ECE 550/650 – Intro to Quantum Computing

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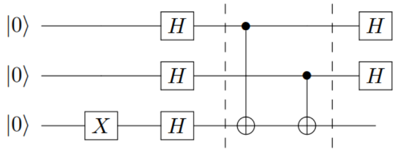
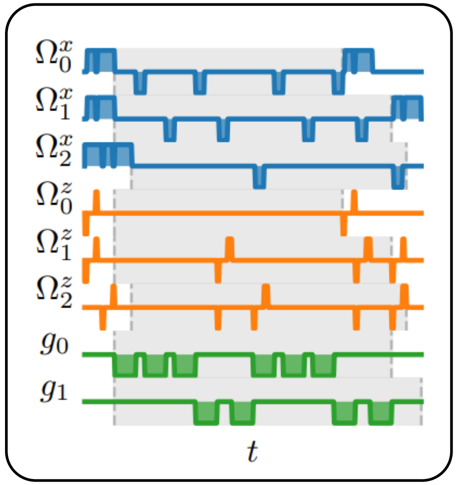
Outline of the course

- Quantum Optics
  - What is interference (classical vs. single particle)
  - Superposition of states
  - Measurement and measurement basis
- Atomic physics
  - Spin states in magnetic fields and spin transitions
  - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
  - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
  - Error sources (dephasing, spontaneous decay)
  - Ramsey pulses and Spin echo pulse sequences
  - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
  - Two qubit interactions – gate speed vs. error rates
  - Entanglement – correlation at a distance
  - Bell states and the Bell basis
  - XX gates, Controlled Phase gates, Swap



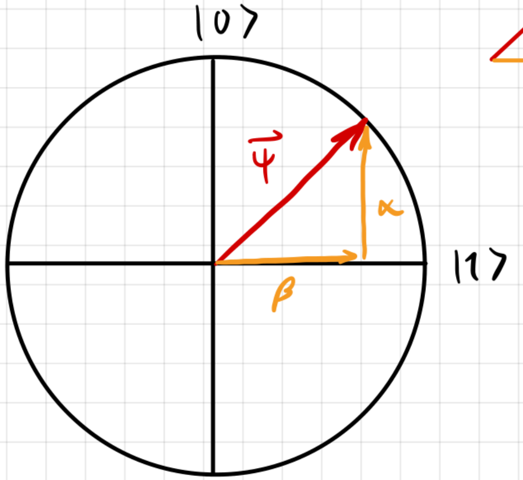

```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```

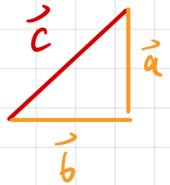


- Quantum Hardware
  - Photonics – nonlinear phase shifts
  - Transmons – charge noise, SWAP gate
- Quantum Circuits
  - Single and two qubit gates
  - Hadamard gate , CNOT gate
- Quantum Algorithms
  - Amplitude amplification
  - Grover's Search
  - Oracle - Deutsch Jozsa
  - Bernstein Vazirani
  - Quantum Fourier Transform and period finding
  - Shor's algorithm

If time permits

- Error Correction
  - Repetition codes
  - Color Codes
  - Surface code





$$\vec{c} = a\hat{y} + b\hat{x}$$

$$c^2 = a^2 + b^2$$

Probability $|\psi|^2 = \alpha^2 + \beta^2 = \langle\psi|\psi\rangle$

state vector $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$|\psi|^2 = \langle\psi|\psi\rangle = (\alpha\langle 0| + \beta\langle 1|)(\alpha|0\rangle + \beta|1\rangle)$$

$$|\psi|^2 = \alpha^2 + \beta^2 = 100\% \text{ (Normalized)}$$

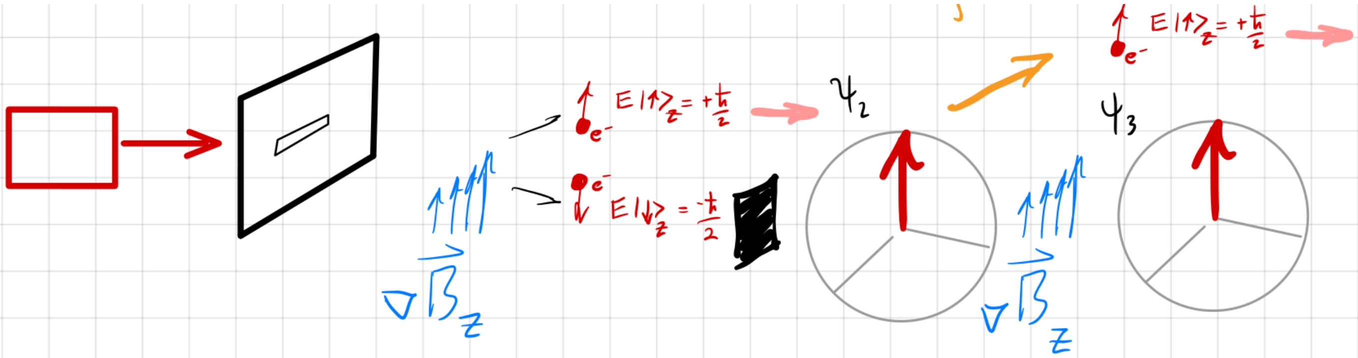
if 25% probability in $|0\rangle$ then $\alpha^2 = 0.25$
and then 75% in $|1\rangle$ so $\beta^2 = 0.75$

$$\alpha = \sqrt{1/4} = 1/2 \qquad \beta = \sqrt{3/4} = \sqrt{3}/2$$

in higher dimensions...

$$\psi = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$$

$$|\psi|^2 = \langle\psi|\psi\rangle = \alpha^2 + \beta^2 + \gamma^2 = 100\%$$



Measuring state along Z twice just gives the same result

$\psi_2 = \psi_3$ (up to global phase)

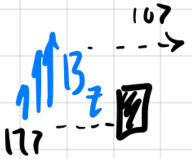
$$\boxed{\text{Projection Operator} = \sum_{i=1}^{n} |\psi_i\rangle\langle\psi_i|}$$

i basis states, $\hat{n}$ basis direction

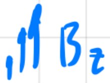
$$|\psi_0\rangle = |\psi_{thermal}\rangle$$

$$|\psi_1\rangle = \sum |\psi_i\rangle_z \langle\psi_i|_z \, |\psi_{thermal}\rangle = |0\rangle\underbrace{\langle 0|\psi_{therm}\rangle}_{1/\sqrt{2}} + |1\rangle\underbrace{\langle 1|\psi_{therm}\rangle}_{1/\sqrt{2}}$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$|\psi_2\rangle = \underbrace{|0\rangle\langle 0|\psi_1\rangle}_{\text{projection only to } |0\rangle} = \frac{1}{\sqrt{2}}|0\rangle \quad \left(\text{polarize } \psi_1 \text{ into } |0\rangle\right)$$



$$|\psi_3\rangle = \sum |\psi_i\rangle_z \langle\psi_i|_z \, |\psi_2\rangle = \frac{1}{\sqrt{2}} \overbrace{\underbrace{|0\rangle\langle 0|0\rangle}_{=1}}^{Z \text{ Basis}} + \frac{1}{\sqrt{2}} \overbrace{\underbrace{|1\rangle\langle 1|0\rangle}_{=0}}^{Z \text{ basis}} = \frac{1}{\sqrt{2}}|0\rangle$$

$$|\psi_3|^2 = \frac{1}{\sqrt{2}}\langle 0|0\rangle\frac{1}{\sqrt{2}} = \frac{1}{2}\langle 0|0\rangle \Rightarrow 50\% \text{ spin up} \checkmark$$

$$\Psi_{therm} \rightarrow \nabla B_z \rightarrow \Psi_1 \rightarrow \text{Polarize} \uparrow_z \rightarrow \Psi_2 \rightarrow \nabla B_x \rightarrow \Psi_3 \rightarrow \text{Polarize} \uparrow_x \rightarrow \Psi_4 \rightarrow \nabla B_y \rightarrow \Psi_5 = ?$$

$\Psi_0 = |\Psi_{therm}\rangle$ = equally all states

∇B_z

$$\Psi_1 = \sum |\psi_i\rangle_z \langle\psi_i|_z \, |\Psi_{therm}\rangle = |0\rangle \underbrace{\langle 0|\Psi_{therm}\rangle}_{=1/\sqrt{2}} + |1\rangle \underbrace{\langle 1|\Psi_{therm}\rangle}_{=1/\sqrt{2}}$$

$$\Psi_1 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

check $|\Psi_1|^2 = 1$ $\quad \langle\Psi_1|\Psi_1\rangle = \frac{1}{\sqrt{2}}(\langle 0| + \langle 1|)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$= \frac{1}{2}(\underbrace{\langle 0|0\rangle}_{=1} + \cancel{\langle 0|1\rangle} + \cancel{\langle 1|0\rangle} + \underbrace{\langle 1|1\rangle}_{=1})$$

$$= \frac{1}{2}(2) = 1 \; \checkmark$$

confirms $\langle 0|\Psi_{therm}\rangle = \frac{1}{\sqrt{2}}$

$\langle 1|\Psi_{therm}\rangle = \frac{1}{\sqrt{2}}$

Polarize to $|\uparrow\rangle_z$ ($|0\rangle_z$)

$$\Psi_2 = |0\rangle\langle 0|\Psi_1\rangle = |0\rangle\langle 0| \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] = \frac{1}{\sqrt{2}}|0\rangle\Big(\underbrace{\langle 0|0\rangle}_{=1} + \underbrace{\cancel{\langle 0|1\rangle}}_{=0}\Big)$$

$$= \frac{1}{\sqrt{2}}|0\rangle \quad \left(\text{also by inspection } \frac{1}{\sqrt{2}}|0\rangle + \cancel{\frac{1}{\sqrt{2}}|1\rangle}\right) \quad |\Psi_2|^2 = \frac{1}{\sqrt{2}}(\langle 0|) \cdot \frac{1}{\sqrt{2}}|0\rangle = \frac{1}{2}\langle 0|0\rangle = 1/2 \; \checkmark$$

∇B_x

$$\Psi_3 = \sum |\psi_i\rangle_x \langle\psi_i|_x \cdot |\Psi_2\rangle = |+\rangle\langle +|\Psi_2\rangle + |-\rangle\langle -|\Psi_2\rangle$$

∇B_x

$= \frac{1}{\sqrt{2}}|0\rangle \left(\frac{1}{\sqrt{2}}|0\rangle + \cancel{\frac{1}{\sqrt{2}}|1\rangle} \right) \quad |\psi_2|^2 = \frac{1}{\sqrt{2}}(\langle 0|) \cdot \frac{1}{\sqrt{2}}|0\rangle = \frac{1}{2}\langle 0|0\rangle = 1/2$ ✓

$$\psi_3 = \sum |\psi_i\rangle_x \langle \psi_i|_x \cdot |\psi_2\rangle = |+\rangle\langle +|\psi_2\rangle + |-\rangle\langle -|\psi_2\rangle$$

$$= |+\rangle\langle +|\left(\frac{1}{\sqrt{2}}|0\rangle\right) + |-\rangle\langle -|\left(\frac{1}{\sqrt{2}}|0\rangle\right)$$

$$= |+\rangle\left(\frac{\langle 0| + \cancel{\langle 1|}^{0}}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}|0\rangle + |-\rangle\left(\frac{\langle 0| - \cancel{\langle 1|}^{0}}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}|0\rangle\right)$$

$$|\psi_3\rangle = |+\rangle \frac{\langle 0|0\rangle}{2} + |-\rangle\frac{\langle 0|0\rangle}{2} = \frac{1}{2}\left(|+\rangle_x + |-\rangle_x\right)$$

$|\psi_3|^2 = 1$?

$$|\psi_3|^2 = \langle \psi_3|\psi_3\rangle = \frac{1}{2}\left(\langle +|_x + \langle -|_x\right) \cdot \frac{1}{2}\left(|+\rangle_x + |-\rangle_x\right)$$

$$= \frac{1}{4}\Big(\underbrace{\langle +|+\rangle}_{=1} + \cancel{\langle +|-\rangle}^{0} + \cancel{\langle -|+\rangle}^{0} + \underbrace{\langle -|-\rangle}_{=1}\Big) = \frac{2}{4} = \frac{1}{2} \; !!!$$

but remember
we Blocked 1/2 !!!
so we expect 1/2 ✓

Polarize to $|-\rangle_x$

$$\psi_4 = |-\rangle_x\langle -|_x\psi_3\rangle = |-\rangle_x\langle -|_x\left(\frac{1}{2}|+\rangle_x + |-\rangle_x\right) = |-\rangle_x \frac{1}{2}\left(\cancel{\langle -|+\rangle}^{0} + \langle -|-\rangle\right)$$

$$= \frac{1}{2}|-\rangle_x$$

(also by inspection ... $\cancel{1/2|+\rangle_x} + 1/2|-\rangle_x$)

∇B_y

$$\psi_5 = \sum |\psi_i\rangle_y \langle \psi_i|_y \cdot |\psi_4\rangle = |{+i}\rangle_y\langle{+i}|_y \frac{|-\rangle_x}{2} + |{-i}\rangle_y\langle{-i}|_y \frac{|-\rangle_x}{2}$$

recall

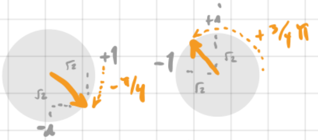
$$|-\rangle_x = \frac{e^{i\pi/4}}{\sqrt2}\left(|i\rangle_y - i|{-i}\rangle_y\right)$$

verify: $\frac{e^{i\pi/4}}{\sqrt2}\left(\frac{|0\rangle + i|1\rangle}{\sqrt2} - i\left(\frac{|0\rangle - i|1\rangle}{\sqrt2}\right)\right)$

$-i\cdot -i = i^2 = -1$

$$\frac{e^{i\pi/4}}{\sqrt2}\left(\frac{|0\rangle - i|0\rangle}{\sqrt2} + \frac{i|1\rangle - |1\rangle}{\sqrt2}\right)$$

$$= \frac{e^{i\pi/4}}{\sqrt2}\left(\underbrace{\frac{(1-i)}{\sqrt2}}|0\rangle + \underbrace{\frac{(i-1)}{\sqrt2}}|1\rangle\right) = \frac{e^{i\pi/4}}{\sqrt2}\left(e^{-i\pi/4}|0\rangle + e^{+i\,3/4\pi}|1\rangle\right)$$



$$= \frac{1}{\sqrt2}\left(e^{0}|0\rangle + e^{i\pi}|1\rangle\right) = \frac{1}{\sqrt2}\left(|0\rangle - |1\rangle\right) = |-\rangle_x \checkmark$$

so $\psi_5 = |{+i}\rangle_y\langle{+i}|_y \frac{e^{i\pi/4}}{\sqrt2}\left(\frac{|i\rangle_y - i\cancel{|{-i}\rangle_y}^{\,0}}{2}\right) + |{-i}\rangle_y\langle{-i}|_y\left(\frac{e^{i\pi/4}}{\sqrt2}\frac{(\cancel{|i\rangle_y}^{\,0} - i|{-i}\rangle_y)}{2}\right)$

$$= |{+i}\rangle_y \frac{e^{i\pi/4}}{\sqrt2}\cdot\frac{\overbrace{\langle +i|+i\rangle}^{=1}}{2} + |{-i}\rangle_y \; -i\cdot\frac{e^{i\pi/4}}{\sqrt2}\,\frac{\overbrace{\langle -i|-i\rangle}^{=1}}{2}$$

$$= \underbrace{\frac{e^{i\pi/4}}{2}}\,\frac{|{+i}\rangle_y}{\sqrt2} + \underbrace{\frac{e^{i\pi/4}}{2}}\,(-i)\,\frac{|{-i}\rangle_y}{\sqrt2}$$

$$= \frac{e^{i\pi/4}}{2}\left(\frac{|{+i}\rangle_y - i|{-i}\rangle_y}{\sqrt2}\right)$$

$$|\psi_5|^2 = \langle\psi_5|\psi_5\rangle = \frac{\cancel{e^{-i\pi/4}}}{2}\left(\frac{\langle +i|_y + i\langle -i|_y}{\sqrt2}\right)\frac{\cancel{e^{+i\pi/4}}}{2}\left(\frac{|{+i}\rangle_y - i|{-i}\rangle_y}{\sqrt2}\right)$$

$$= \frac14\left(\frac{\langle +i|+i\rangle}{2} + \frac{\overbrace{i\cdot(-i)}^{=1}\langle -i|-i\rangle}{2}\right)$$

$1/8$ in $\langle +i|$ and $1/8$ in $\langle -i|$

Each stage splits in half

B_z		B_x		B_y	
$1/2$	$\cdot$	$1/2$	$\cdot$	$1/2$	$= 1/8$

Time Dependent Schrödinger Eq.

$$i\hbar \frac{d\vec{\Psi}}{dt} = E \cdot \vec{\Psi}$$

Describes h
control th
⇒ How we

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

$\hat{H} = $

General Solution:

$$|\Psi(t)\rangle = \underbrace{\boxed{e^{-i\hat{H}t/\hbar}}}_{\text{Rotation!!!}} \cdot \underbrace{|\Psi(t=0)\rangle}_{\text{Initial state}}$$

Arbitrary angle of rotation

We can also rotate the state about the other axes of the Bloch Sphere.

Generically for any axis P = {X,Y,Z} on the Bloch Sphere:

$$\boxed{R_p(\theta) = e^{(-i\theta P/2)}} = \cos(\theta/2)I - i\sin(\theta/2)P$$

Pauli Matrices

X,Y = State (coupling) **Z = Energy/Phase**

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The rotations of the Bloch sphere about the Cartesian axes in the Bloch basis are then given by:

$$R_x(\theta) = e^{(-i\theta X/2)} = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = e^{(-i\theta Y/2)} = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_z(\theta) = e^{(-i\theta Z/2)} = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

The Schrödinger Equation

Wave function

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

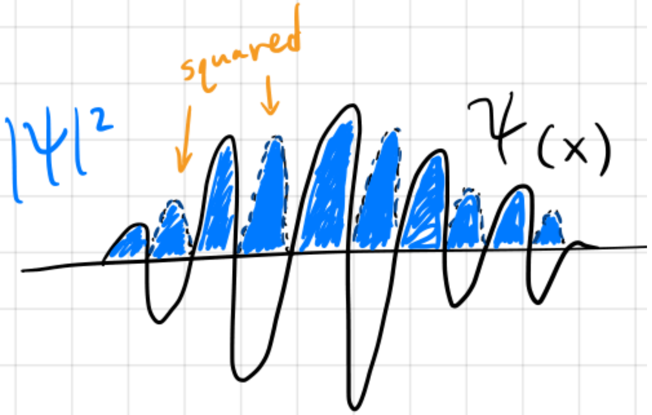
mass

potential Energy (scalar)

$\frac{\partial^2}{\partial x^2}$ $\frac{\partial}{\partial t}$ partial derivatives

$\Psi \equiv$ wavefunction

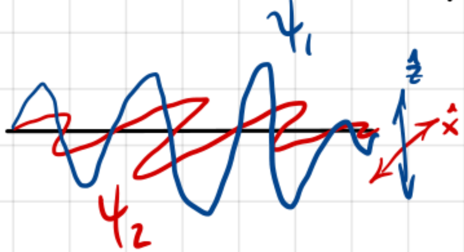
Planck's constant $\equiv \hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}$ J·s



$\langle \Psi_1 | \Psi_2 \rangle = \underbrace{\int \vec{\Psi}_1^* \cdot \vec{\Psi}_2 \, dx}_{\text{Overlap integral!}}$ if $\Psi_1 = \Psi_2$ $\int |\Psi(x,t)|^2 dx =$

How much does each state project (couple) to other.

Normalization $\Rightarrow \int_{-\infty}^{\infty} |\Psi(x,t)|^2 = 1$



$\int \vec{\Psi}_1^* \cdot \vec{\Psi}_2 \, dx = 0 \Rightarrow$ orthogonal! $\langle \Psi_1 | \Psi_2 \rangle = 0$

Seperable Solutions in position & time

if $\Psi(x,t) = \underbrace{\psi(x)}_{\text{position}} \cdot \underbrace{\phi(t)}_{\text{time}}$ if you can separate the wavefunction in space and in time (Not always possible)

$$\underbrace{\frac{\partial \Psi}{\partial t} = \psi \cdot \frac{d\phi}{dt}}_{\text{time}} \qquad \underbrace{\frac{\partial^2 \Psi}{\partial x^2} = \frac{d^2\psi}{dx^2} \cdot \phi}_{\text{position}}$$

Partials become regular derivatives

$$\Rightarrow \frac{i\hbar \cancel{\psi} \frac{d\phi}{dt}}{\cancel{\psi}\phi} = \frac{-\hbar}{2m} \cdot \frac{\frac{d^2\psi}{dx^2}\cancel{\phi}}{\psi\cancel{\phi}} + \frac{V\cancel{\psi\phi}}{\cancel{\psi\phi}} \Rightarrow \overbrace{i\hbar \frac{1}{\phi}\frac{d\phi}{dt}}^{\text{only } \phi} = \overbrace{\frac{-\hbar}{2m}\frac{1}{\psi} \cdot \frac{d^2\psi}{dx^2} + V}^{\text{only } \psi} \Rightarrow \text{separate equations!}$$

(if V const. in t)

$$\Rightarrow i\hbar \frac{1}{\phi} \cdot \frac{d\phi}{dt} = \text{Energy} \Rightarrow \boxed{\frac{d\psi}{dt} = \frac{-iE}{\hbar}\psi}$$

switch from $\phi(t) \Rightarrow \psi(t)$
notation with phi to psi

Time Dependent Schrödinger Eq.

$$i\hbar \frac{d\vec{\Psi}}{dt} = E \cdot \vec{\Psi}$$

Describes how we can control the states in time
$\Rightarrow$ How we will perform gates

solutions $\vec{\Psi}(t) = e^{-iE/\hbar \cdot t} \cdot \vec{\Psi}_0$

Euler $\Rightarrow$ $e^{i\theta} = \cos\theta + i\sin\theta$

Complex wave

$$\Psi(t) = [\cos(-E/\hbar \cdot t) + i\sin(E/\hbar \cdot t)] \cdot \Psi_0$$

$E_{photon} = \hbar\omega = hf$

$\Psi_{photon}(t) = e^{-i\cancel{\hbar}\omega/\cancel{\hbar} \cdot t} \Psi_0 = e^{-i\omega t} \Psi_0 \Rightarrow$ phase of photon wave in time from the energy!!!

Time Independent Schrödinger Eq.

static

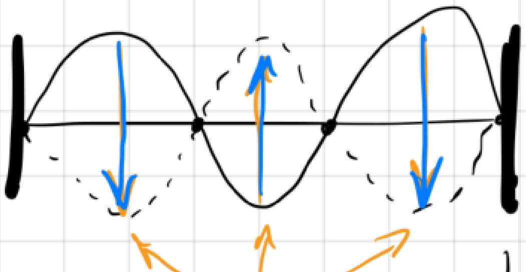
$$\hat{H}\Psi(x) = E\,\Psi(x)$$

Eigen Vector — Eigen Value · Eigen Vector

$$\hat{H} = \underbrace{\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}}_{\text{Kinetic Energy}} + \underbrace{V(x)}_{\text{Potential Energy}}$$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m}$$

$$v = \frac{dx}{dt} \qquad p = -i\hbar\frac{d}{dx}$$



Where is K.E.? ⇒ where 2nd derivative is largest (largest p^2, $\frac{d^2}{dx^2}$)

Solving is important for defining qubits and solving for energies.

However, for this course we will assume they are already two-level systems (well defined) and we have a good idea of their energy

Generalized Pauli Matrices

minus i is 'high'

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Couples states like reflections in MZI

Phase rotations of each state in opposite dir.

Couples states like σ_x but with phase

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T.D.S.E. (Time Dep. Schr. Eq.)

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

$\hat{H} \equiv$ Hamiltonian (Energy operator)
(similar to a Lagrangian)

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

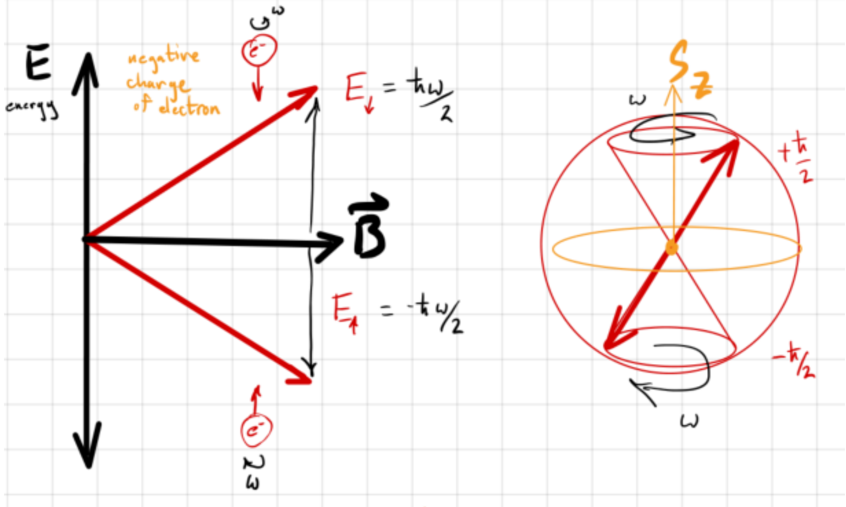
General Solution:

$$|\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\text{Rotation!!!}} \cdot \underbrace{|\psi(t=0)\rangle}_{\text{Initial state}}$$

General Solution:

$$|\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\text{Rotation!!!}} \cdot \underbrace{|\psi(t=0)\rangle}_{\text{Initial state}}$$

$$\hat{H} \equiv \sum E_{ij} \, |E_i\rangle\langle E_j|$$



Energy of the 'wave'!

$$E = \hbar\omega_z/2$$

Phase of the wave!

$$\theta = \omega_z \cdot t$$

Time Dep. Schrodinger Eqn.

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Energy of waves (rotation about Z!)

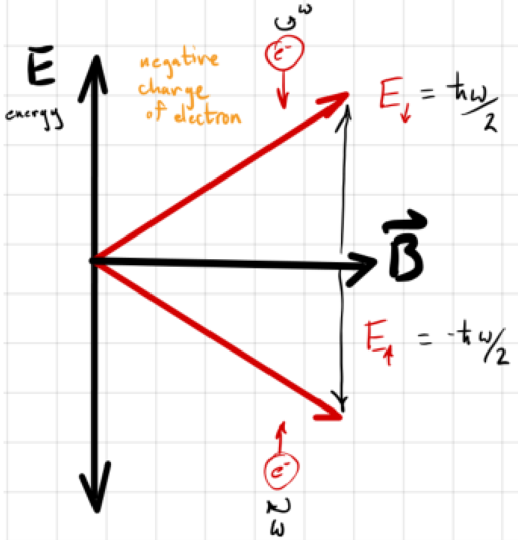
$$H_{00} = \frac{\hbar\omega_z}{2}|0\rangle\langle 0|$$

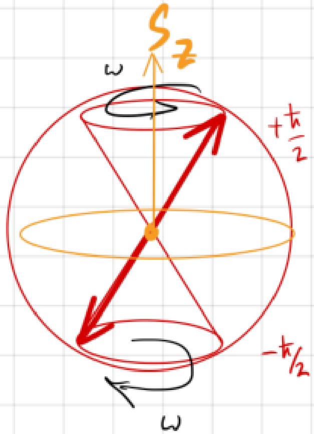
$$H_{11} = \frac{-\hbar\omega_z}{2}|1\rangle\langle 1|$$

$$\hat{H} = \begin{bmatrix} \frac{\hbar\omega_z}{2} & 0 \\ 0 & -\frac{\hbar\omega_z}{2} \end{bmatrix}$$

$$\hat{H} = \frac{\hbar\omega_z}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega_z}{2}\cdot\hat{\sigma_z}$$

$$\hat{H} \equiv \sum E_{ij} |E_i\rangle\langle E_j|$$





$$\hat{H} = \frac{\hbar\omega}{2}|\uparrow\rangle\langle\uparrow| - \frac{\hbar\omega}{2}|\downarrow\rangle\langle\downarrow|$$

$$= \frac{\hbar\omega}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar\omega}{2}\cdot\hat{\sigma}_z$$

$$|\psi(t=0)\rangle = |0\rangle \Rightarrow \psi(t) = e^{i\omega t/2}|0\rangle$$

or

$$|\psi(t=0)\rangle = |1\rangle \Rightarrow \psi(t) = e^{-i\omega t/2}|1\rangle$$

or

$$|\psi(t=0)\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow$$

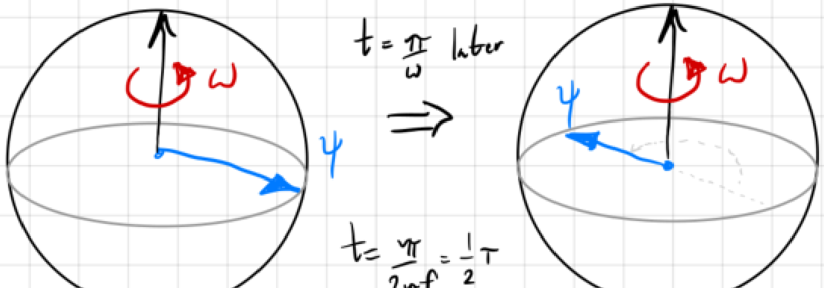
$$\Rightarrow \psi(t) = \frac{1}{\sqrt{2}}\left[e^{i\omega t/2}|0\rangle + e^{-i\omega t/2}|1\rangle\right]$$

σ_z requires ½

$$R_z(\theta) = e^{i\theta\,\hat{\sigma}_z/2} = \underbrace{\cos(\theta/2)\,\hat{\mathbb{1}} - i\sin(\theta/2)\,\hat{\sigma}_z}_{\text{factor of } 1/2}$$

no factor of ½

$$= \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2}\end{bmatrix} \qquad \theta = \omega t \quad \Longrightarrow \quad \psi \text{ rotating about } Z \text{ at } \omega \qquad \omega = 2\pi f$$



Time Dep. Schrodinger Eqn.

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Energy of waves (rotation about Z!)

$$H_{00} = \frac{\hbar\omega_z}{2}|0\rangle\langle 0|$$

$$H_{11} = \frac{-\hbar\omega_z}{2}|1\rangle\langle 1|$$

$$\hat{H} = \begin{bmatrix}\frac{\hbar\omega_z}{2} & 0\\ 0 & -\frac{\hbar\omega_z}{2}\end{bmatrix}$$

Phase of the wave!

$$\boldsymbol{\theta = \omega_z \cdot t}$$

$$\hat{H} = \frac{\hbar\omega_z}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar\omega_z}{2}\cdot\widehat{\sigma_z}$$

T.D.S.E. & Rabi Oscillations

$$|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar} |\Psi(t=0)\rangle$$

Let $\hat{H}$ couple two states $\Rightarrow \hat{H} = \frac{\hbar\Omega}{2}|0\rangle\langle 1| + \frac{\hbar\Omega}{2}|1\rangle\langle 0|$

$$\hat{H} = \frac{\hbar\Omega}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar\Omega}{2}\hat{\sigma}_x$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$R_x(\theta) = e^{i\theta \cdot \hat{\sigma}_x/2} \Rightarrow R_x(\Omega \cdot t) = e^{i\Omega t \hat{\sigma}_x/2}$$

Time Dep. Schrodinger Eqn.

$$i\hbar \frac{\partial \hat{\Psi}}{\partial t} = \hat{H} \cdot \hat{\Psi}$$

Rabi Coupling (coupling states)

$$H_{01} = \frac{\hbar\Omega_{Rabi}}{2}|0\rangle\langle 1|$$

$$H_{10} = \frac{\hbar\Omega_{Rabi}}{2}|1\rangle\langle 0|$$

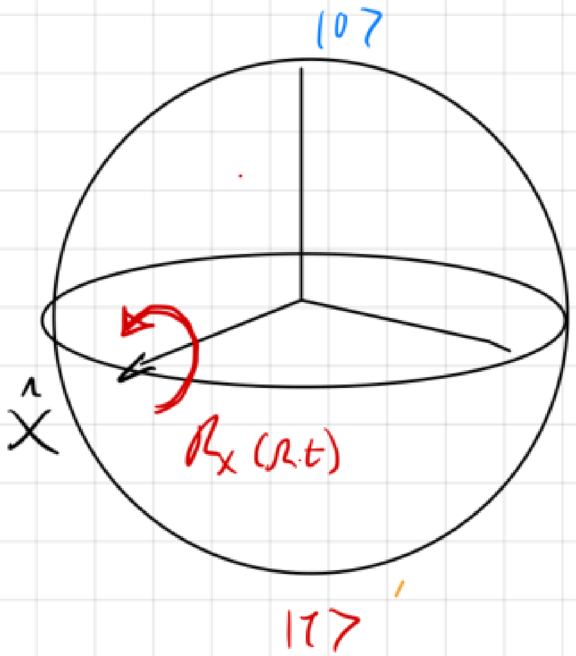
$$\hat{H} = \begin{bmatrix} 0 & \frac{\hbar\Omega_{Rabi}}{2} \\ \frac{\hbar\Omega_{Rabi}}{2} & 0 \end{bmatrix}$$

$$\hat{H} = \frac{\hbar\Omega_{Rabi}}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{\hbar\Omega_{Rabi}}{2} \cdot \hat{\sigma_x}$$

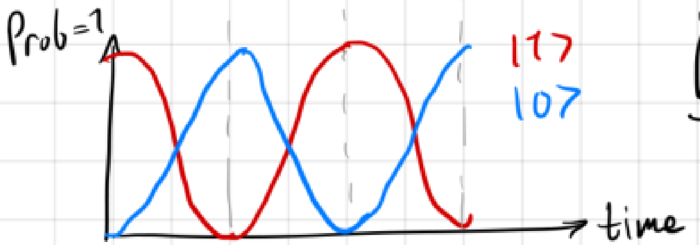
$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(t=0)\rangle$$

$$= e^{-i\cancel{\hbar}\frac{\Omega}{2}\cdot\hat{\sigma}_x\cdot t/\cancel{\hbar}}|\psi(t=0)\rangle$$

$$= \underbrace{e^{-i\Omega t\cdot\hat{\sigma}_x/2}}_{R_x(\Omega\cdot t)\,!!!}|\psi(t=0)\rangle$$



Rotation about Bloch Sphere



Rabi Oscillations

Time Dep. Schrodinger Eqn.

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Rabi Coupling (coupling states)

$$H_{01} = \frac{\hbar\Omega_{Rabi}}{2}|0\rangle\langle 1|$$

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$$\hat{H} = \begin{bmatrix} 0 & \frac{\hbar\Omega_{Rabi}}{2} \\ \frac{\hbar\Omega_{Rabi}}{2} & 0 \end{bmatrix}$$

$$\hat{H} = \frac{\hbar\Omega_{Rabi}}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{\hbar\Omega_{Rabi}}{2}\cdot\widehat{\sigma_x}$$

Time Dep. Schrodinger Eqn.

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \hat{H}\cdot\hat{\Psi}$$

Time Dep. wavefunction

$$\hat{\Psi} = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

Rabi Coupling (coupling states)

$$i\,\hbar\,\frac{\partial\hat{\Psi}}{\partial t} = \begin{bmatrix}0 & \Omega_{Rabi}\\ \Omega_{Rabi} & 0\end{bmatrix}\cdot\hat{\Psi}$$

$$i\,\hbar\,\frac{\partial}{\partial t}\begin{bmatrix}\alpha(t)\\ \beta(t)\end{bmatrix} = \begin{bmatrix}0 & \Omega_{Rabi}\\ \Omega_{Rabi} & 0\end{bmatrix}\cdot\hat{\Psi}$$

case 1: $\delta = 0$ (Resonance)

$\frac{d\alpha}{dt} = \dot{\alpha}$ $\quad \frac{d^2\alpha}{dt^2} = \ddot{\alpha}$

$i\hbar\,\dot{\alpha}(t) = \frac{\hbar\Omega}{2}\beta(t)$

$i\hbar\,\dot{\beta}(t) = \frac{\hbar\Omega}{2}\alpha(t)$

or

$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar\Omega}{2}\sigma_x\cdot\psi$

$\frac{\partial}{\partial t}\left(i\hbar\,\dot{\alpha}(t)\right) = \frac{\partial}{\partial t}\left(\hbar\frac{\Omega}{2}\cdot\beta(t)\right)$

$i\hbar\,\ddot{\alpha}(t) = \hbar\frac{\Omega}{2}\dot{\beta}(t)$

$\dot{\beta} = \frac{i\cancel{\hbar}\ddot{\alpha}}{\cancel{\hbar}\Omega/2} = \frac{i\ddot{\alpha}\cdot 2}{\Omega}$

$i\hbar\dot{\beta} = \frac{\hbar\Omega}{2}\alpha$

$\dot{\beta} = \frac{\cancel{\hbar}\Omega}{i\cancel{\hbar}2}\alpha = \frac{-i\Omega\alpha}{2}$

equal $\Rightarrow$ $\frac{i\ddot{\alpha}\cdot 2}{\Omega} = \frac{-i\Omega\alpha}{2}$

$\ddot{\alpha} = -\frac{\Omega^2}{4}\alpha$

case 1: $\delta = 0$ (Resonance)

$$i\hbar\,\dot{\alpha}(t) = \frac{\hbar\Omega}{2}\beta(t)$$

$$i\hbar\,\dot{\beta}(t) = \frac{\hbar\Omega}{2}\alpha(t)$$

or

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar\Omega\sigma_x}{2}\cdot\psi$$

$$\frac{\partial\alpha}{\partial t} = \dot{\alpha} \qquad \frac{\partial^2\alpha}{\partial t^2} = \ddot{\alpha}$$

$$\frac{\partial}{\partial t}\left(i\hbar\,\dot{\alpha}(t)\right) = \frac{\partial}{\partial t}\left(\hbar\frac{\Omega}{2}\cdot\beta(t)\right)$$

$$i\hbar\,\ddot{\alpha}(t) = \frac{\hbar\Omega}{2}\dot{\beta}(t)$$

$$\dot{\beta} = \frac{\cancel{i\hbar}\,\ddot{\alpha}}{\cancel{\hbar}\Omega/2} = \frac{i\ddot{\alpha}\cdot 2}{\Omega}$$

$$i\hbar\dot{\beta} = \frac{\hbar\Omega}{2}\alpha$$

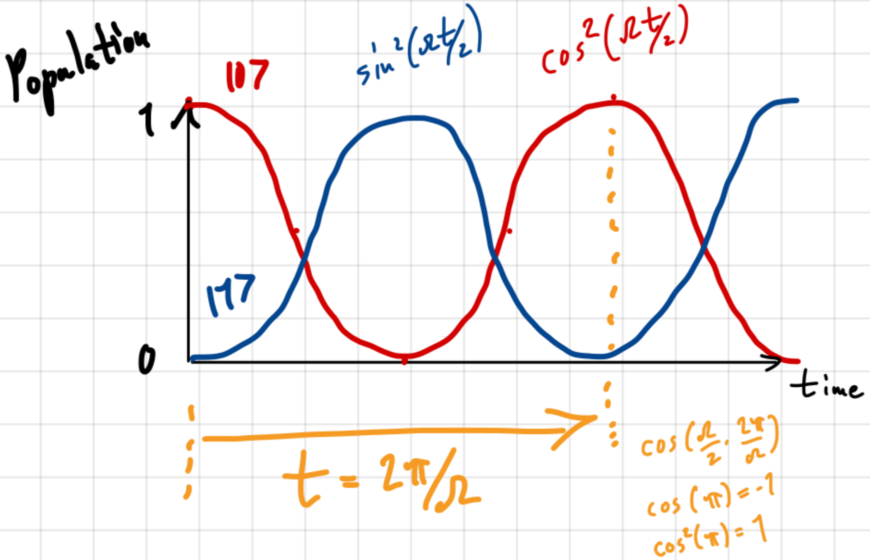
$$\dot{\beta} = \frac{\cancel{\hbar}\Omega}{i\cancel{\hbar}2}\alpha = \frac{-i\Omega\alpha}{2}$$

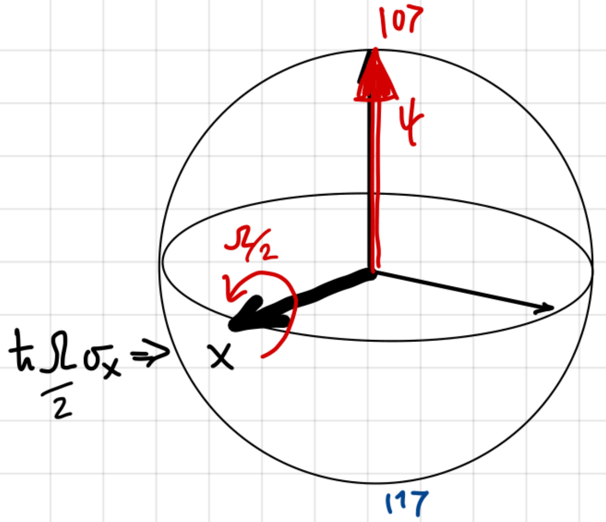
equal $\Rightarrow$ $\dfrac{i\ddot{\alpha}\cdot 2}{\Omega} = \dfrac{-i\Omega\alpha}{2}$

$$\ddot{\alpha} = -\frac{\Omega^2}{4}\alpha$$

if $\alpha(t=0) = 1$ and $\beta(t=0) = 0$ then $\alpha(t) = \cos\left(\frac{\Omega t}{2}\right)$ $\quad \beta = \sin\left(\frac{\Omega t}{2}\right)$

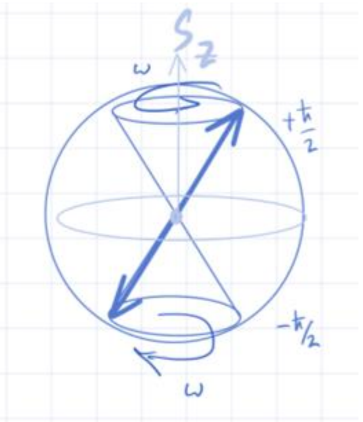
$$\langle\psi|\psi\rangle = \cos^2\left(\frac{\Omega t}{2}\right)|0\rangle + \sin^2\left(\frac{\Omega t}{2}\right)|1\rangle$$





Energy of states (waves) ≡ rotation about Z!

$$\hat{H} = \begin{bmatrix} \frac{\hbar\omega_z}{2} & 0 \\ 0 & -\frac{\hbar\omega_z}{2} \end{bmatrix}$$



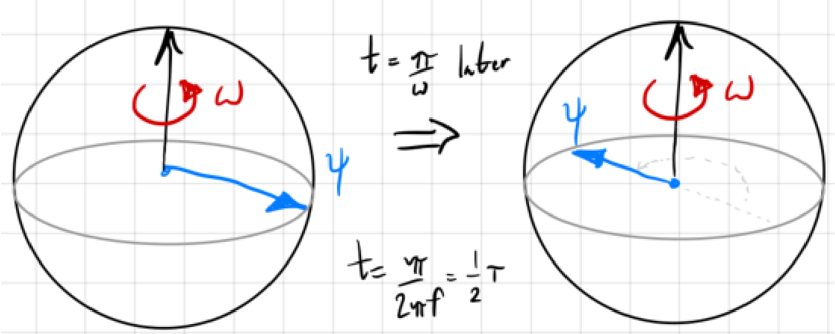
Energy of the 'wave'!

$$\boldsymbol{E = \hbar\omega_z/2}$$

Phase of the wave!

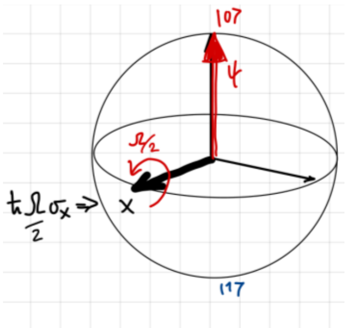
$$\boldsymbol{\theta = \omega_z \cdot t}$$

$$\hat{H} = \frac{\hbar\omega_z}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \frac{\hbar\omega_z}{2} \cdot \widehat{\sigma_z}$$



Rabi Coupling (coupling states) ≡ rotation about X (or Y)

$$\hat{H} = \begin{bmatrix} 0 & \frac{\hbar\Omega_{Rabi}}{2} \\ \frac{\hbar\Omega_{Rabi}}{2} & 0 \end{bmatrix}$$



$$\hat{H} = \frac{\hbar\Omega_{Rabi}}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{\hbar\Omega_{Rabi}}{2} \cdot \widehat{\sigma_x}$$