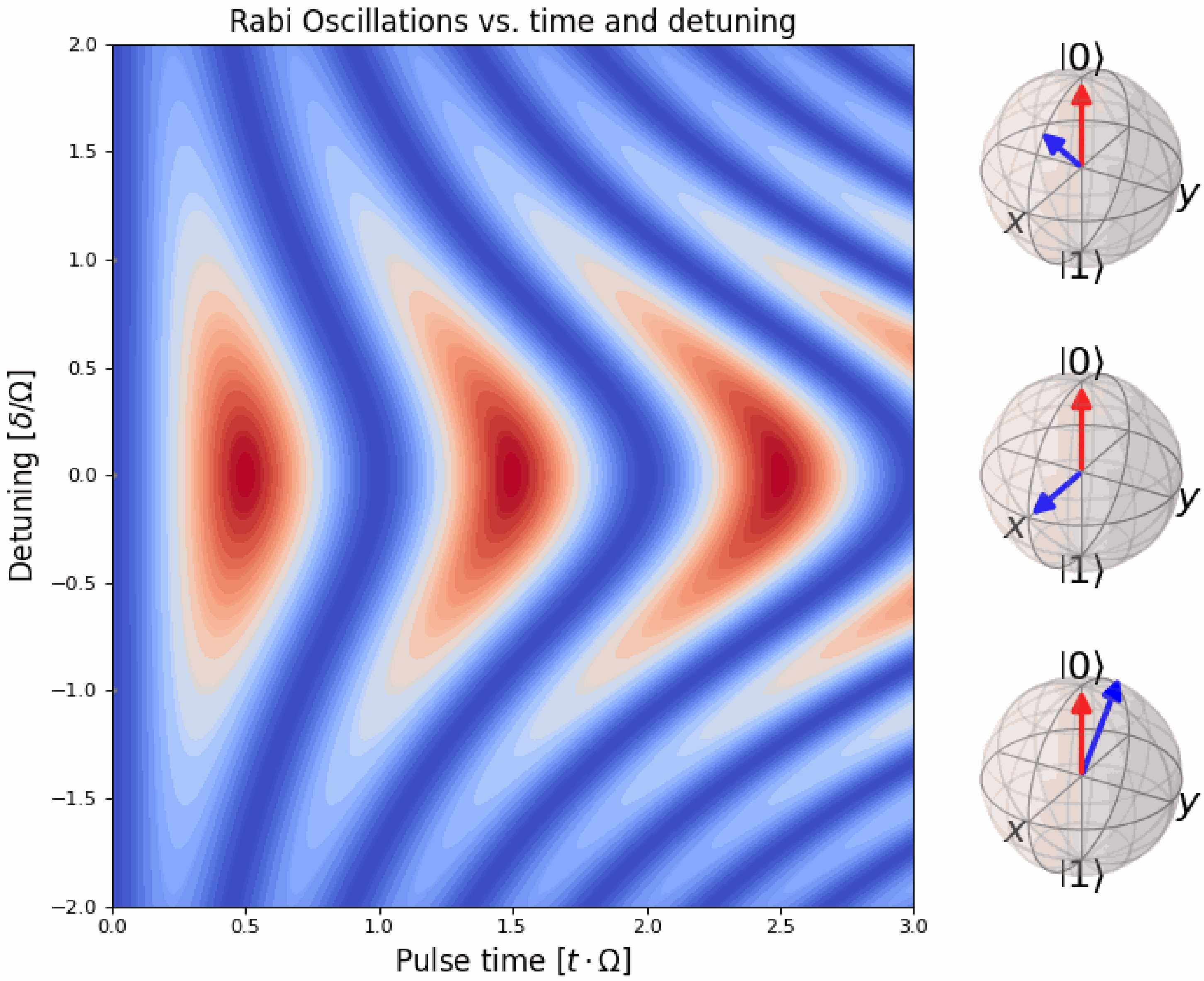

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