ECE 550/650 – Intro to Quantum Computing

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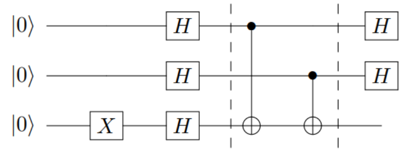
Outline of the course

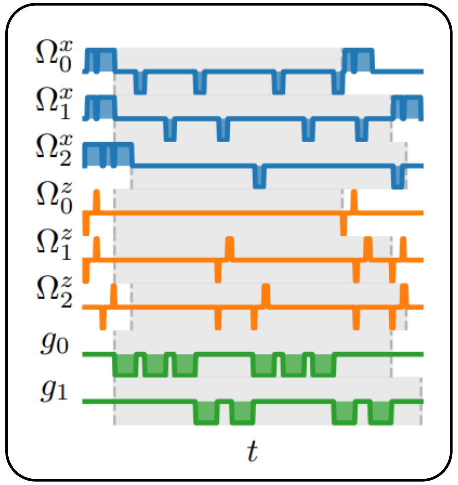
- Quantum Optics
  - What is interference (classical vs. single particle)
  - Superposition of states
  - Measurement and measurement basis
- Atomic physics
  - Spin states in magnetic fields and spin transitions
  - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
  - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
  - Error sources (dephasing, spontaneous decay)
  - Ramsey pulses and Spin echo pulse sequences
  - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
  - Two qubit interactions – gate speed vs. error rates
  - Entanglement – correlation at a distance
  - Bell states and the Bell basis
  - XX gates, Controlled Phase gates, Swap




```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```

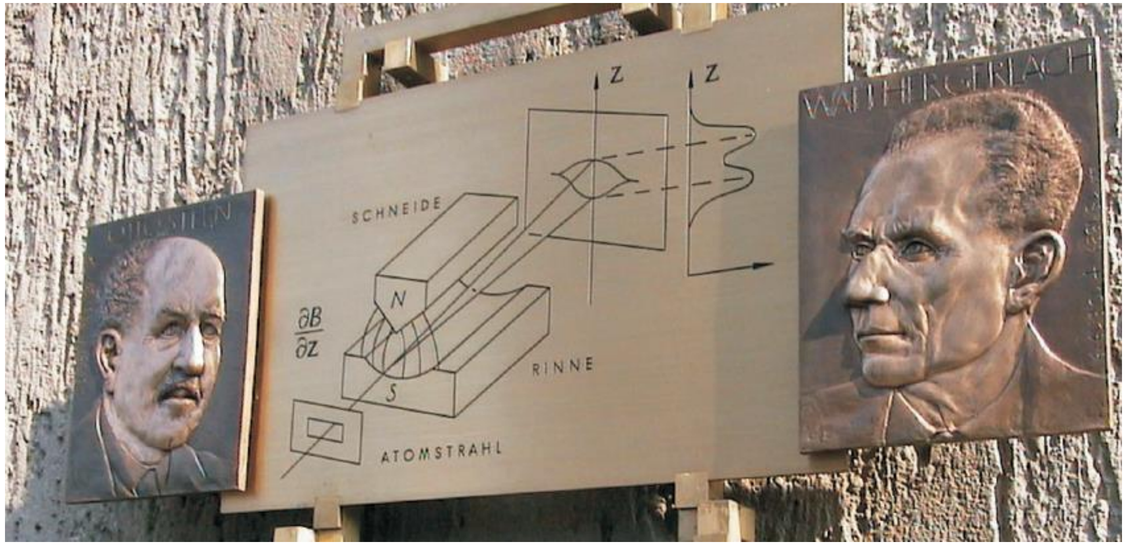


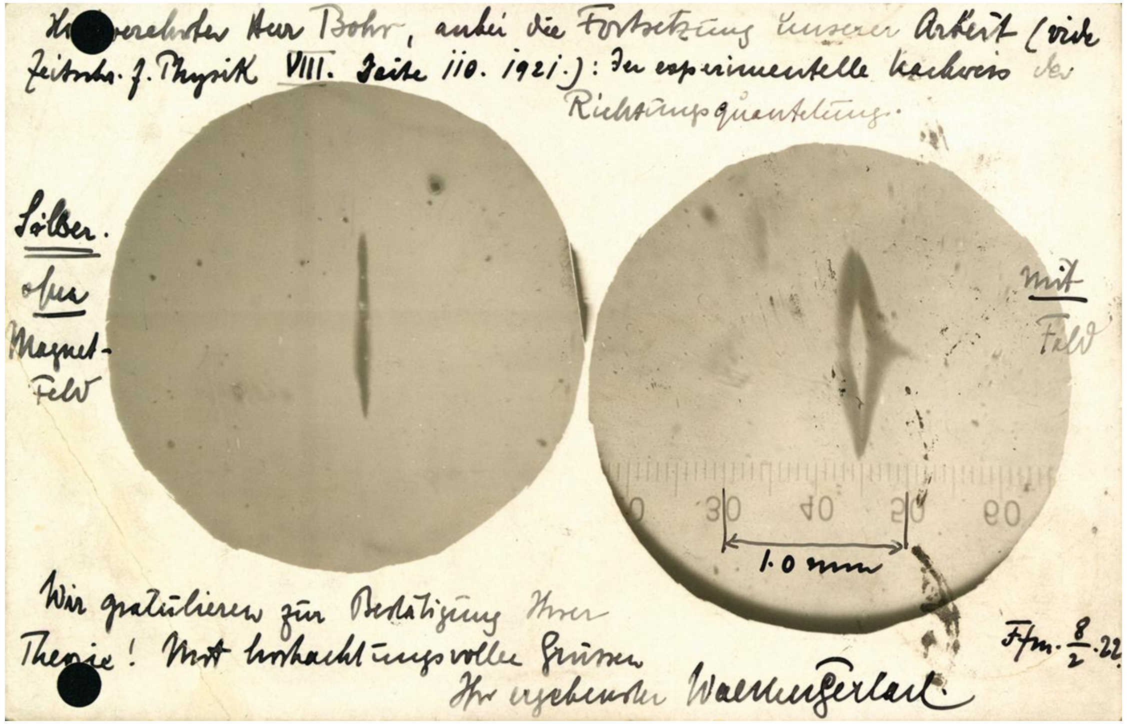
- Quantum Hardware
  - Photonics – nonlinear phase shifts
  - Transmons – charge noise, SWAP gate
- Quantum Circuits
  - Single and two qubit gates
  - Hadamard gate , CNOT gate
- Quantum Algorithms
  - Amplitude amplification
  - Grover's Search
  - Oracle - Deutsch Jozsa
  - Bernstein Vazirani
  - Quantum Fourier Transform and period finding
  - Shor's algorithm

If time permits

- Error Correction
  - Repetition codes
  - Color Codes
  - Surface code

Stern-Gerlach Experiment





After venting to release the vacuum, Gerlach removed the detector flange. But he could see no trace of the silver atom beam and handed the flange to me. With Gerlach looking over my shoulder as I peered closely at the plate, we were surprised to see gradually emerge the trace of the beam…. Finally we realized what [had happened]. I was then the equivalent of an assistant professor. My salary was too low to afford good cigars, so I smoked bad cigars. These had a lot of sulfur in them, so my breath on the plate turned the silver into **silver sulfide**, which is jet black, so easily visible.
It was like developing a photographic film.

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Electron in a magnetic field = $\vec{B}$

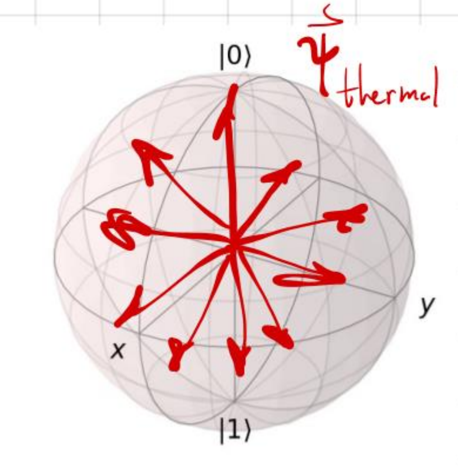
Thermal Source → e^- → hot!! $\vec{v}$

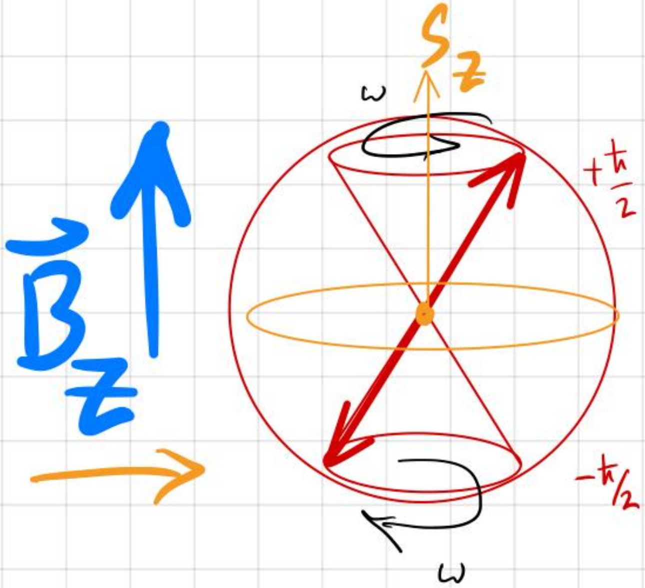
$\Psi_{thermal}$

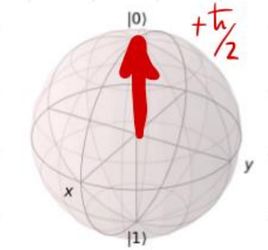
$\vec{B}$ ↑ Ψ_{final} ?

electron must pick a spin!

no partial spin allowed.

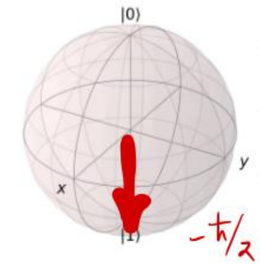






$|0\rangle_z$

↑ or ↓ not both



$|1\rangle_z$

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$$\vec{\mu} = \frac{q}{2m}\vec{S} \Rightarrow \frac{g\cdot q_e}{2\cdot m_e}\cdot\vec{S} = -\frac{g e}{2m_e}\cdot\vec{S} = \vec{\mu}_e = -2\frac{\mu_B}{\hbar}\cdot\vec{S}$$

dipole moment · charge · mass · spin · g-factor (particle dep.) · e^- negative charge · 1.6×10^{-19} C · $g = 2$ for e^- · electron · Bohr

$$\mu_B = \frac{e\hbar}{2m_e}$$

charge e^-

$$\vec{E} = \vec{\mu}\cdot\vec{B} = -2\frac{\mu_B}{\hbar}\cdot\vec{S}\cdot\vec{B} = -2\frac{\mu_B}{\hbar}B\left(\pm\frac{\hbar}{2}\right) = \mp\mu_B\cdot B$$

$S = \pm\hbar/2$

$$\omega = -\mu_B B/\hbar$$

$$E = \hbar\omega = -\cancel{\hbar}\,\mu_B B/\cancel{\hbar}$$

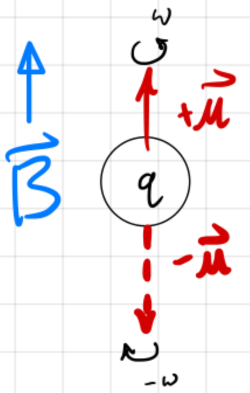
$\omega \equiv$ angular velocity [rad/s]

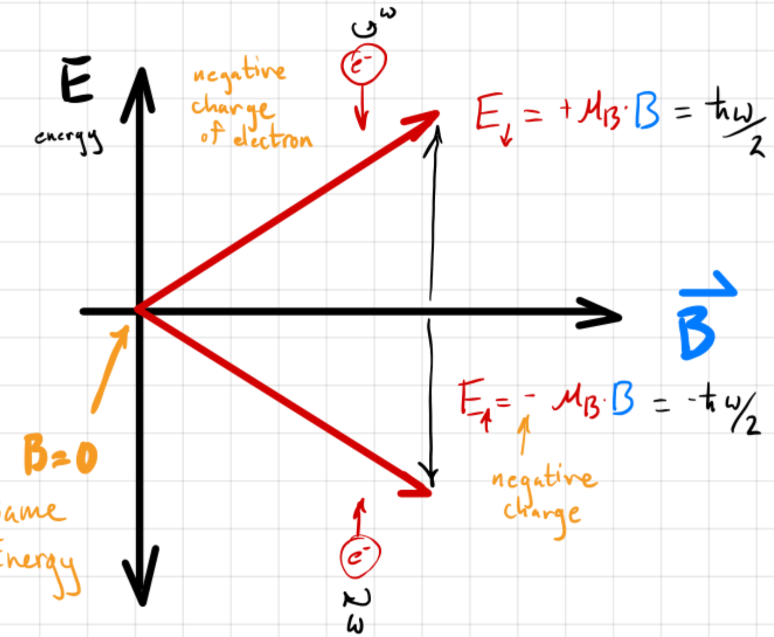
⇒ precession about quantization axis

⇒ **Phase**

opposite spins precess in opposite directions

⇒ acquire opposite phase due to Energy difference



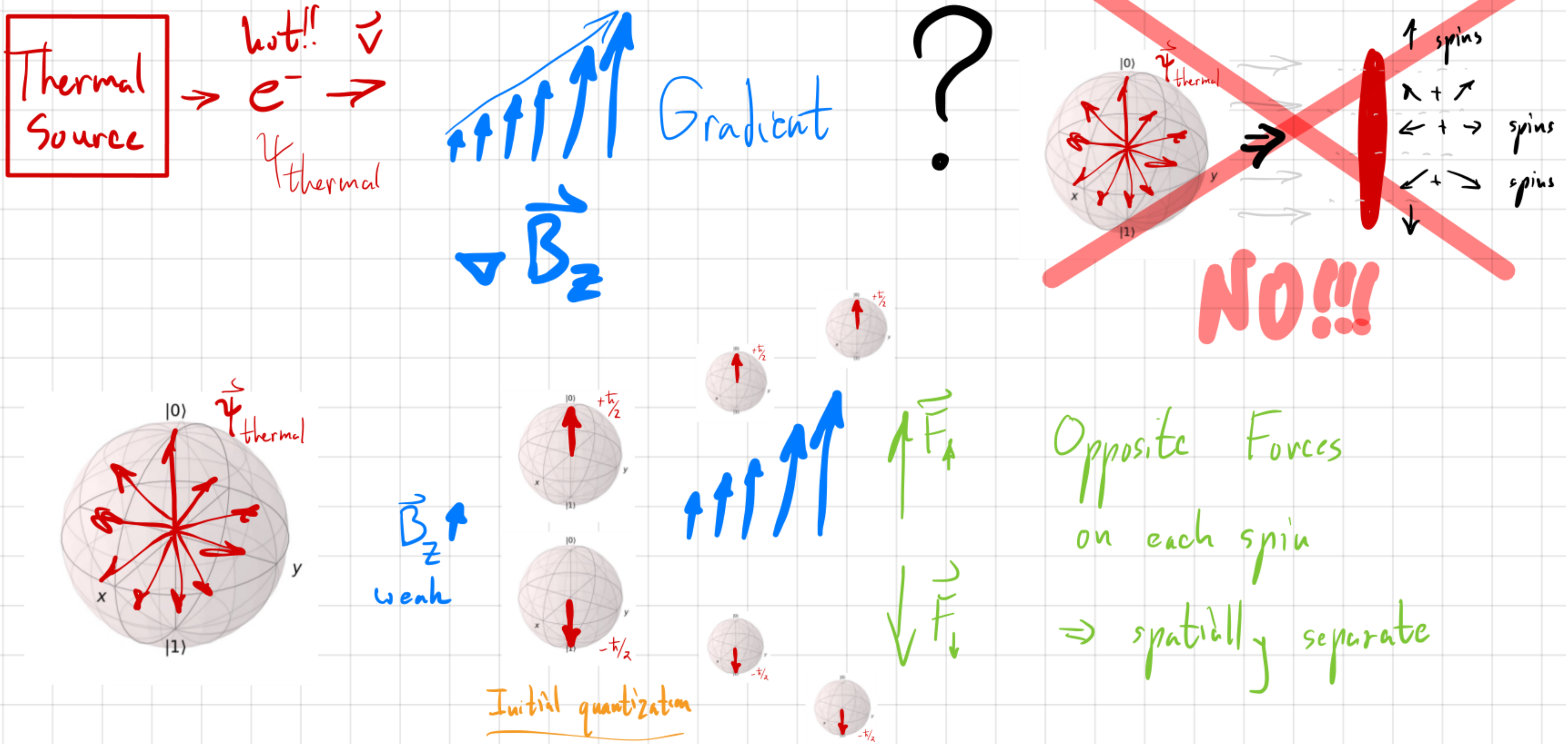


Electrons are spin 1/2

⇒ Fermions

spinning negative charge ⇒ magnetic moment

The Stern Gerlach Experiment



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Gradients in Energy => Forces

so Gradient in $\vec{B}$ => Spin dep. Energy gradient => Spin dependent Force

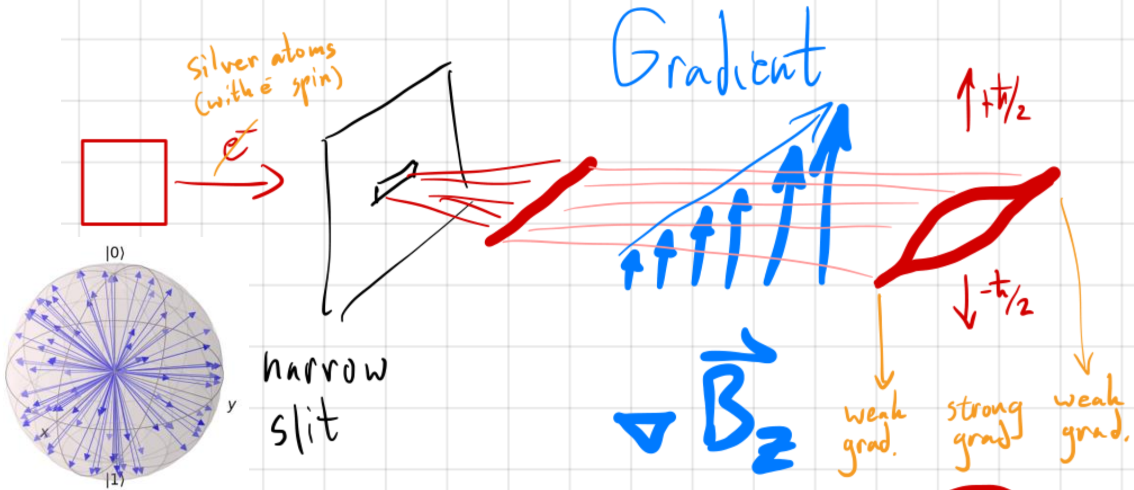
$$\vec{F}_B = -\nabla U_B \qquad F_B = -\frac{d}{dz} U_B = -\frac{d}{dz}\left(\mp \mu_B \cdot B\right) = \pm \mu_B \cdot \frac{dB}{dz}$$

E_B

only non zero along Z

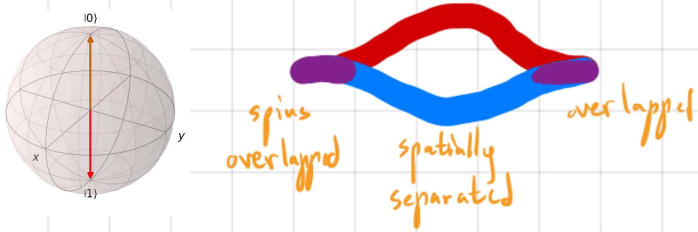
minus, ↑
plus, ↓

plus, ↑
minus ↓



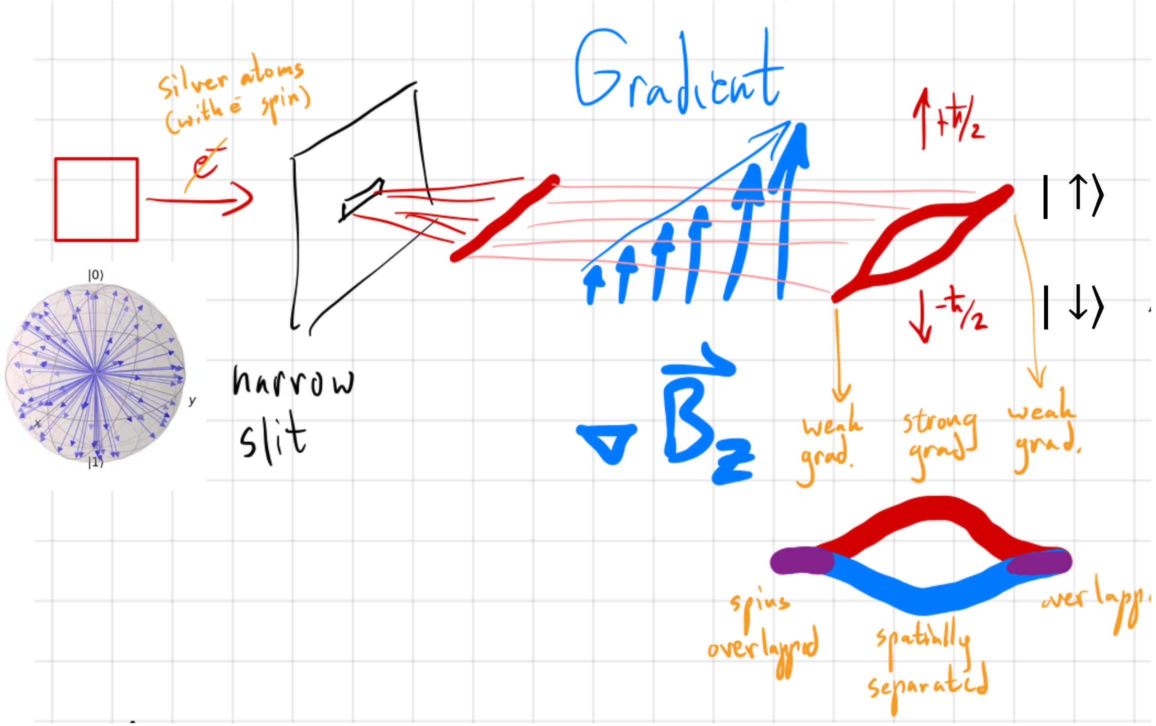
Allows polarization of atoms

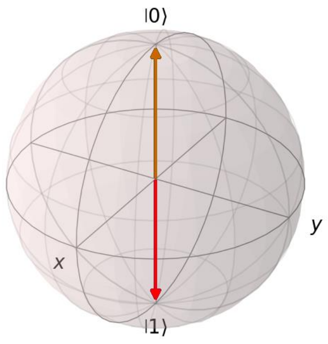
Blocking one path keeps only the other spin

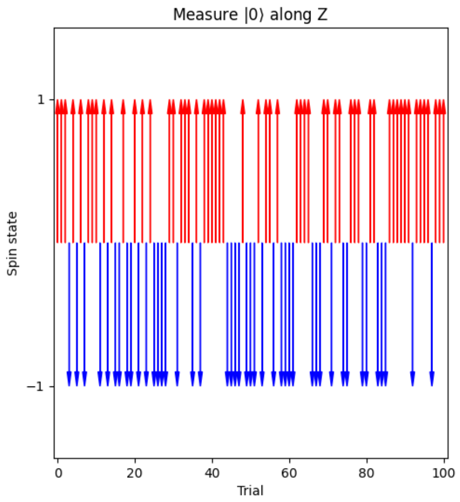


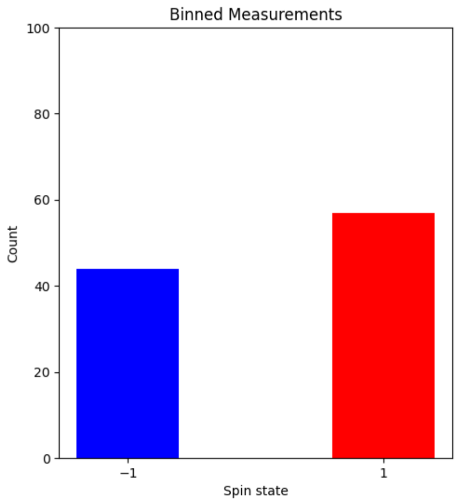
Qubit Measurement

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$









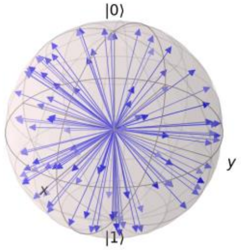
https://nbviewer.org/github/UMassIonTrappers/Introduction-to-Quantum-Computing/blob/main/Lab_02_Measurement_Basis%2C_Spatial_quantization_and_the_Stern_Gerlach_Exp_.ipynb

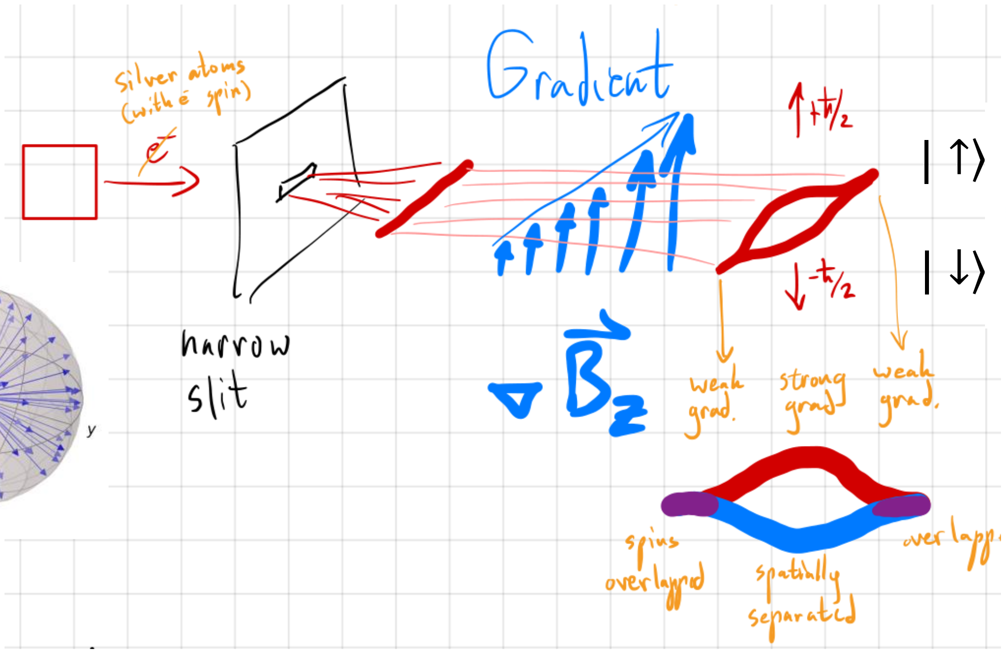
Electrons vs. Photons

But what direction?

$|\Psi_0\rangle = ?$

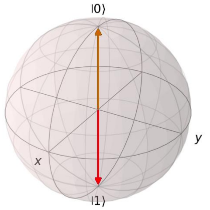
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



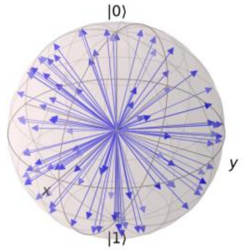


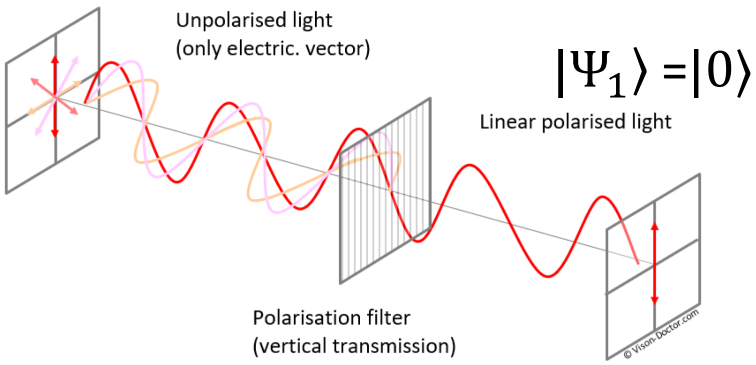
$|\uparrow\rangle$

$|\downarrow\rangle$

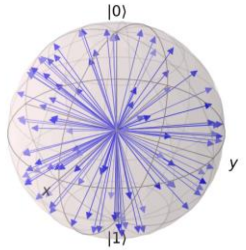


Optics → Polarizer





$|\Psi_1\rangle = |0\rangle$



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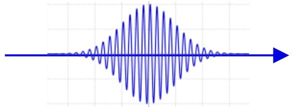
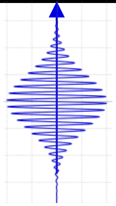


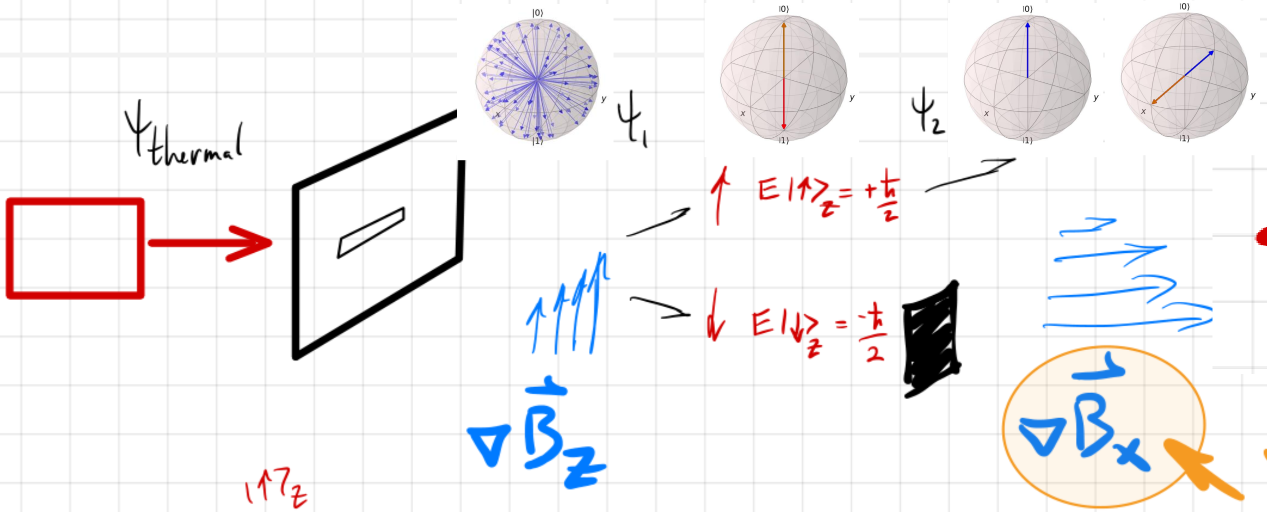
MZI…

Blocked path



Qubit State Preparation!

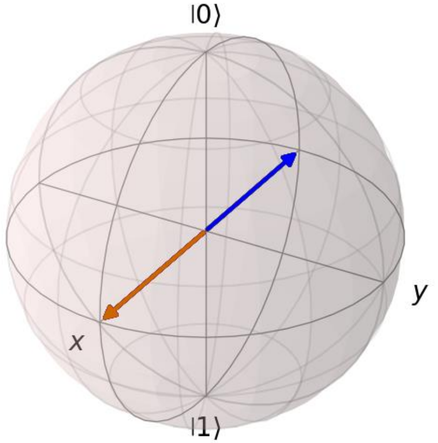
What happens when we apply another gradient? But in another direction?

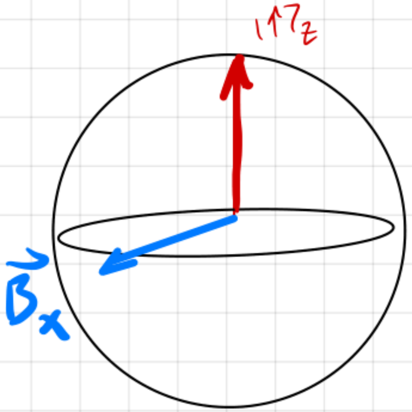


$|+\rangle_x$ or $|-\rangle_x$

but not both

X direction



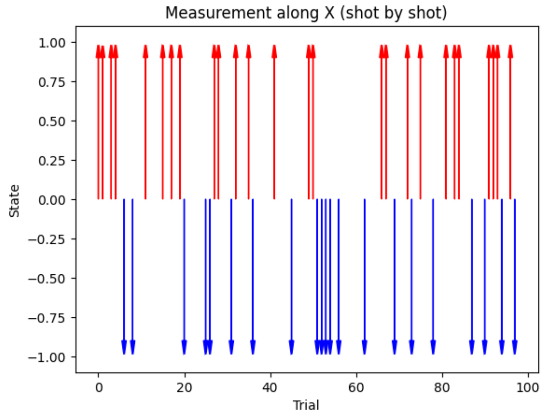


Again only 2 possibilities up or down, but now up along $\hat{x}$ and down along $\hat{x}$, $|\uparrow\rangle_x$ or $|\downarrow\rangle_x$

$$\left.\begin{aligned} |\uparrow\rangle_x &\equiv |+\rangle \\ |\downarrow\rangle_x &\equiv |-\rangle \end{aligned}\right\} \textbf{X Basis States} \qquad \begin{aligned} |+\rangle &\equiv \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ |-\rangle &\equiv \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned}$$

in-phase

out-of-phase



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Let's check what $\langle + | + \rangle = ?$

$$\langle + | + \rangle = \left(\frac{\langle 0 | + \langle 1 |}{\sqrt{2}} \right) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) = \frac{1}{2} \left[\langle 0|0\rangle + \langle 0|1\rangle + \langle 1|0\rangle + \langle 1|1\rangle \right] =$$

$$= \frac{1}{2} \left[1 + 0 + 0 + 1 \right] = 1$$

What happens if we add $|+\rangle + |-\rangle$? (create equal superposition) with no relative phase

$$\frac{|+\rangle + |-\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{|0\rangle + \cancel{|1\rangle}}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left(\frac{|0\rangle - \cancel{|1\rangle}}{\sqrt{2}} \right) = \frac{\cancel{2}|0\rangle}{\cancel{2}} = |0\rangle_z$$

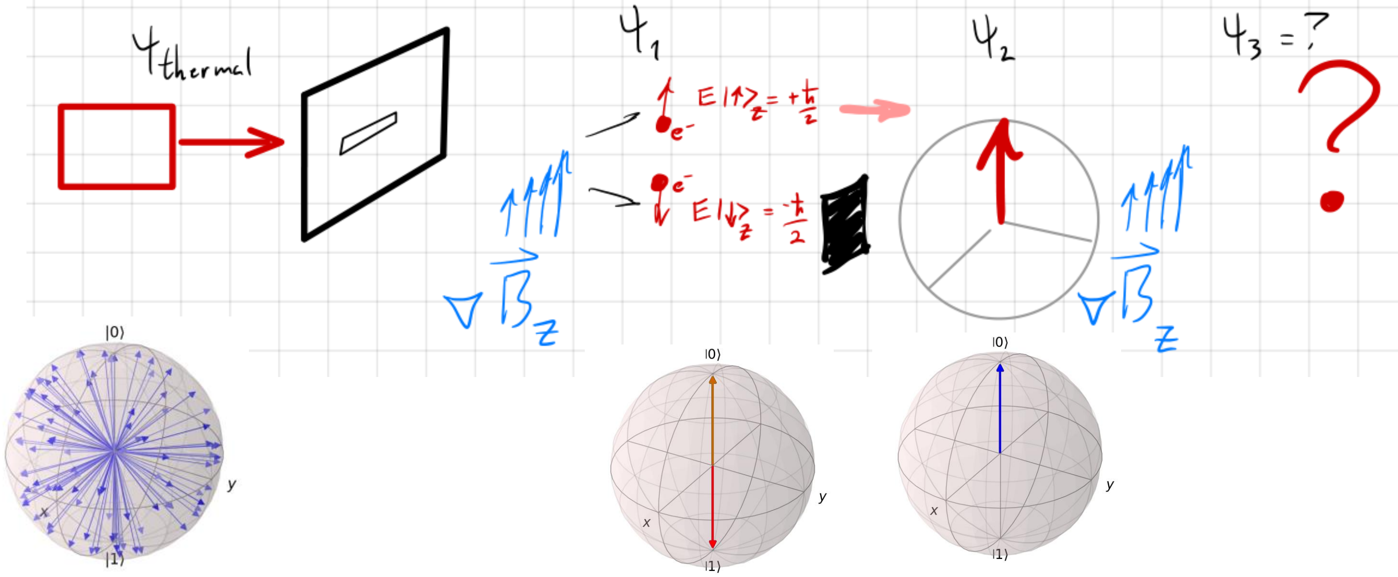
We get $|\uparrow\rangle_z$!!! (the 'plus Z' basis state)

$|0\rangle \equiv |\uparrow\rangle_z$ was already$_z$ a superposition of $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$ ($|+\rangle_x$ and $|-\rangle_x$) a superposition that is **in phase**

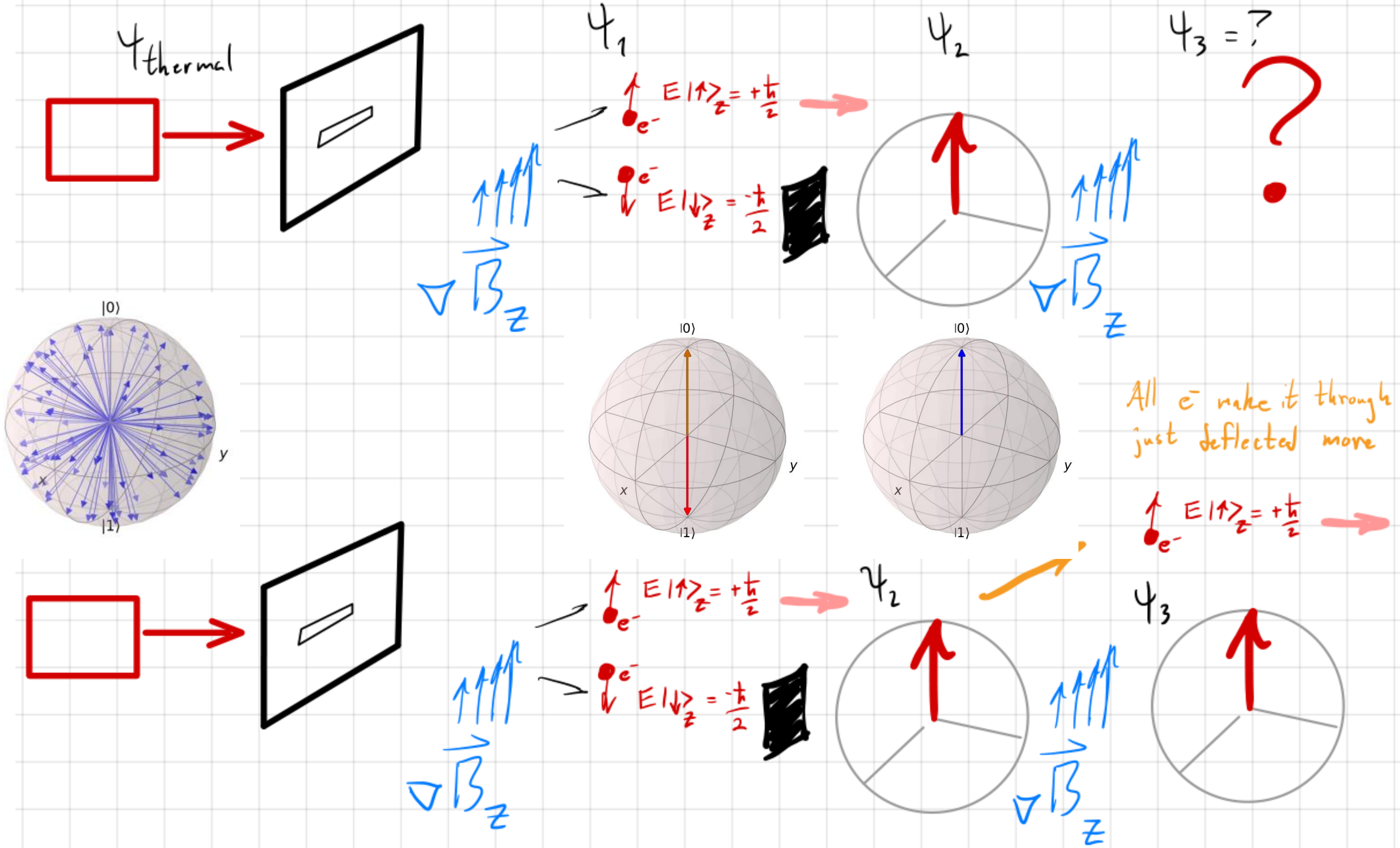
$$\psi_{3_x} = \frac{|+\rangle_x + |-\rangle_x}{\sqrt{2}} = |0\rangle_z = \psi_{2_z}$$

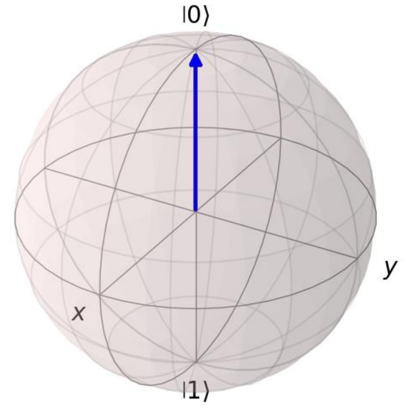
Applying $\vec{B}_x$ just projected that superposition along $\hat{x}$, showing equal probability of $|+\rangle$ and $|-\rangle$

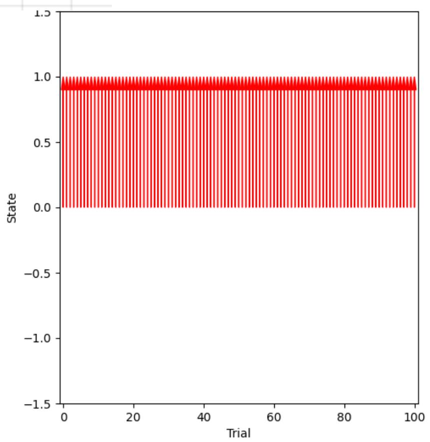
Apply B_z again, after polarizing in Z?

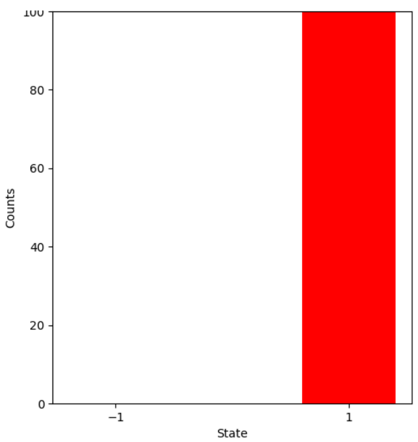


Apply B_z again, after polarizing in Z?





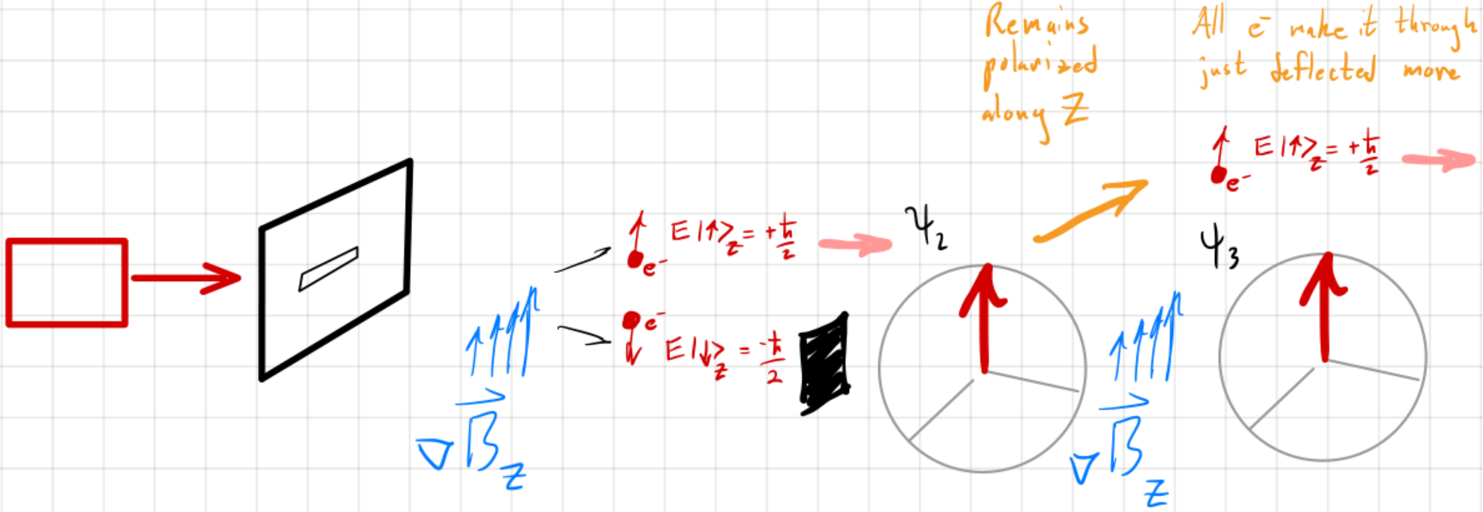




Measuring state along Z
twice just gives the same result

$\psi_2 = \psi_3$ (up to global phase)

$\boxed{\text{Projection operator} = \sum_i |i_n\rangle\langle i_n|}$



Measuring state along Z
twice just gives the same result

$\psi_2 = \psi_3$ (up to global phase)

$$\boxed{\text{Projection operator} = \sum_i |i_{\hat{n}}\rangle\langle i_{\hat{n}}|}$$

i basis states , $\hat{n}$ basis direction

$$|\psi_0\rangle = |\psi_{thermal}\rangle$$

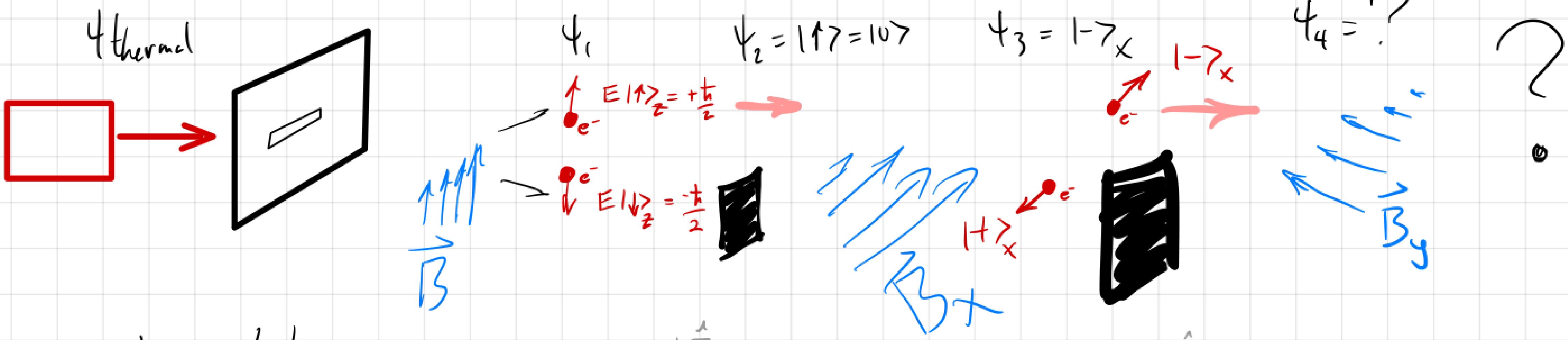
$$|\psi_1\rangle = \sum |i_z\rangle\langle i_z|\,|\psi_{therm}\rangle = |0\rangle\underbrace{\langle 0|\psi_{therm}\rangle}_{1/\sqrt{2}} + |1\rangle\underbrace{\langle 1|\psi_{therm}\rangle}_{1/\sqrt{2}}$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|\psi_2\rangle = \underbrace{|0\rangle\langle 0|\psi_1\rangle}_{\text{projection only to } |0\rangle} = \frac{1}{\sqrt{2}}|0\rangle \xRightarrow{\text{renormalize}} |0\rangle$$ (polarize ψ_1 into $|0\rangle$)

$$|\psi_3\rangle = \sum |i_z\rangle\langle i_z|\psi_2\rangle = |0\rangle\underbrace{\langle 0|0\rangle}_{=1} + |1\rangle\underbrace{\langle 1|0\rangle}_{=0} = |0\rangle$$

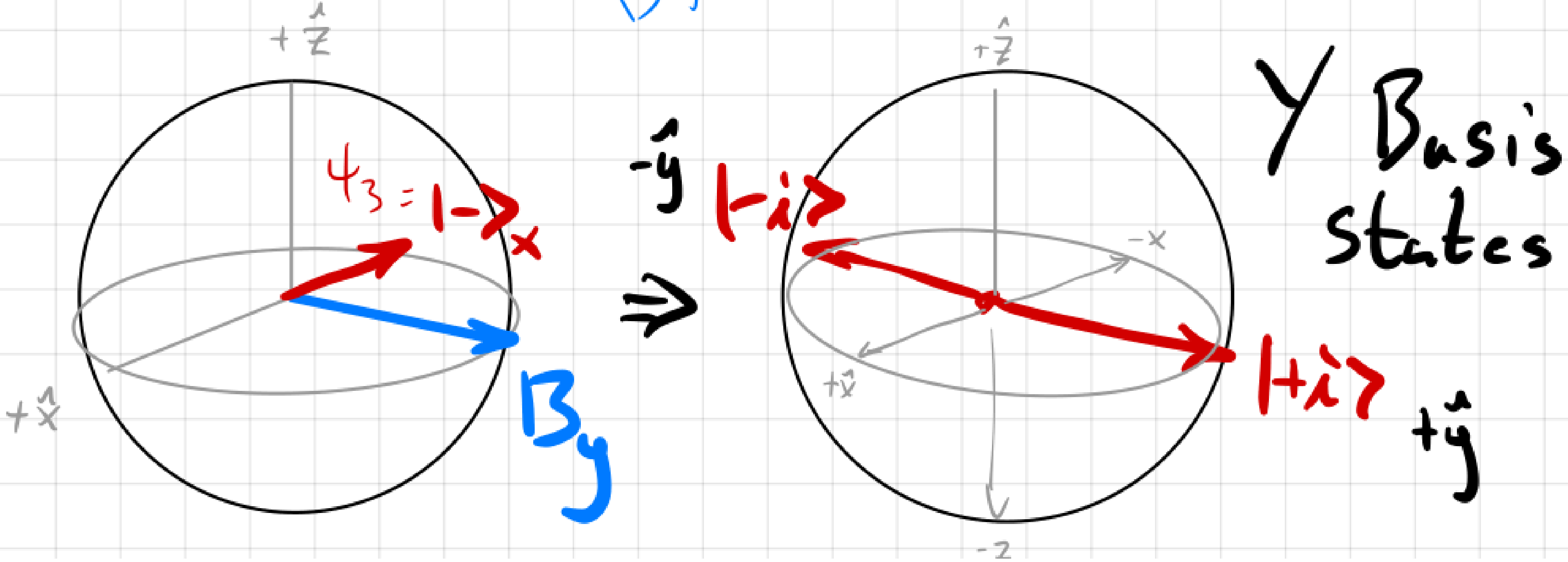
What about Y axis?



y basis states

$$|+i\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}$$

$$|-i\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$



Try guessing that $|-\rangle_x$ must be an out-of-phase superposition of $\pm|i\rangle$

$$\frac{|+i\rangle - |-i\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[\frac{|0\rangle + i|1\rangle}{\sqrt{2}} - \left(\frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right)\right] = \frac{1}{2}\left(\cancel{|0\rangle} + i|1\rangle - \cancel{|0\rangle} + i|1\rangle\right) = i|1\rangle$$

$$\frac{|+i\rangle + |-i\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[\frac{|0\rangle + \cancel{i|1\rangle}}{\sqrt{2}} + \frac{|0\rangle - \cancel{i|1\rangle}}{\sqrt{2}}\right] = |0\rangle$$

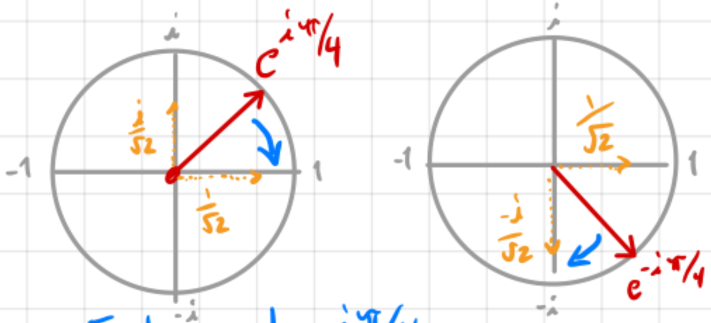
No? Try substituting directly.

Definition

$$|-\rangle_x = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{|0\rangle + i(i|1\rangle)}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[\frac{|i\rangle + |-i\rangle}{\sqrt{2}} + i\left(\frac{|i\rangle - |-i\rangle}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left(\overbrace{\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)}^{\text{phase}}|i\rangle + \overbrace{\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)}^{\text{phase}}|-i\rangle\right) = \frac{1}{\sqrt{2}}\cdot e^{i\pi/4}\cdot\left[\underbrace{|i\rangle - i|-i\rangle}\right] = \mathbf{4_4} \text{ in } Y \text{ basis}$$

Superposition of Y basis states with Global & relative phase!



Factor out $e^{i\pi/4}$

$= 1$ $\quad$ $= e^{-i\pi/2} = -i$

Lab_02_Measurement_Basis,_Spatial_quantization_and_the_Stern_Gerlach_Exp.ipynb

https://nbviewer.org/github/UMassIonTrappers/Introduction-to-Quantum-Computing/blob/main/Lab_02_Measurement_Basis%2C_Spatial_quantization_and_the_Stern_Gerlach_Exp_.ipynb

State Measurement

Measurement is an important concept in quantum mechanics. Imagine that we want to measure the qubit state after a rotation to verify that we have rotated it to another state. What do we **expect** to measure (what is the expectation value)?

We know quantum states are quantized. An electron can only ever be in one state or the other and a photon can only ever be in one cavity or the other. However, it can have a probability of being in both one *and* the other before we measure it.

It is like a coin that can only be heads or tails once it falls (never landing on edge) but while it is in the air has some probability of being both. If the qubit is 'flipped' into an equal superposition of up and down (like a coin) it will be up 50% of the time and down 50% of the time but it can only ever land heads/tails (up/down).

On the Bloch sphere we can see that the probability of being in each state is related to the projection of the state vector along the z axis. If the state (vector) is pointing up then it is more likely to be measured up. If it is pointing down then down. And if it is sideways (no component in the z direction) then it is in an equal superposition of up and down.

However, we need some observable or measureable value to determine which state we were in. For the electron this observable is the spin. The spin project operator is :

$$S_z = \frac{\hbar}{2}\sigma_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle s_z \rangle = \langle \Psi | S_z | \Psi \rangle$$

It projects the state onto the Z basis and multiplies by ±ħ/2 depending on the state. Now the superposition tells us the probability that we'll get ±ħ/2.

Measurement and Expectation Values

Expectation value of spin:
$\langle S_z \rangle = \langle \Psi | \mathrm{S_z} | \Psi \rangle$

Recall photons:
$\langle I \rangle = \langle \vec{\mathrm{E}} | \vec{\mathrm{E}} \rangle$

Plank's Constant
$\frac{h}{2\pi} = \hbar = 1 \times 10^{-34}\, J \cdot s$
Energy (Joules) = $\hbar\omega = hf$

$$\hat{S}_z \equiv \frac{\hbar}{2}\hat{\sigma}_z \equiv \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle 0 | S_z | 0 \rangle = \frac{\hbar}{2}(1 \quad 0)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}(1 \quad 0)\begin{pmatrix} 1 * 1 + 0 * 0 \\ 0 * 1 + (-1) * 0 \end{pmatrix} = \frac{\hbar}{2}(1 \quad 0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}$$

$$\langle 1 | S_z | 1 \rangle = \frac{\hbar}{2}(0 \quad 1)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\hbar}{2}(0 \quad 1)\begin{pmatrix} 1 * 0 + 0 * 1 \\ 0 * 0 + (-1) * 1 \end{pmatrix} = \frac{\hbar}{2}(0 \quad 1)\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \frac{-\hbar}{2}$$

Pauli Matrices (Spin Matrices)

$$\hat{\sigma}_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{\sigma}_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Projection Operators

$$\hat{\sigma}_z \equiv |0\rangle\langle 0| - |1\rangle\langle 1| \quad \hat{\sigma}_x \equiv |0\rangle\langle 1| + |1\rangle\langle 0| \quad \hat{\sigma}_y \equiv -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

Changes phase **Changes state** **Changes state and phase!**

Pauli Matrices and rotations about the Bloch Sphere

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The rotations of the Bloch sphere about the Cartesian axes in the Bloch basis are then given by:

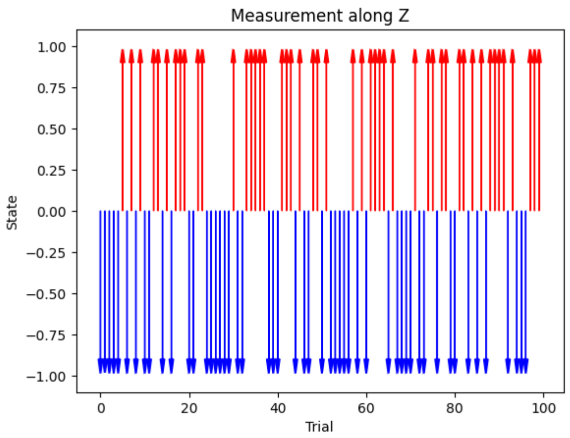
$$R_x(\theta) = e^{(-i\theta X/2)} = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

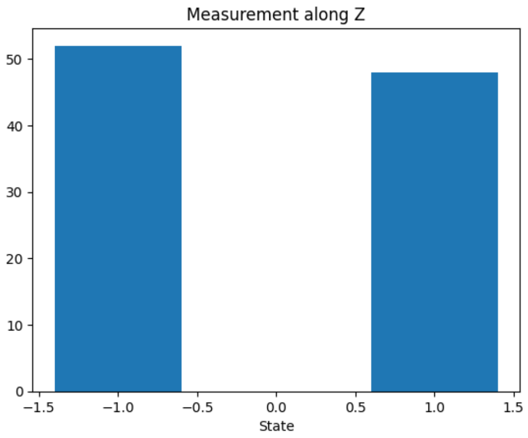
$$R_y(\theta) = e^{(-i\theta Y/2)} = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_z(\theta) = e^{(-i\theta Z/2)} = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

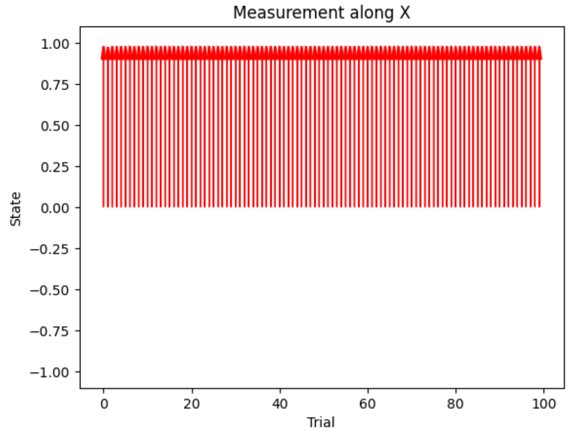
Measurement Basis

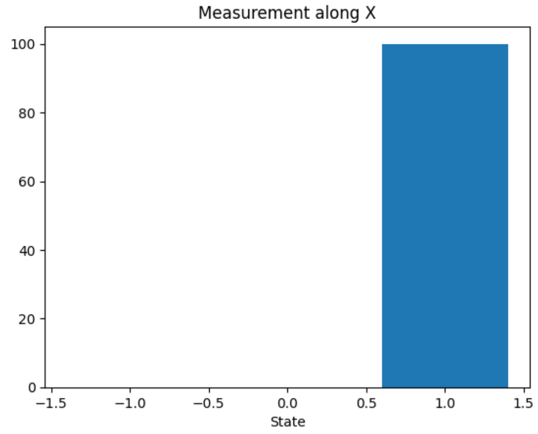
- Measure |+⟩ along Z:





- Measure |+⟩ along X:





Measurement and Expectation Values

Expectation value of spin:
$\langle S_z \rangle = \langle \Psi | \mathrm{S_z} | \Psi \rangle$

Recall photons:
$\langle I \rangle = \langle \vec{\mathrm{E}} | \vec{\mathrm{E}} \rangle$

Plank's Constant
$\frac{h}{2\pi} = \hbar = 1 \times 10^{-34}\, J \cdot s$
Energy (Joules) = $\hbar\omega = hf$

$\hat{S}_z \equiv \frac{\hbar}{2}\hat{\sigma}_z \equiv \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\langle + | S_z | + \rangle = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 * \frac{1}{\sqrt{2}} + 0 * \frac{1}{\sqrt{2}} \\ 0 * \frac{1}{\sqrt{2}} + (-1) * \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = 0$$