ECE 550/650 – Intro to Quantum Computing

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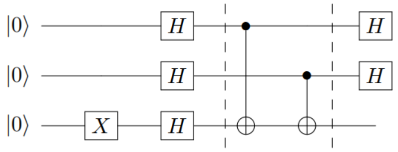
Outline of the course

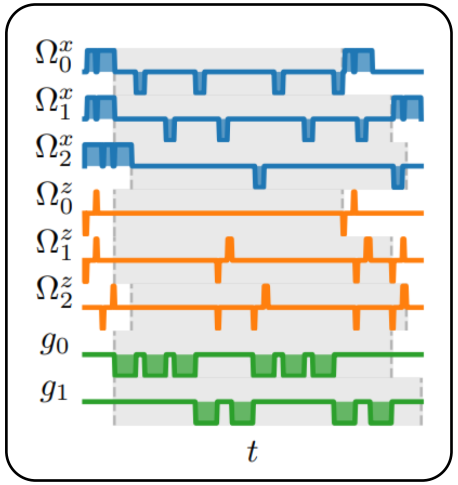
- Quantum Optics
  - What is interference (classical vs. single particle)
  - Superposition of states
  - Measurement and measurement basis
- Atomic physics
  - Spin states in magnetic fields and spin transitions
  - Transitions between atomic states (Rabi oscillations of qubits)
- Single qubits
  - Single qubit gates (electro-magnetic pulses, RF, MW, phase)
  - Error sources (dephasing, spontaneous decay)
  - Ramsey pulses and Spin echo pulse sequences
  - Calibration (finding resonance and verifying pulse time and amplitudes)
- Two qubit gates
  - Two qubit interactions – gate speed vs. error rates
  - Entanglement – correlation at a distance
  - Bell states and the Bell basis
  - XX gates, Controlled Phase gates, Swap




```
qc = QubitCircuit(3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# Oracle function f(x)
qc.add_gate(
    "CNOT", controls=0, targets=2)
qc.add_gate(
    "CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```



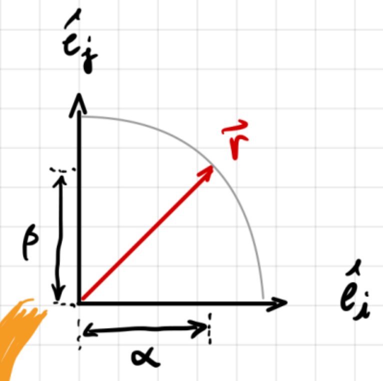
- Quantum Hardware
  - Photonics – nonlinear phase shifts
  - Transmons – charge noise, SWAP gate
- Quantum Circuits
  - Single and two qubit gates
  - Hadamard gate , CNOT gate
- Quantum Algorithms
  - Amplitude amplification
  - Grover's Search
  - Oracle - Deutsch Jozsa
  - Bernstein Vazirani
  - Quantum Fourier Transform and period finding
  - Shor's algorithm

If time permits

- Error Correction
  - Repetition codes
  - Color Codes
  - Surface code

Bloch Sphere - Superposition

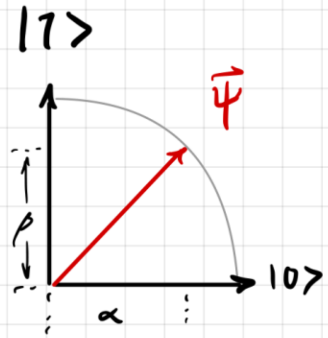
Quantum State Vector & Bloch sphere



$\vec{r} = \alpha \hat{e}_i + \beta \hat{e}_j$

$\vec{\Psi} = \alpha|0\rangle + \beta|1\rangle$

$\vec{\Psi} = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix}$

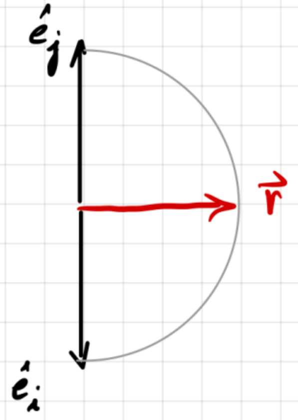


$\hat{e}_j \cdot \hat{e}_i = 0$

$\langle 1|0\rangle = 0$

$|r| = |u|$ = length of unit vector = 1

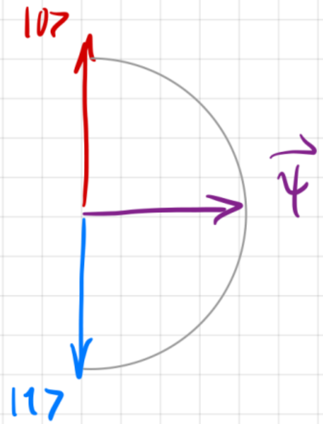
$\langle\Psi|\Psi\rangle = 1$ (100% probability that particle exists)

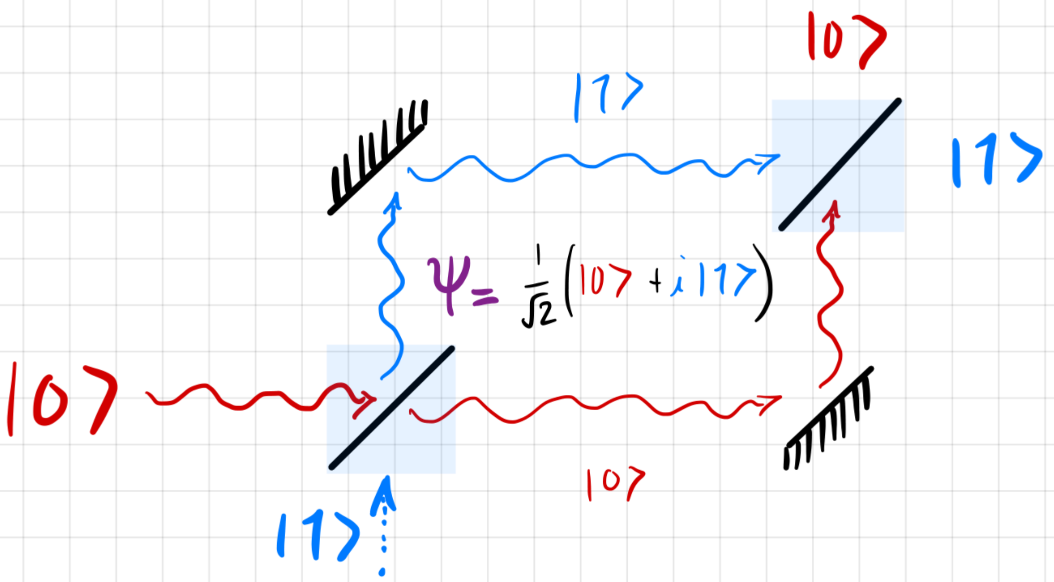


Now basis states along same axis but still orthogonal

$\vec{r} = \alpha \hat{e}_j + \beta \hat{e}_i$

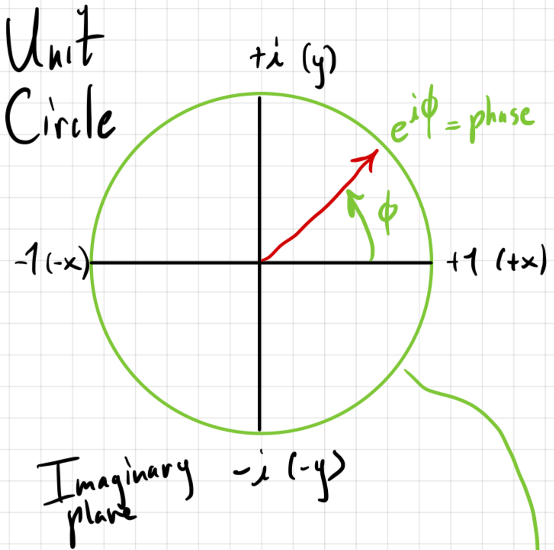
$\vec{\Psi} = \alpha|0\rangle + \beta|1\rangle$

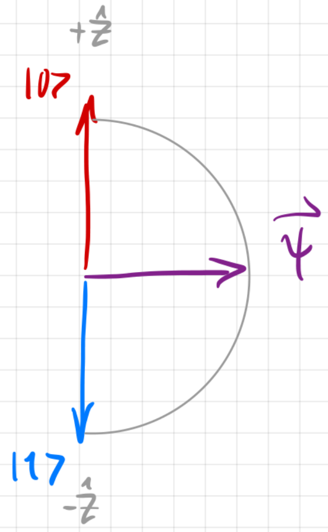




Bloch Sphere

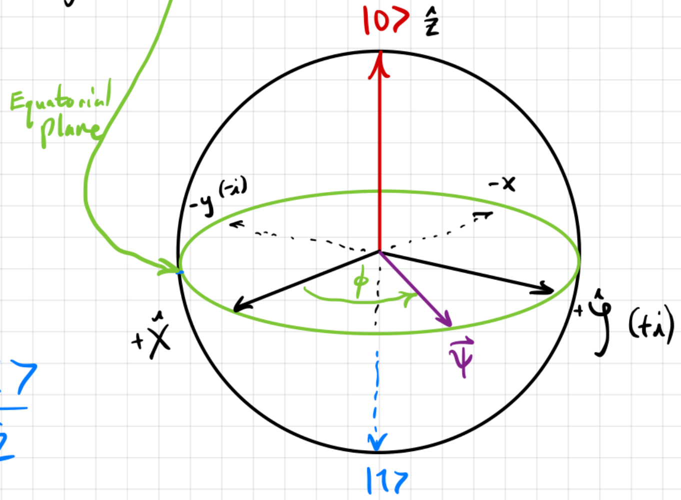
Unit Circle



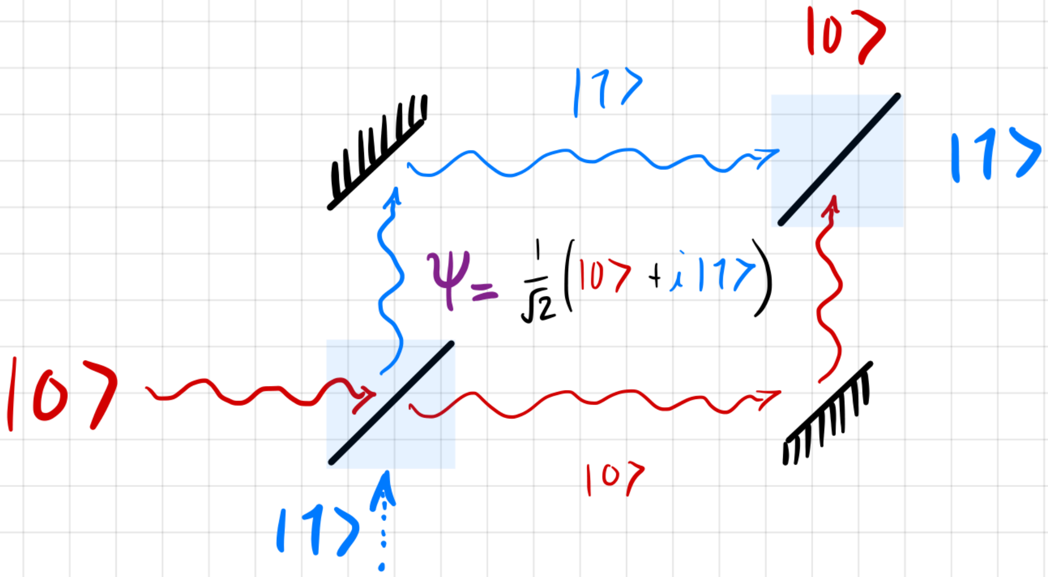


Combine to get

$\boxed{\text{Bloch Sphere}}$



$$\psi = \frac{|0\rangle}{\sqrt{2}} + e^{i\phi}\frac{|1\rangle}{\sqrt{2}}$$



Beam Splitter has two parts

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix} = \frac{1}{\sqrt{2}}\underbrace{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}_{\hat{\mathbb{1}}} + \frac{i}{\sqrt{2}}\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{\hat{\sigma}_x} = \underbrace{\begin{pmatrix}t & 0\\ 0 & t\end{pmatrix}}^{\text{Transmission}} + \underbrace{\begin{pmatrix}0 & r\\ r & 0\end{pmatrix}}^{\text{Reflection}}$$

Transmission $\Rightarrow$ Identity $\cdot \frac{1}{\sqrt{2}}$ = Amplitude to stay in the same mode (same state)

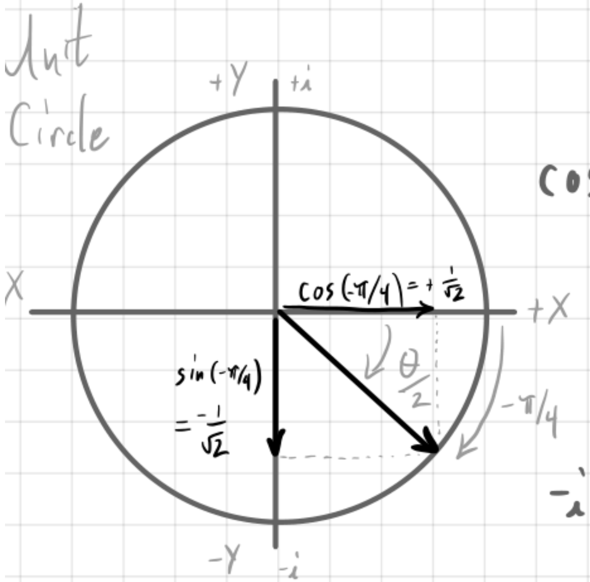
Reflection $\Rightarrow$ $\underbrace{\text{Off Diagonals} \cdot \frac{1}{\sqrt{2}} = \text{Amplitude to couple to the other mode (other state)}}_{\text{Couples states to each other (Outer Products)}}$

Beam Splitter has two parts

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix} = \frac{1}{\sqrt{2}}\underbrace{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}}_{\hat{\mathbb{1}}} + \frac{i}{\sqrt{2}}\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{\hat{\sigma}_x} = \overbrace{\begin{pmatrix}t & 0\\ 0 & t\end{pmatrix}}^{\text{Transmission}} + \overbrace{\begin{pmatrix}0 & r\\ r & 0\end{pmatrix}}^{\text{Reflection}}$$

$$= \cos(-\pi/4)\,\hat{\mathbb{1}} - i\sin(-\pi/4)\,\hat{\sigma}_x = e^{-i\underbrace{\pi/2}_{\text{Twice the angle!!!}}\cdot\,\hat{\sigma}_x/2}$$

Axis 'steals' factor of 1/2

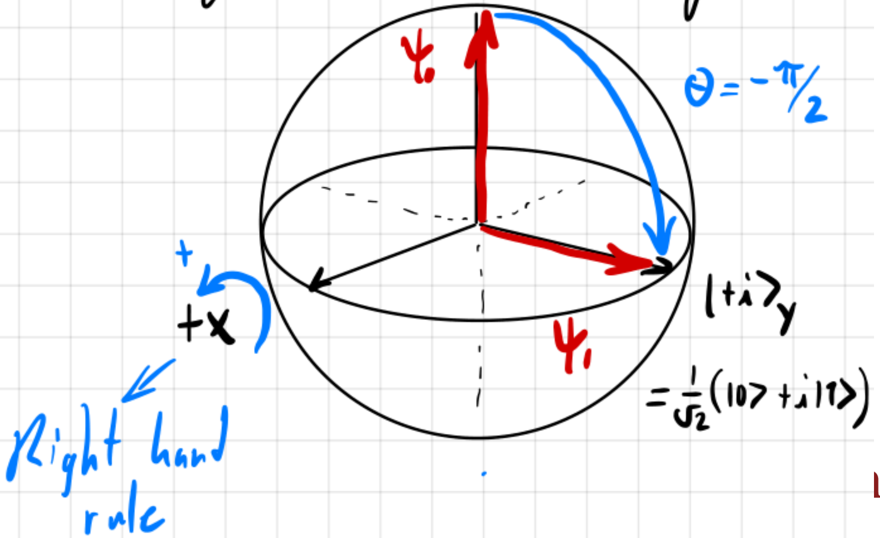


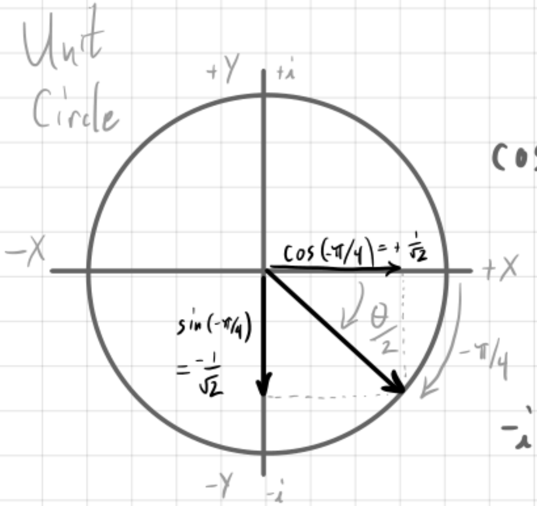
$$\cos(-\pi/4) = +\frac{1}{\sqrt{2}}$$

$$\sin(-\pi/4) = \frac{-1}{\sqrt{2}}$$

$$-i\cdot\sin(-\pi/4) = \frac{+i}{\sqrt{2}} \checkmark$$

Equivalent to rotations on a sphere



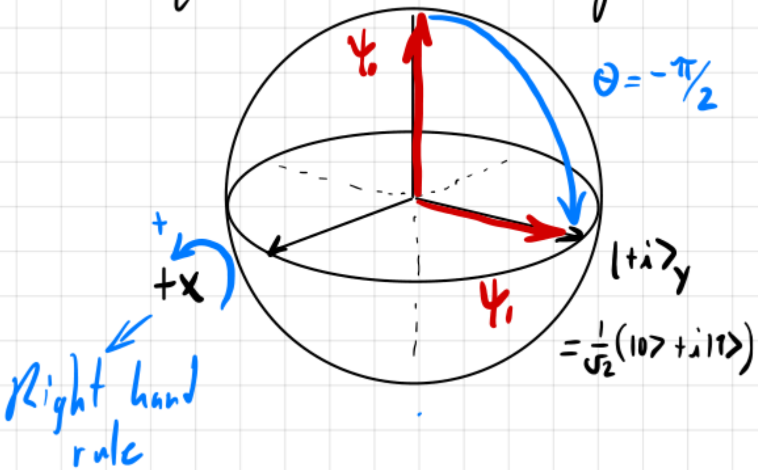


$\cos(-\pi/4) = +\frac{1}{\sqrt{2}}$

$\sin(-\pi/4) = \frac{-1}{\sqrt{2}}$

$-i \cdot \sin(-\pi/4) = \frac{+i}{\sqrt{2}}$ ✓

Equivalent to rotations on a sphere



Euler Formula for Matrices: $e^{-i\theta \cdot \hat{\sigma}_i/2} = \cos(\theta/2) \cdot \hat{\mathbb{1}} - i \sin(\theta/2) \cdot \hat{\sigma}_i$

$$X(\theta = -\pi/2) = e^{-i\theta \cdot \hat{\sigma}_x/2} = \cos(\theta/2)\,\hat{\mathbb{1}} - i \sin(\theta/2)\,\hat{\sigma}_x$$

$$= \cos\left(-\frac{\pi}{2} \cdot \frac{1}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i \sin\left(-\frac{\pi}{2} \cdot \frac{1}{2}\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\left(\frac{-1}{\sqrt{2}}\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} = \boxed{BS}$$ ✓

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Euler Formula for Matrices: $e^{-i\theta \cdot \hat{\sigma}_i/2} = \cos(\theta/2) \cdot \hat{\mathbb{1}} - i \sin(\theta/2) \cdot \hat{\sigma}_i$

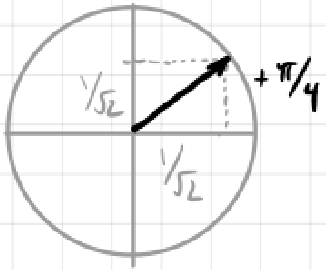
$$\boxed{Y(\theta = \tfrac{\pi}{2})} = e^{-i(\pi/2)\hat{\sigma}_y/2}$$

$$\hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

(minus i is high)

$$= \cos\left(\tfrac{\pi}{2} \cdot \tfrac{1}{2}\right) \hat{\mathbb{1}} - i \sin\left(\tfrac{\pi}{2} \cdot \tfrac{1}{2}\right) \hat{\sigma}_y$$

$$= \underbrace{\cos(\pi/4)}_{1/\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i \underbrace{\sin\left(\tfrac{\pi}{4}\right)}_{= 1/\sqrt{2}} \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}$$



$$\cos(\pi/4) = \frac{1}{\sqrt{2}} \qquad \sin(\pi/4) = \frac{1}{\sqrt{2}}$$

Euler Formula for Matrices: $e^{-i\theta\cdot\hat{\sigma}_i/2} = \cos(\theta/2)\cdot\hat{\mathbb{1}} - i\sin(\theta/2)\cdot\hat{\sigma}_i$

$$\boxed{Y(\theta=\tfrac{\pi}{2})} = e^{-i(\pi/2)\hat{\sigma}_y/2} \qquad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

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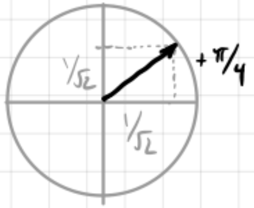
$$= \cos\left(\tfrac{\pi}{2}\cdot\tfrac{1}{2}\right)\hat{\mathbb{1}} - i\sin\left(\tfrac{\pi}{2}\cdot\tfrac{1}{2}\right)\hat{\sigma}_y$$

$$= \underbrace{\cos(\pi/4)}_{1/\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\underbrace{\sin\left(\tfrac{\pi}{4}\right)}_{=1/\sqrt{2}}\begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}$$



$$\cos(\pi/4) = \frac{1}{\sqrt{2}} \qquad \sin(\pi/4) = \frac{1}{\sqrt{2}}$$

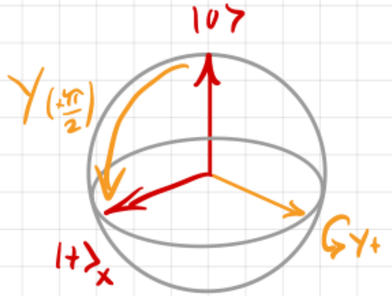
$$Y(\pi/2) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & +i^2 \\ -i^2 & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$Y(\pi/2)|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right] = |+\rangle_x$$



Euler Formula for Matrices: $e^{-i\theta\cdot\hat{\sigma}_i/2} = \cos(\theta/2)\cdot\hat{\mathbb{1}} - i\sin(\theta/2)\cdot\hat{\sigma}_i$

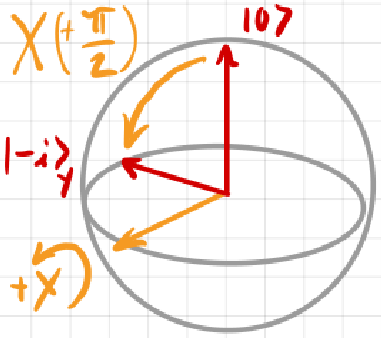
$$X\left(\theta=\tfrac{\pi}{2}\right) = e^{-i(\pi/2)\,\hat{\sigma}_x/2} \qquad \hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \underbrace{\cos\left(\pi/2\cdot\tfrac{1}{2}\right)}_{\cos(\pi/4)=\frac{1}{\sqrt{2}}}\hat{\mathbb{1}} - i\,\underbrace{\sin\left(\pi/2\cdot\tfrac{1}{2}\right)}_{\sin(\pi/4)=\frac{1}{\sqrt{2}}}\hat{\sigma}_x$$

$$X\left(\theta=\tfrac{\pi}{2}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\cdot\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$X\left(\theta=\tfrac{\pi}{2}\right)\cdot|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)$$



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Euler Formula for Matrices: $e^{-i\theta\cdot\hat{\sigma}_i/2} = \cos(\theta/2)\cdot\hat{\mathbb{1}} - i\sin(\theta/2)\cdot\hat{\sigma}_i$

$\boxed{Z(\theta=\pi)}$

$$= e^{-i\theta\hat{\sigma}_Z/2}$$

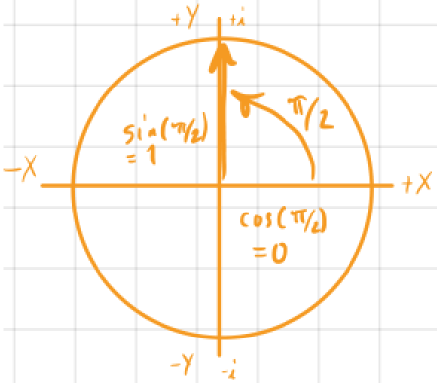
$$= \underbrace{\cos(\pi/2)}_{=0}\,\hat{\mathbb{1}} - i\,\underbrace{\sin(\pi/2)}_{=1}\,\hat{\sigma}_Z$$

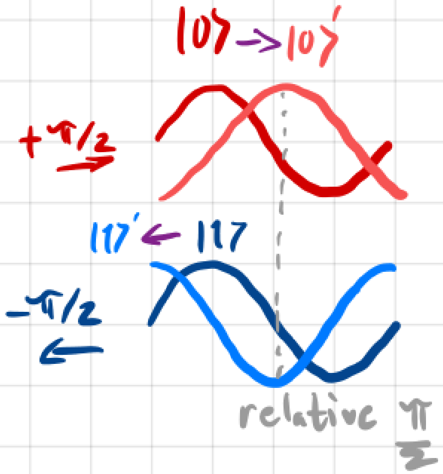
$$= \cancel{0\cdot\begin{bmatrix}1&0\\0&1\end{bmatrix}} - i\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$= \begin{bmatrix}-i&0\\0&+i\end{bmatrix} = \begin{bmatrix}e^{i\pi/2}&0\\0&e^{-i\pi/2}\end{bmatrix}$$

$\pm\pi/2$ to both states

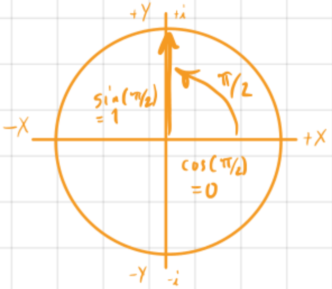
$$\hat{\sigma}_Z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$$





$$\boxed{Z(\theta=\pi)} = e^{-i\theta\hat{\sigma}_z/2}$$

$$\hat{\sigma}_z = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

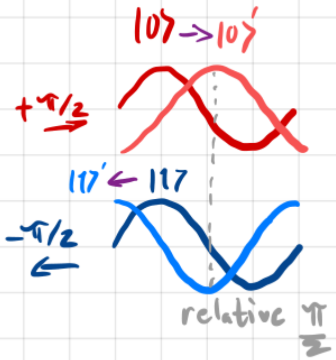


$$= \underbrace{\cos(\pi/2)}_{=0}\,\hat{\mathbb{1}} - i\underbrace{\sin(\pi/2)}_{=1}\,\hat{\sigma}_z$$

$$= \cancel{0\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}} - i\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}-i & 0\\ 0 & +i\end{bmatrix} = \begin{bmatrix}e^{i\pi/2} & 0\\ 0 & e^{-i\pi/2}\end{bmatrix}$$

± π/2 to both states



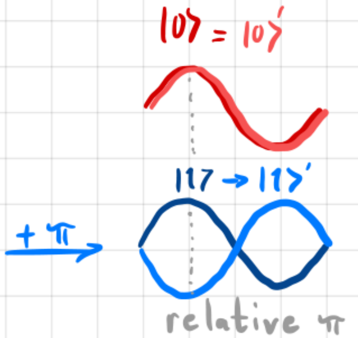
$$\Rightarrow \underbrace{e^{+i\pi/2}}_{\text{Global phase (rotate both states)}}\begin{bmatrix}-i & 0\\ 0 & +i\end{bmatrix} = \begin{bmatrix}e^{+i\pi/2 - i\pi/2} & 0\\ 0 & e^{i\pi/2 + i\pi/2}\end{bmatrix}$$

$e^{i\pi/2} = +i$

$e^{-i\pi/2} = -i$

$$= \begin{bmatrix}e^{0} & 0\\ 0 & e^{i\pi}\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

π phase shift! to just one state



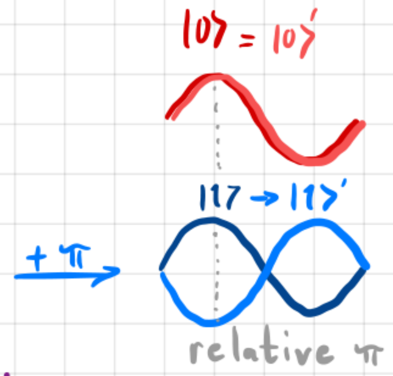
$$\Psi_1 = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

$$\Rightarrow \underbrace{e^{+i\pi/2}}_{\substack{\text{Global}\\ \text{phase}\\ \text{(rotate both states)}}} \begin{bmatrix} -i & 0 \\ 0 & +i \end{bmatrix} = \begin{bmatrix} e^{+i\pi/2 - i\pi/2} & 0 \\ 0 & e^{i\pi/2 + i\pi/2} \end{bmatrix}$$

$$e^{i\pi/2} = +i$$
$$e^{-i\pi/2} = -i$$

$$= \begin{bmatrix} e^{0} & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

π phase shift! to just one state



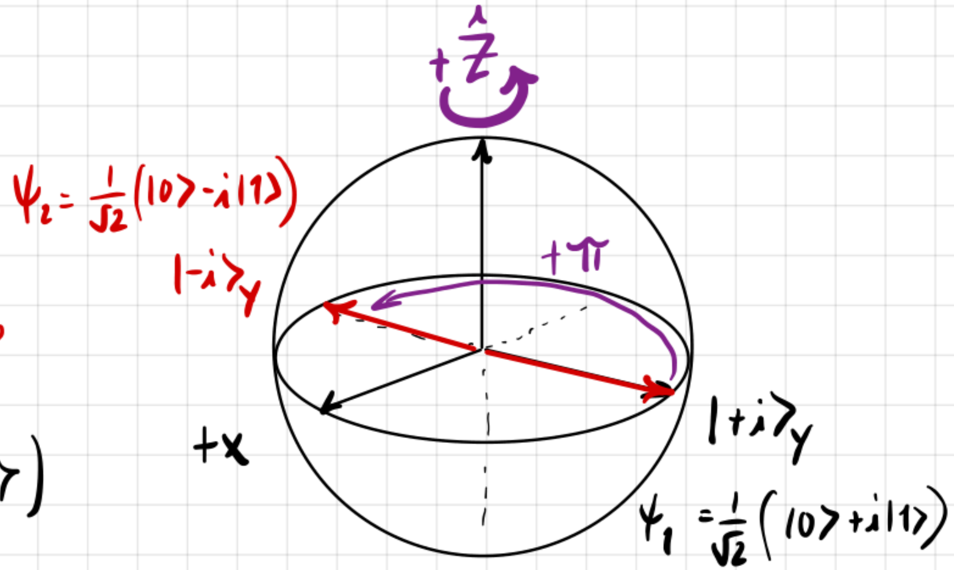
$$\psi_1 = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle \right)$$

$$|\psi_2\rangle = \boxed{Z(\pi)} \cdot \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle \right)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ +i \end{bmatrix}$$

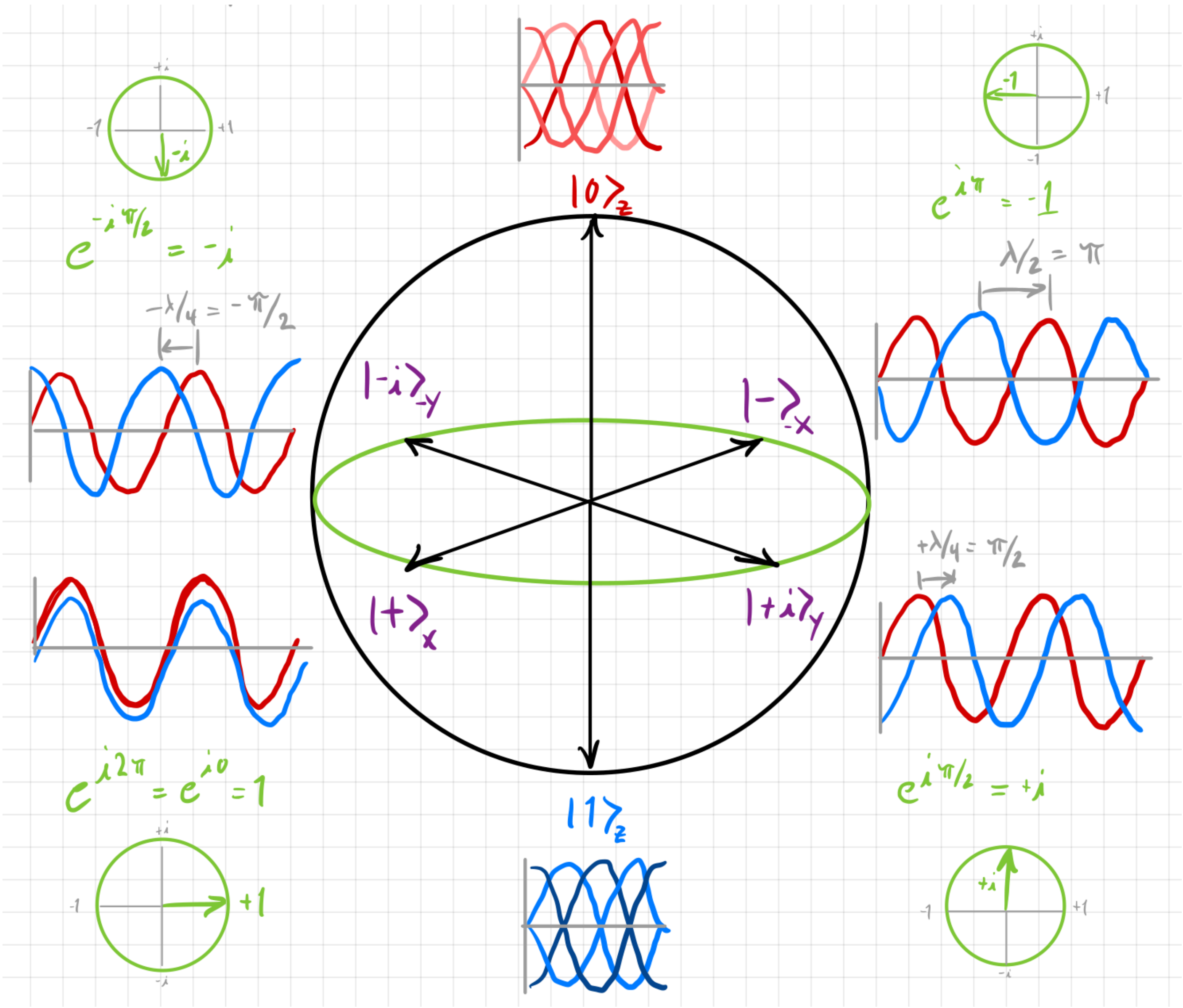
Flip

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \left(|0\rangle - i|1\rangle \right)$$



Bloch Sphere Basis States

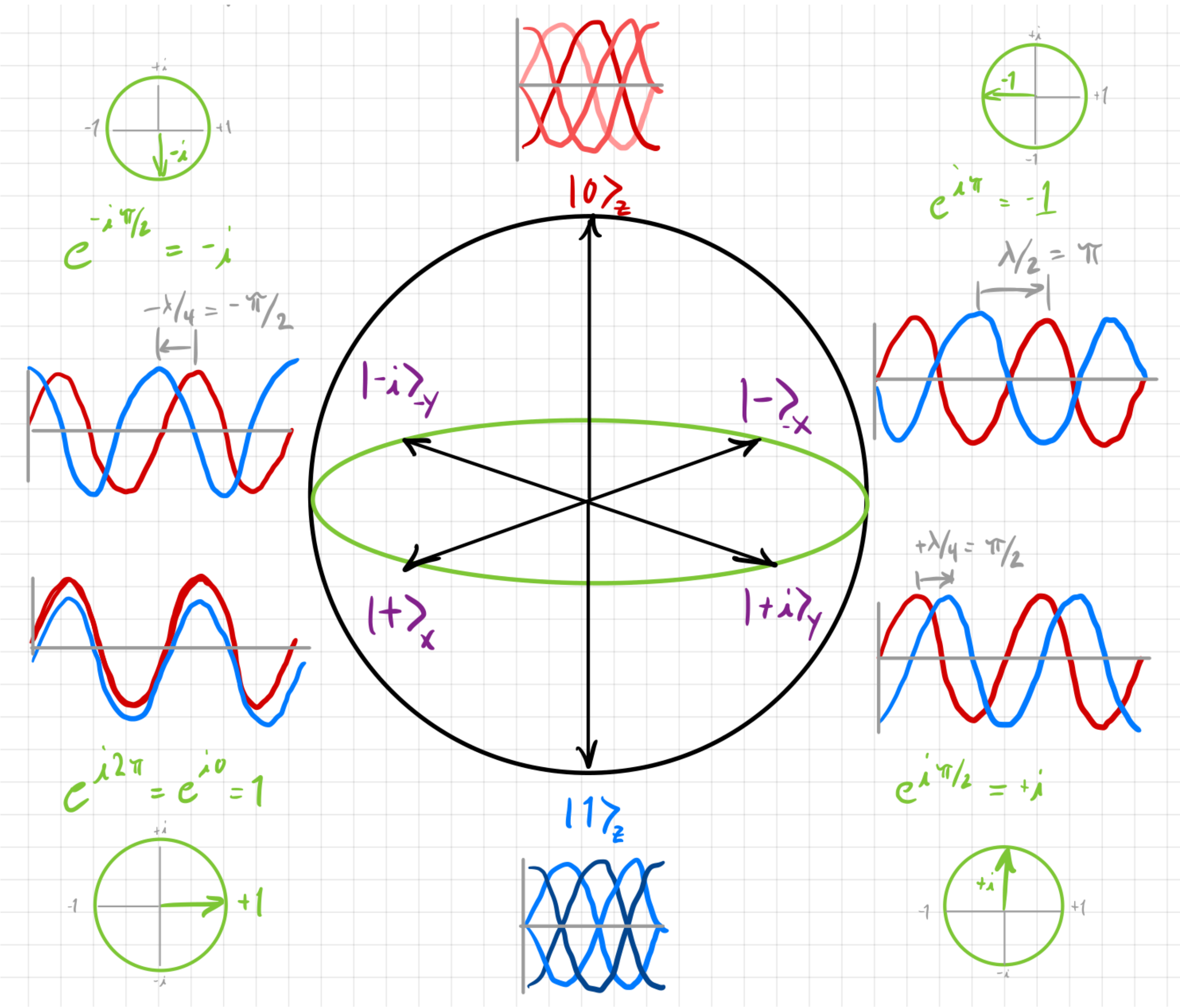
X Basis	$\|+\rangle_x = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$\|-\rangle_x = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$
Y Basis	$\|i\rangle_y = \frac{1}{\sqrt{2}}(\|0\rangle + i\|1\rangle)$	$\|-i\rangle_y = \frac{1}{\sqrt{2}}(\|0\rangle - i\|1\rangle)$
Z Basis	$\|0\rangle_z$	$\|1\rangle_z$



|0⟩z
|1⟩z
|-i⟩-y
|-⟩-x
|+⟩x
|+i⟩y
e^{-iπ/2} = -i
-λ/4 = -π/2
e^{iπ} = -1
λ/2 = π
+λ/4 = π/2
e^{i2π} = e^{i0} = 1
e^{iπ/2} = +i
+1
-1
+i
-i

Bloch Sphere Basis States

X Basis	$\lvert + \rangle_x = \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)$	$\lvert - \rangle_x = \frac{1}{\sqrt{2}}(\lvert 0\rangle - \lvert 1\rangle)$
Y Basis	$\lvert i \rangle_y = \frac{1}{\sqrt{2}}(\lvert 0\rangle + i\lvert 1\rangle)$	$\lvert -i \rangle_y = \frac{1}{\sqrt{2}}(\lvert 0\rangle - i\lvert 1\rangle)$
Z Basis	$\lvert 0\rangle_z$	$\lvert 1\rangle_z$



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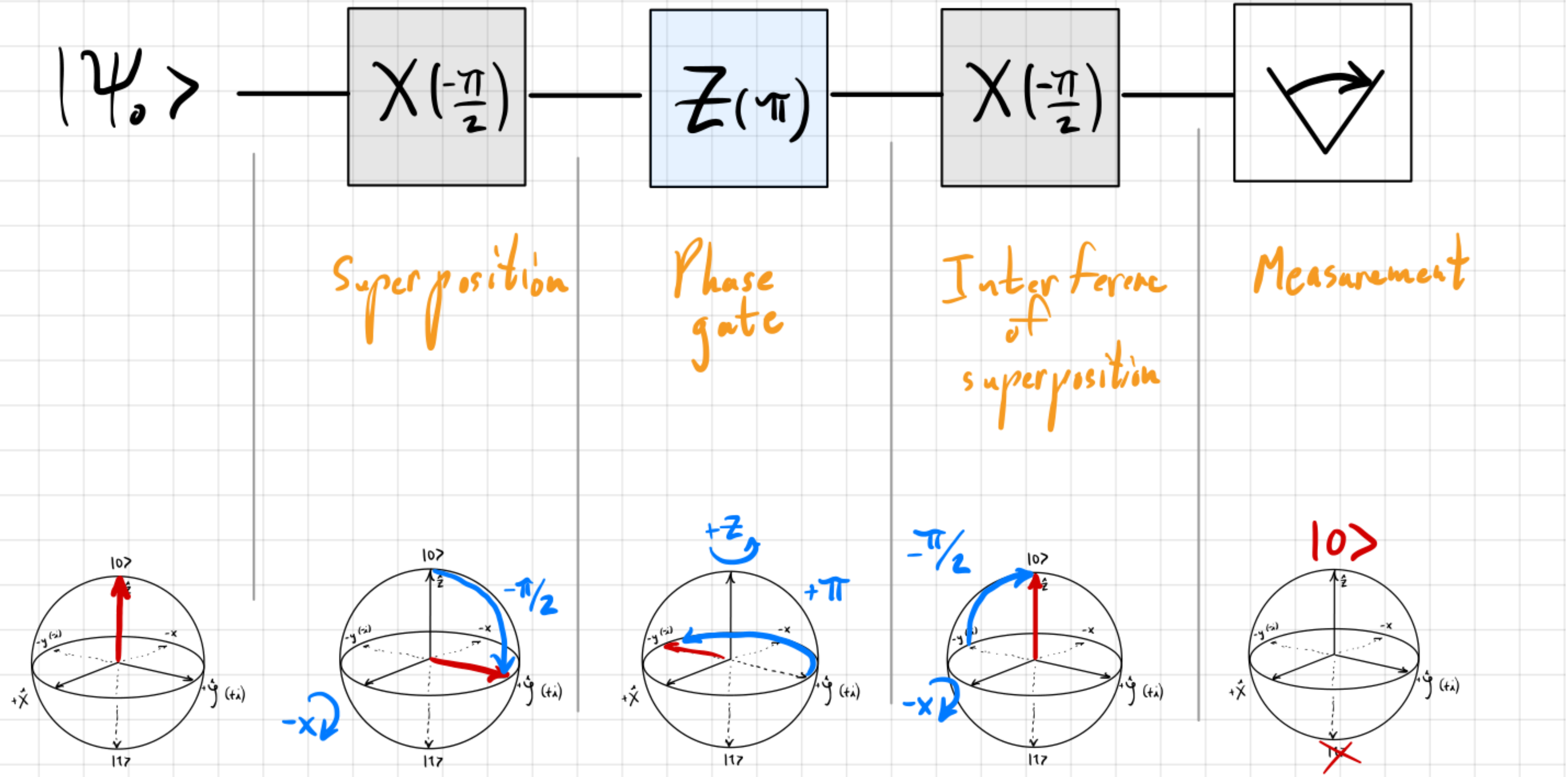


$\psi_0 = |0\rangle$

$\psi_1 = X(-\pi/2)\psi_0$

$\psi_1 = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$

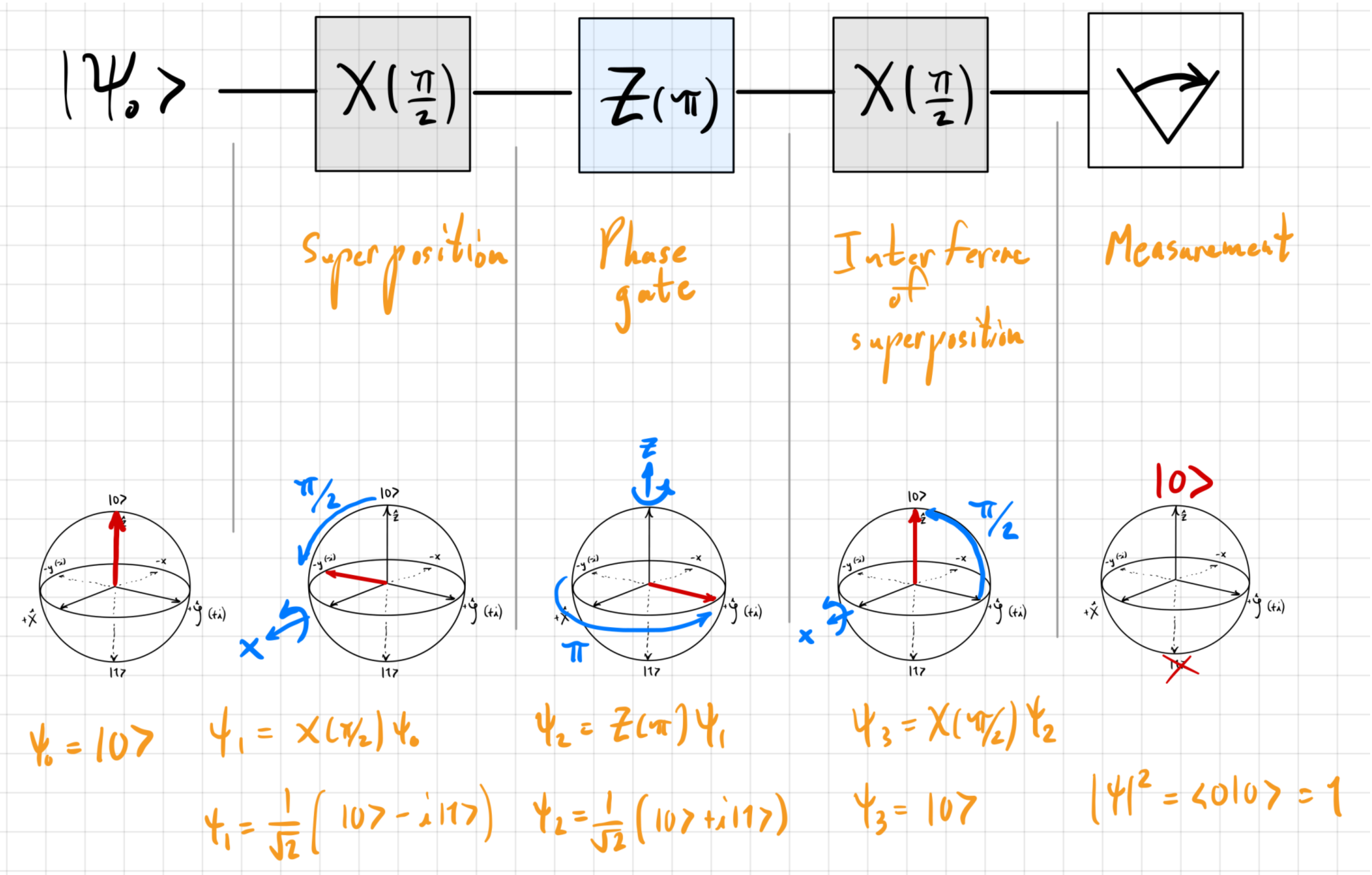
$\psi_2 = Z(\pi)\psi_1$

$\psi_2 = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)$

$\psi_3 = X(-\pi/2)\psi_2$

$\psi_3 = |0\rangle$

$|\psi|^2 = \langle 0|0\rangle = 1$


$|\psi_0\rangle$
$X(\frac{\pi}{2})$
$Z(\pi)$
$X(\frac{\pi}{2})$
Superposition
Phase gate
Interference of superposition
Measurement
$\pi/2$
Z
π
$\pi/2$
$|0\rangle$
$\psi_0 = |0\rangle$
$\psi_1 = X(\pi/2)\,\psi_0$
$\psi_1 = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$
$\psi_2 = Z(\pi)\,\psi_1$
$\psi_2 = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$
$\psi_3 = X(\pi/2)\,\psi_2$
$\psi_3 = |0\rangle$
$|\psi|^2 = \langle 0|0\rangle = 1$

Pauli Matrices (Spin Matrices)

$$\hat{\sigma}_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{\sigma}_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Projection Operators

$$\hat{\sigma}_z \equiv |0\rangle\langle 0| - |1\rangle\langle 1| \qquad \hat{\sigma}_x \equiv |0\rangle\langle 1| + |1\rangle\langle 0| \qquad \hat{\sigma}_y \equiv -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

No change in state
but changes phase (identity adds 'i')

Changes state
(and phase from identity)

Changes state
(phase cancels in identity)

Pauli Matrices and rotations about the Bloch Sphere

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The rotations of the Bloch sphere about the Cartesian axes in the Bloch basis are then given by:

$$R_x(\theta) = e^{(-i\theta X/2)} = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = e^{(-i\theta Y/2)} = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_z(\theta) = e^{(-i\theta Z/2)} = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

Time Dependent Schrödinger Eq.

$$i\hbar \frac{d\vec{\Psi}}{dt} = E \cdot \vec{\Psi}$$

Describes h
control th
$\Rightarrow$ How we

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

$\hat{H} =$

General Solution:

$$|\psi(t)\rangle = \underbrace{\boxed{e^{-i\hat{H}t/\hbar}}}_{\text{Rotation!!!}} \cdot \underbrace{|\psi(t=0)\rangle}_{\text{Initial state}}$$

Arbitrary angle of rotation

We can also rotate the state about the other axes of the Bloch Sphere.

Generically for any axis P = {X,Y,Z} on the Bloch Sphere:

$$\boxed{R_p(\theta) = e^{(-i\theta P/2)}} = \cos(\theta/2)I - i\sin(\theta/2)P$$

Pauli Matrices

X,Y = State (coupling) **Z = Energy/Phase**

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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Measurement and Expectation Values

Expectation value of spin:
$\langle S_z \rangle = \langle \Psi | \mathrm{S_z} | \Psi \rangle$

Recall photons:
$\langle I \rangle = \langle \vec{\mathrm{E}} | \vec{\mathrm{E}} \rangle$

Plank's Constant
$\frac{h}{2\pi} = \hbar = 1 \times 10^{-34}\, J \cdot s$
Energy (Joules) = $\hbar\omega = hf$

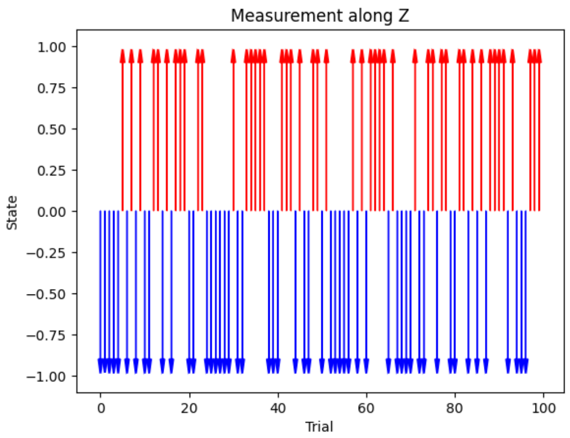
$$\hat{S}_z \equiv \frac{\hbar}{2}\hat{\sigma}_z \equiv \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

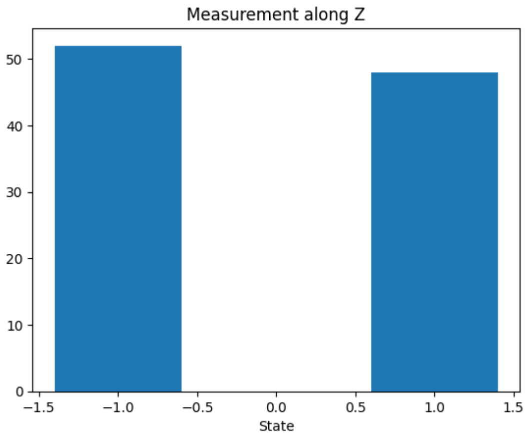
$$\langle 0|S_z|0\rangle = \frac{\hbar}{2}\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1*1+0*0\\ 0*1+(-1)*0\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{\hbar}{2}$$

$$\langle 1|S_z|1\rangle = \frac{\hbar}{2}\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}1*0+0*1\\ 0*0+(-1)*1\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\ -1\end{pmatrix} = \frac{-\hbar}{2}$$

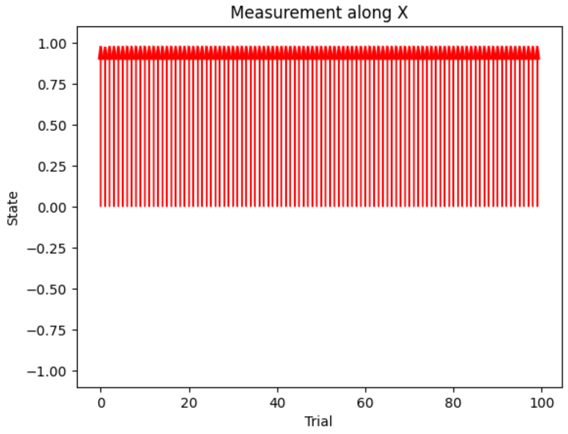
Measurement Basis

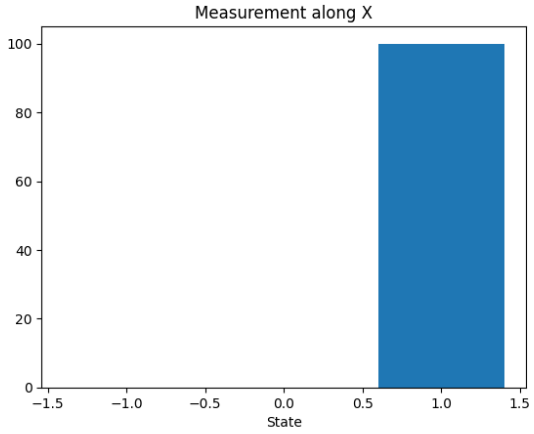
- Measure |+⟩ along Z:





- Measure |+⟩ along X:





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$$\langle + | S_z | + \rangle = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 * \frac{1}{\sqrt{2}} + 0 * \frac{1}{\sqrt{2}} \\ 0 * \frac{1}{\sqrt{2}} + (-1) * \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = 0$$