UMass Physics Textbook 131

By: David Nguyen
Online: <**https://legacy.cnx.org/content/col12181/1.21**>

Collection structure revised: 2017/06/23
PDF Generated: 2017/06/23 11:36:09

Table of Contents

UNIT 1 OVERVIEW

UMASS AMHERST Instructor's Notes

Things to Consider as You Read:

This section provides an overview of the first unit on mathematical tools and foundational concepts for Physics 131; we will be using concepts from this unit throughout the class.

There are also a few hints and tips in the section on how to do the homework efficiently; I would suggest developing good homework habits early.

This overview is also available as a video on Youtube at **https://www.youtube.com/watch?v=ikLsqv2dhY8 (https://www.youtube.com/watch?v=ikLsqv2dhY8)** .

In this unit, we will explore some of the fundamental mathematical tools and basic concepts that we will need throughout the rest of the course in our study of physics, including:

- An introduction to what physics is as a discipline and how that might be similar to or different from some of the other sciences you may have studied,
- A review of the basic idea of units
- My policy on significant figures
- Introduce the basic ideas of mean and standard deviation for use in the laboratory exercises within this course
- The definition of displacement, velocity, and acceleration; in particular, how velocity and acceleration are similar to and different from distance and speed
- How to use iterative methods to predict the motion of objects that move with non-uniform acceleration
- On a purely mathematical note, you will be exploring what a vector is and how they can be added and subtracted.

General Notes About Homework

The homework in this course is intended to provide you with some basic information. The material in the preparation will be the starting point for what we discuss in class. This helps to make sure that everyone with their varying backgrounds in physics is starting at the same point. We will then build upon this preparation in class, using in-class activities to get you ready for exams. This is somewhat different probably from your other courses where the purpose of the homework is to provide additional practice on in-class material to help you get ready for exams. **In this course, the homework gets you ready for class, and class is what gets you ready for the exams.**

How to be Successful

Each homework is divided up into sections. Within each section, the first question is your readings to do for that particular section, followed by a set of problems. The information you need to complete a set of problems will be in the readings at the beginning of that section. The readings are presented in terms of a checklist. This problem is not for a grade, it's just presented as a checklist to make sure that you get everything done. So, you may have various readings within the OpenStax textbook UMass edition which is on Perusall, and you may also have some videos. The videos are embedded directly within Mastering and you should be able to play them right there, but if you cannot, you can go and click this link and it will take you to the course YouTube page, and you can watch the videos there. The transcripts for all of the videos are also included in the textbook themselves, so if you want to go and read the text because you prefer to read or you want to add some sort of Perusall comment to some of the content of the video, you can do that within the textbook in Perusall, so each video has an associated section in the textbook and in Perusall with the transcript of that video for you to comment.

Once you've completed the readings, you're now ready to move on to the actual homework problems. These problems are there to help you check that you understood what was in the various readings and videos, and to help you refine your understanding. Most of the individual parts of each problem are one-step. If you find yourself doing long chains of calculations, come get help in the consultation room. You're probably approaching the problem in a way that's not very efficient. When doing the homework, don't skip the readings and the videos. Your comments on the actual readings in Perusall are graded in accordance with the policy in the syllabus and form a part of your homework grade. We acknowledge that doing all of these readings and all of this homework is hard work, and we are here to help; we've provided quite a few resources to help you be successful in completing this assignment. Moreover, since it is so much work, the preparation is your entire homework for this course. There is no required end of chapter homework assignments; you only need to do this preparation. This is your big focus for your homework.

What to focus on in the Unit 1 Preparation Homework

I want you to focus on, while doing this homework, the definitions of the terms **position**, **velocity**, and **acceleration**, the few basic equations such as $\vec{v} = \frac{\Delta x}{\Delta t}$ and $\vec{a} = \frac{\Delta v}{\Delta t}$, including what all the symbols mean and when these equations can be applied. Many people in studying physics for the first time understand they need to know what the symbols mean, but they tend to skip over this second element, which is just as important, if not more so, because not every equation can be applied in every situation. I will also ask you to learn how to just “turn the crank” for various types of calculations, such as iterative calculations, and vector arithmetic. Don’t worry if you don’t really understand what you’re doing when you do these calculations. If conceptually it doesn’t make sense, that’s okay; we will spend time in class working with these ideas and getting an understanding of what you’re doing. I just want you to know how to do these calculations.

Finally, I would like to have a quick philosophical comment regarding motion with constant acceleration. If you have had any physics before, you may have seen the so-called kinematic equations, which are these two here:

$$d = v_0 t + \frac{1}{2} a t^2 \quad v^2 = {v_0}^2 + 2ad$$

We will NOT be using these equations in this class. We will be approaching the subject, and many others, in ways that may be different from how you may have seen them in a previous physics class. We believe that physics is not about memorizing equations and learning how to piece those equations together. We believe instead that physics is about fundamental ideas, and we will teach this course from this perspective. Occasionally, this will result in physics homework very different from what you may expect. A good example is the homework for the second unit, where you have some actual fill-in-the-blank type of questions. If you try to learn physics as a set of ideas instead of a set of equations to be pieced together, and start your analysis of situations from fundamental physical principles, then your physics experience will enrich and enhance your understanding of your other courses, as opposed to just being a course that you just “have to take for your major”.

1 INTRODUCTION: THE NATURE OF SCIENCE AND PHYSICS

Figure 1.1 Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

Chapter Outline

Chapter Overview

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

1.1 An Introduction to Physics

UMASS AMHERST Instructor's Notes

Things to Consider as You Read:

- What *is* science? Where does physics fit into this definition?
- How does the scientific process work?
- What is a scientific theory? What makes a theory credible?
- How will physics help you as a non-physics major?
- How are models used in science?
- What are some of the limits of classical physics?

Figure 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics.

Consider a smart phone (**Figure 1.3**). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

Figure 1.3 The Apple “iPhone” is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See **Figure 1.4** and **Figure 1.5**.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car’s ignition system as well as the transmission of electrical signals through our body’s nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (**Figure 1.6** and **Figure 1.7**). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

UMASS AMHERST Instructor's Notes

While it might feel like you'll never need to use physics later on, knowing the physics of what's happening will give you a deeper understanding, or at least a deeper appreciation, of it. As we move forward in the course, try to connect the material in physics to your outside studies; it might be a lot easier than you expect!

Figure 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

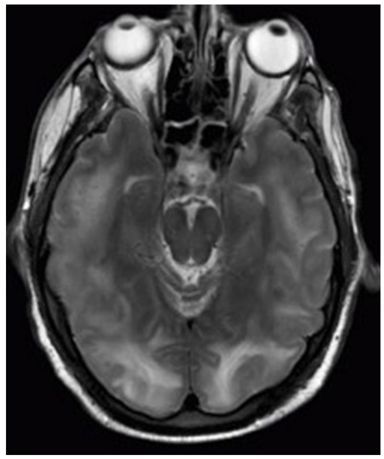


Figure 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)

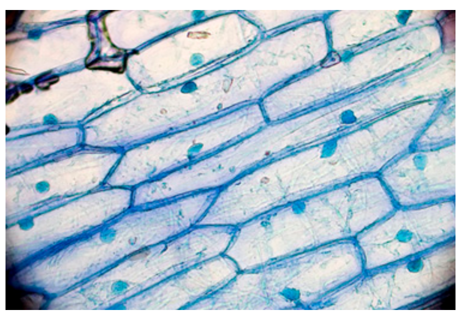


Figure 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

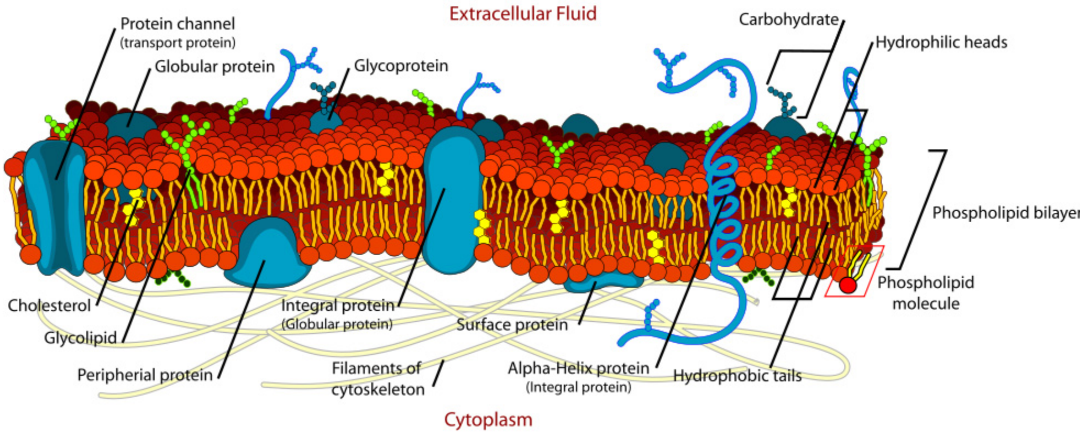


Figure 1.7 An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See **Figure 1.8** and **Figure 1.9**.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

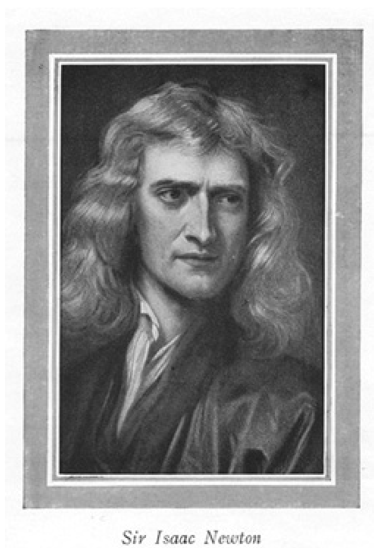


Figure 1.8 **Isaac Newton** (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Figure 1.9 **Marie Curie** (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See **Figure 1.10**.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

UMASS AMHERST Instructor's Notes

Thinking in terms of laws and principles will be an important aspect of this course. In terms of problem solving, one of the first steps in solving a problem will be to think about which laws and principles apply. Keep this in mind as we move forward; taking the time to stop and take this step will help you in the long run.

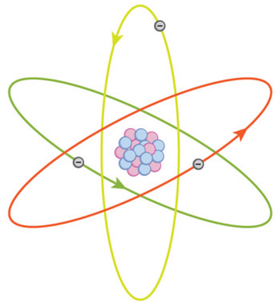


Figure 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See **Figure 1.11**, **Figure 1.12**, and **Figure 1.13**.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

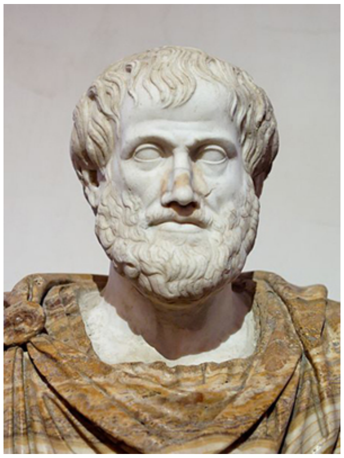


Figure 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)

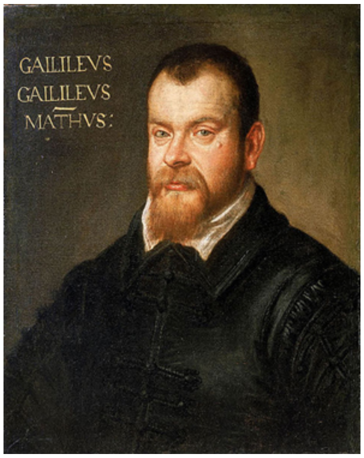


Figure 1.12 **Galileo Galilei** (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)

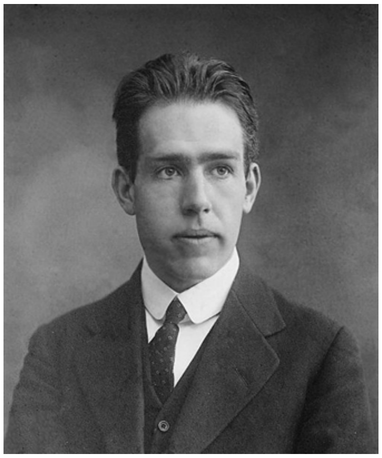


Figure 1.13 **Niels Bohr** (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

UMASS AMHERST Instructor's Notes

This idea of creating models will be relevant in 132; we will be dealing with electrons and photons, which don't behave the way most things do. Being able to wrap your head around these ideas will be challenging, but being able to model this strangeness will help you over this hurdle.

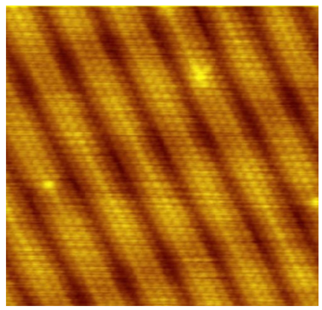


Figure 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

PhET Explorations: Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

PhET Interactive Simulation

Figure 1.15 Equation Grapher (http://legacy.cnx.org/content/m64001/1.35/equation-grapher_en.jar)

1.2 Physical Quantities and Units

UMASS AMHERST Instructor's Notes

This section should be a refresher for most of you, and so reading this section is optional. However, if this is unfamiliar territory for you, I would suggest reading through this section, and please come and see me if you are still uncomfortable with this material and we will work something out.

Things to Consider as You Read:

- Familiarizing yourself with the prefixes, such as nano-, milli-, centi-, will save you some trouble down the road.

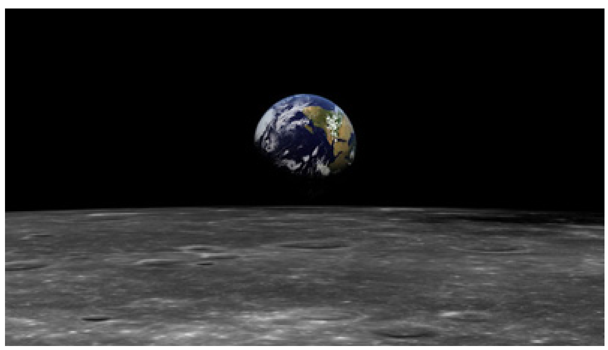


Figure 1.16 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See **Figure 1.17**.)

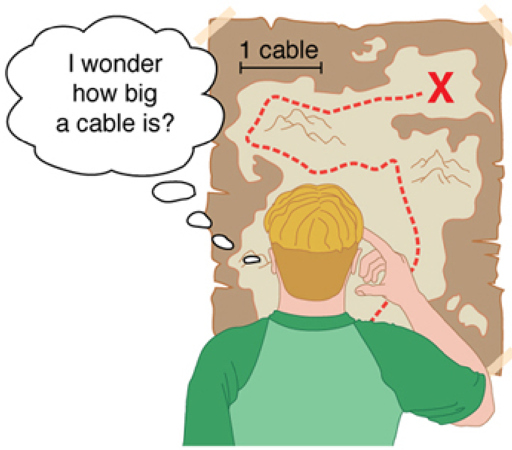


Figure 1.17 Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth’s rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See **Figure 1.18**.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

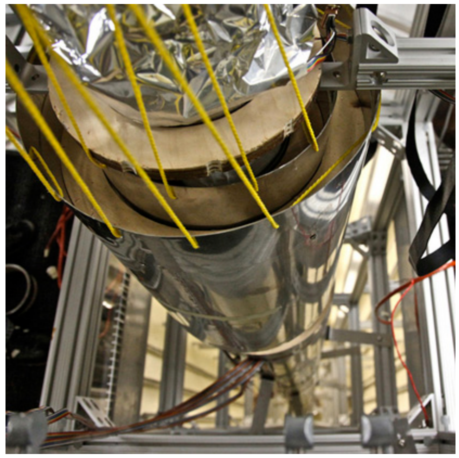


Figure 1.18 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See **Figure 1.19**.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.

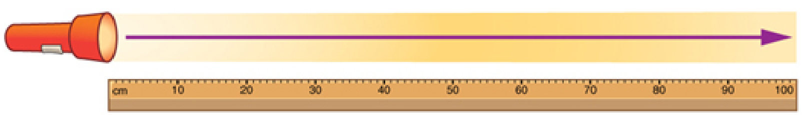


Figure 1.19 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in **Introduction to Electric Current, Resistance, and Ohm's Law (https://legacy.cnx.org/content/m42339/latest/)** when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. **Table 1.2** gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}\text{ m},$ while the diameter of the Sun is on the order of $10^9\text{ m}.$

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Table 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value[1]	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^9	gigahertz	GHz	10^9 Hz	a microwave frequency
mega	M	10^6	megacurie	MCi	10^6 Ci	high radioactivity
kilo	k	10^3	kilometer	km	10^3 m	about 6/10 mile
hecto	h	10^2	hectoliter	hL	10^2 L	26 gallons
deka	da	10^1	dekagram	dag	10^1 g	teaspoon of butter
—	—	10^0 (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in **Table 1.3**. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See **Figure 1.20** and **Figure 1.21**.)

1. See Appendix A (https://legacy.cnx.org/content/m42699/latest/) for a discussion of powers of 10.

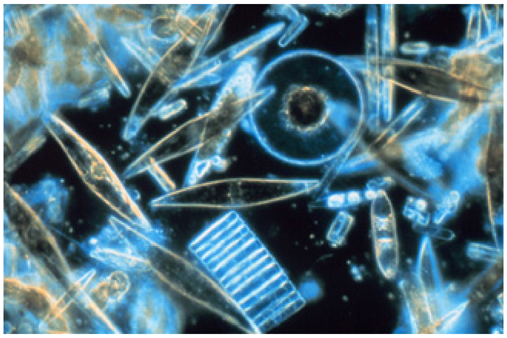


Figure 1.20 Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

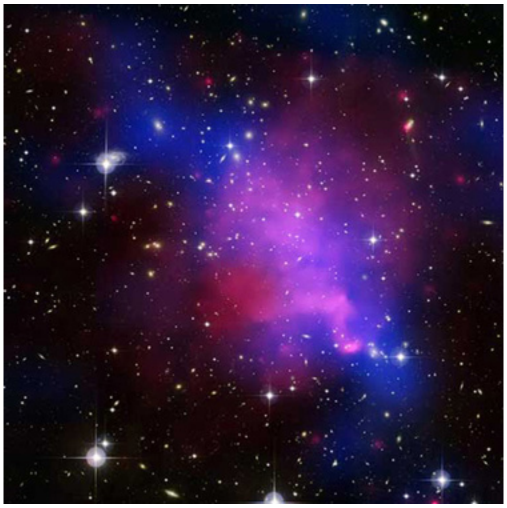


Figure 1.21 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\,\cancel{\text{m}}\times\frac{1\text{ km}}{1000\,\cancel{\text{m}}}=0.080\text{ km}. \tag{1.1}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click **m42720 (https://legacy.cnx.org/content/m42720/latest/)** for a more complete list of conversion factors.

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $\left(9.11\times10^{-31}\text{ kg}\right)$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $\left(1.67\times10^{-27}\text{ kg}\right)$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $\left(8.64\times10^{4}\text{ s}\right)$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $\left(3.16\times10^{7}\text{ s}\right)$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $\left(7.35\times10^{22}\text{ kg}\right)$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $\left(5.97\times10^{24}\text{ kg}\right)$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $\left(1.99\times10^{30}\text{ kg}\right)$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Example 1.1 Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}. \tag{1.3}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr . Thus,

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}. \tag{1.4}$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2}, \tag{1.5}$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}}, \tag{1.6}$$

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}. \tag{1.7}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module **Accuracy, Precision, and Significant Figures (https://legacy.cnx.org/content/m42120/latest/)** will help you answer these questions.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution
The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

1.3 Writing Numbers

UMASS AMHERST Instructor's Notes

Your Quiz would Cover
- Estimating the number of appropriate digits for any calculation. One big thing is that we will use properties of data to predict/determine the number of decimal points that are appropriate.

Other Things to Consider as You Read:
- While we won't be using sig-fig rules, there are some guidelines on the appropriate amount of digits to include in a result.

The following section is based upon:
Denker, J. Uncertainty as Applied to Measurements and Calculations. Uncertainty as Applied to Measurments and Calculations (2011). Available at: **http://www.av8n.com/physics/uncertainty.htm (http://www.av8n.com/physics/uncertainty.htm)** . (Accessed: 26th August 2016)

How Many Digits To Include

Here are some simple rules that apply whenever you are writing down a number:

1. Use many enough digits to avoid unintended loss of information.

2. Use few enough digits to be reasonably convenient.

Important note: The previous two sentences tell you everything you need to know for most purposes, including real-life situations as well as academic situations at every level from primary school up to and including introductory college level.

3. When using a calculator, it is good practice to leave intermediate results in the machine. This is simultaneously more accurate and more convenient than writing them down and then keying them in again.

Seriously: The primary rule is to use plenty of digits. You hardly even need to think about it. Too many is vastly better than too few. To say the same thing the other way: If you ever have more digits than you need *and* they are causing major inconvenience, then you can think about reducing the number of digits.

When to Write Down Uncertainty

In many cases, when you write down a number, you need not *and should not* associate it with any notion of uncertainty.

- One way this can happen is if you have a number with zero uncertainty. If you roll a pair of dice and observe five spots, the number of spots is 5. This is a raw data point, with no uncertainty whatsoever. So just write down the number. Similarly, the number of centimeters per inch is 2.54, by definition, with no uncertainty whatsoever. Again: just write down the number.
- Another possibility is that there is a cooked data blob, which in principle must have “some” uncertainty, but the uncertainty is too small to be interesting. It is insignificant. It is unimportant. It is immaterial. There are plenty of situations a moderately rough approximation is sufficient. There are even some situations where an *extremely* rough approximation is called for.

Along the same lines, here is a less-extreme example that arises in the introductory chemistry class. Suppose the assignment is to balance the equation for the combustion of gasoline, namely

$a\ C_8H_{18} + b\ O_2 \rightarrow x\ CO_2 + y\ H_2O$

by finding numerical values for the coefficients *a*, *b*, *x*, and *y*. The conventional answer is (*a*, *b*, *x*, *y*) = (2, 25, 16, 18). The outcome of the real reaction must have “some” uncertainty, because there will generally be some non-idealities, including the presence of other molecules such as CO or C_{60}, not to mention NO_2 or whatever. However, my point is that we don’t necessarily care about these non-idealities. We can perfectly well find the idealized solution to the idealized equation and postpone worrying about the non-idealities and uncertainties until much, much later.

As another example, suppose you use a digital stopwatch to measure some event, and the reading is 1.234 seconds. We call this number the *indicated* time, and we distinguish it from the *true* time of the event. In principle, there is no chance that the indicated time will be exactly equal to the true time; true time is a *continuous variable*, which means that it can take on an infinite amount of values, such as 1.234, 1.2341, 1.23406, 1.2360000009, and so on, whereas the indicated time is *quantized*, which means that it can only take on certain values, such as, if you have a stopwatch with only three decimal places, you can have 1.234 and 1.235, but no values in between, such as 1.2347. However, in many cases you may decide that it is close enough, in which case you should just write down the indicated reading and not worry about the quantization error.

Let us continue with the stopwatch example. Suppose we make two observations. The first reading is 1.234 seconds, and the second reading is just the same, namely 1.234 seconds. Meanwhile, however, you may believe that if you repeated the experiment many times, the resulting set of readings would have some amount of scatter, namely ± 0.01 seconds. The two observations that we actually have don't show any scatter at all, so your estimate of the uncertainty remains hypothetical and theoretical. Theoretical information is still information, and should be written down in the lab book, plain and simple. For example, you might write a sentence that says "Intuition suggests the timing data is reproducible ± 0.01 seconds." It would be even better to include some explanation of why you think so. The principle is simple: Write down what you know. Say what you mean, and mean what you say. The same principle applies to the indicated values. The recommend practice is to write down each indicated value, as-is, plain and simple.

You are not trying write down the true values. You don't know the true values (except insofar as the indicated values represent them, indirectly). You don't need to know the true values, so don't worry about it. The rule is: *Write down what you know.* So write down the indicated value. Also: You are not obliged to attribute any uncertainty to the numbers you write down. Normal lab-book entries do not express an uncertainty using $A \pm B$ notation or otherwise, and they do not "imply" an uncertainty using sig figs or otherwise. We are always uncertain about the true value, but we aren't writing down the true value, so that's not a concern.

Some people say there must be some uncertainty "associated" with the number you write down, and of course there is, indirectly, in the sense that the indicated value is "associated" with some range of true values. We are always uncertain about the true value, but that does not mean we are uncertain about the indicated value. These things are "associated" ... but they are not the same thing.

In a well-designed experiment, things like readability and quantization error usually do not make a large contribution to the overall uncertainty anyway. Please do not confuse such things with "the" uncertainty.

It is usually a good practice to keep all the original data. When reading an instrument, read it as precisely as the instrument permits, and write down the reading "as is" ... without any conversions, any roundoff, or anything else.

Why We're Not Using Significant Figures

No matter what you are trying to do, significant figures are the wrong way to do it.

When writing, do not use the number of digits to imply anything about the uncertainty. If you want to describe a distribution, describe it explicitly, perhaps using expressions such as 1.234 ± 0.055.

When reading, do not assume the number of digits tells you anything about the overall uncertainty, accuracy, precision, tolerance, or anything else, unless you are absolutely sure that's what the writer intended ... and even then, beware that the meaning is very unclear.

People who care about their data don't use sig figs.

Significant-digit dogma destroys your data and messes up your thinking in many ways, including:

- Given data that can be described by an expression such as $A \pm B$, such as 1.234 ± 0.055, converting it to sig figs gives you an excessively crude and erratic representation of the uncertainty, B.
- Converting to sig figs can cause excessive roundoff error in the nominal value, A. This is a big problem.
- Sig figs cause people to misunderstand the distinction between roundoff error and uncertainty.
- Sig figs cause people to misunderstand the distinction between uncertainty and significance. Sig figs cause people to misunderstand the distinction between the *indicated value* and the corresponding range of *true values*.
- Sig figs cause people to misunderstand the distinction between distributions and numbers. Distributions have width, whereas numbers don't. Uncertainty is necessarily associated with some distribution, not with any particular point that might have been drawn from the distribution.
- As a consequence, sig figs make people hesitate to write down numbers. They think they need to know the amount of supposedly "associated" uncertainty before they can write the number, when in fact they don't. Very commonly, there simply isn't any "associated" uncertainty anyway.
- Sig figs weaken people's understanding of the axioms of the decimal numeral system.
- Sig figs provide no guidance as to the appropriate decimal representation for repeating decimals such as 80 ÷ 81, or irrational numbers such as √2 or π.

UMASS AMHERST Instructor's Notes

For the problems in class and on exams, the three-digit rule discussed in that section is what we will go with. For homework on mastering physics, however, answers will be judged as correct if they are within two percent of the correct value, so my suggestion is to just put in plenty of digits. You may get a message that says something about incorrect number of significant figures if you do this.. However, don't worry about it, the problem has been graded correctly. Mastering physics is just telling you that you did your significant figures wrong. However, since I don't care about significant figures, it's not really a big deal; you will get full credit either way.

In regards to how to present data in the labs, section 1.4 has all the information on how people present data, and so you should follow these guidelines for the labs as well..

1.4 Introduction to Statistics

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Mean and Standard Deviation: Know the definition of both of these as well as how to calculate them for a given data set.
- Using mean and standard deviation in calculations.

This section focuses on some basic statistics facts that we will need for this class. This section is also available as a video. *Link to the video:* **https://www.youtube.com/watch?v=sGjq350QY7c (https://www.youtube.com/watch?v=sGjq350QY7c)**

How to present data for laboratory exerciese

How do people actually present data? A common method is $\mu \pm \sigma$, where μ is the **mean**, and σ is the **standard deviation**. Before we talk about mean and standard deviation, we have to discuss a little bit about measurement. Most objects have some variation. People come in a variety of heights, for example, and even manufactured objects like, say pencils, will have a variety of lengths. If you can measure them precisely enough, this variation may be very small, for objects made by machines for example, but it will still be there. Another example that's not just lengths or heights is the number of blood cells passing through a capillary per second. This quantity will, of course, vary from second to second. Another example might be that if you have a spring based launcher of a ball, the ball will travel slightly different distances each time, if for no other reason than the spring coils slightly different ways on the molecular level with each launch. These types of variation are intrinsic, and result in variation in your measurement. However, sometimes measuring something directly is tough, and you need to use indirect methods, like we do for our library lab. One way to get a feel for the precision of your method is to make the measurement with a few different methods, with what we expect to have similar levels of precision. The variation in the results of the different methods can give a sense of the precision of your measurements; this is how we will evaluate our methods for the library lab.

Mean and Standard Deviation

To talk about the ideas of mean and standard deviation, it's helpful to have an example. Say we took the height of 20 men from the United States and presented the data in the table below.

Table 1.4

Person	Height [cm]
1	177.7
2	181.4
3	179.4
4	164.9
5	180.4
6	174.0
7	178.6
8	176.1
9	181.9
10	179.7
11	175.8
12	175.0
13	180.9
14	181.9
15	181.0
16	180.5
17	169.2
18	173.3
19	171.8
20	176.5

Note that there are no uncertainties listed in this table. Yes, the ruler that we're using has some limit of precision, which is apparently .1 cm according to this table, but the variation between measurements is much larger than this, so the precision of the ruler won't be too important. In a well-designed experiment the precision of your instruments should be much less than the intrinsic variation that you are trying to measure.

Mean

The most complete way to report this data would be to report the entire table as we've done here. However, this becomes impractical as more and more data are collected. Moreover, it becomes very difficult to see trends when you just got big lists of numbers. Therefore, we need ways to characterize our data. If you're looking for a way to characterize the data, the first thing that you might think of to do would be to take the average. Now in mathematics, the word average is replaced with the word mean; they're synonyms. There are many different symbols for mean and each discipline seems to have their own one, so I'm going to present you with all of them. I wish we could agree on which symbol to use, but we can't, so I'm just going to show you all of what's out there. The Greek letter μ here is a very general symbol for mean. A general tip I would have is that you learn your Greek alphabet. Another way to represent mean is, let's say we're using the variable H to represent the height of a man, then you might see $\bar{H}$ or $\langle H \rangle$ to represent the mean. The formula for the mean is given by

$$\mu = \frac{1}{N}\sum_i x_i$$

Many of the equations that you might see in this section can get pretty ugly looking, but they are manageable if you stop and parse them down and read what the equation is trying to tell you. A good tip for questions is to read actually right to left. So, let's give that a shot with this equation. The letter i represents an index over the measurements. Here we have 20 measurements, so i is an integer that runs from 1 to 20. x_i is one specific measurement, so x_5 is the height of the fifth person which, according to our table, is 180.4 cm. To calculate the mean, we add up all the measurements, and then divide by the number of measurements. Let's calculate the mean. For these data, when we add up all of the measurements, we get a sum of 3540 cm. We divide by the number of measurements; take $\frac{3540cm}{20cm}$, which gives us a result of 177 cm. This is our average or mean.

Standard Deviation

The mean provides a great starting point for characterizing data, but it's insufficient because it's missing a key feature. Just representing the mean gives us no clue on how spread out these data are. Phrased differently, we don't have any information on what is the average distance for a random data point to the mean. So if we're looking at this question, let's try to translate this question into mathematics. Well, the distance from a given data point x_i to the mean would be $\langle H \rangle - x_i$, and the average distance would be, well, take all of these different distances, $\mu - x_i$, add them all up and divide by the number of measurements.

However, this idea has a problem. Some distances are lower than the mean. For example, person 6 is slightly shorter than our average. So, his distance to the mean will be positive, while some people are taller than the average, for example person 2, so their distance to the mean will be negative. If I add up positive numbers and negative numbers, I'll probably get a result that's very close to zero due to the cancellation. So how can we get around this problem? Well, you might think absolute values, but for calculus reasons, absolute values have some problems, so a better way to get around having negative numbers is to look at squaring them, because no matter what, when I take a number and square it, the result is positive. So, let's look at the average squared distance from the mean. Mathematically, the average squared distance from the mean would be, take the distance from the data point to the mean, just mean minus data point, square it, add them all up and divide by the number of measurements.

$$\sigma^2 = \frac{1}{n}\sum_{i}^{n} (x_i - \mu)^2$$

Now all the numbers being added are positive, so there's no cancellation. This quantity is called the variance, and we will label it with the variable σ^2, for reasons that will become apparent in a moment. Let's calculate the variance for this data. Again, variance is kind of an ugly formula, so you really got to slow down and take it one piece at a time. So let's take an entry *i*=1 and what do we do, take the entry and subtract it from the mean. This for *i*=1 is -0.7 cm. We repeat this for all of our data. We get these results. Next in variance, we see we should square so for *i*=1 the result is .49 cm^2. I want to point out that we've now moved from cm to cm^2, because when you square a number with units, you got to square the units too, and when we repeat it for all of our data and get these results to calculate variance, we take all of these numbers and add them up, which gives us 403.5 cm2. To get the variance divided by the number of measurements, which in this case is 20 giving us a variance of 20.2 cm^2.

Now variance has different units than mean, as we've already seen. The mean for this data set is 177.0 cm while the variance is 20.2 cm ^{2}. It's very difficult to compare numbers with different units. To deal with this we, instead of looking at the variance, look at the standard deviation, which we represent by σ. So, the standard deviation is the square root of the variance. This is why we represent variance with a σ^2. In this example to get the standard deviation, we take square root of the variance so the square root of 20.2 cm^2, to give us 4.49 cm. Now we have two quantities that are both in cm, and allows us to compare them.

So how do we report numbers in the laboratory exercises? In this class, most of the labs in this course will have multiple measurements. We can use these different trials to calculate a mean and a standard deviation, and we can use this standard deviation as an uncertainty and use it to tell us how many decimals we should record. In our height example, we had a mean of 177 cm and a standard deviation a 4.49 cm. An appropriate way to represent this result would be 177 plus or minus 4.5 cm. This representation has a lot of advantages; it represents the average 177, and we have the standard deviation, which gives the person reading the number a sense of the spread of the data, and we have a reasonable number of digits. I've gone with one digit past the decimal point, and I did this based upon the standard deviation. While our standard deviation is officially 4.49, I rounded it to 4.5, because .01 is very small relative to our standard deviation, so I can't really trust that .01. So while I removed some certainty of nice hard sig-fig rules, this is how numbers are actually reported in research, and this is how we'll do it in class. Part of the point of the laboratory exercises to get you some experience with this sort of usage.

The following section is based upon:
Denker, J. Uncertainty as Applied to Measurements and Calculations. Uncertainty as Applied to Measurments and Calculations (2011). Available at: **http://www.av8n.com/physics/uncertainty.htm (http://www.av8n.com/physics/uncertainty.htm)** . (Accessed: 26th August 2016)

Incorporating Mean and Standard Deviation into Calculations - Crank Three Times

Here's a simple yet powerful way of estimating the uncertainty of a result, given the uncertainty of the thing(s) it depends on.

Here's the procedure, in the simple case when there is only one input variable with appreciable uncertainty:

a. Set up the calculation. Do it once in the usual way, using the nominal, best-estimate values for all the input variables.
b. Then re-do the calculation with the uncertain variable at the end of its upper error bar.
c. Then re-do the calculation with the uncertain variable at the end of its lower error bar.

I call this the *Crank Three Times* method. Here is an example:

Table 1.5

x	$\frac{1}{x}$
2.02 (high case)	.495
2 (nominal case)	.5
1.98	.505

Equation 35 (https://www.av8n.com/physics/uncertainty.htm#eq-crank-3-linear) tells us that if *x* is distributed according to *x*=2±.02 then $\frac{1}{x}$ is distributed according to $\frac{1}{x}$=.5±.005. The Crank Three Times method is by no means an exact error analysis. It is an approximation. The nice thing is that you can understand the nature of the approximation. One of the glories of the Crank Three Times method is that in cases where it doesn't work, it will tell you it isn't working, provided you listen to what it's trying to tell you. If you get asymmetrical error bars, you need to investigate further. Something bad is happening, and you need to check closely to see whether it is a little bit bad or very, very bad.

As far as I can tell, for every flaw that this method has, the sig-figs method has the same flaw plus others ... which means Crank Three Times is therefore superior.

Crank Three Times shouldn't require more than a few minutes of labor. Once a problem is set up, turning the crank should take only a couple of minutes; if it takes longer than that you should have been doing it on a spreadsheet all along. And if you are using a spreadsheet, Crank Three Times is super-easy and super-quick.

If you have *N* variables that are (or might be) making a significant contribution to the uncertainty of the result, the Crank Three Times method could more precisely be called the Crank 2*N*+1 Times method. Here's the procedure: Set up the spreadsheet and wiggle each variable in turn, and see what happens. Wiggle them *one* at a time, leaving the other *N*−1 at their original, nominal values.

For example, let's say you're looking for the area of a rectangle, and the length and width of the rectangle are measured to be 5±.6 and 2±.4, respectively. Using $A = lw$, the nominal crank is 10 cm^2. Wiggling the length results in 10±1.2 cm^2 and wiggling the width results in 10±2 cm^2. Note that even though the width has a lower uncertainty associated with it, its uncertainty it creates in the result is higher than that of the length.

If you are worried about what happens when two of the input variables are simultaneously at the ends of their error bars, you can check that case if you want. However, beware that if there are many variables, checking all the possibilities is exponentially laborious. Furthermore, it is improbable that many variables would simultaneously take on extreme values, and checking extreme cases can lead you to overestimate the uncertainty. For these reasons, and others, if you have numerous variables and need to study the system properly, at some point you need to give up on the Crank Three Timesmethod and do something more sophisticated called a Monte Carlo analysis which we will not discuss in this class. The Crank Three Times method can be considered an ultra-simplified variation of the Monte Carlo method, suitable for introductory reconnaissance.

In the *rare* situation where you want a worst-case analysis, you can move each variable to whichever end of its error bar makes a positive contribution to the final answer, and then flip them all so that each one makes a negative contribution. In most cases, however, a worst-case analysis is wildly over-pessimistic, especially when there are more than a few uncertain variables.

Remember: there are many cases, especially when there are multiple uncertain variables and/or correlations among the variables and/or nonlinearities for which you will need to be more sophisticated.. The Crank Three Times method can be considered an ultra-simplified variation of the Monte Carlo method, suitable for introductory reconnaissance.

Here is another example, which is more interesting because it exhibits nonlinearity:

Table 1.6

x	$\frac{1}{x}$
2.9 (high case)	.35
2 (nominal case)	.5
1.1 (low case)	.91

Equation 36 (https://www.av8n.com/physics/uncertainty.htm#eq-crank-3-nonlinear) tells us that if *x* is distributed according to *x*=2±.9 then $\frac{1}{x}$ is distributed according to $\frac{1}{x}=5^{-0.16}_{+0.42}$. Even though the error bars on *x* are symmetric, the error bars on $\frac{1}{x}$ are markedly lopsided.

Lopsided error bars are fairly common in practice. Sometimes they are merely a symptom of a harmless nonlinearity, but sometimes they are a symptom of something much worse. As an example, let's say you had a calculation that was $\frac{1}{x\text{-}2}$, and the value of x was found to be 3±2.When you do the crank three times method, the nominal crank is 1, the upper crank is $\frac{1}{3}$, and the lower crank is -1. Both the upper and lower cranks give values less than the nominal; the result is $1^{-2/3}_{-2}$, which doesn't make much sense. The absurdity arises because at x=2, the function $\frac{1}{x\text{-}2}$ is equal to $\frac{1}{0}$ and the function is undefined, i.e. has a *singularity***.** Here's a graph of the function and the data, which is the value of x and its uncertainty:

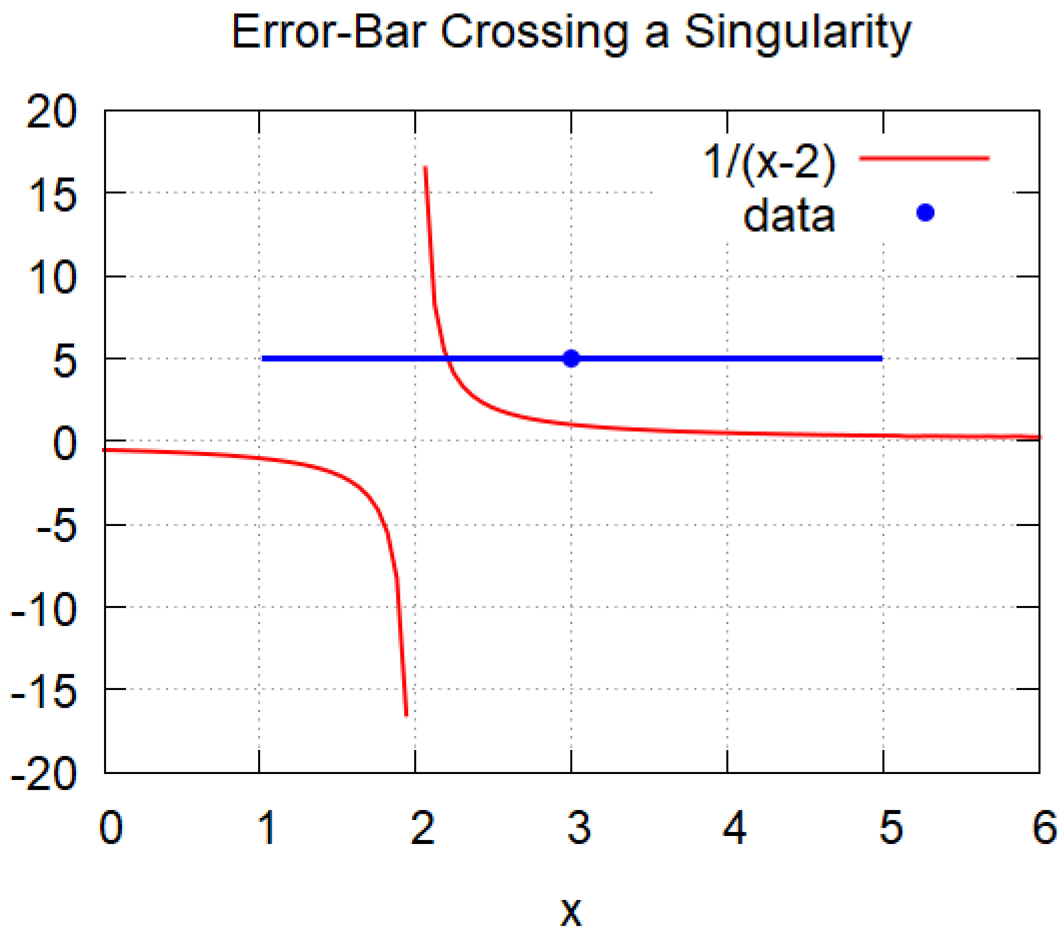


Figure 1.22

Notice that for all the values above the nominal value of x (indicated by the point), the function behaves normally, but for the values below, the function has a ‘break’ in it at x=2, where the function becomes a division by zero. Notice as well that the function spikes up around the point x=2 as well. If we were to continue the function as it approaches closer and closer to 2, we would see that the function would go up to infinity and to negative infinity, and infinite values tend to break uncertainty calculations. What the nonsense result is trying to tell you is that your error bars contain a problem point, such as the one above. Results such as these are the ones you should be wary of.

1.5 Accuracy and Precision

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Intepreting data and results. This ties into understanding how mean and standard deviation relate to uncertainty.

Figure 1.23 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

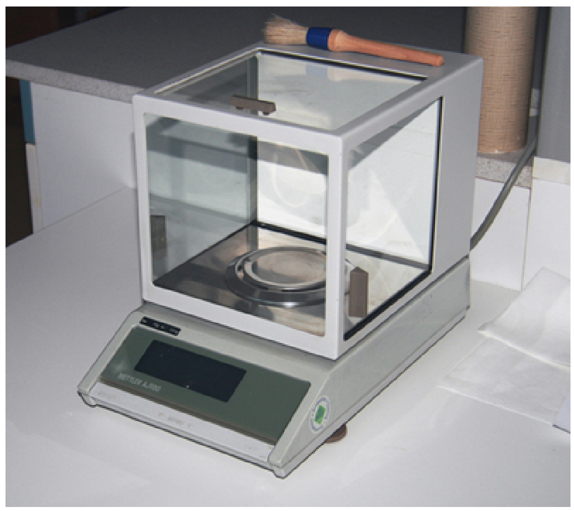


Figure 1.24 Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull’s-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In **Figure 1.25**, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in **Figure 1.26**, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

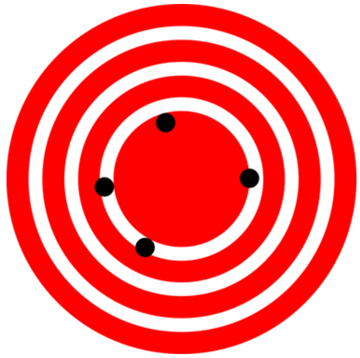


Figure 1.25 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)

Figure 1.26 In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A , is often denoted as δA ("delta A "), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2.$

Precision and accuracy is also is through mean and standard deviation. If your mean is close to the actual value, it is considered to be *accurate*, and if your standard deviation is small, it is considered to be *precise*. For example, let's say you were measuring the speed of a car driving by at 30 m/s. If the measured result is 60±0.1 m/s, the mean is far too high but your standard deviation is fairly low, so this result would be precise but not accurate. If the measured result is 29±40 m/s, the mean is fairly close to the actual speed, but the standard deviation is fairly high, so this result would be accurate, but not precise.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. Irregularities in the object being measured,
3. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C ? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0°C to a dangerously high 40.0°C . A thermometer with an uncertainty of $3.0^{\circ}\mathrm{C}$ would be useless.

Check Your Understanding

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of $\pm 0.05\ \mathrm{s}$. Runners on the track coach's team regularly clock 100-m sprints of $11.49\ \mathrm{s}$ to $15.01\ \mathrm{s}$. At the school's last track meet, the first-place sprinter came in at $12.04\ \mathrm{s}$ and the second-place sprinter came in at $12.07\ \mathrm{s}$. Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?

Solution
No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

PhET Explorations: Estimation

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

Figure 1.27 Estimation (http://legacy.cnx.org/content/m64071/1.2/estimation_en.jar)

Glossary

accuracy: the degree to which a measured value agrees with correct value for that measurement

classical physics: physics that was developed from the Renaissance to the end of the 19th century

conversion factor: a ratio expressing how many of one unit are equal to another unit

derived units: units that can be calculated using algebraic combinations of the fundamental units

English units: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units: units that can only be expressed relative to the procedure used to measure them

kilogram: the SI unit for mass, abbreviated (kg)

law: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter: the SI unit for length, abbreviated (m)

method of adding percents: the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

metric system: a system in which values can be calculated in factors of 10

model: representation of something that is often too difficult (or impossible) to display directly

modern physics: the study of relativity, quantum mechanics, or both

order of magnitude: refers to the size of a quantity as it relates to a power of 10

percent uncertainty: the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

physical quantity : a characteristic or property of an object that can be measured or calculated from other measurements

physics: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

precision: the degree to which repeated measurements agree with each other

quantum mechanics: the study of objects smaller than can be seen with a microscope

relativity: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

scientific method: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second: the SI unit for time, abbreviated (s)

SI units : the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

significant figures: express the precision of a measuring tool used to measure a value

theory: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

uncertainty: a quantitative measure of how much your measured values deviate from a standard or expected value

units : a standard used for expressing and comparing measurements

Section Summary

1.1 An Introduction to Physics

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

1.1 An Introduction to Physics

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

2. How does a model differ from a theory?

3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

4. What determines the validity of a theory?

5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?

8. When is it *necessary* to use relativistic quantum mechanics?

9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

1.5 Accuracy and Precision

11. What is the relationship between the accuracy and uncertainty of a measurement?

12. Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

2. A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

3. Show that $1.0 \text{ m/s} = 3.6 \text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0 \text{ m/s} = 3.6 \text{ km/h}.$

4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8. The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

10. (a) Refer to **Table 1.3** to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

1.5 Accuracy and Precision

Express your answers to problems in this section to the correct number of significant figures and proper units.

11. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

12. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

13. (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. $(1 \text{ km} = 0.6214 \text{ mi})$

14. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

15. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

16. A can contains 375 mL of soda. How much is left after 308 mL is removed?

17. State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2) / (46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60\times10^{-19}\right)(3712)$.

18. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

19. (a) If your speedometer has an uncertainty of 2.0 km/h at a speed of 90 km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60 km/h, what is the range of speeds you could be going?

20. (a) A person's blood pressure is measured to be $120 \pm 2 \text{ mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80 mm Hg?

21. A person measures his or her heart rate by counting the number of beats in 30 s. If 40 ± 1 beats are counted in $30.0 \pm 0.5 \text{ s}$, what is the heart rate and its uncertainty in beats per minute?

22. What is the area of a circle 3.102 cm in diameter?

23. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

24. A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

25. The sides of a small rectangular box are measured to be $1.80 \pm 0.01 \text{ cm}$, $2.05 \pm 0.02 \text{ cm, and } 3.1 \pm 0.1 \text{ cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

26. When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1 \text{ lbm} = 0.4539 \text{ kg}$. (a) If there is an uncertainty of 0.0001 kg in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

27. The length and width of a rectangular room are measured to be $3.955 \pm 0.005 \text{ m}$ and $3.050 \pm 0.005 \text{ m}$. Calculate the area of the room and its uncertainty in square meters.

28. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002 \text{ cm}$ diameter a distance of $3.250 \pm 0.001 \text{ cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

2 KINEMATICS

Figure 2.1 The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Chapter Outline

Introduction to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics and **Two-Dimensional Kinematics (https://legacy.cnx.org/content/m42126/latest/)** we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In **Two-Dimensional Kinematics (https://legacy.cnx.org/content/m42126/latest/)** , we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

2.1 Displacement

UMASS AMHERST Instructor's Notes

Your Quiz would Cover
- How position differs from displacement, and how distance differs from displacement
- When to use position and when to use displacement
- Identify that displacement has a direction

Figure 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

UMASS AMHERST Instructor's Notes

While *position* and *displacement* might seem like they mean the same thing, there is a difference in what they mean that is important to understand. It's helpful to keep this in mind as you read about the two, and perhaps try to put what the difference is in your own words.

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor’s position could be described in terms of where she is in relation to the nearby white board. (See **Figure 2.3**.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See **Figure 2.4**.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object’s position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_f - x_0, \tag{2.1}$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f.

UMASS AMHERST Instructor's Notes

We will be using Δ quite a lot in this class. Whenever you see Δ in this class, it will always mean the change in whatever the Δ is in front of, or final minus initial.

Note that the SI unit for displacement is the meter (m) (see **Physical Quantities and Units (https://legacy.cnx.org/content/m42091/latest/)**), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

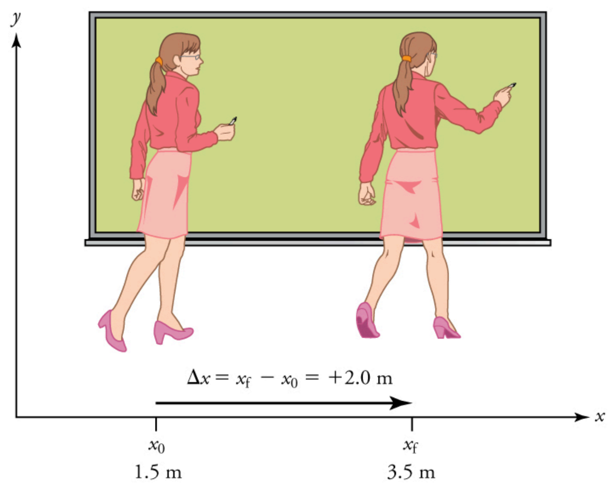


Figure 2.3 A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0$ m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

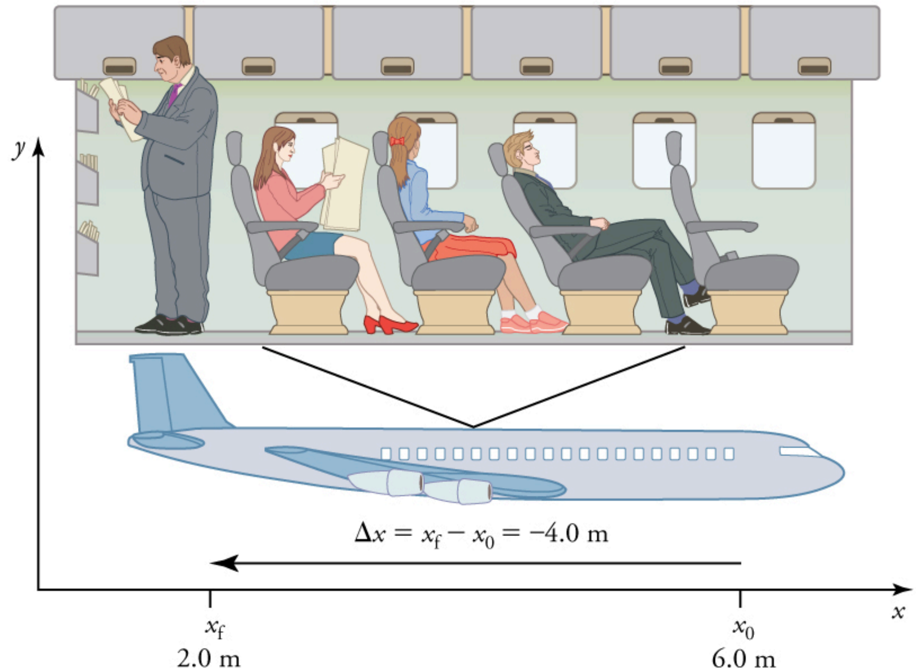


Figure 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x . The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in **Figure 2.3**.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\text{ m}$ and her final position is $x_f = 3.5\text{ m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}. \tag{2.2}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{ m}$ and his final position is $x_f = 2.0\text{ m}$, so his displacement is

$$\Delta x = x_f - x_0 = 2.0\text{ m} - 6.0\text{ m} = -4.0\text{ m}. \tag{2.3}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

UMASS AMHERST Instructor's Notes

Just like with position and displacement,there is a difference between the meaning of*distance* and *displacement*,even though they may seem similar, and it's important that you understand this difference. The following note discusses this in detail.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution

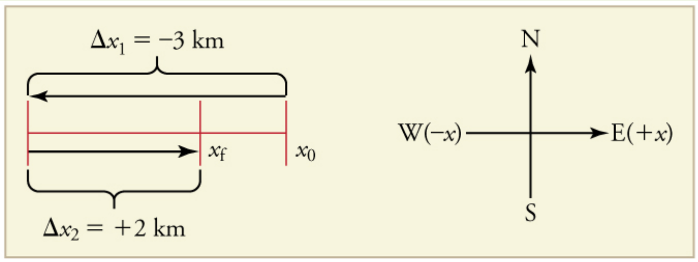


Figure 2.5

(a) The rider's displacement is $\Delta x = x_f - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km.

2.2 Vectors, Scalars, and Coordinate Systems

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Definitions of vectors and scalars, and the difference between them

Other Things to Consider as You Read:

- In physics, often times you get to change the coordinate system to match the situation, such as deciding where the origin will be, so knowing how to manipulate the coordinate system is a useful tool. This idea also has a fundamental connection to physics that we'll explore later on.

Figure 2.6 The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

UMASS AMHERST Instructor's Notes

You may begin to notice that this chapter has a lot of basic definitions. It's important for you to understand these definitions in a fundamental way; they will play an important role in our discussion of physics. Also, a lot of these terms you may have already heard of, such as acceleration, but these words also have a more precise physics definition, so make sure that you understand these definitions in regards to physics.

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in **Figure 2.6**, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

UMASS AMHERST Instructor's Notes

Keep in mind that last sentence; before you start working, you can change your coordinate system and what you define as positive and negative directions however you want, but once you start working with that system, you *cannot* change it.

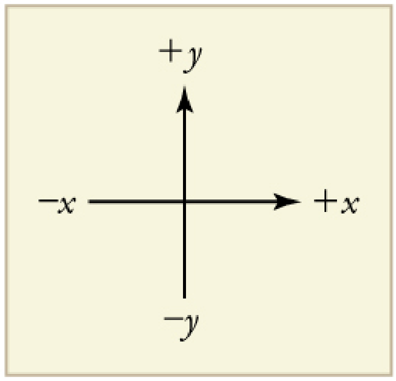


Figure 2.7 It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(-)$.

Check Your Understanding

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2.3 Time, Velocity, and Speed

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Describing how velocity is different from speed
- Identifying that velocity has a direction
- Identifying that velocity is always parallel to the path
- Solving for position as a function of time, given velocity as a function of time. Eventually, we will be using this idea with simulations.

Other Things to Consider as You Read:

- As with the previous sections, there are a couple of definitions to pay attention to. There are some terms you may have heard before, but, again, it's the specific physics definition that's important to know.

Figure 2.8 The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in **Physical Quantities and Units (https://legacy.cnx.org/content/m42091/latest/)** , the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple— **time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** $\boldsymbol{\Delta t}$ is the difference between the ending time and beginning time,

$$\Delta t = t_{\text{f}} - t_0, \tag{2.4}$$

where Δt is the change in time or elapsed time, t_{f} is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ , means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_{\text{f}} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta t = t_{\text{f}} \equiv t)$

UMASS AMHERST Instructor's Notes

We will also be using this simplification; t will always mean Δt , and initial time will always be zero, unless stated otherwise

Velocity

UMASS AMHERST Instructor's Notes

It's difficult to understand physics without understanding velocity, so try to internalize what velocity is, and think about what negative velocity means as well.

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}, \tag{2.5}$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_{f} and x_0 are the final and beginning positions at times t_{f} and t_0 , respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{2.6}$$

UMASS AMHERST Instructor's Notes

The equation above is a mathematical definition of velocity. If you think about physics as a series of different ideas, the equation is just another representation of the idea of velocity, along with the definition with words. It's important to know what velocity is in both words and the mathematics, as well as to connect those two definitions together.

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\text{ m/s}. \tag{2.7}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

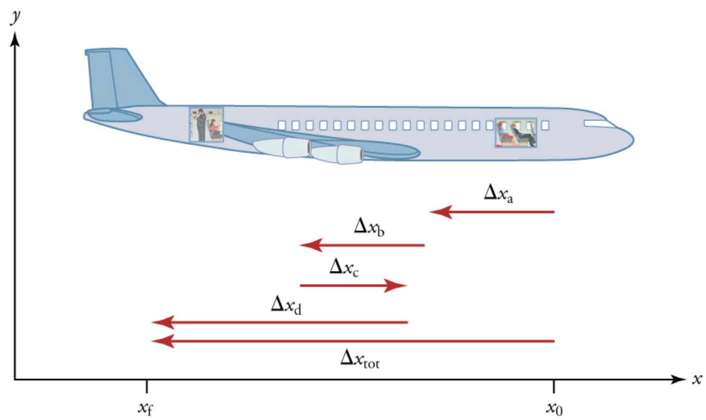


Figure 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

UMASS AMHERST Instructor's Notes

As with position and displacement, speed and velocity are also two similar ideas but with distinct differences, so focus on what each means and how the differ in meaning.

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

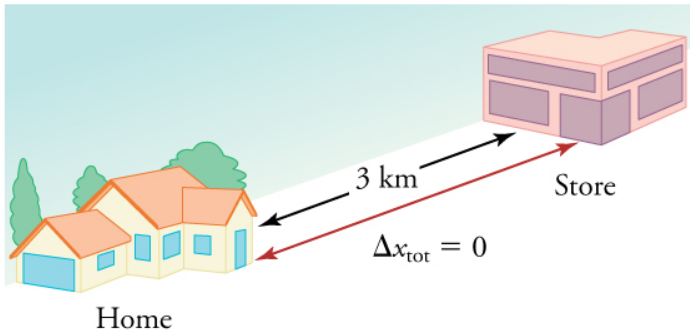


Figure 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in **Figure 2.11**. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

UMASS AMHERST Instructor's Notes

The idea of modelling a situation with a more simplified version is particularly useful. For example, if you want to model the velocity of a person walking forward at a speed, you could treat their motion as a single point moving forward in a straight line. Realistically, the person could be walking in a line that's slightly off being straight, or they could be walking at slightly varying speeds, and different parts of their body could be shifting around, like their arms could be swinging, but often times you don't really have to care. When you do care, you can factor in these facts later, but often times the model is close enough to reality, and is usually only off by a few percent. This idea of modeling will come up later in this course as well, especially with forces.

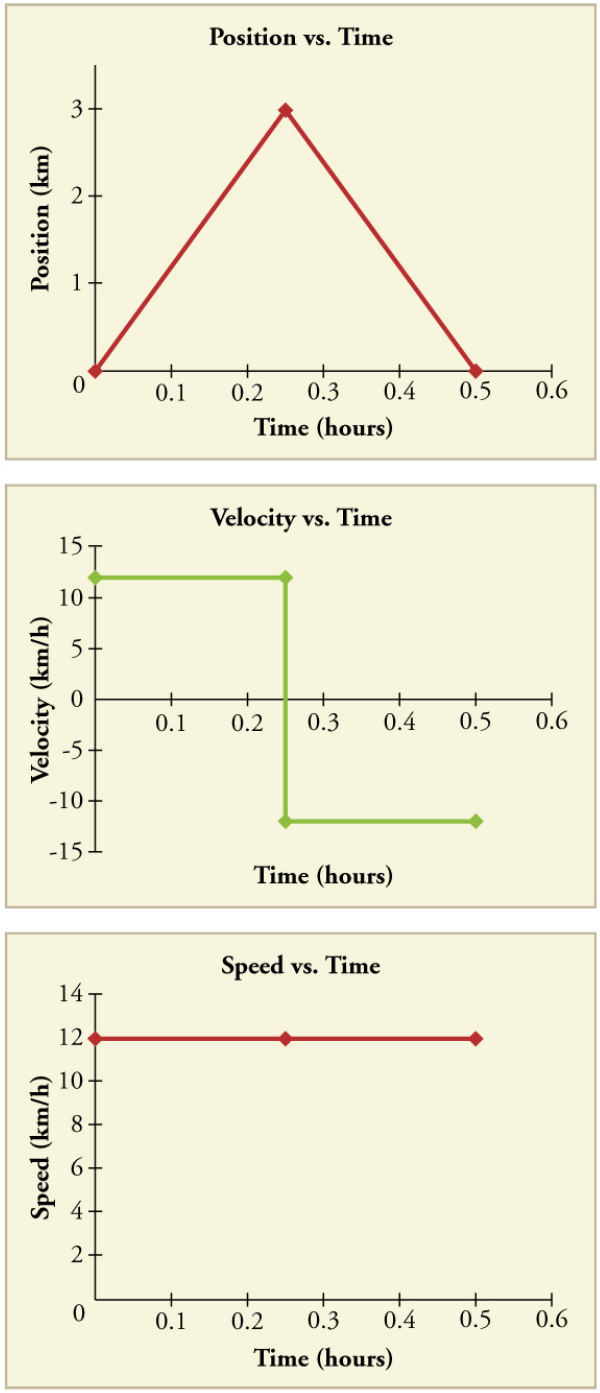


Figure 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

UMASS AMHERST Instructor's Notes

Just like how there is a mathematical representation of velocity and a representation of velocity in words, the above is a graphical represntation of velocity. This will apply to other concepts that we will discuss in class, and being able to connect all of these representations together to the idea will help your understanding.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

UMASS AMHERST Instructor's Notes

I would highly suggest trying out these 'Making Connections' sections; seeing physics in the real world will not only help you better understand the physics in this class, but will make you understanding and appreciation of nature much richer.

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution

(a) The average velocity of the train is zero because $x_\text{f} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}} \tag{2.8}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s} \tag{2.9}$$

2.4 Acceleration

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Identifying that acceleration has a direction
- Using the relationship between average velocity and position to solve problems about the motion of objects.
- Describing how velocity will change given acceleration.

Other Things to Consider as You Read:

- Acceleration is a more abstract concept to wrap your head around than velocity and position, and it might take some time and practice to understand.

Figure 2.12 A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\mathrm{f} - v_0}{t_\mathrm{f} - t_0}, \tag{2.10}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathrm{m/s^2}$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

UMASS AMHERST Instructor's Notes

Like with velocity, try to internalize what average acceleration means, its mathematical representation above, and try to connect these two definitons.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

Figure 2.13 A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider **Figure 2.14**.

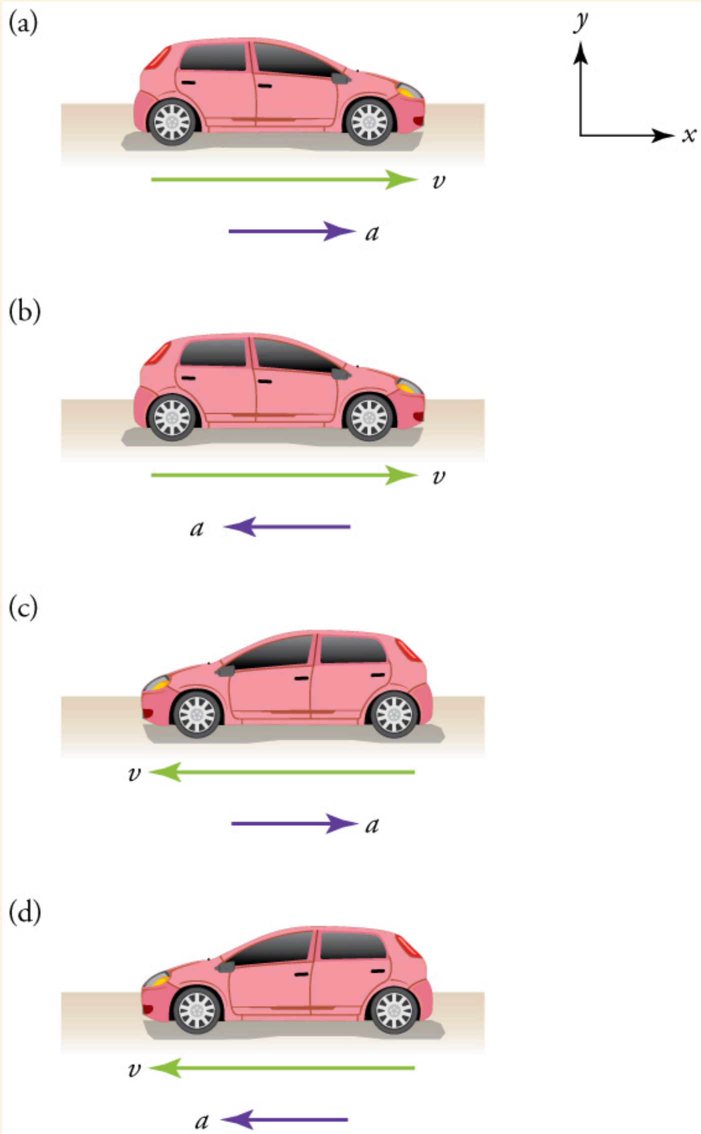


Figure 2.14 (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

UMASS AMHERST Instructor's Notes

This misconception is a pit many students fall into. Remember, there are precise definitions for terms such as acceleration that we want you to know, and knowing these will help you avoid these misunderstandings.

Example 2.1 Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 2.15 (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

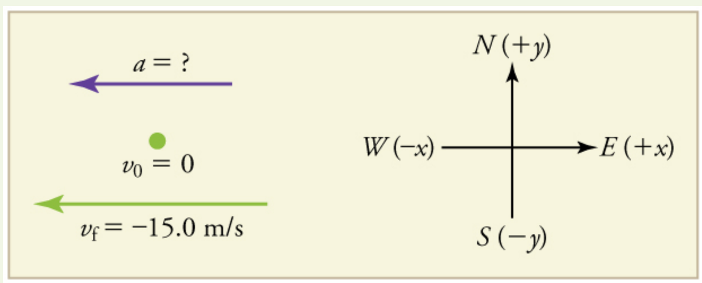


Figure 2.16

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2. \tag{2.11}$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in **Time, Velocity, and Speed (https://legacy.cnx.org/content/m42096/latest/)** —that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. **Figure 2.17** shows graphs of instantaneous acceleration versus time for two very different motions. In **Figure 2.17**(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In **Figure 2.17**(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and -2.0 m/s^2 , respectively.

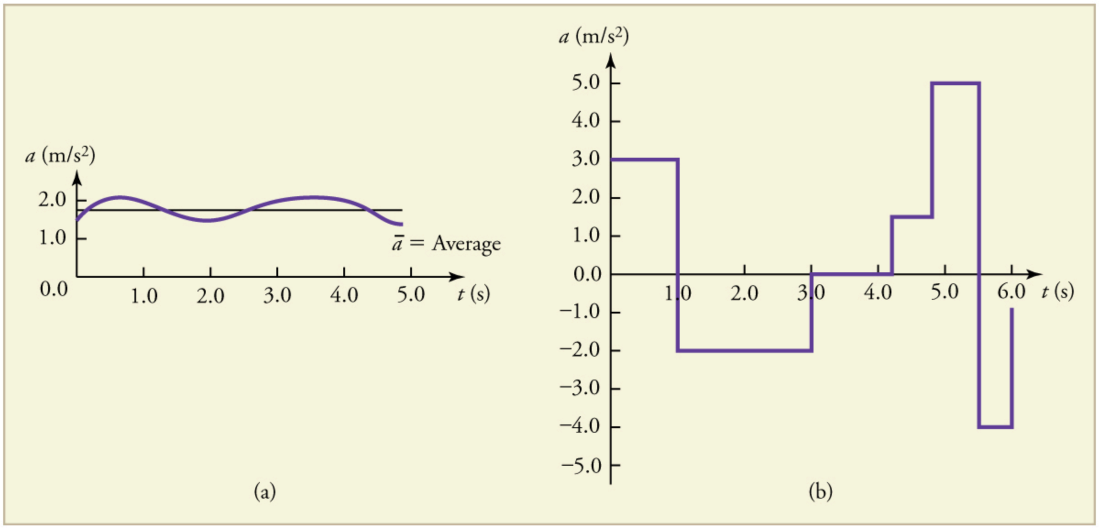


Figure 2.17 Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in **Figure 2.18**. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

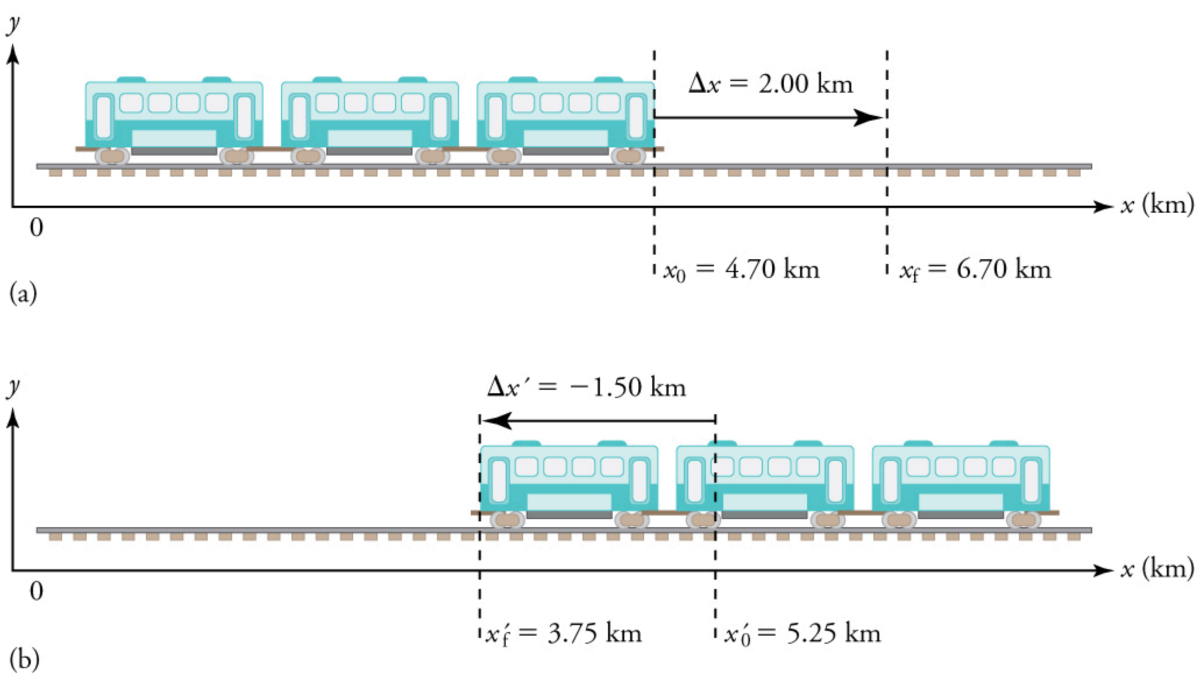


Figure 2.18 One-dimensional motion of a subway train considered in **Example 2.2**, **Example 2.3**, **Example 2.4**, **Example 2.5**, **Example 2.6**, and **Example 2.7**. Here we have chosen the x -axis so that + means to the right and $-$ means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_{f} . Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_{f} . Its displacement $\Delta x'$ is -1.5 km . (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example 2.2 Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 2.18?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_\text{f} - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_\text{f} = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x'_\text{f} = 3.75\text{ km}$ and $x'_0 = 5.25\text{ km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_\text{f} - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km} \tag{2.12}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_\text{f} - x'_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km} \tag{2.13}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 2.3 Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 2.18?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2.2. Distance traveled is the total length of the path traveled between the two positions. (See Displacement (https://legacy.cnx.org/content/m42033/latest/) .) In the case of the subway train shown in Figure 2.18, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5\text{ km}.$ Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.4 Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 2.18(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

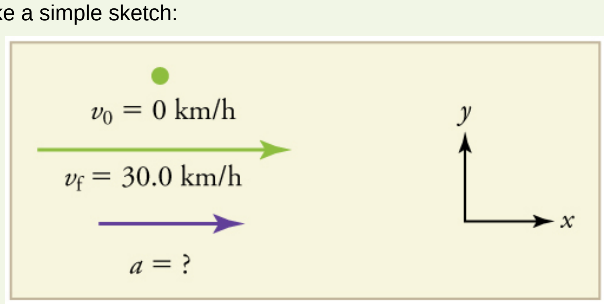


Figure 2.19

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_\text{f} = 30.0\text{ km/h}$, and $\Delta t = 20.0\text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\text{ km/h}}{20.0\text{ s}} \tag{2.14}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See **Physical Quantities and Units (https://legacy.cnx.org/content/m42091/latest/)** for more guidance.)

$$\bar{a} = \left(\frac{+30\text{ km/h}}{20.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.417\text{ m/s}^2 \tag{2.15}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 2.5 Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in **Figure 2.18**(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

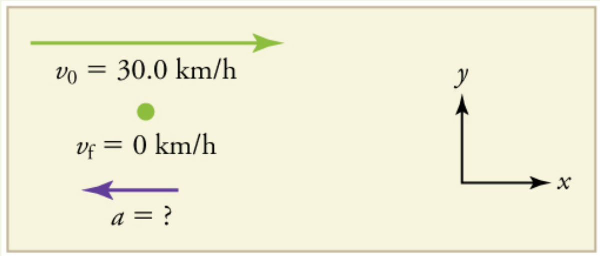


Figure 2.20

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\text{ km/h}$, $v_\text{f} = 0\text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\text{ s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_\text{f} - v_0 = 0 - 30.0\text{ km/h} = -30.0\text{ km/h} \tag{2.16}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\text{ km/h}}{8.00\text{ s}} \tag{2.17}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\text{ km/h}}{8.00\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -1.04\text{ m/s}^2. \tag{2.18}$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in **Example 2.4** and **Example 2.5** are displayed in **Figure 2.21**. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

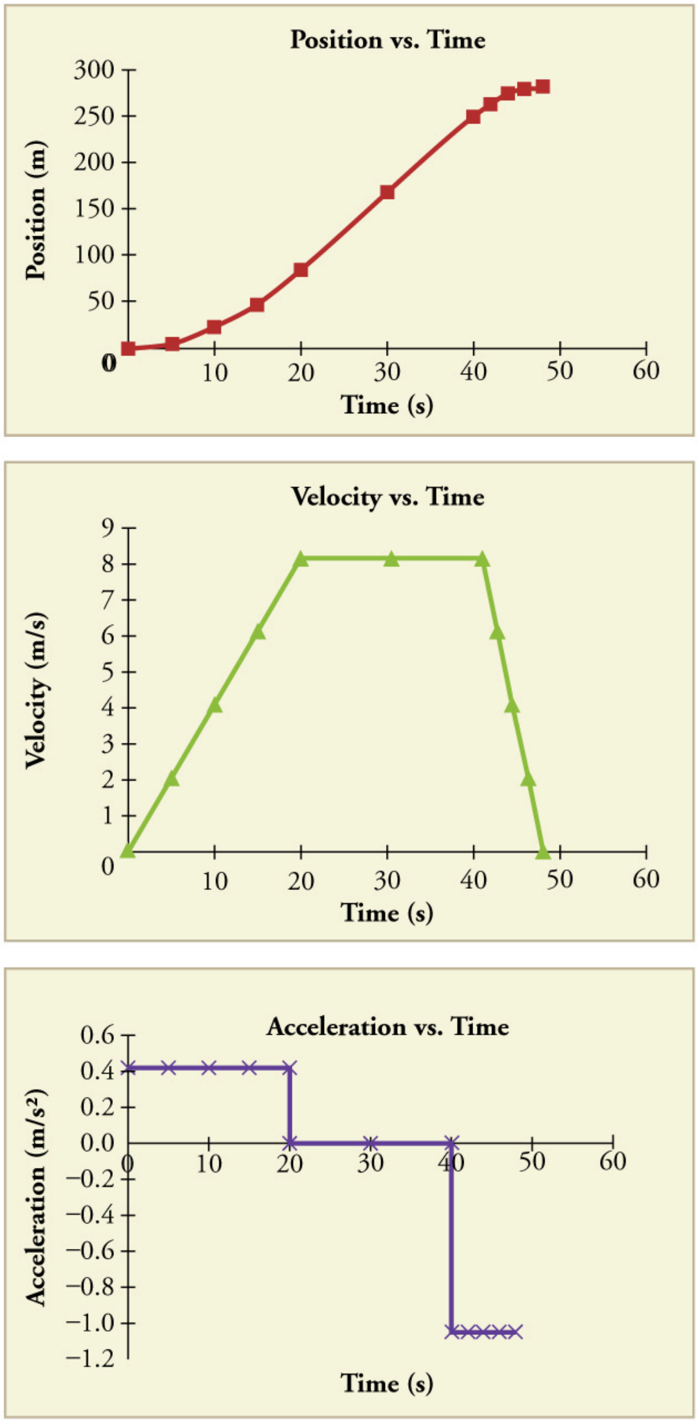


Figure 2.21 (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 2.6 Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of **Example 2.2**, and shown again below, if it takes 5.00 min to make its trip?

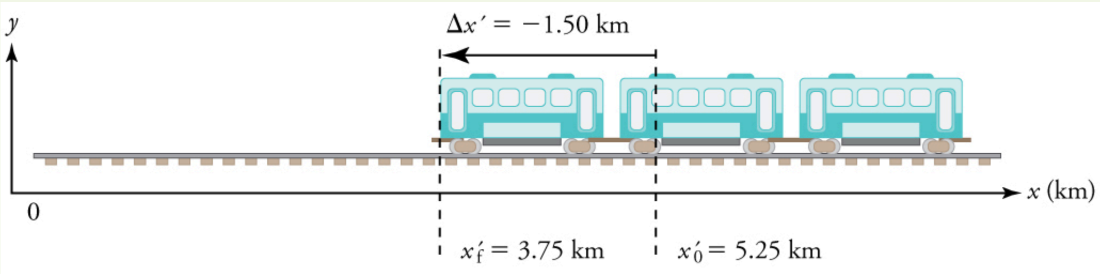


Figure 2.22

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_f = 3.75\text{ km}$, $x'_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in **Example 2.2**.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}} \tag{2.19}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h} \tag{2.20}$$

Discussion

The negative velocity indicates motion to the left.

Example 2.7 Calculating Deceleration: The Subway Train

Finally, suppose the train in **Figure 2.22** slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

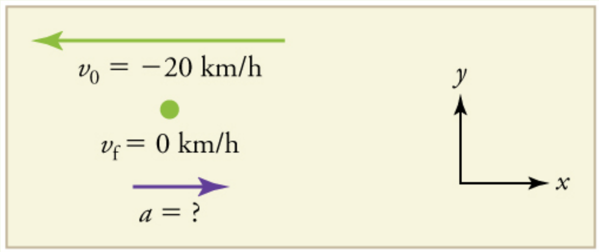


Figure 2.23

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\text{ km/h}$, $v_f = 0\text{ km/h}$, $\Delta t = 10.0\text{ s}$.

2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20\text{ km/h}) = +20\text{ km/h}. \tag{2.21}$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\text{ km/h}}{10.0\text{ s}} \tag{2.22}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\text{ km/h}}{10.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = +0.556\text{ m/s}^2 \tag{2.23}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in **Example 2.5**, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in **Example 2.7**, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in **Figure 2.22** is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Solution

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PhET Explorations: Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

PhET Interactive Simulation

Figure 2.24 Moving Man (http://legacy.cnx.org/content/m64042/1.4/moving-man_en.jar)

2.5 Graphical Analysis of One-Dimensional Motion

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Describing the units of the value, slope, and integral of any graph
- Using slope of a position vs. time graph to sketch a velocity vs. time graph
- Using slope of a velocity vs. time graph to sketch an acceleration vs. time graph
- Using area under an acceleration vs. time graph to sketch a velocity vs. time graph
- Using the area under a velocity vs. time graph to sketch a position vs. time graph

Other Things to Consider as You Read:

- As with the other sections, there are definitions of basica concepts that we will be using throughout the course. We will also be working with graphs a lot as well.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in **Figure 2.25**, a straight-line graph has the general form

$$y = mx + b. \tag{2.24}$$

Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

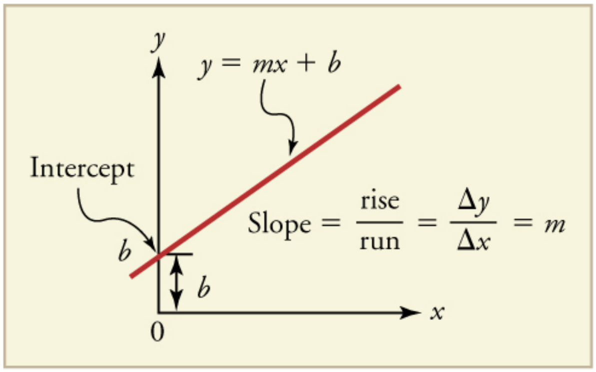


Figure 2.25 A straight-line graph. The equation for a straight line is $y = mx + b$.

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have x on the vertical axis and t on the horizontal axis. **Figure 2.26** is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada.

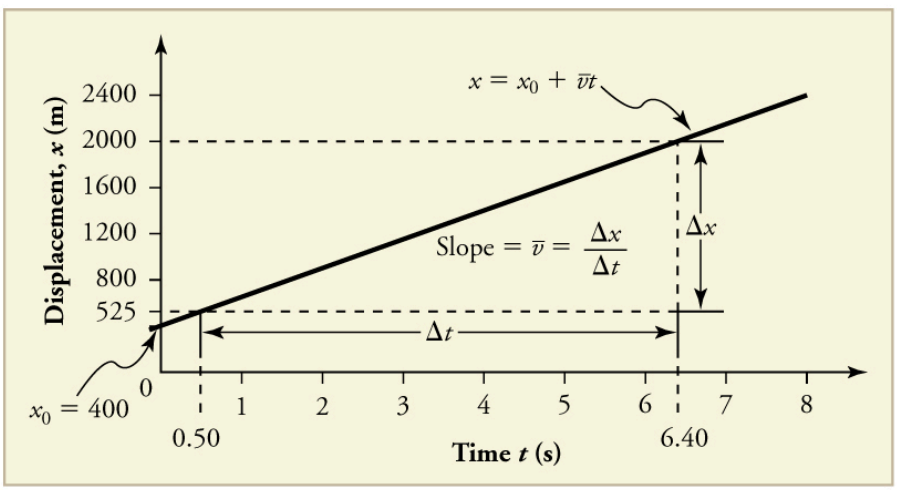


Figure 2.26 Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{2.25}$$

or

$$x = x_0 + \bar{v}t. \tag{2.26}$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of *x* vs. *t*

The slope of the graph of displacement x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = v \tag{2.27}$$

Notice that this equation is the same as that derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension (https://legacy.cnx.org/content/m42099/latest/)** .

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 2.8 Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in **Figure 2.26**.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}\,. \tag{2.28}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}}, \tag{2.29}$$

yielding

$$\bar{v} = 250\text{ m/s}. \tag{2.30}$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in **Figure 2.27** below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

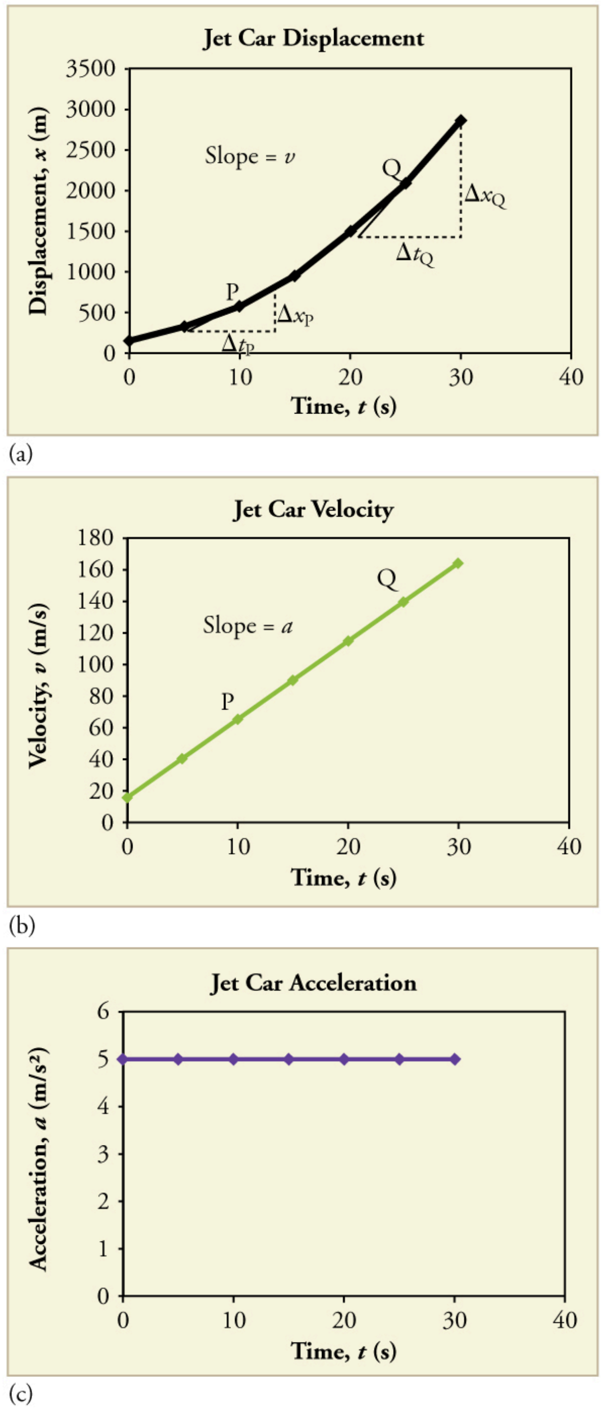


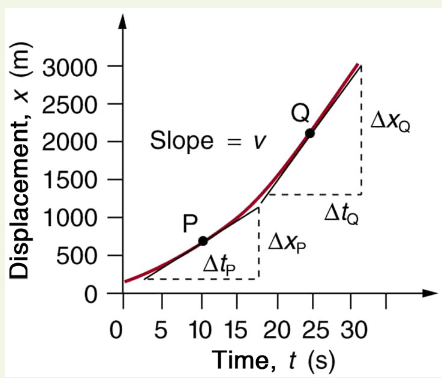
Figure 2.27 Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of 5.0 m/s^2 over the time interval plotted.

Figure 2.28 A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of displacement versus time in **Figure 2.27**(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in **Figure 2.27**(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in **Figure 2.27**(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in **Figure 2.27**(c).

Example 2.9 Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

Figure 2.29 The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in **Figure 2.29**, where Q is the point at $t = 25\text{ s}$.

Solution

1. Find the tangent line to the curve at $t = 25\text{ s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_\text{Q} = \frac{\Delta x_\text{Q}}{\Delta t_\text{Q}} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})} \tag{2.31}$$

Thus,

$$v_\text{Q} = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}. \tag{2.32}$$

Discussion

This is the value given in this figure's table for v at $t = 25\text{ s}$. The value of 140 m/s for v_Q is plotted in **Figure 2.29**. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of *v* vs. *t*

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a \tag{2.33}$$

Since the velocity versus time graph in **Figure 2.27**(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in **Figure 2.27**(c).

Additional general information can be obtained from **Figure 2.29** and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + at. \tag{2.34}$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in **Motion Equations for Constant Acceleration in One Dimension (https://legacy.cnx.org/content/m42099/latest/)**.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in **Figure 2.30**. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in **Figure 2.27**.) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55\text{ s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

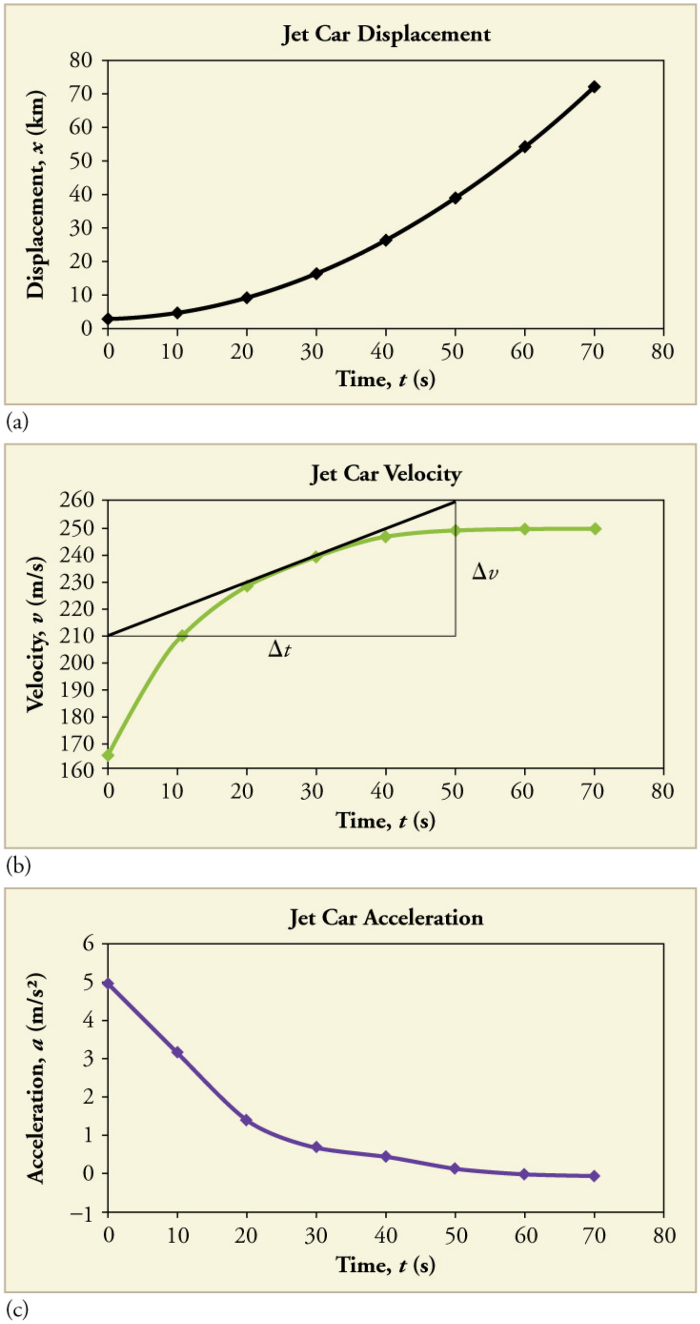


Figure 2.30 Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in **Figure 2.27** ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 2.10 Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in **Figure 2.30**(b).

Strategy

The slope of the curve at $t = 25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in **Figure 2.30**(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\text{ m/s} - 210\text{ m/s})}{(51\text{ s} - 1.0\text{ s})} \tag{2.35}$$

$$a = \frac{50\text{ m/s}}{50\text{ s}} = 1.0\text{ m/s}^2. \tag{2.36}$$

Discussion

Note that this value for a is consistent with the value plotted in **Figure 2.30**(c) at $t = 25\text{ s}$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

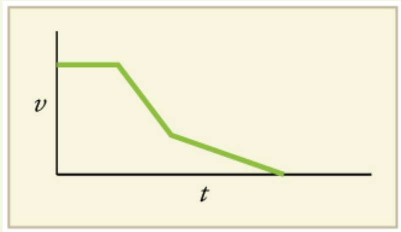


Figure 2.31

Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

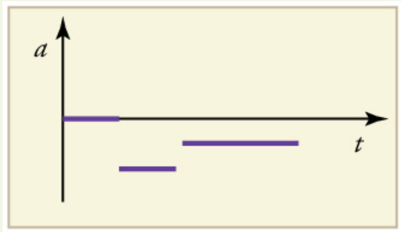


Figure 2.32

2.6 SImulations

Notes from your UMass Instructors

Your Quiz would Cover

- Given a velocity as a function of time, be able to solve for the position as a function of time iteratively.
- Use iterative methods to solve for the motion of an object given an arbitrary (non-constant) acceleration.

This section is based on videos from the UMass Physics 131 YouTube page. The video really adds something beyond this text though, and so for this section, it is **highly recommended** that you watch the videos instead. Here are the links to the videos:

- Theory on Solving Problems with Simulation: **https://www.youtube.com/watch?v=i1elRzD7HYQ (https://www.youtube.com/watch?v=i1elRzD7HYQ)**
- First Example of Solving a Problem with Simulation: **https://www.youtube.com/watch?v=j_HBtITs948 (https://www.youtube.com/watch?v=j_HBtITs948)**
- A More Complex Example of Solving a Problem with Simulation**https://www.youtube.com/watch?v=6FoKBtxuEhg (https://www.youtube.com/watch?v=6FoKBtxuEhg)**

Theory on Solving Problems Using Simulation

Simulation is going to be a tool that we use frequently in this class to solve problems. Why? Well, many physics problems are not solvable with just algebra. Now, you might have suspected this, and figured that, okay, maybe not with algebra, but maybe if I invoke some higher math such as calculus, I can start to do real physics problems. Well, that helps, but even with calculus and some of the most sophisticated math out there, you still can't solve most of the interesting real-world problems that we have. You can solve models, you can solve simplifications, but to try and get everything, the problems are actually undoable.

For example, just to make an extreme example, think about the Earth, the Sun, and the moon. The force on these three objects can be described by a force law from the 17th century, $F = \frac{gmM}{r^2}$, that says the force of gravity between two masses, say the earth and the moon, is their masses and the distance between them squared. That's it. For two objects, say, just the Earth and the moon, you can write an equation that describes their motion using this force law. However, you *cannot* write down an equation that describes the motion of the Sun, the Earth, and the moon; you need go to simulation. Moreover, the ideas of simulation that we're going to be discussing here and throughout this class are being used more and more in essentially all fields of science to solve complex problems. In the Fall 2015 semester, one of the SIs for Physics 131 was using these same ideas to solve problems in his life science based senior honors thesis as an undergraduate. Hopefully, this impresses upon you the relevance of this technique to all fields of science and medicine in the modern day.

What's the philosophy, the premise, behind the idea of simulation? Well, this is perhaps best done in the context of an example. Let's say we have a runner, and we're looking at the runner at some instant, and then some small time later, say, 0.000001 seconds later. Well, if we want to model the motion of the runner, we would think about the average velocity and how that's related to their position and the change in time. If we imagine a really small amount of time, then this runner's velocity from, say the first instant to the next instant, does not change very much. We're going to say that these two are super close together. If we make our time interval small enough, then the velocity essentially between both instants; it's essentially constant. The runner could be running at, say, 5 m/s, at the first instant, and 5.0001 at the next instant, but the change is small enough where we could just say that the runner is running at the average between those two velocities, which is essentially 5 m/s anyways.

Now we're going to do a little bit of algebra. Using $\langle v \rangle = \frac{\Delta x}{\Delta t}$ multiplying Δt to the other side gives us

$$\langle v \rangle \Delta t = \Delta x$$

or

$$\langle v \rangle \Delta t = x_f - x_i$$

Bringing over the initial position of the runner gives us the final position

$$\langle v \rangle \Delta t + x_i = x_f$$

We're assuming that the speed is essentially constant, so we can use the runners initial speed for average speed, and so we can solve for x_f, where they are at the end of this time interval. In other words, if we know where I am now, and my speed now, and I'm free to assume that my speed won't change because I'm considering just a tiny time interval, then I can use that information to predict where I'm going to be in the future. This is the idea of simulation. I use what I know about the system at any given instant to predict how things are going to be in the next small time, little bit of time later.

The same philosophy holds true for acceleration. I could have repeated the entire series of steps with acceleration. Let's start with the definition of acceleration, $\langle a \rangle = \frac{\Delta v}{\Delta t}$. If I'm thinking about a very small time interval then the average acceleration is going to be the acceleration; the acceleration is not going to change very much as long as this Δt is really, really small. Doing a little bit of algebra,

$$\langle a \rangle \Delta t = \Delta v$$

$$\langle a \rangle \Delta t = v_f - v_i$$

$$\langle a \rangle \Delta t + v_i = v_f$$

Again, if I know my speed at some instant, my acceleration at some instant, and my Δt is small enough, then I can use that information to predict what my Δv is going to be some small time later. We'll be doing this throughout the course with a wide variety of concepts, from forces to temperature to entropy. In all cases, you're using what you know now to predict what will happen a small time later.

Example of Solving a Problem Using Simulation

Let's take a very simple situation and work our way up.

A ball is dropped from 10 meters above the ground, and as we'll see later in the course, a falling object undergoes a constant acceleration of $-9.81\frac{m}{s^2}$. We'll discuss this in more detail later, and we'll get into the physics of freefall at length, but for now, we'll just show you how to run the mechanics of the simulation. How far above the ground is the ball 0.02 seconds later?

We're dealing with very small amounts of time, so our assumption that things are constant will be true. To get started, we'll set up a table:

Table 2.1

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	10	0	-9.81

Our table has position, velocity, acceleration, and time, as these will all play into the motion of the ball. We'll define the beginning of the drop at t=0, with an initial position of 10 meters, and the initial velocity of 0 m/s, as the ball isn't moving at the very beginning of the drop. For the acceleration, as stated earlier, it's $-9.81\frac{m}{s^2}$. Again, don't get too engrossed in the physics right now, we'll talk about why it's a negative and a positive much later. For now, I just want to go through the mechanics.

Now we'll move on to some time later. We're going to go some small amount of time later, so let's pick our change in time to be 0.01, since it's small and fits into 0.02 seconds nicely. Remember, this whole idea is predicated on the assumption that velocity and acceleration aren't changing very much over the time, and the only way that can be true is if the time is small, so we need to take small time steps. For the next time step, the time is the previous time plus the change in time, or $t + \Delta t$, which comes out to be 0.01. Acceleration won't change, so we can leave that as is.

Now what about the velocity and the position? From above, we solved for final position and final velocity as

$$\langle v\rangle\Delta t + x_i = x_f$$

$$\langle a\rangle\Delta t + v_i = v_f$$

We can use the initial position, initial velocity, and acceleration to solve for the final velocity and final position. Remember, since we're taking a small time step, the velocity won't change much, so we can replace the average velocity with just the initial velocity. Acceleration doesn't change, so the average acceleration is just what it started as in the beginning. Plugging in the numbers gives us: $\left(0\frac{m}{s}\right)(0.01s) + (10m) = 10m$

$$\left(-9.81\frac{m}{s^2}\right)(0.01s) + 0\frac{m}{s} = -0.0981\frac{m}{s}$$

We can use these numbers to continue our table:

Table 2.2

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	10	0	-9.8
0.01	10	-0.0981	-9.8

We can repeat this process for the next step in time, using the values we found for the next time step as our new initial conditions. So, plugging in the numbers into the equations:

$$\left(-0.0981\frac{m}{s}\right)(0.01s) + (10m) = 9.99902m$$

$$\left(-9.81\frac{m}{s^2}\right)(0.01s) + -0.0981\frac{m}{s} = -0.1962\frac{m}{s}$$

And putting out new values into the table:

Table 2.3

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	10	0	-9.81
0.01	10	-0.0981	-9.81
0.02	9.99902	9.99902	-9.81

So the answer to our initial problem is that the ball is 9.99902 meters off the ground 0.02 seconds later.

The ball doesn't move very far in the short amount of time; we could have figured that out probably by intuition. However, using simulations can better your understanding of concepts such as acceleration and velocity. For example, notice that the ball's position doesn't change in the first time step. Through this simulation, you can see that the acceleration does not result in a change in position immediately, but rather a change in velocity, and it's this velocity that causes the change in position.

More Complex Example

Let's say you have a car, starting at rest, with an acceleration defined to be: $a(t) = (5\frac{m}{s^2})t^2$

A. How fast is it moving after 0.02s?
B. Where is it after 5s?
C. What if it changed to $a(t) = (2\frac{m}{s^2})t^2$, where would it be after 10s?

This example has a few different parts, where our acceleration is not the nice simple constant. In the previous example, the acceleration was constant, so you could solve it without using simulation. However, with an acceleration that's not constant, the only way you can solve it is through simulation.

Let's get started with part a). First, we make our table and put in our initial conditions. Again, we use time, position, velocity, and acceleration for our table. We can set time to start at 0 and position to start at 0, since the problem doesn't specify a starting position. Initial velocity is 0, since the car starts at rest. To find the initial acceleration, we plug in our time into the acceleration given by the problem, so:

$$a(0) = \left(5\frac{m}{s^2}\right)0^2 = 0\frac{m}{s^2}$$

So the initial acceleration is 0 as well.

Table 2.4

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	0	0	0

Now let's take our first step and go to 0.01 seconds, where we're still going to be using these two expressions.

$$\langle v\rangle\Delta t + x_i = x_f$$

$$\langle a\rangle\Delta t + v_i = v_f$$

I don't want you to memorize these now, remember, these just come from the definitions of velocity and acceleration. These just the definitions of these quantities rewritten, so it's not a new equation, it's the same definitions we've been exploring in this entire preparation. Again, we plug in the initial values into the two expressions, so: $\left(0\frac{m}{s}\right)(0.01s) + (0m) = 0m$

$$\left(0\frac{m}{s^2}\right)(0.01s) + 0\frac{m}{s} = 0\frac{m}{s}$$

We also have to solve for our new acceleration using the formula given in the problem. Before we move on, notice that there's a Δt in our expressions for final position and velocity, but there is a t in the formula for acceleration. Normally, these are interchangeable, but in this problem, they mean two separate things. The change in time Δt , so the time steps, while t is the total time up to that step, so it's important to keep in mind this distinction. Solving for the new acceleration gives us:

$$a(0.01) = \left(5\frac{m}{s^2}\right)0.01^2 = 0.0005\frac{m}{s^2}$$

And our new table looks like:

Table 2.5

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	0	0	0
0.01	0	0	0.0005

Repeating the process for 0.02 gives us:

$$\left(0\frac{m}{s}\right)(0.01s) + (0m) = 0m$$

$$\left(0.0005\frac{m}{s^2}\right)(0.01s) + 0\frac{m}{s} = 0.000005\frac{m}{s}$$

$$a(0.02) = \left(5\frac{m}{s^2}\right)0.02^2 = 0.002\frac{m}{s^2}$$

Notice that 0.02 is used for the time in the acceleration. Again, keep in mind the distinction between Δt and t in the problem. Filling out the table gives us:

Table 2.6

Time (t)	Position (x)	Velocity (v)	Acceleration (a)
0	0	0	0
0.01	0	0	0.0005
0.02	0	0.000005	0.002

So the answer to the question is that the car is moving 0.000005 m/s after 0.02 seconds.

Alright, so now we've solved this problem by hand for up to .02 seconds, great. The next question is, where is it after five seconds? Well, doing this by hand in one one-hundredth of a second increments for five seconds is going to take us a really long time. You could do it, but you'd be at it for quite a while. This is where the benefit of using a computer to solve the problem will come into play. Since simulation is mostly just a process, you can have a computer program, like Excel or Google Spreadsheets, run through the process for you. We'll be going over how to do this in class.

Glossary

acceleration: the rate of change in velocity; the change in velocity over time

average acceleration: the change in velocity divided by the time over which it changes

average speed: distance traveled divided by time during which motion occurs

average velocity: displacement divided by time over which displacement occurs

deceleration: acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

dependent variable: the variable that is being measured; usually plotted along the y -axis

displacement: the change in position of an object

distance: the magnitude of displacement between two positions

distance traveled: the total length of the path traveled between two positions

elapsed time: the difference between the ending time and beginning time

independent variable: the variable that the dependent variable is measured with respect to; usually plotted along the x -axis

instantaneous acceleration: acceleration at a specific point in time

instantaneous speed: magnitude of the instantaneous velocity

instantaneous velocity: velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics: the study of motion without considering its causes

model: simplified description that contains only those elements necessary to describe the physics of a physical situation

position: the location of an object at a particular time

scalar: a quantity that is described by magnitude, but not direction

slope: the difference in y -value (the rise) divided by the difference in x -value (the run) of two points on a straight line

time: change, or the interval over which change occurs

vector: a quantity that is described by both magnitude and direction

y-intercept: the y- value when x = 0, or when the graph crosses the y -axis

Section Summary

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_{\text{f}} - x_0,$$

where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_{\text{f}} - t_0,$$

where t_{f} is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t .
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0}.$$

- The SI unit for acceleration is m/s^2 .
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5 Graphical Analysis of One-Dimensional Motion

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v .
- The slope of a graph of velocity v vs. time t graph is acceleration a .
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu m/s\ \left(50 \times 10^{-6}\ m/s\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in **Exercise 2.4**?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be $-5^{\circ}C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.5 Graphical Analysis of One-Dimensional Motion

18. (a) Explain how you can use the graph of position versus time in **Figure 2.33** to describe the change in velocity over time. Identify (b) the time (t_a , t_b , t_c , t_d , or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

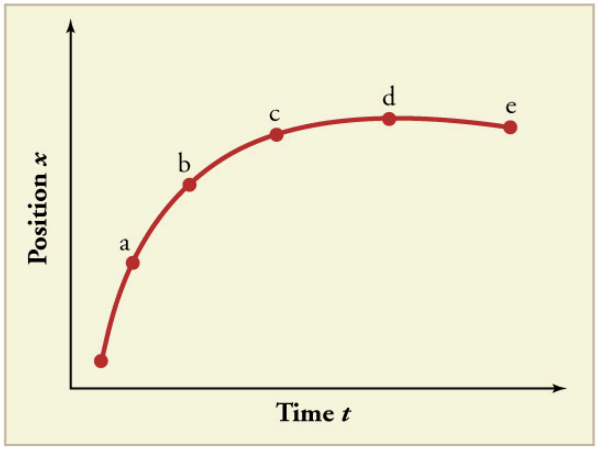


Figure 2.33

19. (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in **Figure 2.34**. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

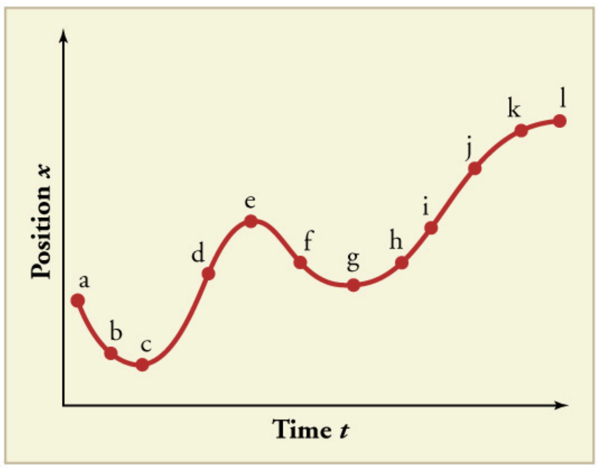


Figure 2.34

20. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in **Figure 2.35**. (b) Based on the graph, how does acceleration change over time?

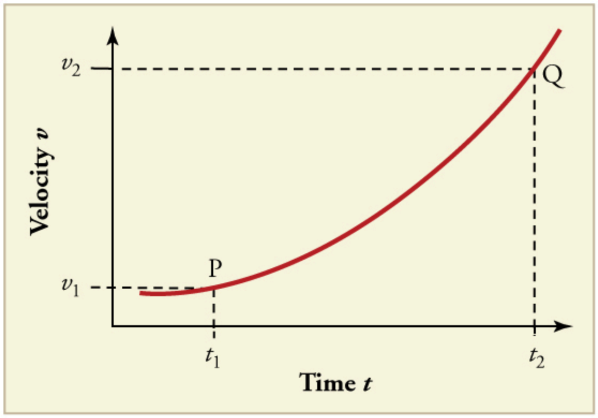


Figure 2.35

21. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in **Figure 2.36**. (b) Identify the time or times (t_a , t_b , t_c , etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

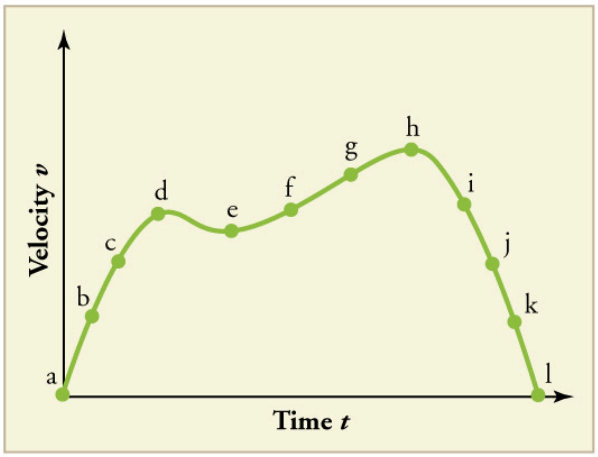


Figure 2.36

22. Consider the velocity vs. time graph of a person in an elevator shown in **Figure 2.37**. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from **Motion Equations for Constant Acceleration in One Dimension (https://legacy.cnx.org/content/m42099/latest/)** for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

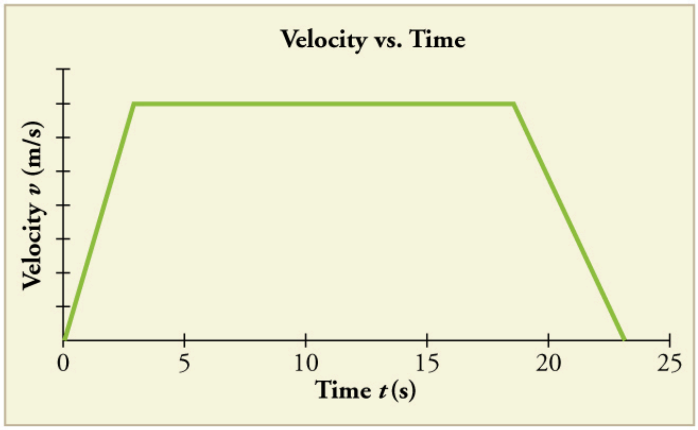


Figure 2.37

23. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1 Displacement

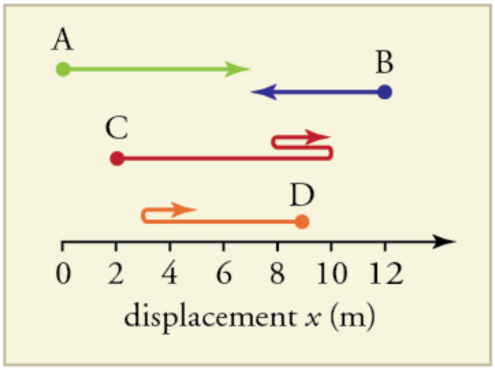


Figure 2.38

1. Find the following for path A in **Figure 2.38**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2. Find the following for path B in **Figure 2.38**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3. Find the following for path C in **Figure 2.38**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4. Find the following for path D in **Figure 2.38**: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84\times10^{6}\ \text{m}$ (1%)?

11. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00\times10^{8}\ \text{m/s})$.

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06\times10^{-10}\ \text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20\times10^{6}\ \text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

17. Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g $(9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g $(9.80\ \text{m/s}^2)$?

2.5 Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

20. (a) By taking the slope of the curve in Figure 2.39, verify that the velocity of the jet car is 115 m/s at $t = 20\ \text{s}$. (b) By taking the slope of the curve at any point in Figure 2.40, verify that the jet car's acceleration is $5.0\ \text{m/s}^2$.

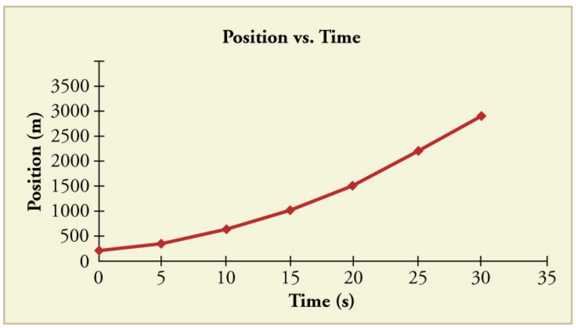


Figure 2.39

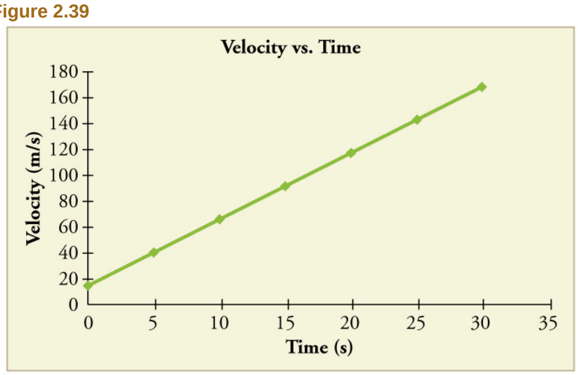


Figure 2.40

21. Using approximate values, calculate the slope of the curve in Figure 2.41 to verify that the velocity at $t = 10.0\ \text{s}$ is 0.208 m/s. Assume all values are known to 3 significant figures.

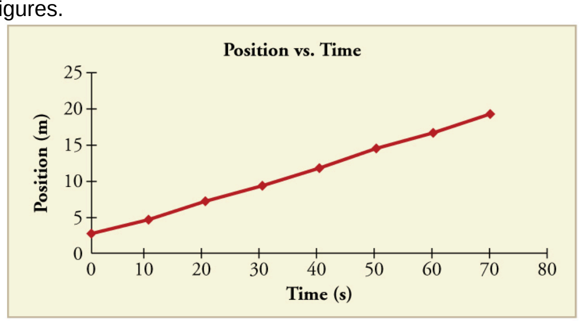


Figure 2.41

22. Using approximate values, calculate the slope of the curve in Figure 2.41 to verify that the velocity at $t = 30.0\ \text{s}$ is 0.238 m/s. Assume all values are known to 3 significant figures.

23. By taking the slope of the curve in Figure 2.42, verify that the acceleration is $3.2\ \text{m/s}^2$ at $t = 10\ \text{s}$.

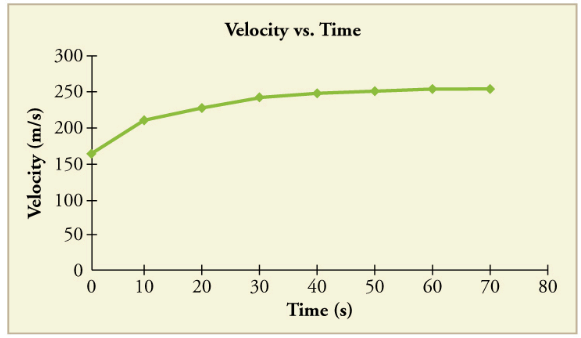


Figure 2.42

24. Construct the displacement graph for the subway shuttle train as shown in m42100 (https://legacy.cnx.org/content/m42100/latest/#import-auto-id2590556) (a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

25. (a) Take the slope of the curve in **Figure 2.43** to find the jogger's velocity at $t = 2.5 \text{ s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in **Figure 2.44**.

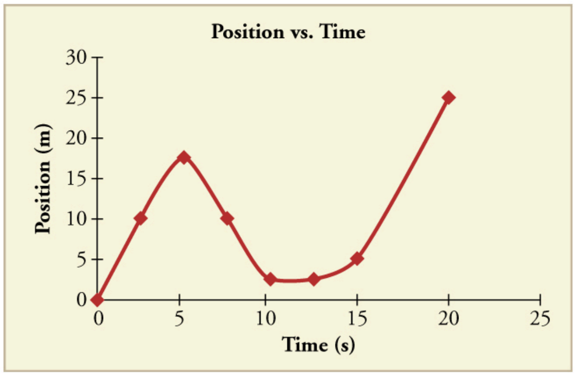


Figure 2.43

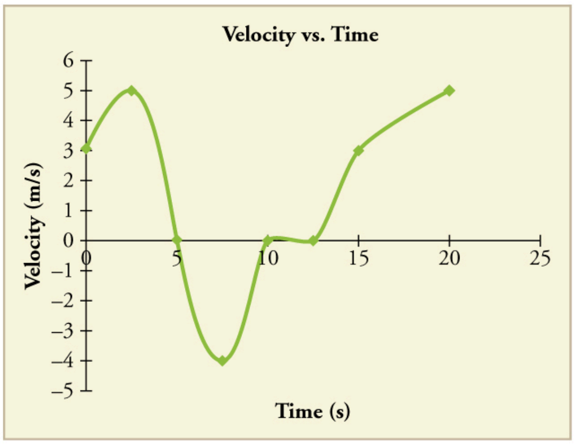


Figure 2.44

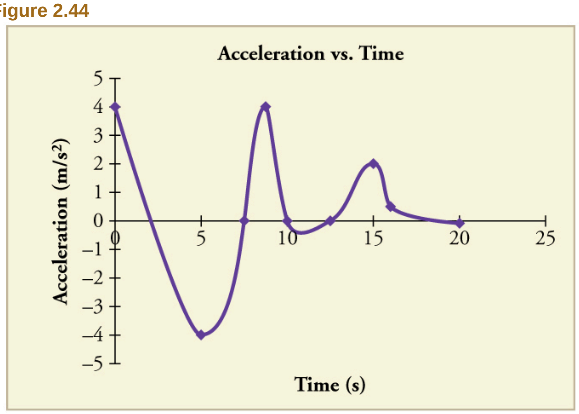


Figure 2.45

26. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See **Figure 2.46**). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5 \text{ s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

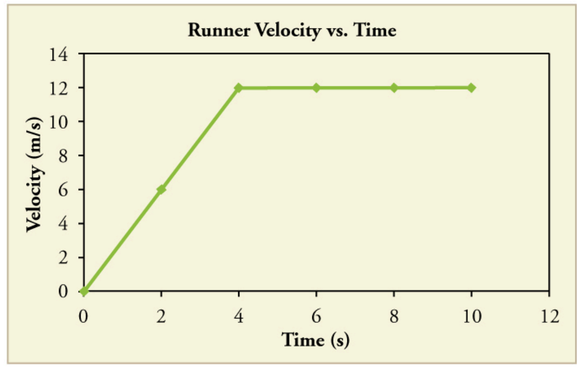


Figure 2.46

27. **Figure 2.47** shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

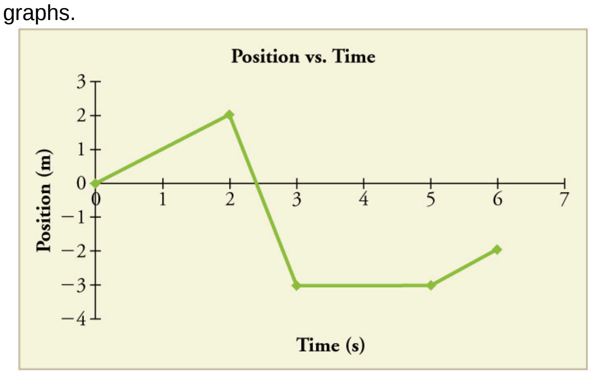


Figure 2.47

3 TWO-DIMENSIONAL KINEMATICS

Figure 3.1 Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons)

Chapter Outline

3.1. Kinematics in Two Dimensions-An introduction
- A vector is a quantity with a magnitude and direction
- Converting between magnitude/direction and the component form for any vector. This ties into the Pythagorean Theorem

3.2. Vector Addition and Subtraction: Graphical Methods
- Given two graphical representations of vectors, be able to draw the sum or difference.
- Describe both visually and mathematically what happens when a scalar is multiplied by a vector.
- Convert between magnitude/direction and component form for any vector

3.3. Vector Addition and Subtraction-Analytical Methods
- Adding vectors by components

3.4. Addition of Velocities

Introduction to Two-Dimensional Kinematics

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

3.1 Kinematics in Two Dimensions-An introduction

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- A vector is a quantity with a magnitude and direction
- Converting between magnitude/direction and the component form for any vector. This ties into the Pythagorean Theorem

Figure 3.2 Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in **Figure 3.3**.

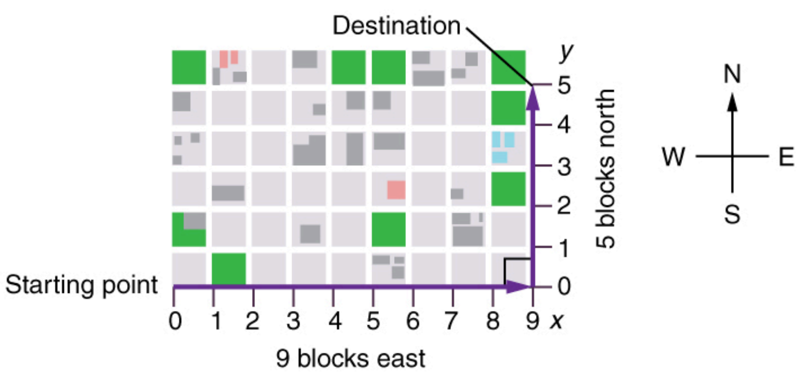


Figure 3.3 A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

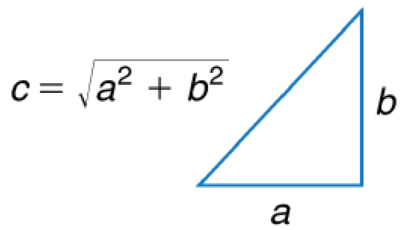


Figure 3.4 The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for c : $c = \sqrt{a^2 + b^2}$.

UMASS AMHERST Instructor's Notes

We will be using the pythagorean theorem all throughout two-dimensional kinematics, as well as throughout this entire course. If you are uncomfortable or unfamiliar with the Pythagorean Theorem, or even if it's just been a long time since you've used it, please come see your instructor as soon as possible and they will get you up to speed.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)

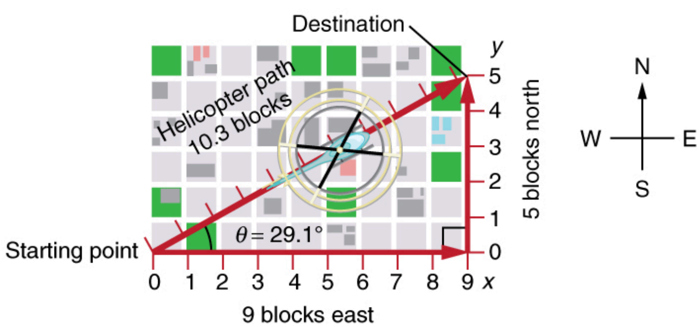


Figure 3.5 The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in **Figure 3.5** is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector’s magnitude. The arrow’s length is indicated by hash marks in **Figure 3.3** and **Figure 3.5**. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in **Figure 3.5**. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in **Vector Addition and Subtraction: Graphical Methods (https://legacy.cnx.org/content/m42127/latest/)** and **Vector Addition and Subtraction: Analytical Methods (https://legacy.cnx.org/content/m42128/latest/)** .)

The Independence of Perpendicular Motions

UMASS AMHERST Instructor's Notes

The idea of the independence of perpendicular motion is a fundamental one that you should take some time to think about, and there are some questions about this on the homework.

The person taking the path shown in **Figure 3.5** walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

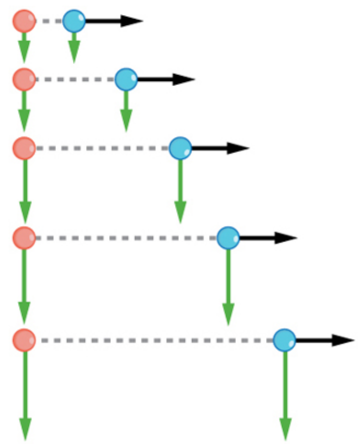


Figure 3.6 This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

UMASS AMHERST Instructor's Notes

This graphic displays this concept quite nicely; notice how both balls fall downward at the same speed at each point, even though one of the balls has a horizontal velocity. Basically, the velocity of the ball in the x-direction has no effect on the velocity in the y-direction, and vice-versa. This will be an important idea, especially when working with vectors.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in **Vector Addition and Subtraction: Graphical Methods (https://legacy.cnx.org/content/m42127/latest/)** and **Vector Addition and Subtraction: Analytical Methods (https://legacy.cnx.org/content/m42128/latest/)** . We will find such techniques to be useful in many areas of physics.

PhET Explorations: Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

PhET Interactive Simulation

Figure 3.7 Ladybug Motion 2D (http://legacy.cnx.org/content/m64153/1.2/ladybug-motion-2d_en.jar)

3.2 Vector Addition and Subtraction: Graphical Methods

UMASS AMHERST Instructor's Notes

Your Quiz will Cover

- Given two graphical representations of vectors, be able to draw the sum or difference. There are some simple procedures to follow. Solidify your understanding of these procedures and we can work on why this makes sense in class
- Describe both visually and mathematically what happens when a scalar is multiplied by a vector. If I give you a vector and a number, you should be able to turn the crank and multiply them mathematically. I am NOT expecting you to be able to do this graphically and will not ask you what it means. Just focus on the mechanics of how to do it.
- Convert between magnitude/direction and component form for any vector

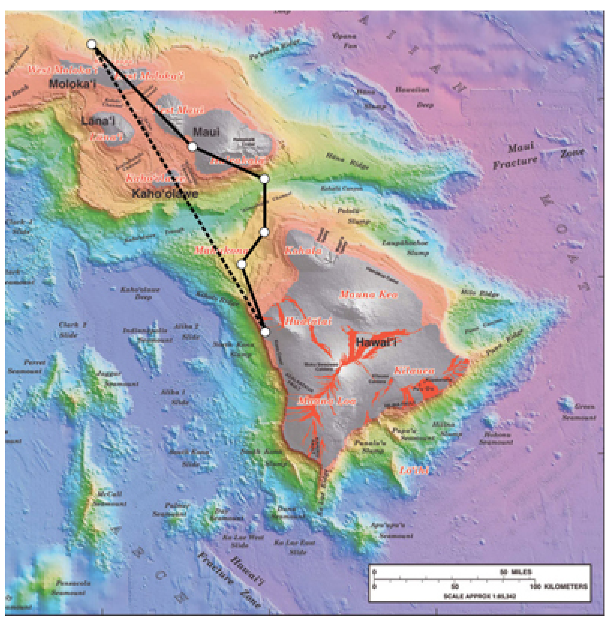


Figure 3.8 Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector’s magnitude and pointing in the direction of the vector.

Figure 3.9 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in **Kinematics in Two Dimensions: An Introduction (https://legacy.cnx.org/content/m42104/latest/)** . We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D , and its direction by θ .

UMASS AMHERST Instructor's Notes

There's some notation in the following note that would be useful to pay attention too.

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F , and the direction of the variable will be given by an angle θ .

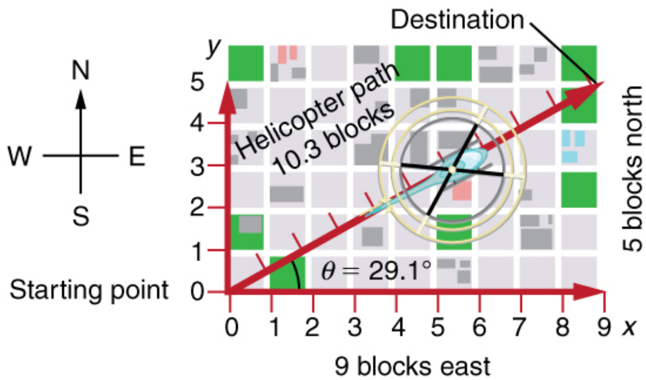


Figure 3.9 A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

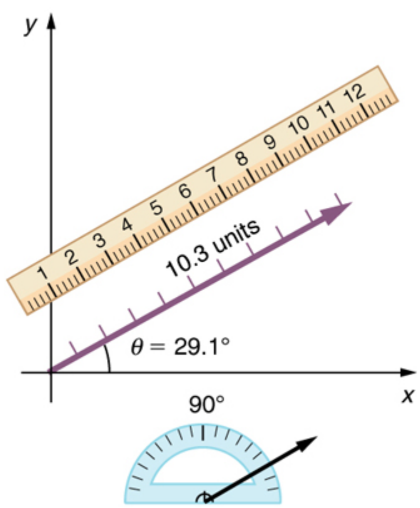


Figure 3.10 To describe the resultant vector for the person walking in a city considered in **Figure 3.9** graphically, draw an arrow to represent the total displacement vector $\mathbf{D}$. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector’s magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

UMASS AMHERST **Instructor's Notes**

Taking some time to understand and practice the head-to-tail method is recommended, you'll notice that there's a series of algorithmic steps, so you just need to learn the process, and it will work for any two vectors.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in **Figure 3.11** below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

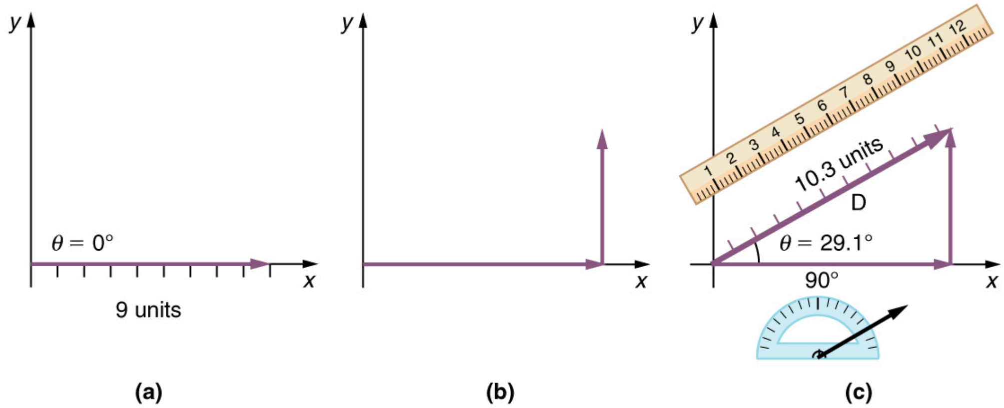


Figure 3.11 Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in **Figure 3.9**. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** $\mathbf{D}$. The length of the arrow $\mathbf{D}$ is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1° .

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.

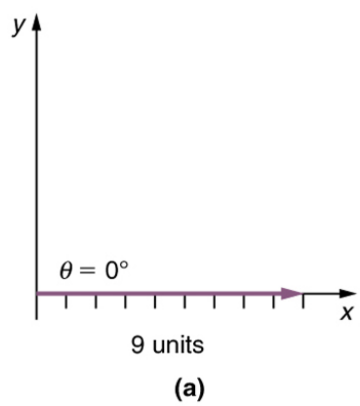


Figure 3.12

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.

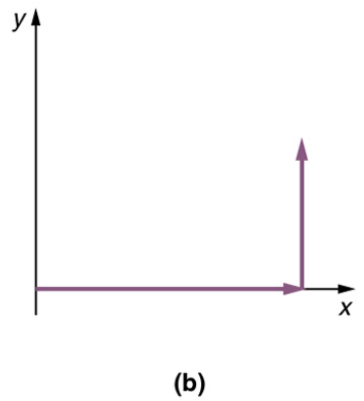


Figure 3.13

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail*.

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.

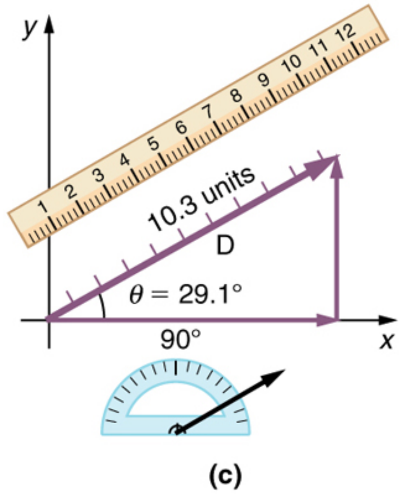


Figure 3.14

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 3.1 Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0^{o} north of east. Then, she walks 23.0 m heading 15.0^{o} north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first $\mathbf{A}$, the second $\mathbf{B}$, and the third $\mathbf{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\mathbf{R}$.

Solution

(1) Draw the three displacement vectors.

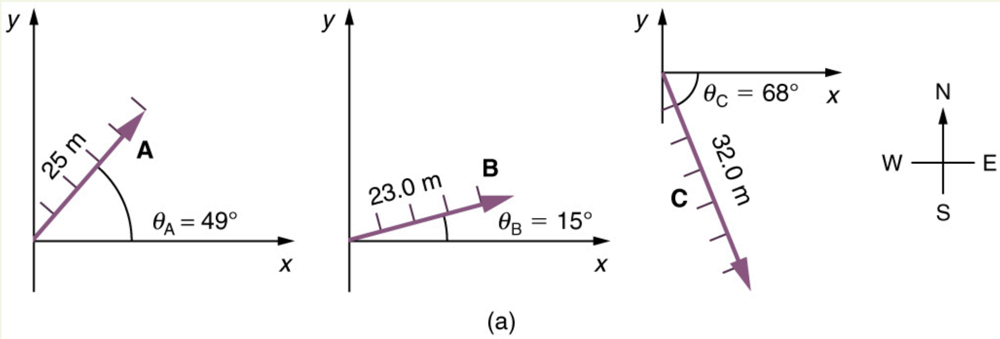


Figure 3.15

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

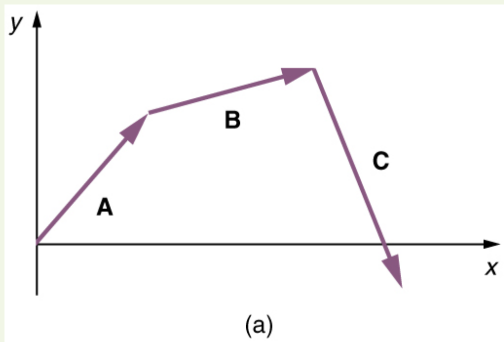


Figure 3.16

(3) Draw the resultant vector, $\mathbf{R}$.

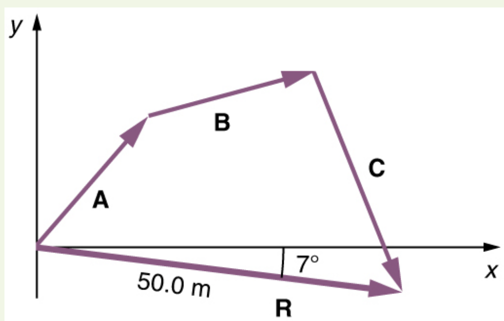


Figure 3.17

(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of $\mathbf{R}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

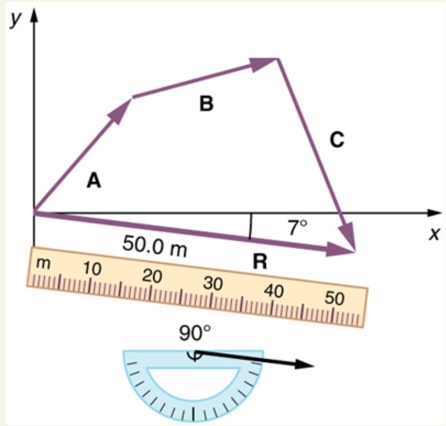


Figure 3.18

In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0\text{ m}$ and $\theta = 7.0^{\circ}$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in **Figure 3.19** and we will still get the same solution.

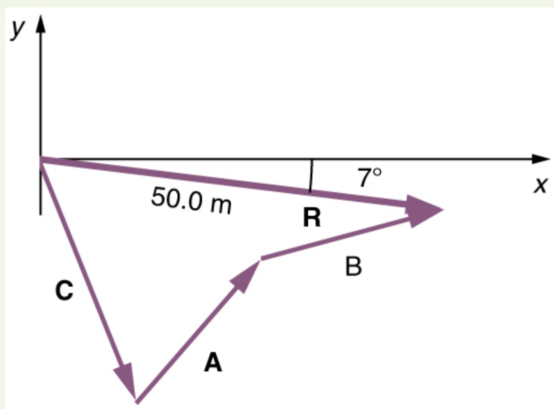


Figure 3.19

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}. \tag{3.1}$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $\mathbf{2 + 3}$ or $\mathbf{3 + 2}$, for example).

Instructor's Notes

Understanding vector subtraction is necessary to understand other physics ideas. For example, acceleration is Δv/Δt, and Δv is v_f - v_i. Velocity is a vector, so you're looking at a vector subtraction whenever you're working with acceleration.

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A} - \mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $-\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in **Figure 3.20**. In other words, $\mathbf{B}$ has the same length as $-\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

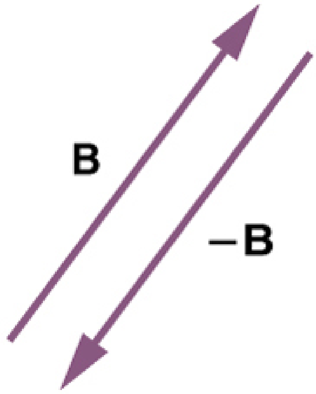


Figure 3.20 The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $-\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}). \tag{3.2}$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 3.2 Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

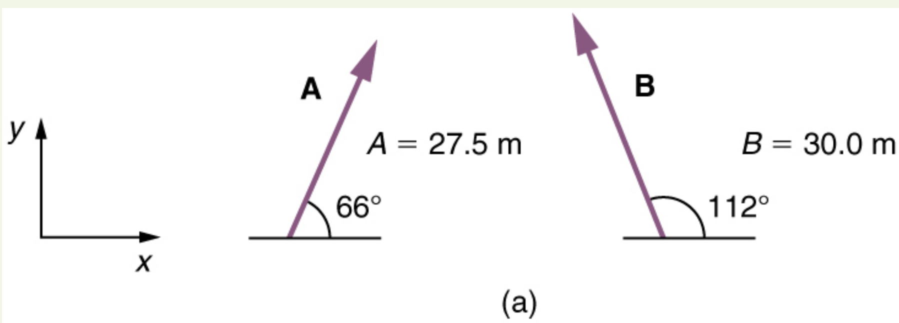


Figure 3.21

Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A} + \mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^\circ - 112^\circ = 68^\circ$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A} + (-\mathbf{B})$, or $\mathbf{A} - \mathbf{B}$.

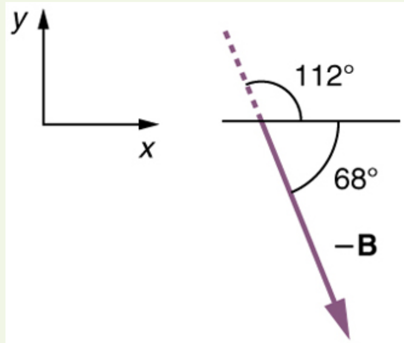


Figure 3.22

We will perform vector addition to compare the location of the dock, $\mathbf{A} + \mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathbf{A} + (-\mathbf{B})$.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.

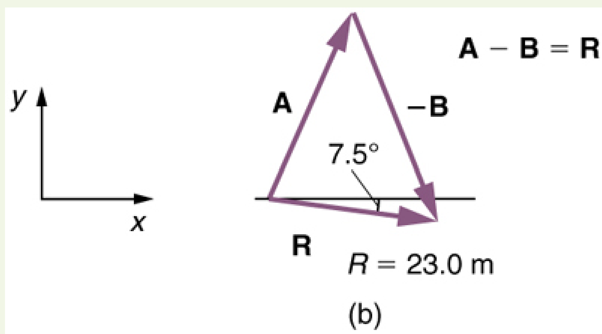


Figure 3.23

In this case, $R = 23.0\text{ m}$ and $\theta = 7.5^\circ$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}'$:

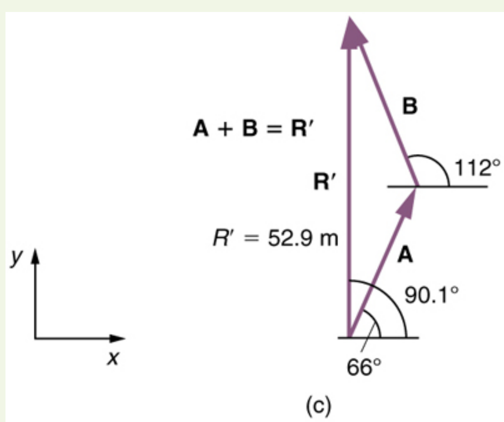


Figure 3.24

In this case $R = 52.9\text{ m}$ and $\theta = 90.1^\circ$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

UMASS AMHERST Instructor's Notes

If you've taken another physics course, you've probably seen **F** = *m***a**. This equation will play a significant role in this class, and you'll notice that mass is a scalar, and acceleration is a vector, so understanding how scalars and vectors multiply will be important.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5\text{ m}$, or 82.5 m, in a direction 66.0^{o} north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of $c\ A$,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5\text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

UMASS AMHERST Instructor's Notes

When dealing with vectors using analytic methods (which is covered in the next section), you need to break down vectors into essentially x-components and y-components. This next part covers this idea, so try to familiarize yourself with breaking down vectors as you read.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y-*components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0^{o} north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in **Projectile Motion (https://legacy.cnx.org/content/m42042/latest/)** , and much more when we cover **forces** in **Dynamics: Newton's Laws of Motion (https://legacy.cnx.org/content/m42129/latest/)** . Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in **Vector Addition and Subtraction: Analytical Methods (https://legacy.cnx.org/content/m42128/latest/)** are ideal for finding vector components.

PhET Explorations: Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

PhET Interactive Simulation

Figure 3.25 Maze Game (http://legacy.cnx.org/content/m64160/1.1/maze-game_en.jar)

3.3 Vector Addition and Subtraction-Analytical Methods

UMASS AMHERST Instructor's Notes

Your Quiz would Cover

- Adding vectors by components. Don't focus too much on what it means to add vectors. Just learn the mechanics of how to do it. We will talk about the meaning in class.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in **Figure 3.26**, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

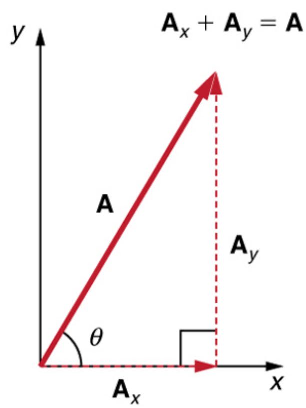


Figure 3.26 The vector $\mathbf{A}$, with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the *x*- and *y*-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

$$\mathbf{A}_x + \mathbf{A_y} = \mathbf{A}. \tag{3.3}$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\text{ m}$ east, $\mathbf{A}_y = 4\text{ m}$ north, and $\mathbf{A} = 5\text{ m}$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathbf{A_y} = \mathbf{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3\text{ m} + 4\text{ m} \neq 5\text{ m} \tag{3.4}$$

Thus,

$$A_x + A_y \neq A \tag{3.5}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*- and *y*-components, we use the following relationships for a right triangle.

$$A_x = A\cos\theta \tag{3.6}$$

and

$$A_y = A\sin\theta. \tag{3.7}$$

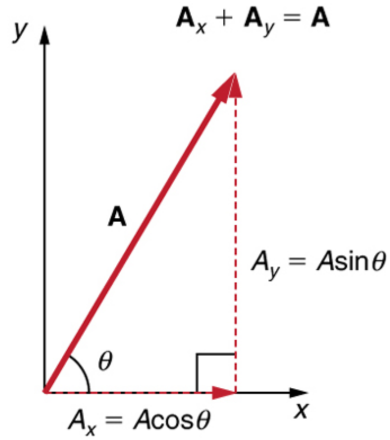


Figure 3.27 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in **Kinematics in Two Dimensions: An Introduction (https://legacy.cnx.org/content/m42104/latest/)** and **Vector Addition and Subtraction: Graphical Methods (https://legacy.cnx.org/content/m42127/latest/)** .

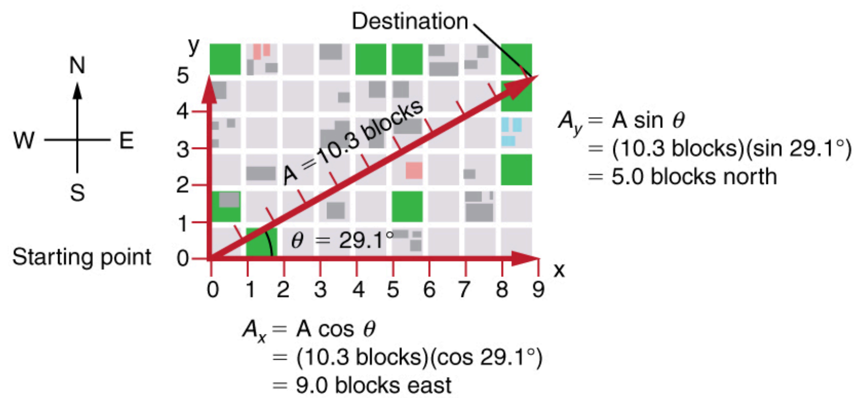


Figure 3.28 We can use the relationships $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3$ blocks and $\theta = 29.1^\circ$, so that

$$A_x = A\cos\theta = (10.3\text{ blocks})(\cos 29.1^\circ) = 9.0\text{ blocks} \tag{3.8}$$

$$A_y = A\sin\theta = (10.3\text{ blocks})(\sin 29.1^\circ) = 5.0\text{ blocks}. \tag{3.9}$$

Calculating a Resultant Vector

If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, we use the following relationships:

$$A = \sqrt{A_x{}^2 + A_y{}^2} \tag{3.10}$$

$$\theta = \tan^{-1}(A_y / A_x). \tag{3.11}$$

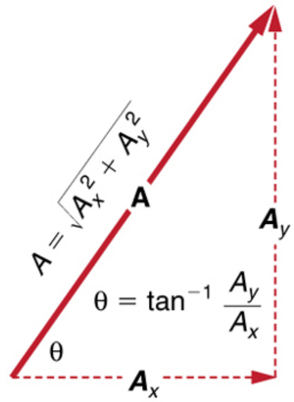


Figure 3.29 The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1^\circ$, as before.

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

UMASS AMHERST Instructor's Notes

Now that you know how to break down vectors into components, here's a procedure to adding vectors analytically. There's some trigonometry involved, so, again, if you're not familiar or comfortable with trigonometry, come see your instructor. You should be familiar with both methods. you should be able to add two vectors given their x and y components, and you should be able to draw the resulting vector of two added vectors. Also, we will go over how to use these to solve problems, so focus primarily on the methods of adding vectors.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider **Figure 3.30**, in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.

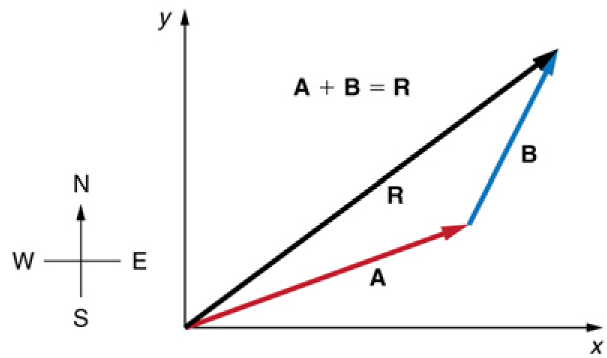


Figure 3.30 Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}.$ There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y / A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to find the components. In Figure 3.31, these components are A_x , A_y , B_x , and B_y . The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B , respectively.

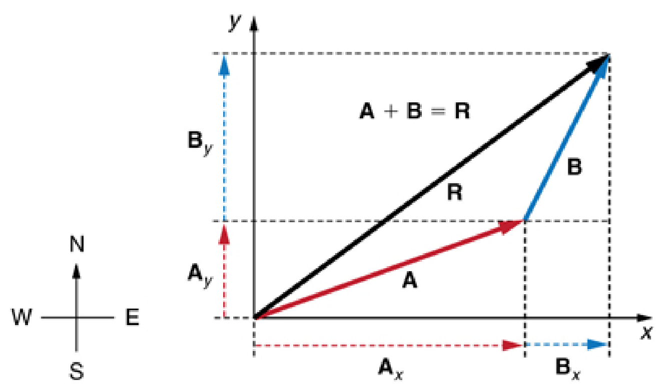


Figure 3.31 To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in Figure 3.32,

$$R_x = A_x + B_x \tag{3.12}$$

and

$$R_y = A_y + B_y. \tag{3.13}$$

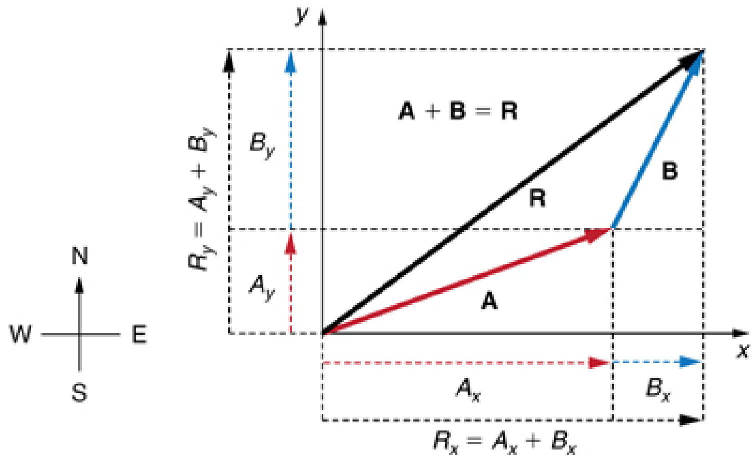


Figure 3.32 The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude* R *of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2}. \tag{3.14}$$

Step 4. *To get the direction of the resultant:*

$$\theta = \tan^{-1}(R_y / R_x). \tag{3.15}$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example 3.3 Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in **Figure 3.33**, using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0° north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of 34.0 m in a direction 63.0° north of east.

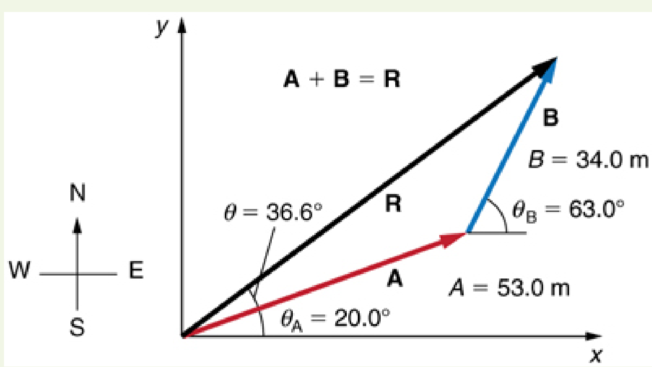


Figure 3.33 Vector $\mathbf{A}$ has magnitude 53.0 m and direction 20.0° north of the *x*-axis. Vector $\mathbf{B}$ has magnitude 34.0 m and direction 63.0° north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. Note that $A = 53.0\text{ m}$, $\theta_\text{A} = 20.0^\circ$, $B = 34.0\text{ m}$, and $\theta_\text{B} = 63.0^\circ$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_\text{A} = (53.0\text{ m})(\cos 20.0^\circ) \\ &= (53.0\text{ m})(0.940) = 49.8\text{ m} \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} B_x &= B\cos\theta_\text{B} = (34.0\text{ m})(\cos 63.0^\circ) \\ &= (34.0\text{ m})(0.454) = 15.4\text{ m.} \end{aligned} \tag{3.17}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_\text{A}$:

$$\begin{aligned} A_y &= A\sin\theta_\text{A} = (53.0\text{ m})(\sin 20.0^\circ) \\ &= (53.0\text{ m})(0.342) = 18.1\text{ m} \end{aligned} \tag{3.18}$$

and

$$\begin{aligned} B_y &= B\sin\theta_\text{B} = (34.0\text{ m})(\sin 63.0\,^\circ\,) \\ &= (34.0\text{ m})(0.891) = 30.3\text{ m.} \end{aligned} \tag{3.19}$$

The *x*- and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m} \tag{3.20}$$

and

$$R_y = A_y + B_y = 18.1\text{ m}+30.3\text{ m} = 48.4\text{ m.} \tag{3.21}$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2\text{ m}} \tag{3.22}$$

so that

$$R = 81.2\text{ m.} \tag{3.23}$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(R_y / R_x) = +\tan^{-1}(48.4 / 65.2). \tag{3.24}$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6\,^\circ\,. \tag{3.25}$$

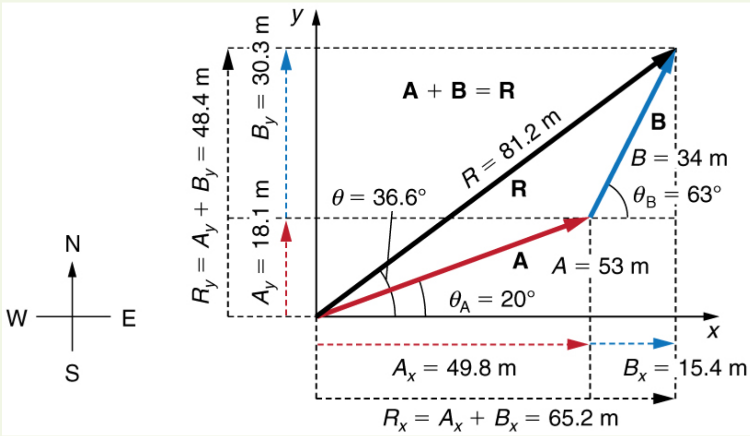


Figure 3.34 Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*- and *y*-components of the resultant $\mathbf{A} - \mathbf{B} = \mathbf{R}$ are thus

$$R_x = A_x + (-B_x) \tag{3.26}$$

and

$$R_y = A_y + (-B_y) \tag{3.27}$$

and the rest of the method outlined above is identical to that for addition. (See **Figure 3.35**.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, **Projectile Motion (https://legacy.cnx.org/content/m42042/latest/)** , is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

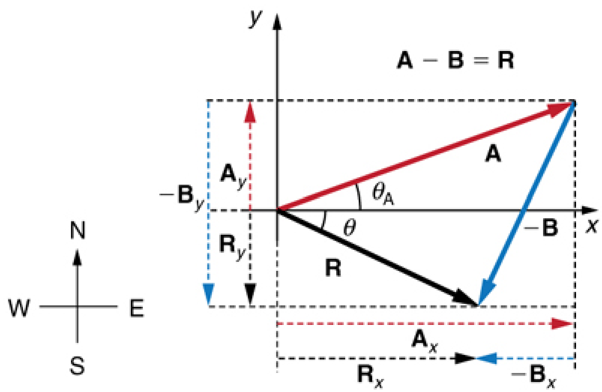


Figure 3.35 The subtraction of the two vectors shown in **Figure 3.30**. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

PhET Explorations: Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

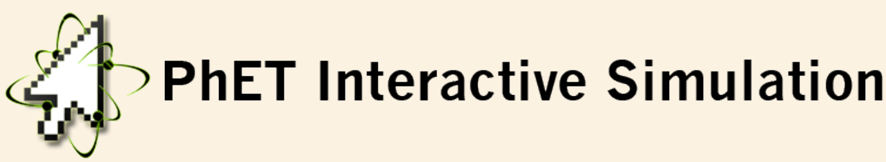


Figure 3.36 Vector Addition (http://legacy.cnx.org/content/m64164/1.2/vector-addition_en.jar)

3.4 Addition of Velocities

UMASS AMHERST Instructor's Notes

We will not being discussing this section in detail. This section is here primarily for your reference.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in **Figure 3.37**. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in **Figure 3.38**. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

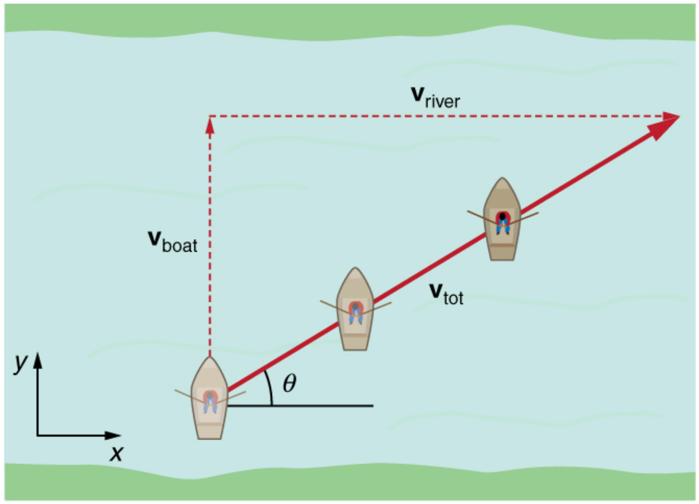


Figure 3.37 A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

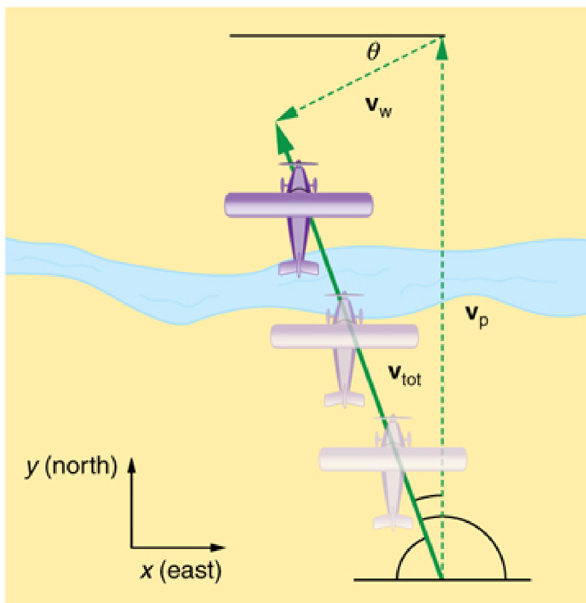


Figure 3.38 An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in Figure 3.37 and Figure 3.38. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in **Vector Addition and Subtraction: Graphical Methods (https://legacy.cnx.org/content/m42127/latest/)** and **Vector Addition and Subtraction: Analytical Methods (https://legacy.cnx.org/content/m42128/latest/)** apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*- and *y*-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta \tag{3.28}$$

$$v_y = v\sin\theta \tag{3.29}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.30}$$

$$\theta = \tan^{-1}(v_y / v_x). \tag{3.31}$$

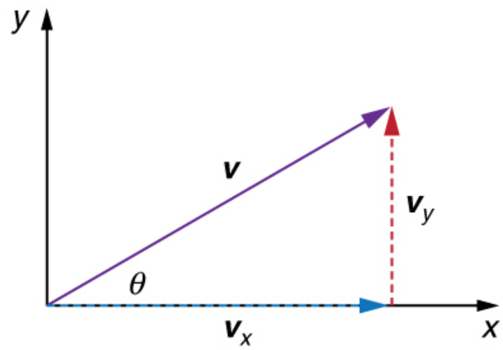


Figure 3.39 The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example 3.4 Adding Velocities: A Boat on a River

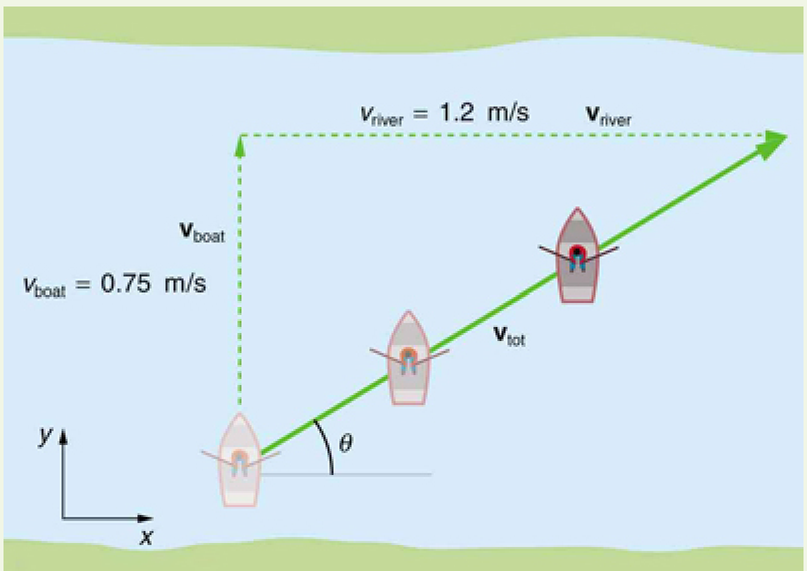


Figure 3.40 A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to **Figure 3.40**, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in **Figure 3.40**. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y / v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}, \tag{3.32}$$

where

$$v_x = v_{\text{river}} = 1.20 \text{ m/s} \tag{3.33}$$

and

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}. \tag{3.34}$$

Thus,

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2} \tag{3.35}$$

yielding

$$v_{\text{tot}} = 1.42 \text{ m/s}. \tag{3.36}$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(0.750 / 1.20). \tag{3.37}$$

This equation gives

$$\theta = 32.0^{\circ}. \tag{3.38}$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with **Figure 3.40**. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0°) the total velocity has relative to the riverbank.

Example 3.5 Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in **Figure 3.41**. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0° west of north.

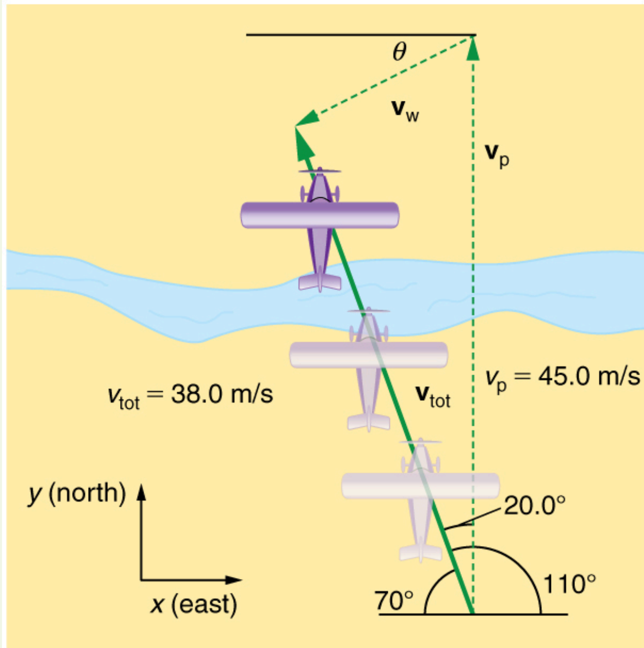


Figure 3.41 An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in Figure 3.41, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in **Vector Addition and Subtraction: Analytical Methods (https://legacy.cnx.org/content/m42128/latest/)** .)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{\text{p}x} = 0$ and $v_{\text{p}y} = v_{\text{p}}$. That is,

$$v_{\text{tot}x} = v_{\text{w}x} \tag{3.39}$$

and

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}}. \tag{3.40}$$

We can use the first of these two equations to find $v_{\text{w}x}$:

$$v_{\text{w}y} = v_{\text{tot}x} = v_{\text{tot}}\cos 110^{\text{o}}. \tag{3.41}$$

Because $v_{\text{tot}} = 38.0\text{ m/s}$ and $\cos 110^{\text{o}} = -0.342$ we have

$$v_{\text{w}y} = (38.0\text{ m/s})(–0.342) = –13\text{ m/s}. \tag{3.42}$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find $v_{\text{w}y}$ we note that

$$v_{\text{tot}y} = v_{\text{w}y} + v_{\text{p}} \tag{3.43}$$

Here $v_{\text{tot}y} = v_{\text{tot}}\sin 110^{\text{o}}$; thus,

$$v_{\text{w}y} = (38.0\text{ m/s})(0.940) - 45.0\text{ m/s} = -9.29\text{ m/s}. \tag{3.44}$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity $v_{\mathrm{w}x}$ and v_{wy} are known, we can find the magnitude and direction of $\mathbf{v}_{\mathrm{w}}$. First, the magnitude is

$$\begin{aligned} v_{\mathrm{w}} &= \sqrt{v_{\mathrm{w}x}^2 + v_{\mathrm{w}y}^2} \\ &= \sqrt{(-13.0\text{ m/s})^2 + (-9.29\text{ m/s})^2} \end{aligned} \tag{3.45}$$

so that

$$v_{\mathrm{w}} = 16.0\text{ m/s}. \tag{3.46}$$

The direction is:

$$\theta = \tan^{-1}(v_{\mathrm{wy}} / v_{\mathrm{wx}}) = \tan^{-1}(-9.29 / -13.0) \tag{3.47}$$

giving

$$\theta = 35.6^{\mathrm{o}}. \tag{3.48}$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in **Figure 3.41**. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3{,}000\text{ km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See **Figure 3.42**.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in **Figure 3.42**. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

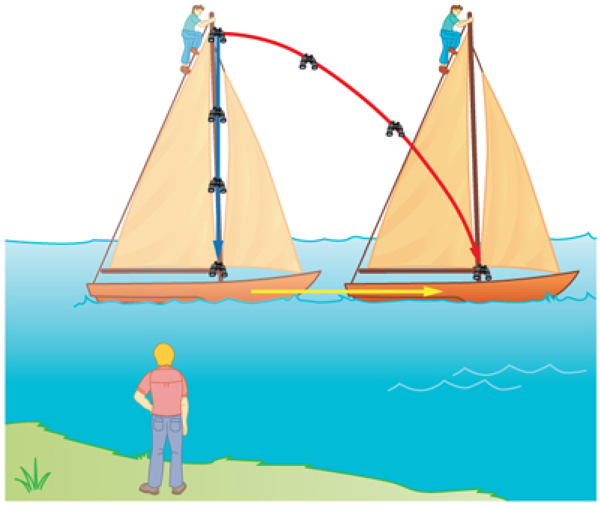


Figure 3.42 Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example 3.6 Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

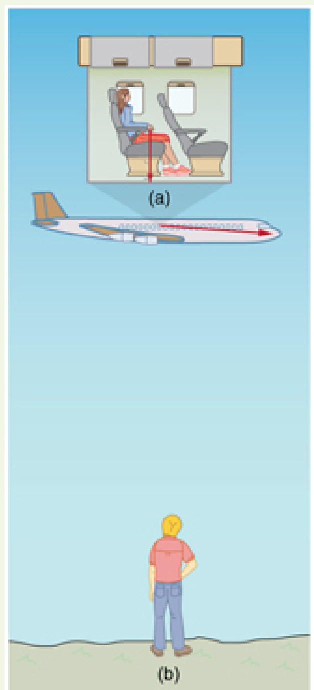


Figure 3.43 The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y{}^2 = v_{0y}{}^2 - 2g(y - y_0). \tag{3.49}$$

Substituting known values into the equation, we get

$$v_y{}^2 = 0^2 - 2(9.80\ \text{m/s}^2)(-1.50\ \text{m} - 0\ \text{m}) = 29.4\ \text{m}^2/\text{s}^2 \tag{3.50}$$

yielding

$$v_y = -5.42\ \text{m/s}. \tag{3.51}$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\ \text{m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260\ \text{m/s}$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{v_x{}^2 + v_y{}^2}. \tag{3.52}$$

Thus,

$$v = \sqrt{(260\ \text{m/s})^2 + (-5.42\ \text{m/s})^2} \tag{3.53}$$

yielding

$$v = 260.06\ \text{m/s}. \tag{3.54}$$

The direction is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(-5.42 / 260) \tag{3.55}$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^\circ. \tag{3.56}$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 - 5.42)\ \text{m/s}$; rather, it is $260.06\ \text{m/s}$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See **Figure 3.43**.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PhET Explorations: Motion in 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

PhET Interactive Simulation

Figure 3.44 Motion in 2D (http://legacy.cnx.org/content/m64143/1.2/motion-2d_en.jar)

Glossary

analytical method: the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

classical relativity: the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

commutative: refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

component (of a 2-d vector): a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

direction (of a vector): the orientation of a vector in space

head (of a vector): the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"

head-to-tail method: a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

magnitude (of a vector): the length or size of a vector; magnitude is a scalar quantity

relative velocity: the velocity of an object as observed from a particular reference frame

relativity: the study of how different observers moving relative to each other measure the same phenomenon

resultant: the sum of two or more vectors

resultant vector: the vector sum of two or more vectors

scalar: a quantity with magnitude but no direction

tail: the start point of a vector; opposite to the head or tip of the arrow

vector: a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

vector addition: the rules that apply to adding vectors together

velocity: speed in a given direction

Section Summary

3.1 Kinematics in Two Dimensions-An introduction

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2 Vector Addition and Subtraction: Graphical Methods

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.

- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

3.3 Vector Addition and Subtraction-Analytical Methods

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

$$\begin{aligned} A_x &= A \cos\theta \\ B_x &= B \cos\theta \end{aligned}$$

  and

$$\begin{aligned} A_y &= A \sin\theta \\ B_y &= B \sin\theta. \end{aligned}$$

  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

$$R_x = A_x + B_x$$

  and

$$R_y = A_y + B_y.$$

  Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

$$R = \sqrt{R_x^2 + R_y^2}.$$

  Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:

$$\theta = \tan^{-1}(R_y / R_x).$$

3.4 Addition of Velocities

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = v \cos\theta$$
$$v_y = v \sin\theta$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta = \tan^{-1}(v_y / v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

3.2 Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2. Give a specific example of a vector, stating its magnitude, units, and direction.

3. What do vectors and scalars have in common? How do they differ?

4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

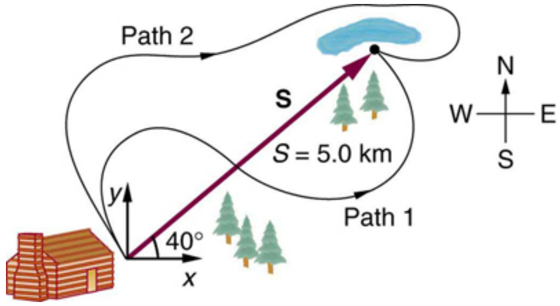


Figure 3.45

5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 3.46. What other information would he need to get to Sacramento?

Figure 3.46

6. Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A} + \mathbf{B}$ the sum of the lengths of the two steps?

7. Explain why it is not possible to add a scalar to a vector.

8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3 Vector Addition and Subtraction-Analytical Methods

9. Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

10. Give an example of a nonzero vector that has a component of zero.

11. Explain why a vector cannot have a component greater than its own magnitude.

12. If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

3.4 Addition of Velocities

13. What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

14. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn’t he need to keep his eyes on the ball?

15. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

16. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger’s frame of reference. Draw its path as viewed by a stationary observer.

17. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

3.2 Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

1. Find the following for path A in **Figure 3.47**: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

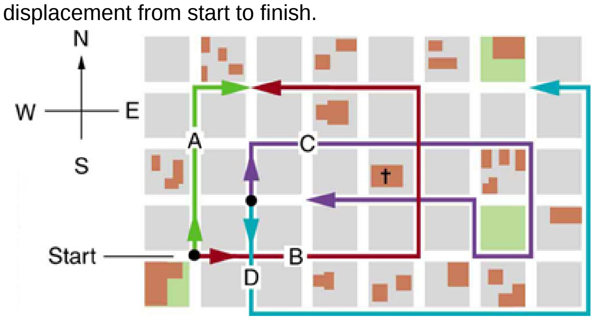


Figure 3.47 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

2. Find the following for path B in **Figure 3.47**: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

3. Find the north and east components of the displacement for the hikers shown in **Figure 3.45**.

4. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in **Figure 3.48**, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

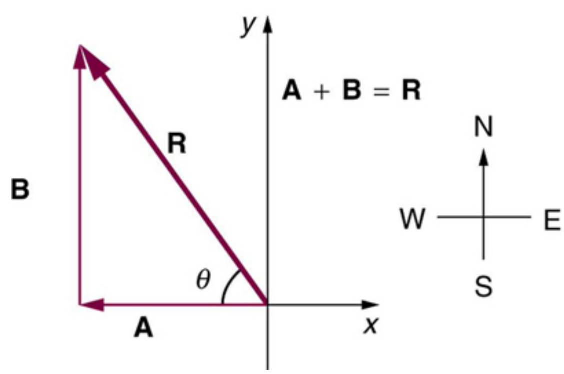


Figure 3.48 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

5. Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in **Figure 3.49**, then this problem finds their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

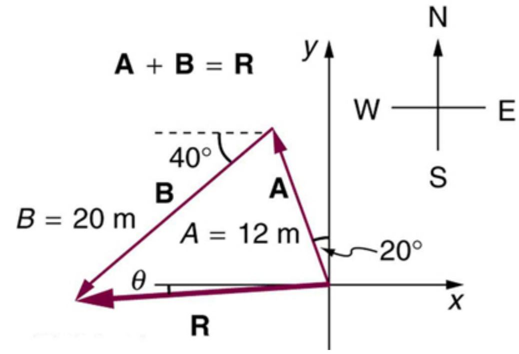


Figure 3.49

6. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

7. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.

8. Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathbf{A} + \mathbf{B} + \mathbf{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

9. Show that the sum of the vectors discussed in **Example 3.2** gives the result shown in **Figure 3.24**.

10. Find the magnitudes of velocities v_A and v_B in Figure 3.50

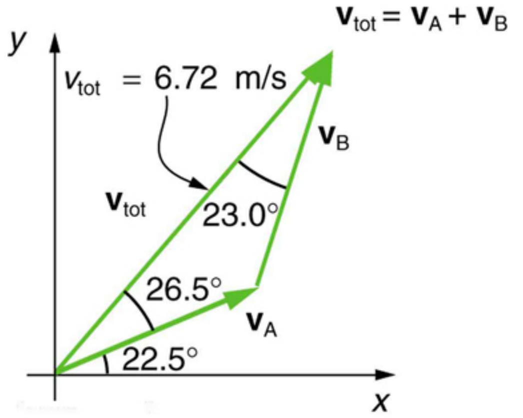


Figure 3.50 The two velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ add to give a total $\mathbf{v}_{tot}$.

11. Find the components of v_{tot} along the *x*- and *y*-axes in Figure 3.50.

12. Find the components of v_{tot} along a set of perpendicular axes rotated 30° counterclockwise relative to those in Figure 3.50.

3.3 Vector Addition and Subtraction-Analytical Methods

13. Find the following for path C in Figure 3.51: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

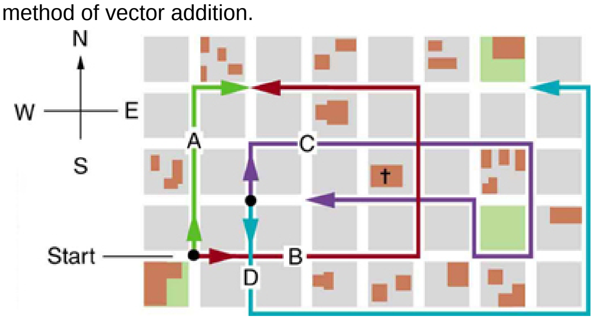


Figure 3.51 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

14. Find the following for path D in Figure 3.51: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

15. Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure 3.52.

Figure 3.52

16. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.53, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

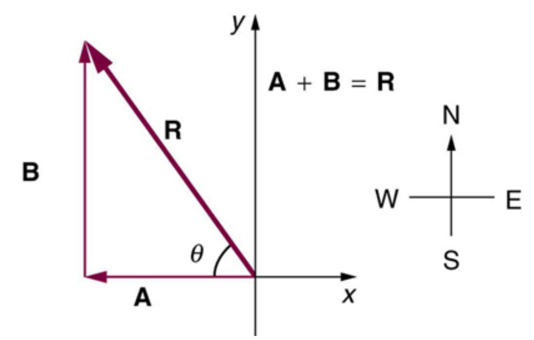


Figure 3.53 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

17. Repeat Exercise 3.16 using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

18. You drive $7.50\ \text{km}$ in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

19. Do **Exercise 3.16** again using analytical techniques and change the second leg of the walk to $25.0\ \text{m}$ straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk $25.0\ \text{m}$ north and then $18.0\ \text{m}$ east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

20. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in **Figure 3.54**. She then correctly calculates the length and orientation of the third side $\mathbf{C}$. What is her result?

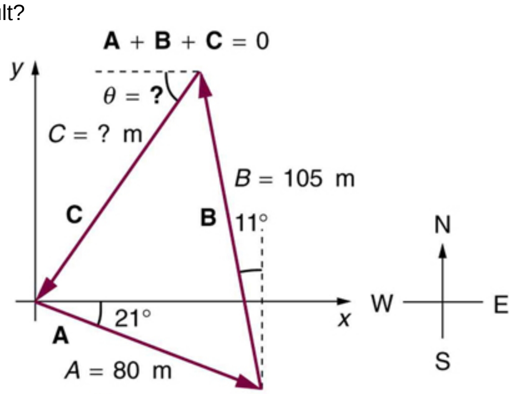


Figure 3.54

21. You fly $32.0\ \text{km}$ in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

22. A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $\mathbf{A},\ \mathbf{B},$ and $\mathbf{C}$ in **Figure 3.55**, and then correctly calculates the length and orientation of the fourth side $\mathbf{D}$. What is his result?

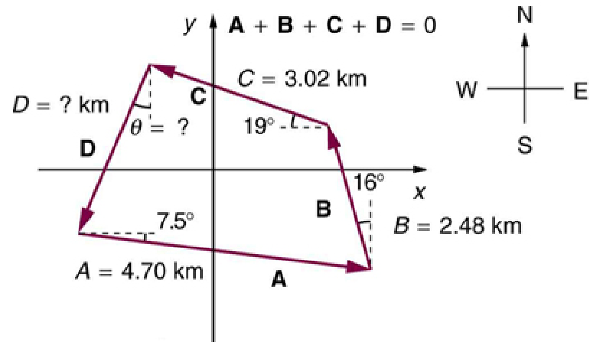


Figure 3.55

23. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: $2.50\ \text{km}$ 45.0° north of west; then $4.70\ \text{km}$ 60.0° south of east; then $1.30\ \text{km}$ 25.0° south of west; then $5.10\ \text{km}$ straight east; then $1.70\ \text{km}$ 5.00° east of north; then $7.20\ \text{km}$ 55.0° south of west; and finally $2.80\ \text{km}$ 10.0° north of east. What is his final position relative to the island?

24. Suppose a pilot flies $40.0\ \text{km}$ in a direction 60° north of east and then flies $30.0\ \text{km}$ in a direction 15° north of east as shown in **Figure 3.56**. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

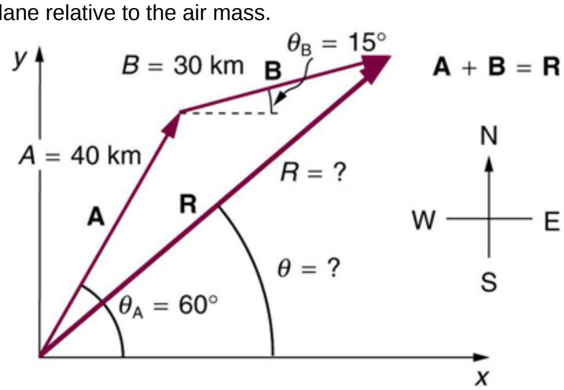


Figure 3.56

3.4 Addition of Velocities

25. Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45° south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

26. A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

27. Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

28. Verify that the coin dropped by the airline passenger in the **Example 3.6** travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

29. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

30. A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

31. (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

32. (a) In what direction would the ship in **Exercise 3.30** have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s ? (b) What would its speed be relative to the Earth?

33. (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in **Exercise 3.31**). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

34. A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

35. The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

36. The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. **Figure 3.57** illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.

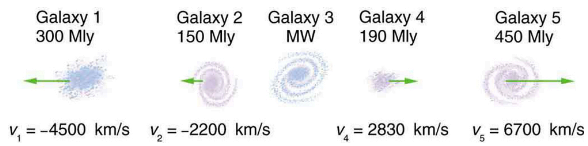


Figure 3.57 Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

37. (a) Use the distance and velocity data in **Figure 3.57** to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

38. An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

39. A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00° west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

40. An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in **Figure 3.58**. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?

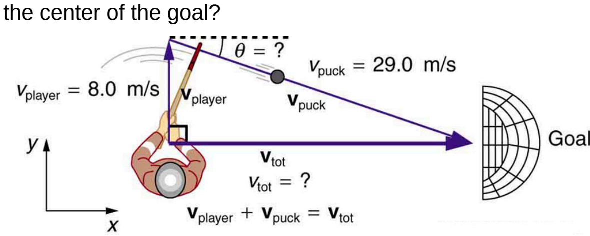


Figure 3.58 An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

41. Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

42. Unreasonable Results A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

43. Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Index

ATTRIBUTIONS

Used here as: Introduction to One-Dimensional Kinematics
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64085/1.2/**

Module: **2.1 Displacement**
Used here as: Displacement
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64041/1.4/**

Module: **2.2 Vectors, Scalars, and Coordinate Systems**
Used here as: Vectors, Scalars, and Coordinate Systems
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64047/1.4/**

Module: **2.3 Time, Velocity, and Speed**
Used here as: Time, Velocity, and Speed
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64044/1.4/**

Module: **2.4 Acceleration**
Used here as: Acceleration
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64042/1.4/**

Module: **2.5 Graphical Analysis of One-Dimensional Motion**
Used here as: Graphical Analysis of One-Dimensional Motion
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64043/1.4/**

Module: **2.6 Simulations**
Used here as: SImulations
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64191/1.1/**

Module: **3.0-Introduction to Two-Dimensional Kinematics**
Used here as: Introduction to Two-Dimensional Kinematics
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64148/1.2/**

Module: **3.1 Kinematics in Two Dimensions-An introduction**
Used here as: Kinematics in Two Dimensions-An introduction
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64153/1.2/**

Module: **3.2 Vector Addition and Subtraction: Graphical Methods**
Used here as: Vector Addition and Subtraction: Graphical Methods
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64160/1.1/**

Module: **3.3 Vector Addition and Subtraction-Analytical Methods**
Used here as: Vector Addition and Subtraction-Analytical Methods
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64164/1.2/**

Module: **3.4 Addition of Velocities**
Used here as: Addition of Velocities
By: David Nguyen
URL: **https://legacy.cnx.org/content/m64143/1.2/**

ABOUT CONNEXIONS