ECE 550/650

Homework 2 – XYZ Basis States and Stern Gerlach (20pts)

Write out solutions by hand and submit them via Gradescope

1. Show that the two basis states $|+\rangle_x$ and $|-\rangle_x$ are orthogonal.

2. Calculate the state Ψ after each rotation within the MZI with a phase shift. $\Psi_0 = |0\rangle$
   $|\Psi_3\rangle = X\left(\frac{-\pi}{2}\right) \cdot Z(\pi) \cdot X\left(\frac{-\pi}{2}\right) \cdot |\Psi_0\rangle$. Draw Ψ on the Bloch sphere after each rotation.
   i. $\Psi_1 =$

   ii. $\Psi_2 =$

   iii. $\Psi_3 =$

3. Compare the probability of getting $|0\rangle_Z$ if you measure the $|+\rangle_X$ and $|-\rangle_X$ states in the Z basis. Are they different? Why/why not?
   Bonus: How would you measure the difference between those states?

4. What state Ψ has 1/5 probability of measuring $|0\rangle$ and 4/5 measuring $|1\rangle$, with phase $\phi = \pi$? Verify it is normalized so $|\Psi|^2 = \langle\Psi|\Psi\rangle = 1$.

5. Draw the diagram for the Stern Gerlach Experiment below starting from a thermal source (draw the state on the Bloch sphere at each step):
   I. Z quantization / polarization to $|0\rangle_Z$
   II. Y quantization / polarization to $|-i\rangle_Y$
   III. X quantization / polarization to $|-\rangle_X$

   How many qubits make it through?

6. Calculate the wavefunction Ψ_i after each polarization.
   (wavefunction at each quantization not needed)