ECE 550/650

Homework 2 – XYZ Basis States and Stern Gerlach (20pts)

Please write out your solutions by hand and submit them via Gradescope.

1. Show that the two basis states $|0\rangle \text{ and } |1\rangle$ are orthogonal.

2. Write out a state Ψ that is a superposition state with ¼ in $|0\rangle$ and ¾ in $|1\rangle$ with phase $\phi = \pi$ (Make sure it is normalized to unit probability by checking if $\langle\Psi|\Psi\rangle = 1$)

3. Calculate the state Ψ after each rotation within the MZI with a phase shift. $\Psi_0 = |0\rangle$ $|\Psi_3\rangle = X\left(\frac{-\pi}{2}\right) \cdot Z(\pi) \cdot X\left(\frac{-\pi}{2}\right) \cdot |\Psi_0\rangle$. Draw Ψ on the Bloch sphere after each rotation.
   i. $\Psi_1 =$

   ii. $\Psi_2 =$

   iii. $\Psi_3 =$

4. Compare the probability of getting $|0\rangle_Z$ if you measure the $|+\rangle_X$ and $|-\rangle_X$ states in the Z basis. Bonus: How do you measure the difference between those states?

5. Draw the diagram for the Stern Gerlach Experiment below starting from a thermal source (draw the state on the Bloch sphere at each step):
   I. Z quantization then polarization to $|0\rangle_Z$
   II. X quantization then polarization to $|-\rangle_X$
   III. Y quantization then polarization to $|-i\,\rangle_Y$

6. How many qubits make it through your Stern Gerlach apparatus?