

ROB101-OPTIMIZATION

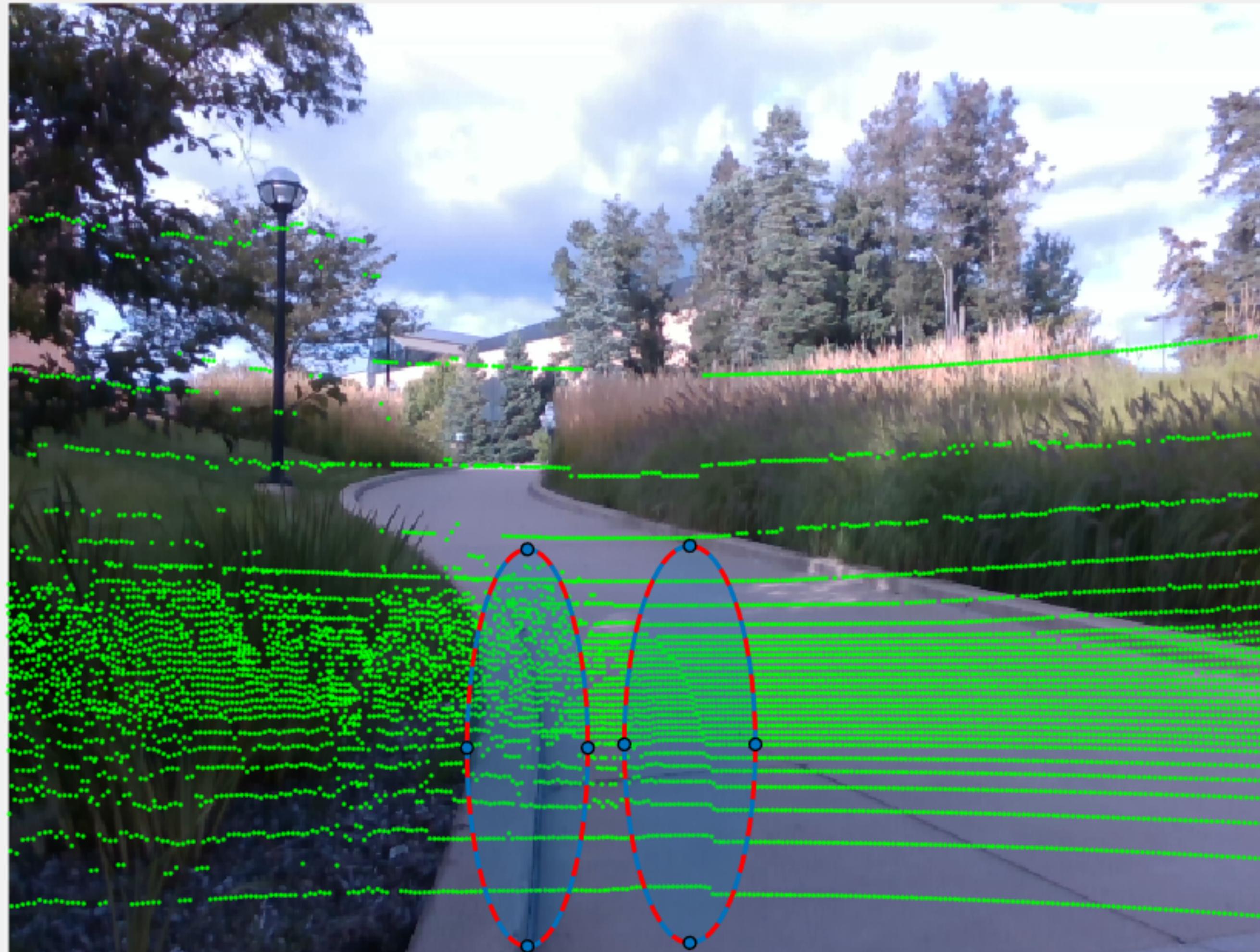
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University of Michigan, Ann Arbor

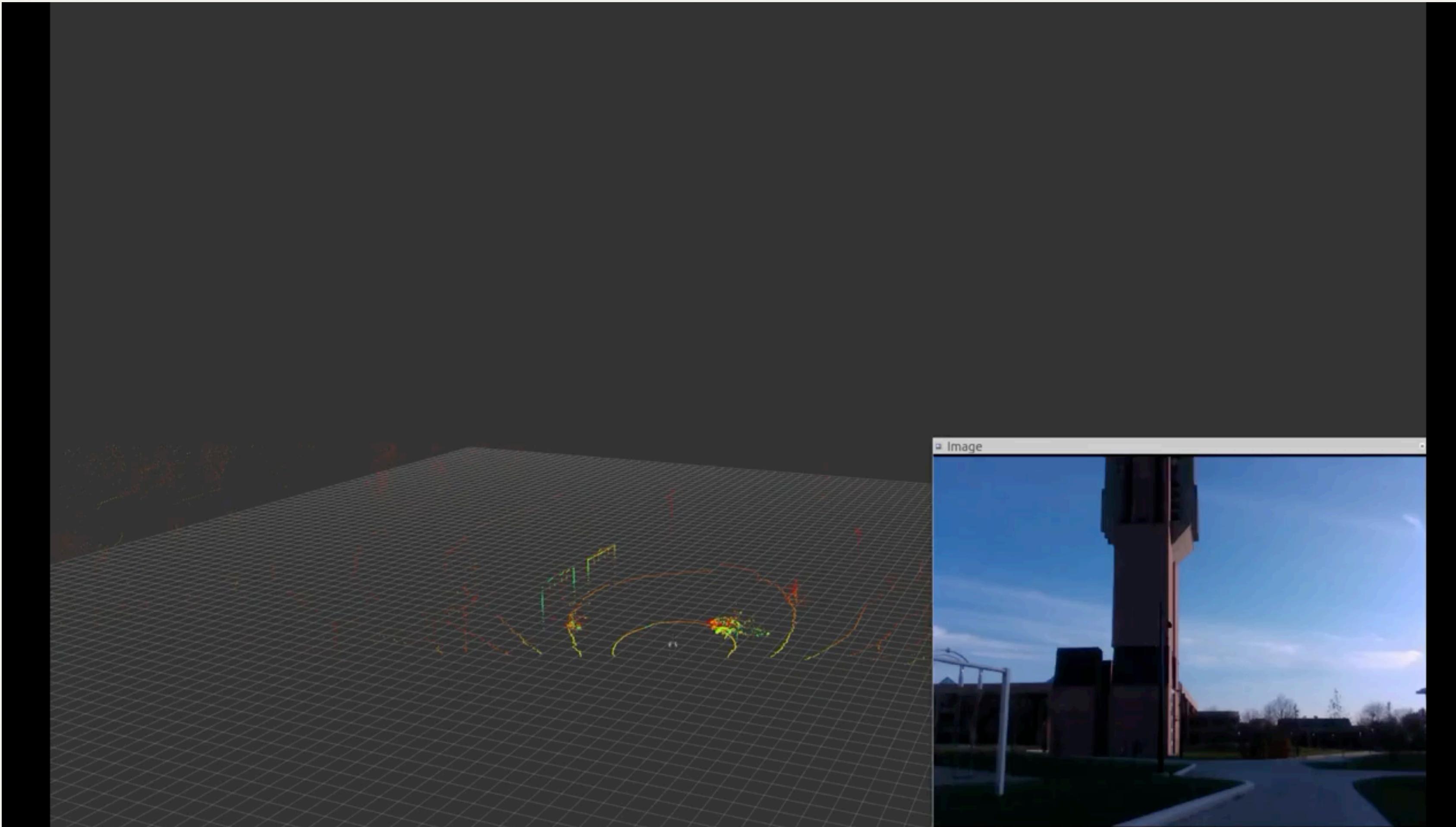
ACCURATE AND PRECISE!!

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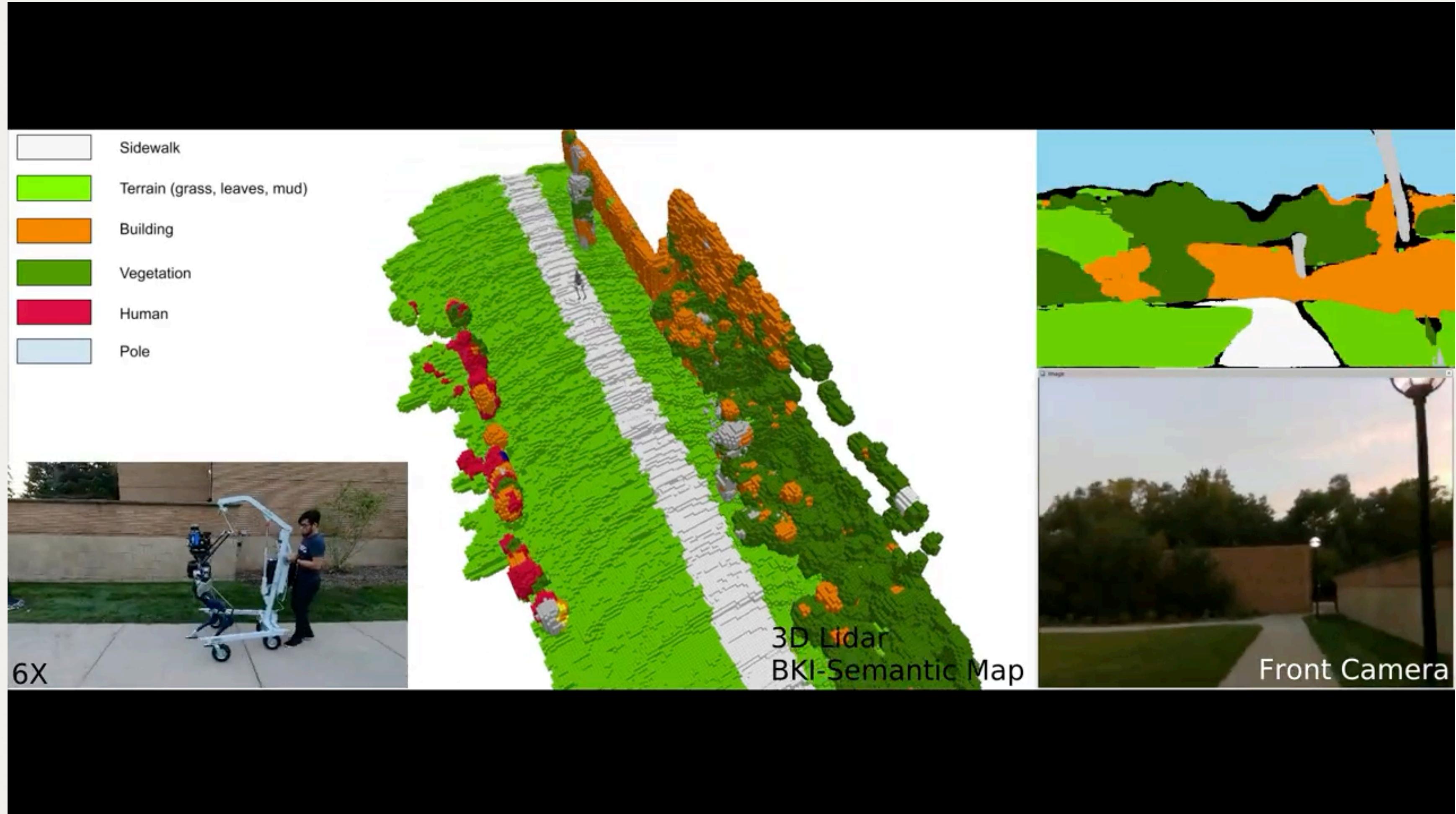
MOTIVATION AND WHY

3



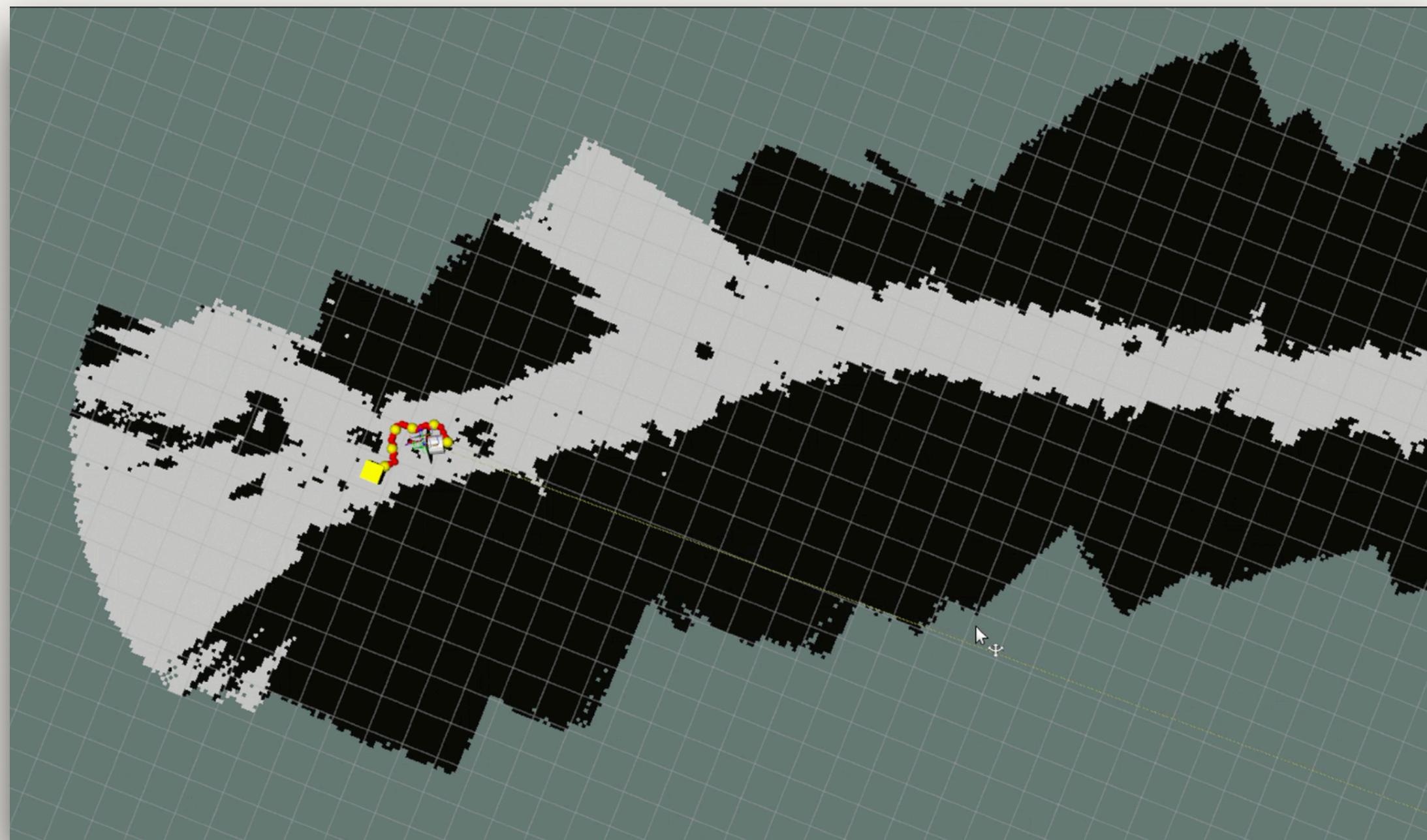
MOTIVATION AND WHY

4



MOTIVATION AND WHY

5



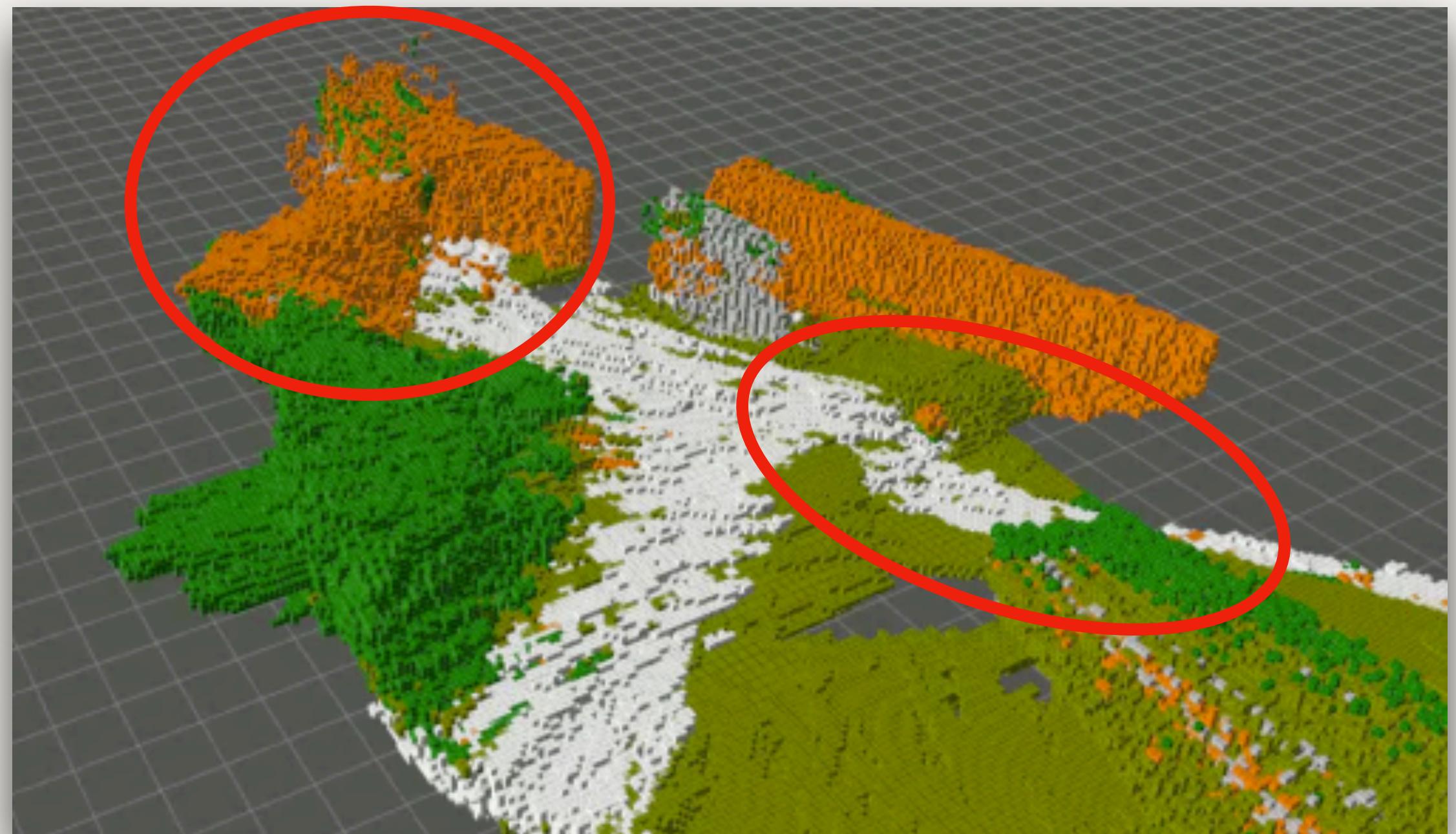
Without LiDAR (Only use RGB-D camera)



With LiDAR and camera calibrated

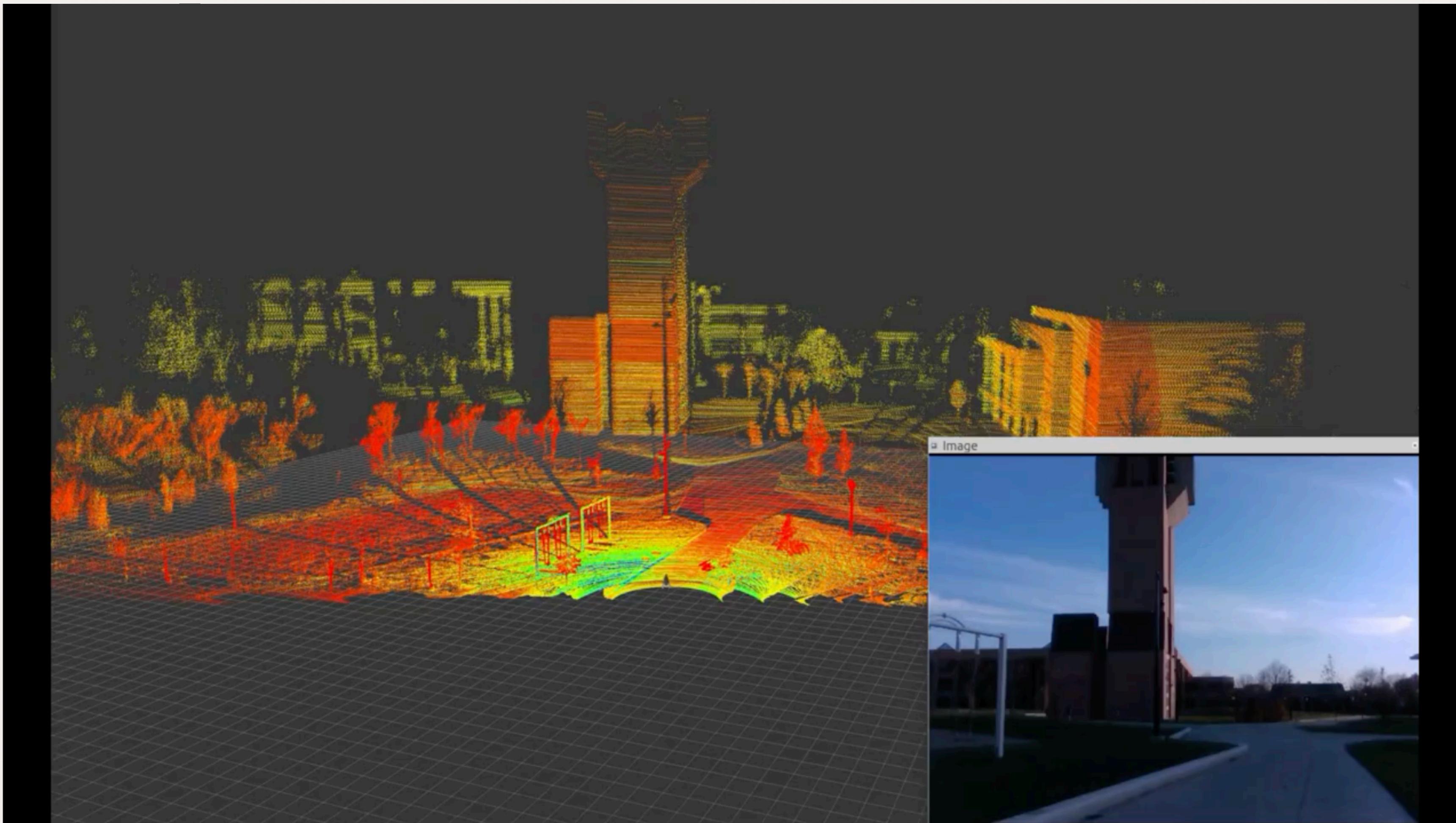
MOTIVATION AND WHY

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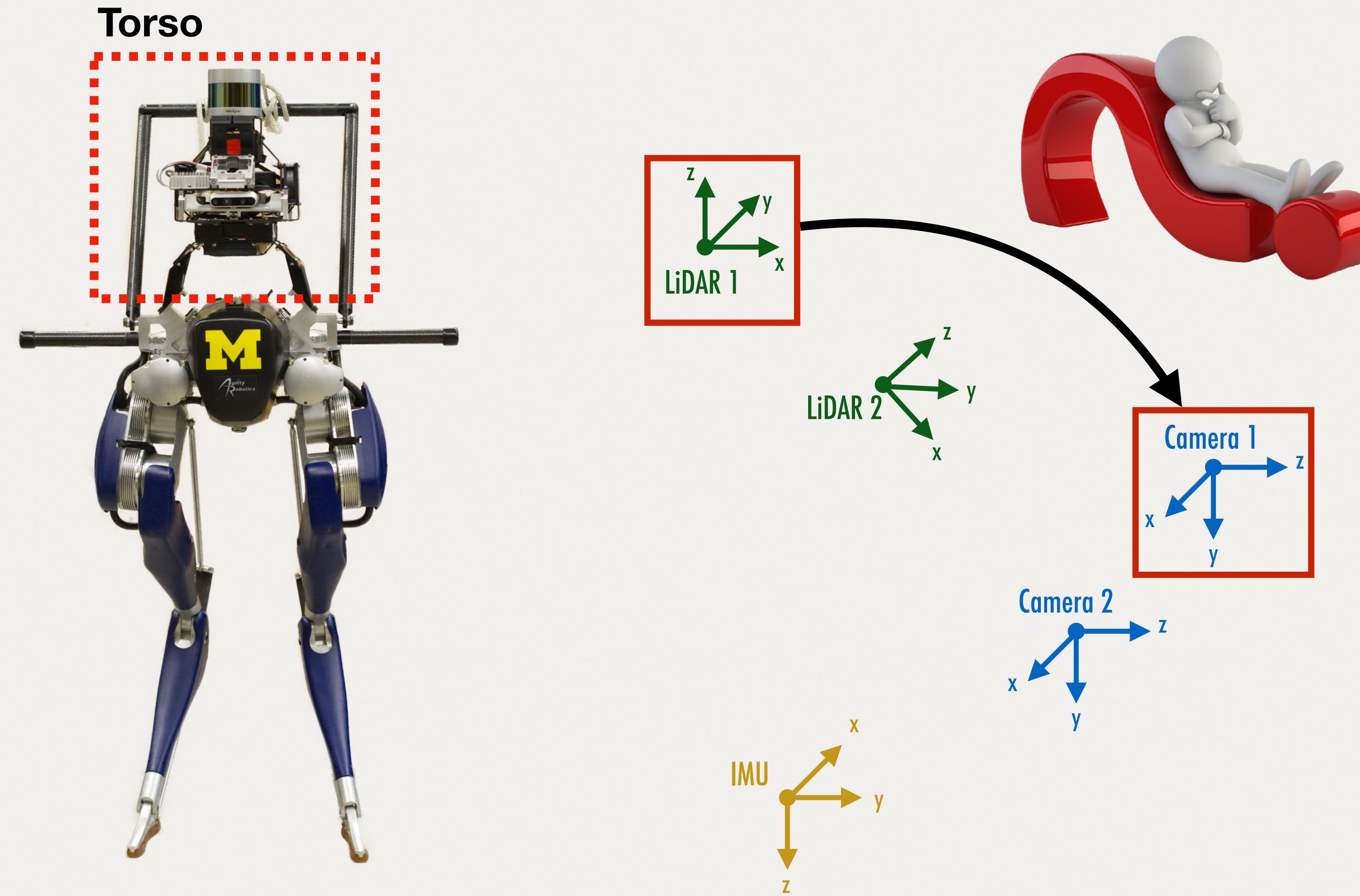
WHAT IS THE PROBLEM

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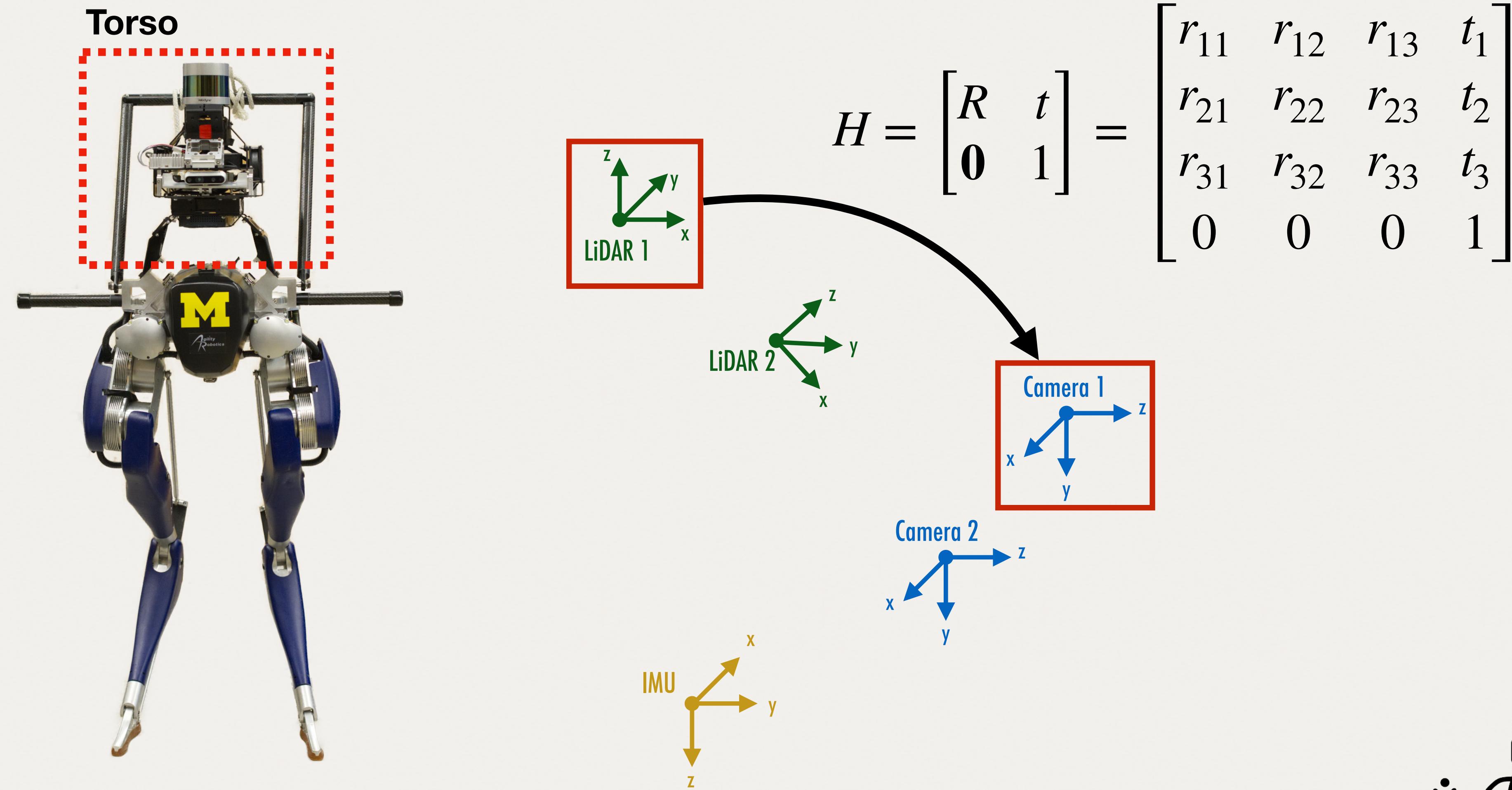
WHAT IS THE PROBLEM

8



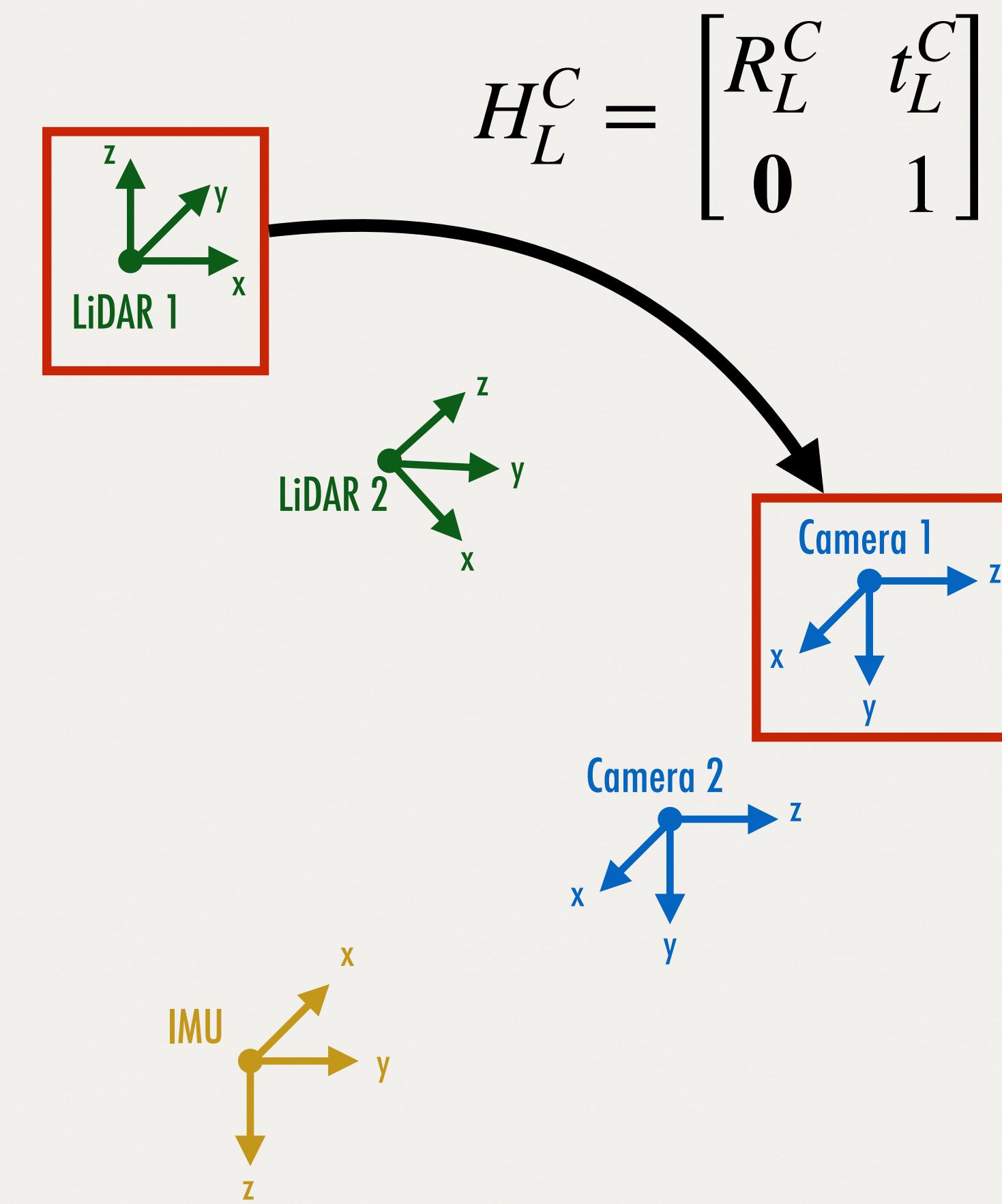
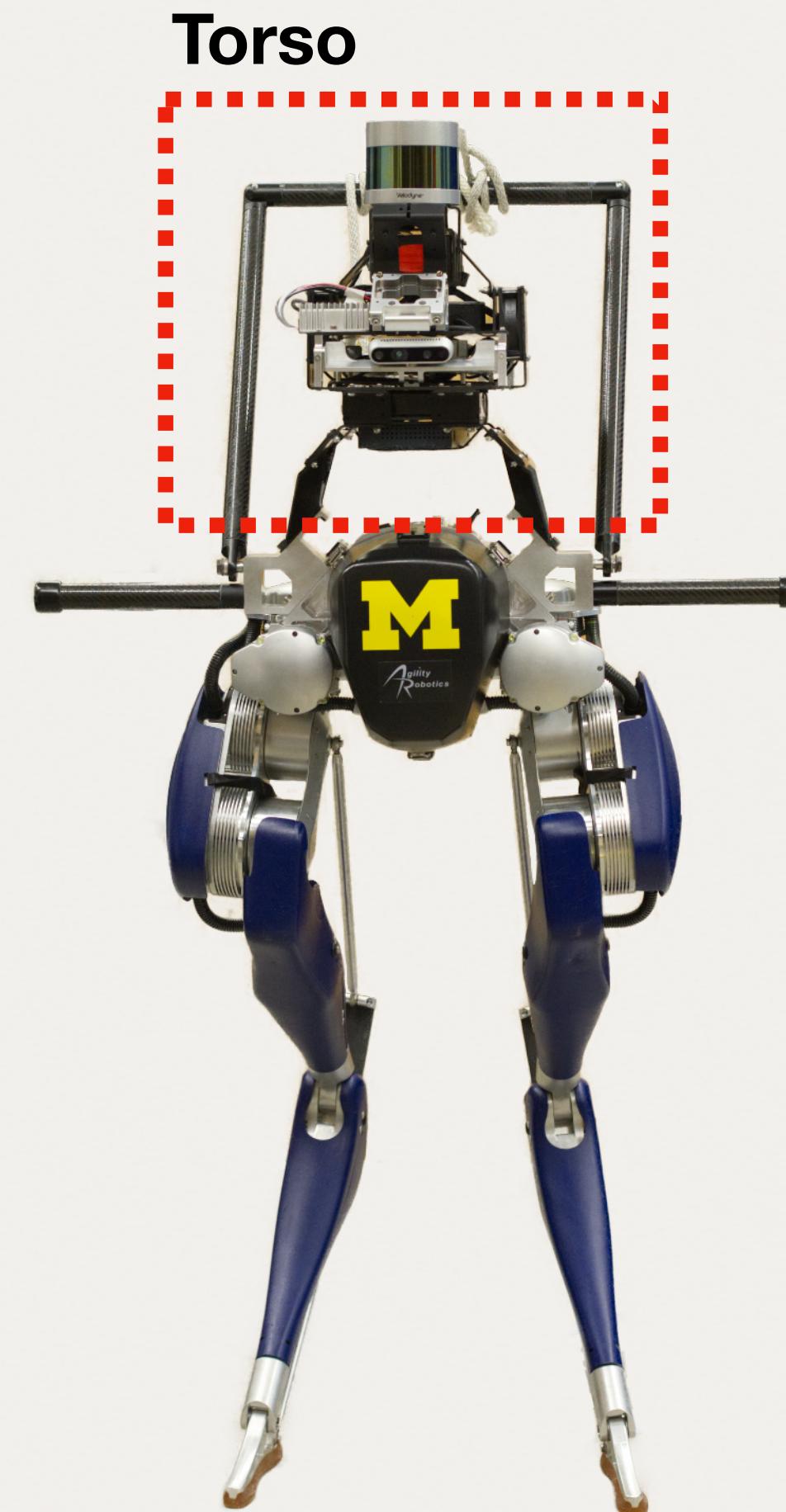
RIGID-BODY TRANSFORMATION

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RIGID-BODY TRANSFORMATION

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WHAT DO WE NEED?

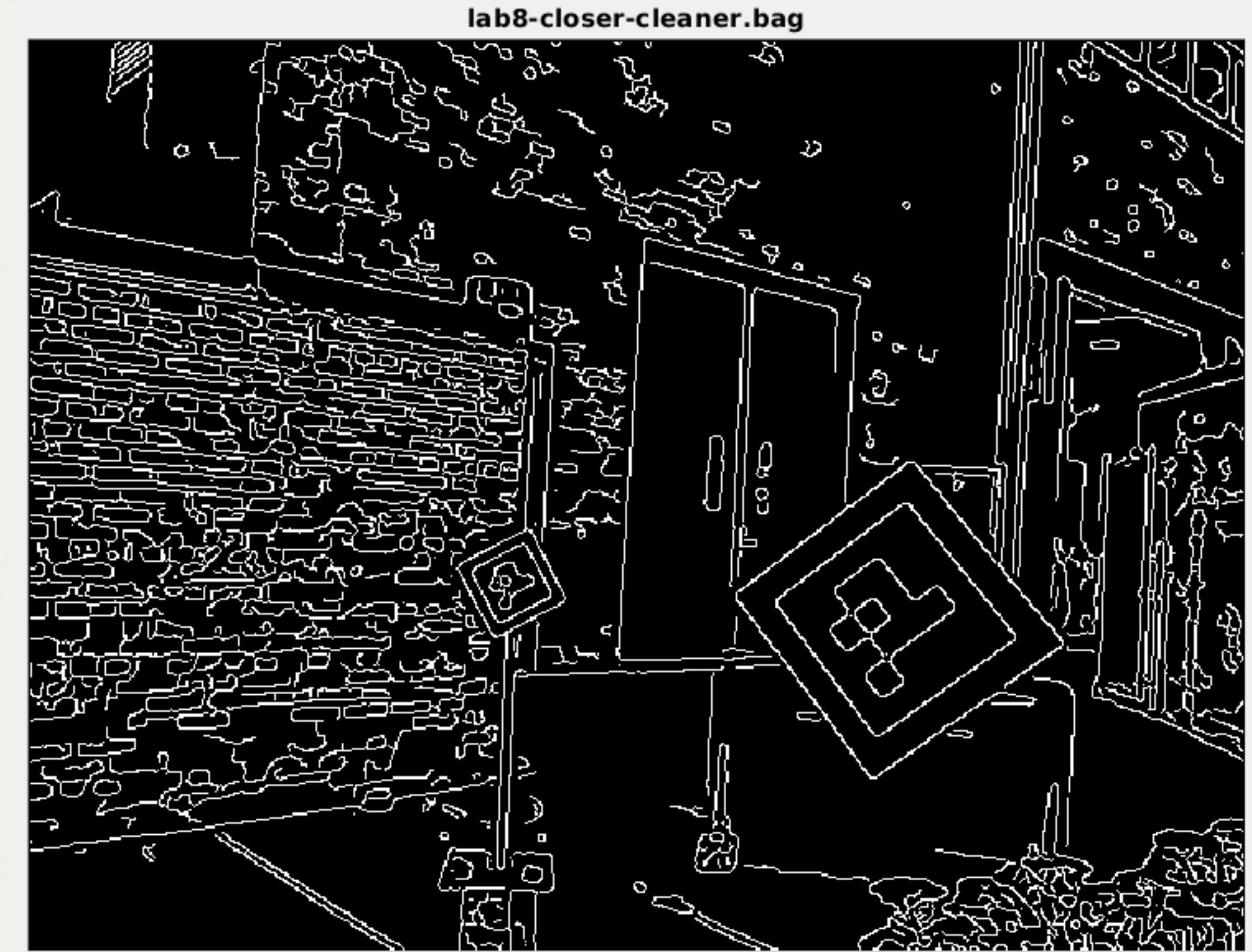
- ▶ Ingredients:
 - ▶ LiDAR features (X)
 - ▶ Camera features (Y)
- ▶ Goal:
 - ▶ Overlaying points from the LiDAR to in image from the camera (R_L^C, t_L^C)

WHAT DO WE NEED?

- ▶ Ingredients:
 - ▶ LiDAR features (X)
 - ▶ Camera features (Y)
 - ▶ Goal:
 - ▶ Overlaying points from the LiDAR to in image from the camera (R_L^C, t_L^C)
- } Specific structures in an environment

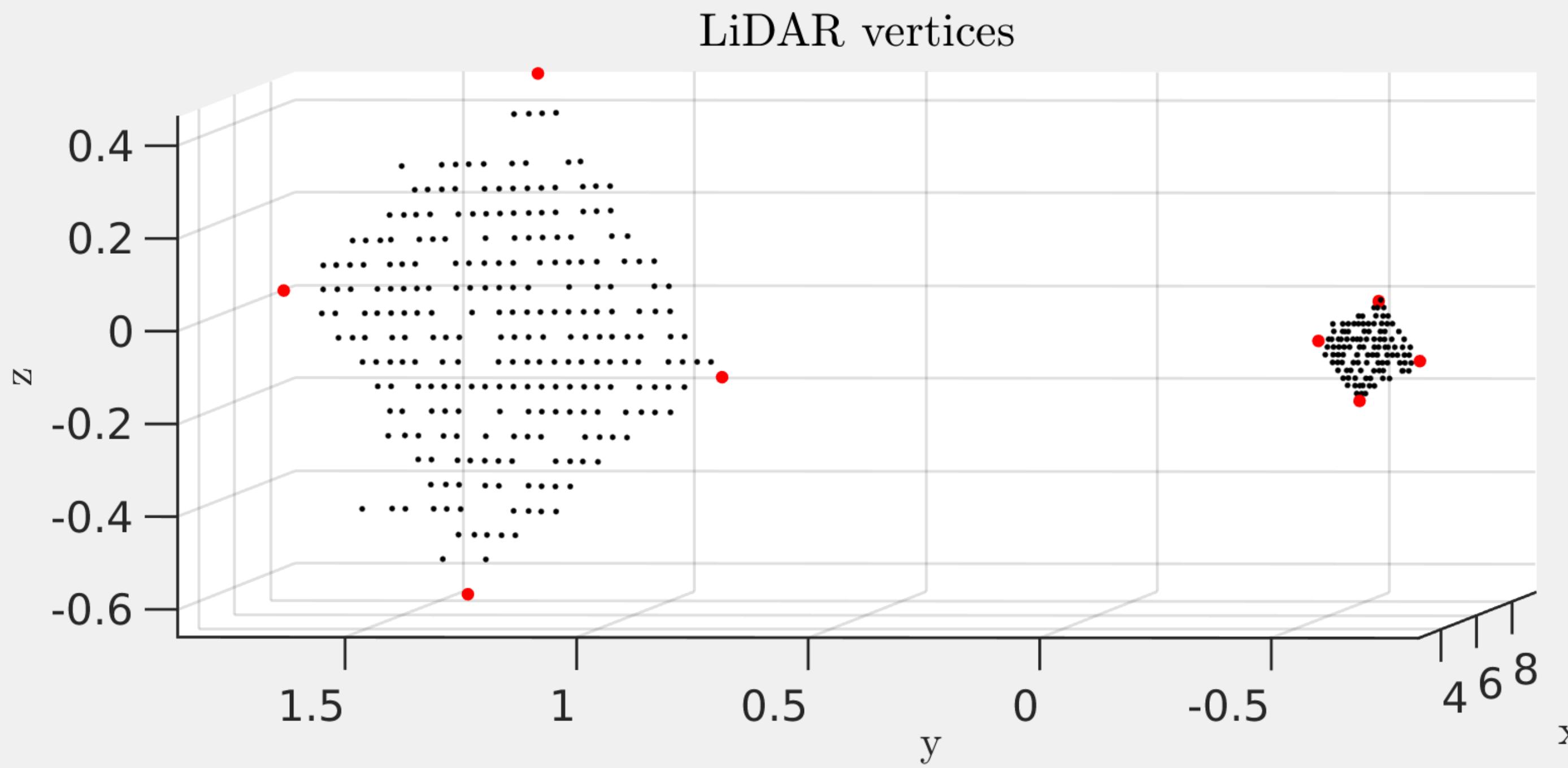
FEATURES: EDGES

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FEATURES: CORNERS

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WHAT DO WE NEED?

- ▶ Ingredients:
 - ▶ LiDAR features (X): 3D
 - ▶ Camera features (Y): 2D
 - ▶ 3D to 2D?
- ▶ Goal:
 - ▶ Overlaying points from the LiDAR to in image from the camera (R_L^C, t_L^C)



WHAT DO WE NEED?

- ▶ Ingredients:
 - ▶ LiDAR features (X): 3D
 - ▶ Camera features (Y): 2D
 - ▶ 3D to 2D?
- ▶ Goal:
 - ▶ Overlaying points from the LiDAR to in image from the camera (R_L^C, t_L^C)

$$(R_L^C, t_L^C) := \arg \min_{R,t} f(R, t, X, Y)$$

Project points, minimize distance

HOW TO FORMULATE

Problem:

$$\left(R_L^{C^*}, t_L^{C^*} \right) := \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance}$$

HOW TO FORMULATE

Problem:

$$\begin{aligned} \left(R_L^{C^*}, t_L^{C^*} \right) &:= \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance} \\ &:= \arg \min_{R,t} \quad \Pi(X_i; R, t) \end{aligned}$$

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Problem:

$$\begin{aligned} \left(R_L^{C^*}, t_L^{C^*} \right) &:= \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance} \\ &:= \arg \min_{R,t} \quad \Pi(X_i; R, t) - Y_i \end{aligned}$$

HOW TO FORMULATE

Problem:

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HOW TO FORMULATE

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$$\begin{aligned} \left(R_L^{C^*}, t_L^{C^*} \right) &:= \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance} \\ &:= \arg \min_{R,t} \sum_{i=1}^{4n} \|\Pi(X_i; R, t) - Y_i\|_2^2 \end{aligned}$$

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Problem:

$$\begin{aligned} \left(R_L^{C^*}, t_L^{C^*} \right) &:= \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance} \\ &:= \arg \min_{R,t} \sum_{i=1}^{4n} \|\Pi(X_i; R, t) - Y_i\|_2^2 \end{aligned}$$

$$H_{k+1} = H_k - s[\nabla f(H_k, X, Y)]^\top : \text{the gradient descent}$$

HOW TO FORMULATE

Problem:

$$\begin{aligned} \left(R_L^{C^*}, t_L^{C^*} \right) &:= \arg \min_{R,t} f(R, t, X, Y) : \text{Project points, minimize distance} \\ &:= \arg \min_{R,t} \sum_{i=1}^{4n} \|\Pi(X_i; R, t) - Y_i\|_2^2 \end{aligned}$$

$$H_{k+1} = H_k - s[\nabla f(H_k, X, Y)]^\top : \text{the gradient descent}$$

$$H_{k+1} = H_k - s[\nabla^2 f(H_k, X, Y)]^{-1}[\nabla f(H_k, X, Y)]^\top : \text{the Hessian}$$

LIVE DEMO

1. [GitHub](#)
2. Download [dataset](#)
3. Put them under ROB101_data folder.
4. Run [rob101_optimization.m](#)
5. Why not Julia?

HOW TO REPRESENT R?

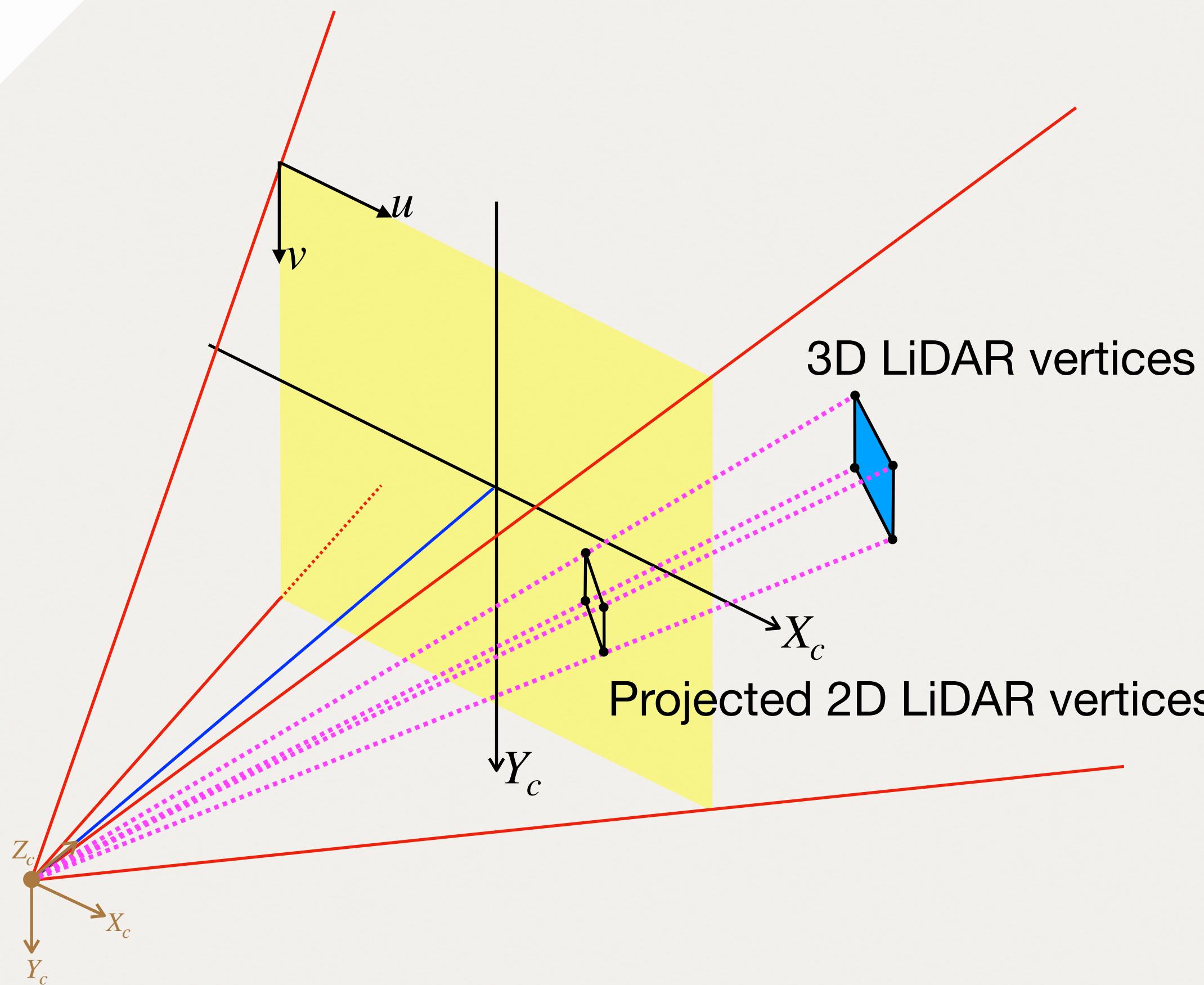
If you dig in more...

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \iff R = \exp(\text{skew}([\omega_1, \omega_2, \omega_3]))$$

Power of Lie group!

PROJECTION MAP: 3D POINTS TO 2D POINTS

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$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsic parameters}} \begin{bmatrix} 1_{3 \times 3} \\ 0_{1 \times 3} \end{bmatrix}^\top \underbrace{\begin{bmatrix} R_L^C & t_L^C \\ 0_{1 \times 3} & 1 \end{bmatrix}}_{\text{extrinsic parameters}} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

$$Y_i = [u \quad v \quad 1]^\top = \left[\frac{u'}{w'} \quad \frac{v'}{w'} \quad 1 \right]^\top,$$

$$\Pi(X_i; R, t) := Y_i$$

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Lab website!



YouTube Channel!