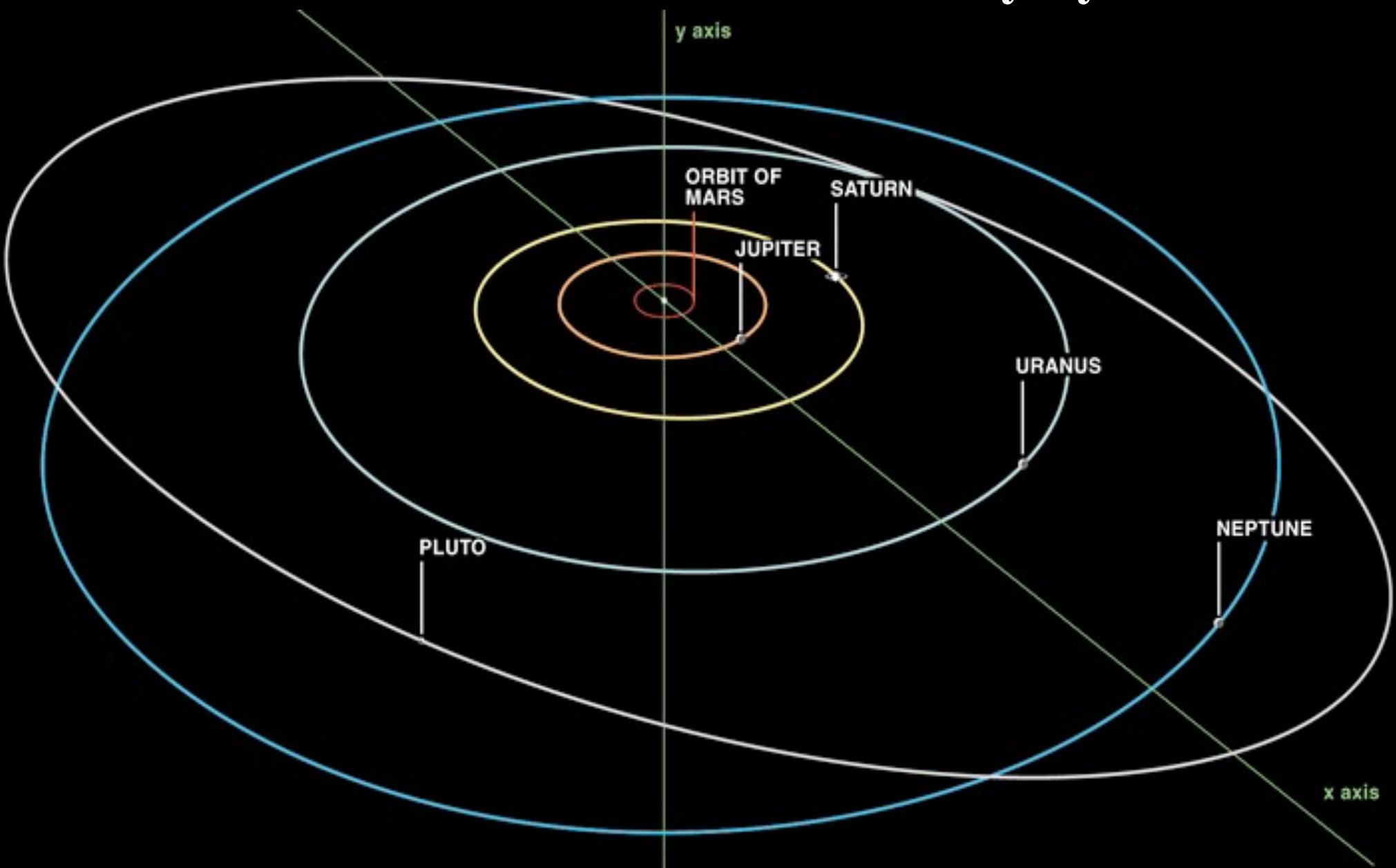


Dynamics:

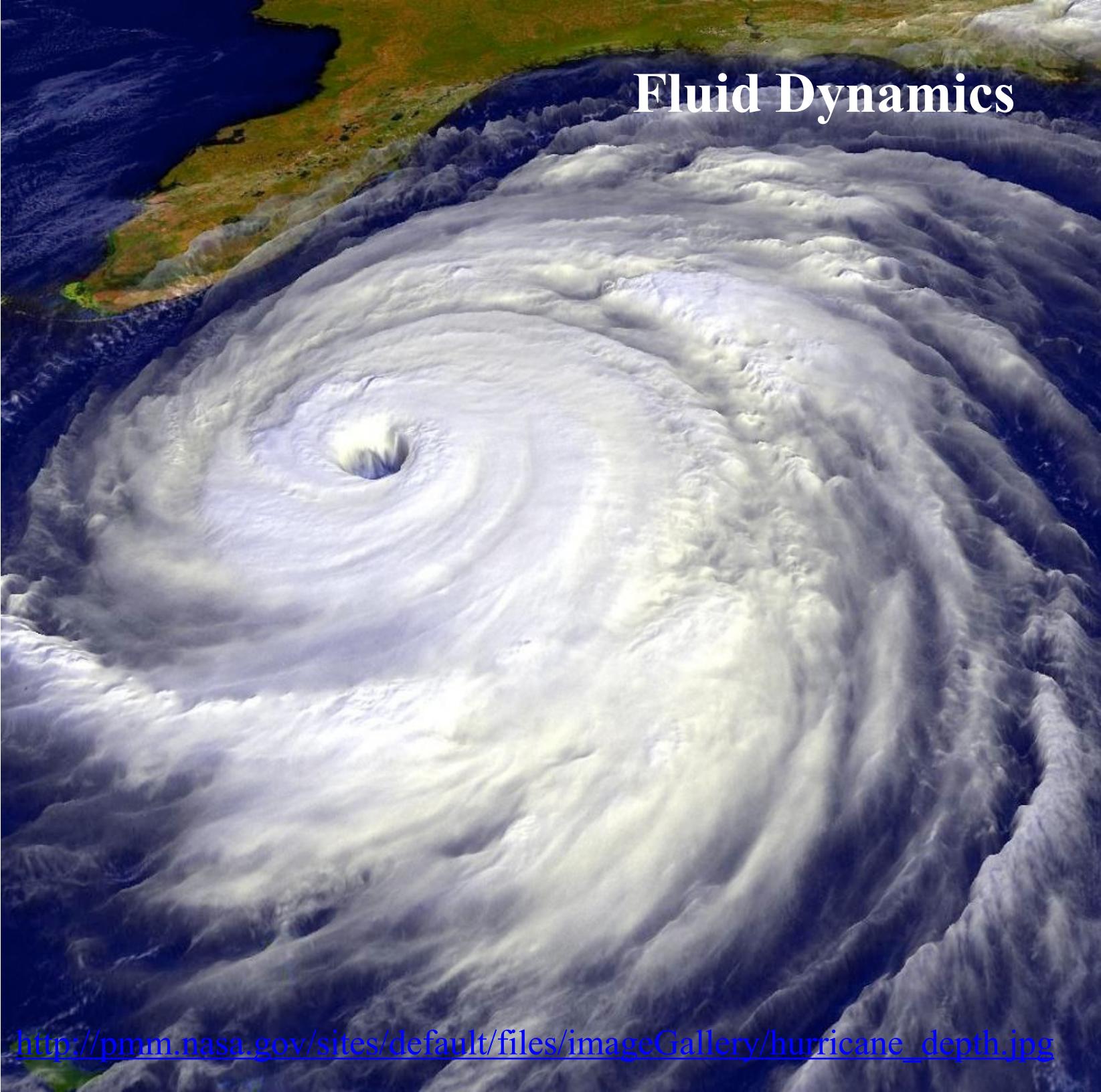
The general study of how systems change over time

How the complex behaviour unfolds and how it changes over time....

Planetary dynamics



Fluid Dynamics

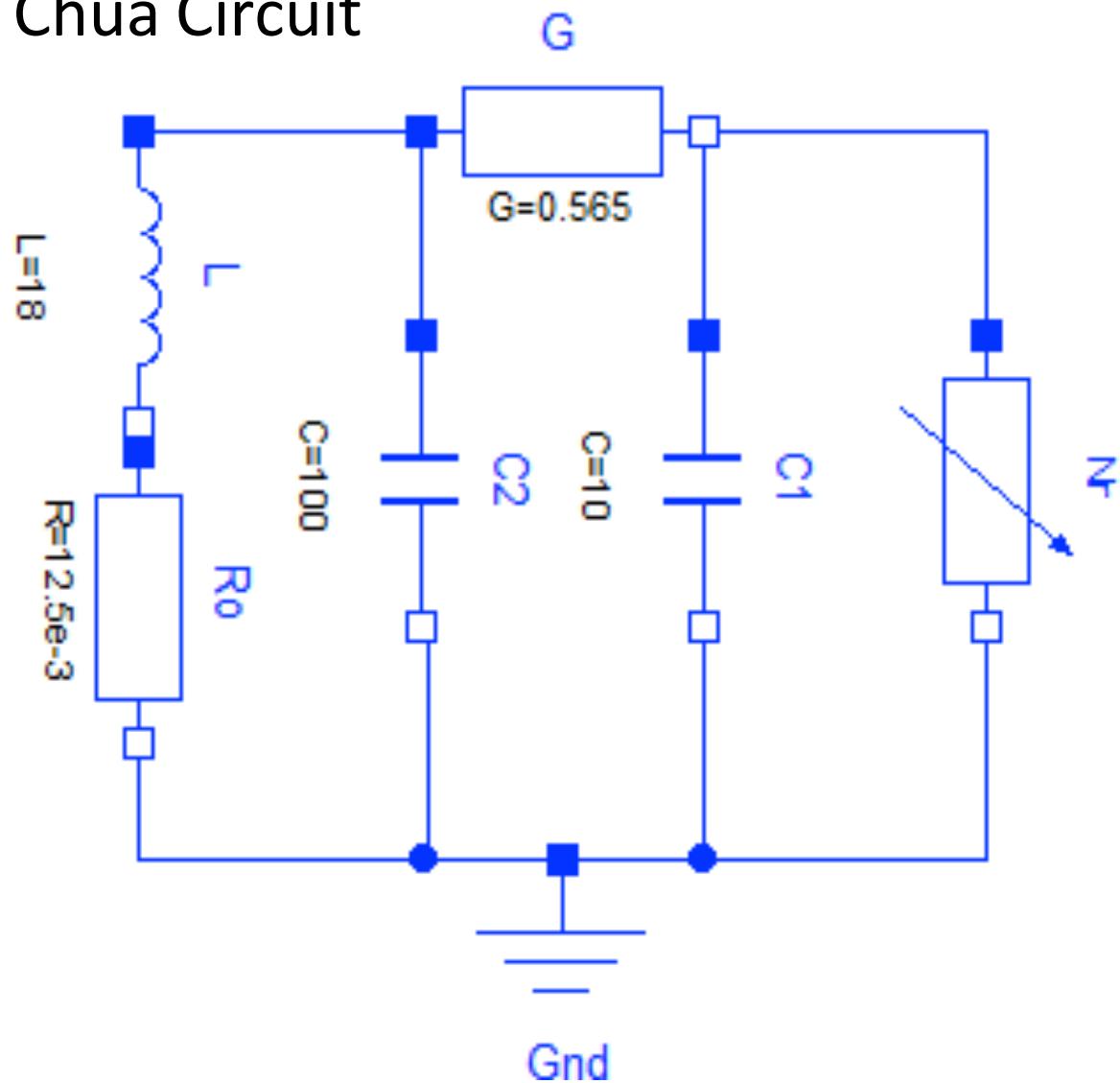


http://pmm.nasa.gov/sites/default/files/imageGallery/hurricane_depth.jpg

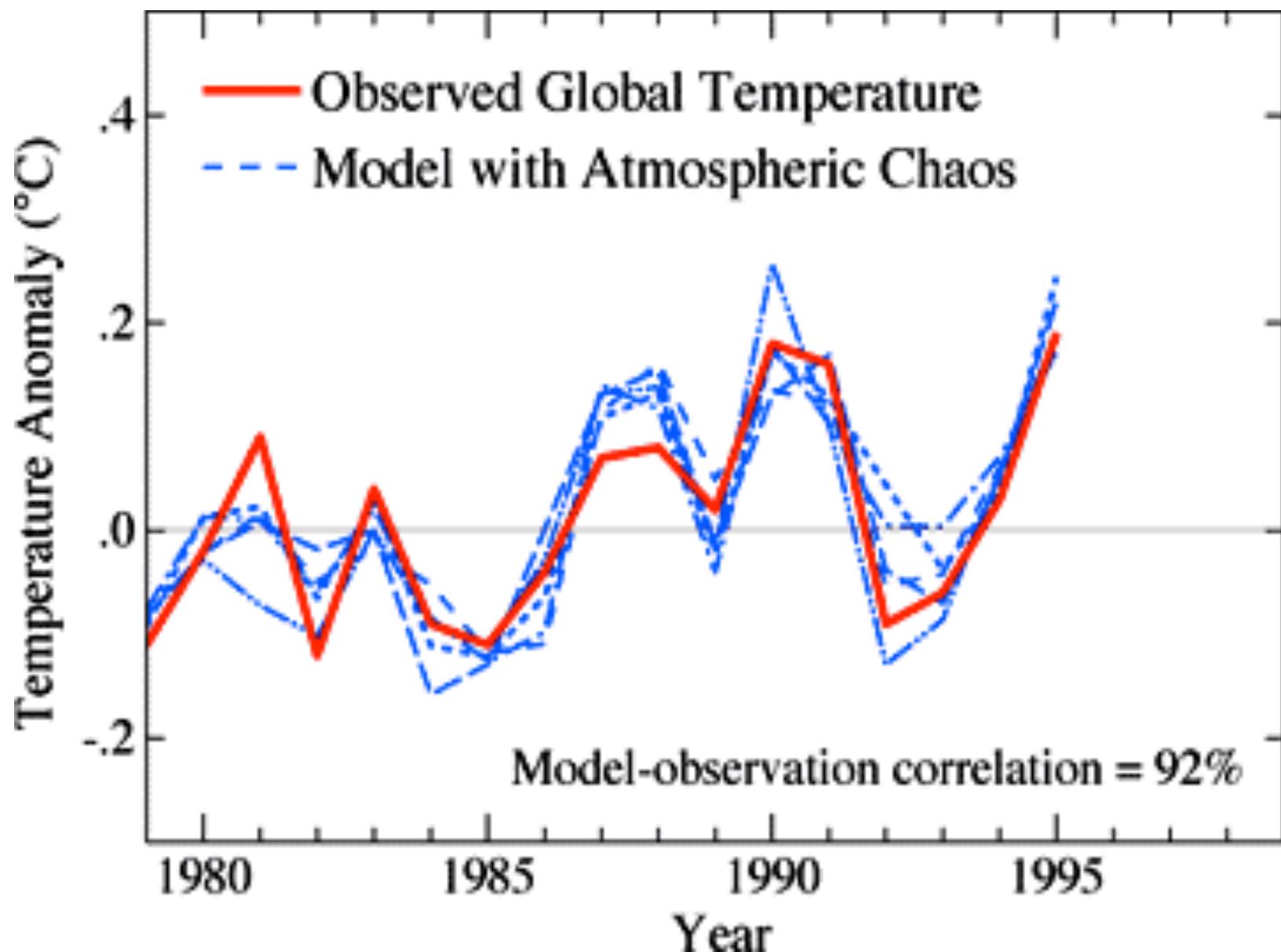
Dynamics of Turbulence

<http://www.noaanews.noaa.gov/stories/images/hurricaneflying2.jpg>

Chua Circuit



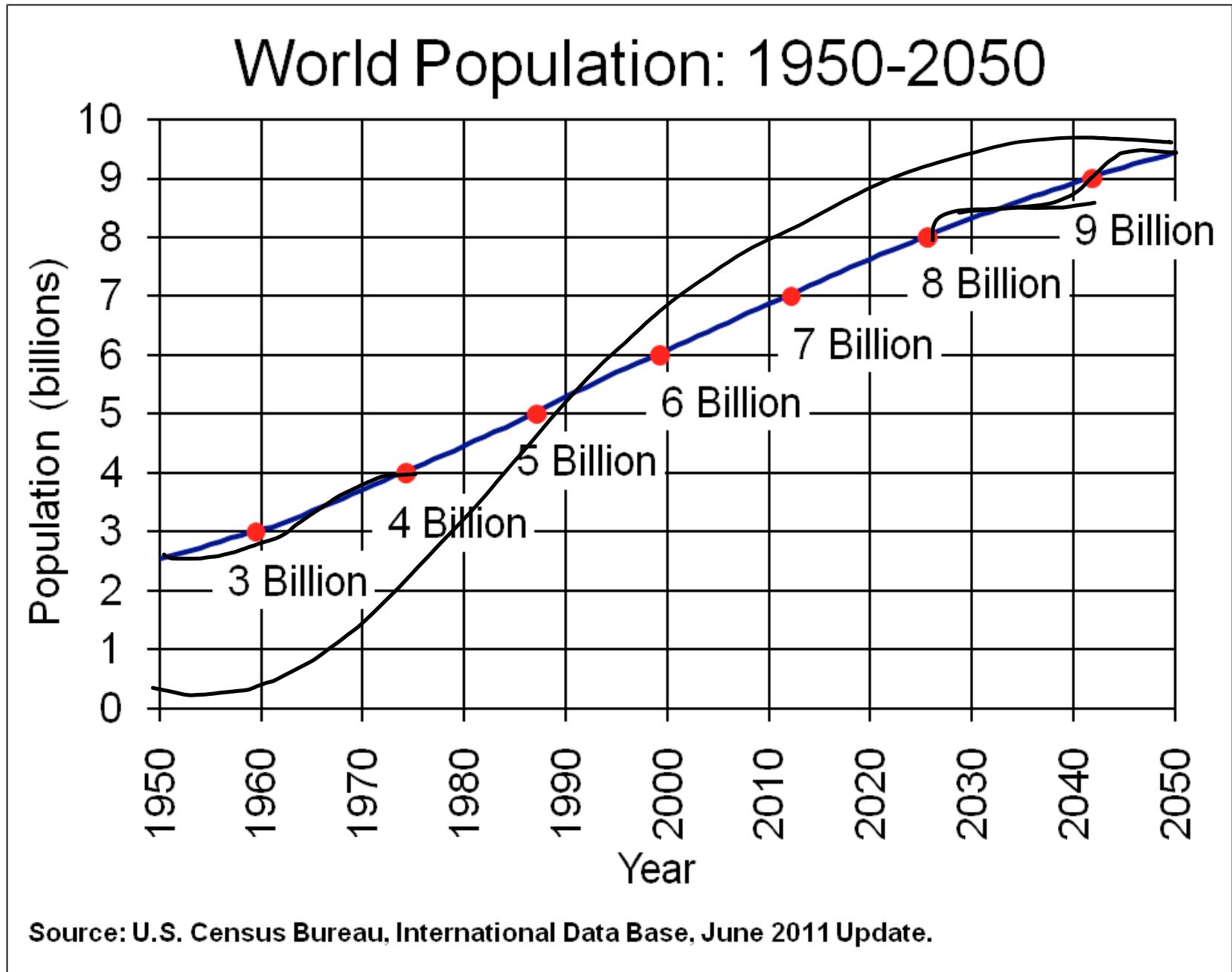
Climate dynamics



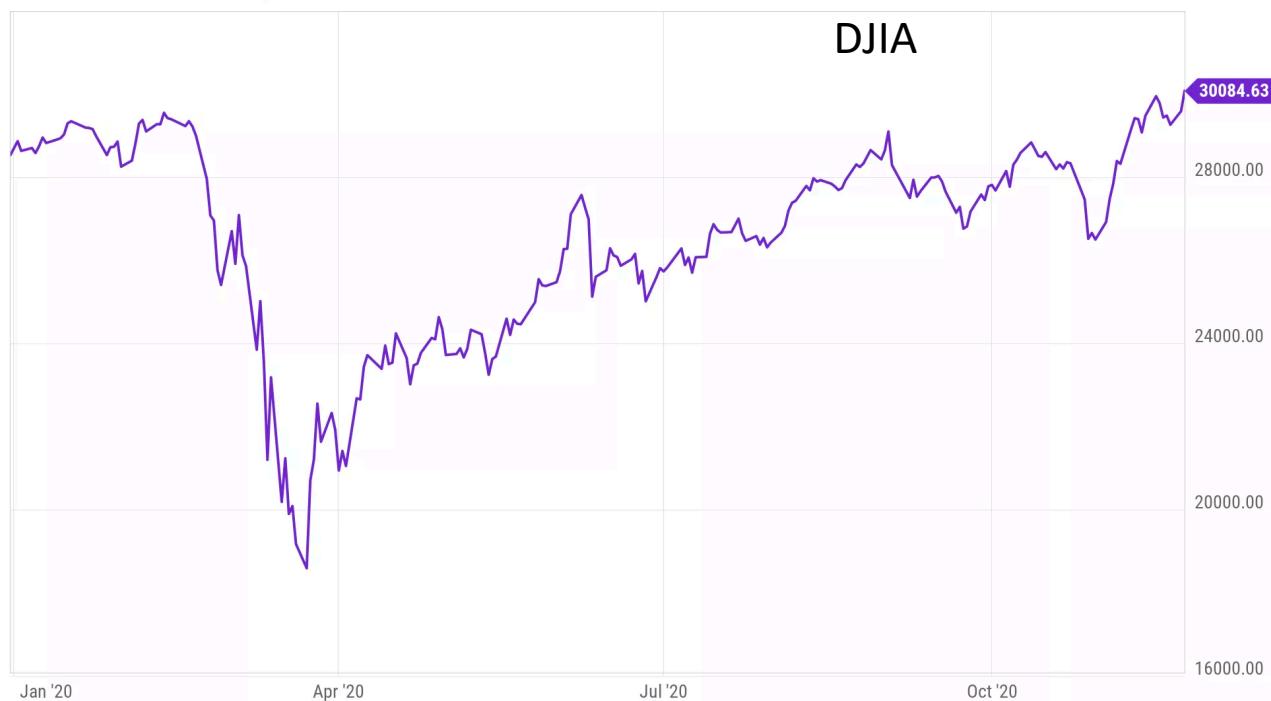
Crowd dynamics



Population dynamics



Dow Jones Industrial Average Level

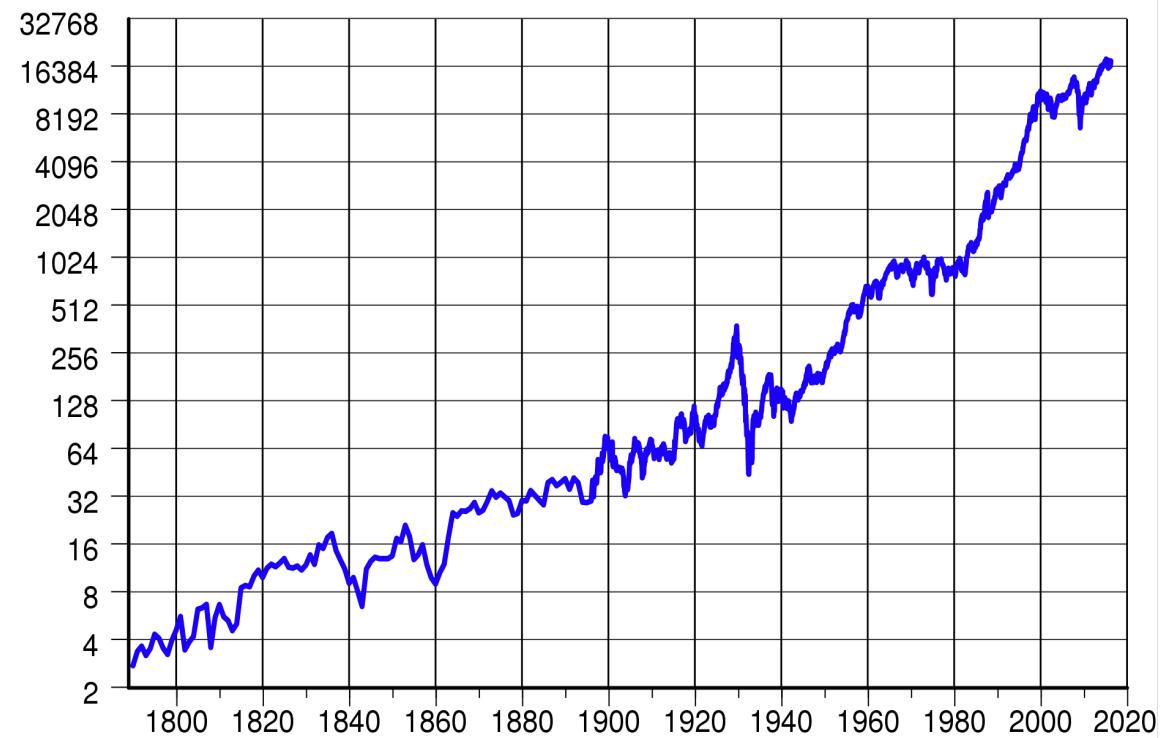


 **Investopedia**

Nov 24 2020, 12:35PM EST. Powered by 

Financial dynamics

Dynamics of stock prices





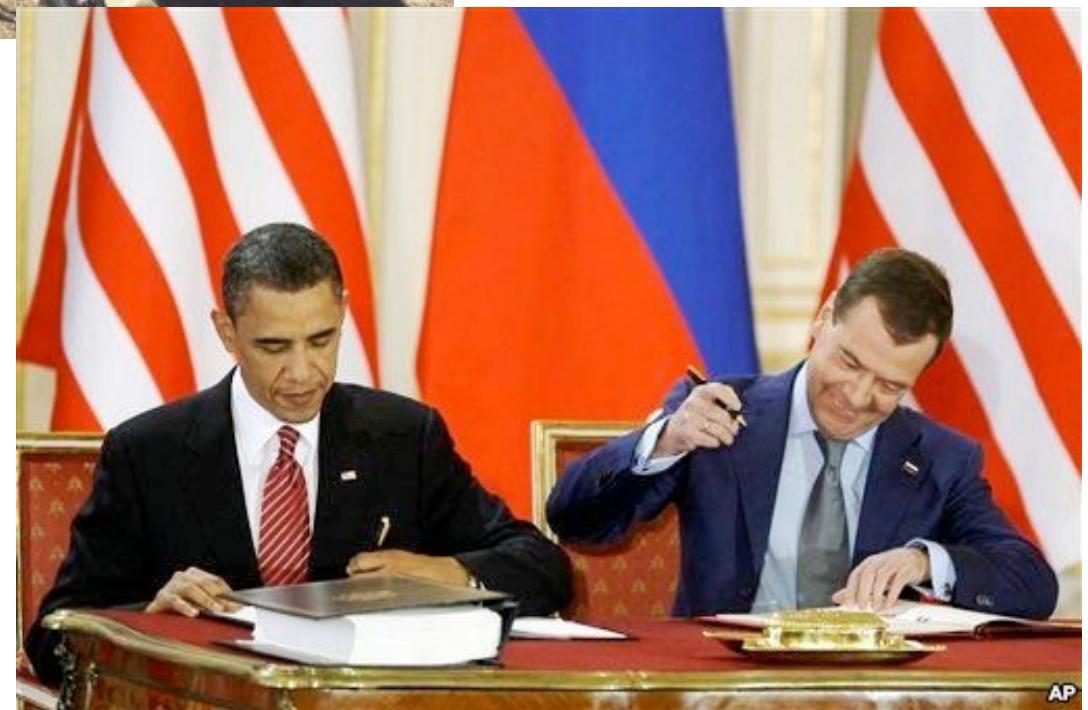
Group dynamics





Dynamics of conflicts

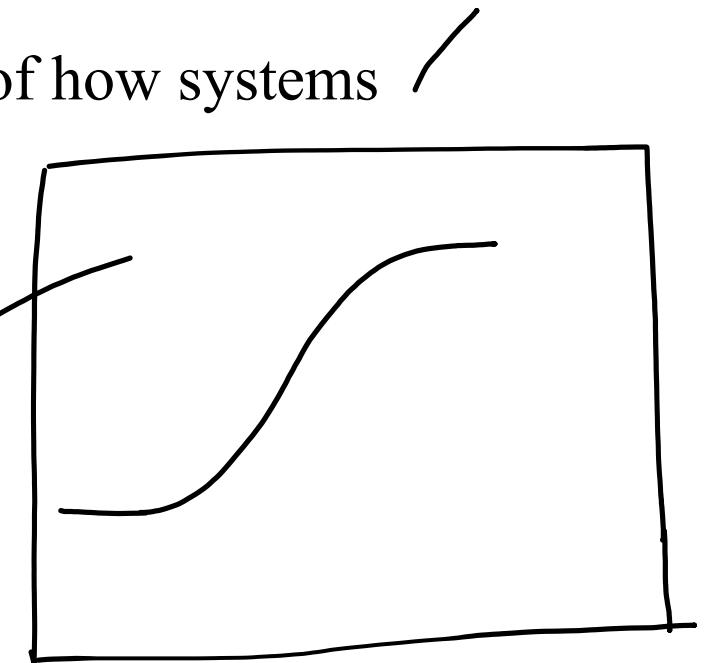
Dynamics of cooperation



Dynamical Systems Theory:

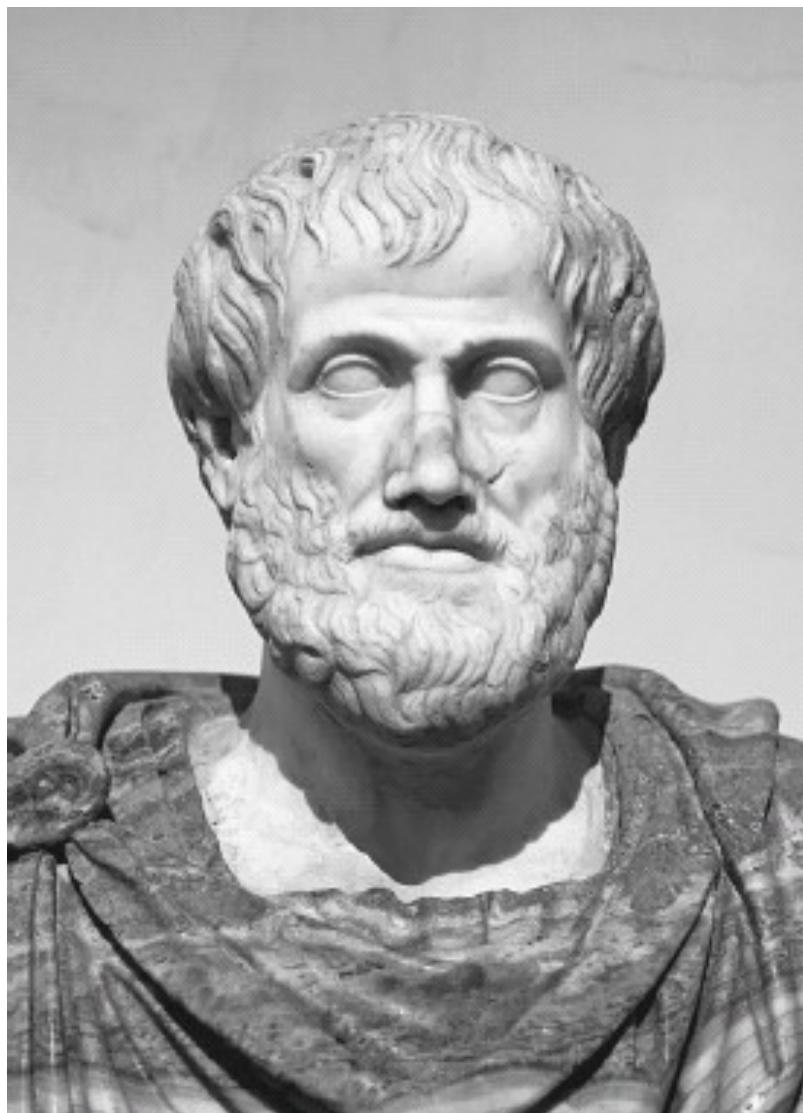
- The branch of mathematics of how systems change over time

- Calculus
- Differential equations
- Iterated maps
- Algebraic topology
- etc

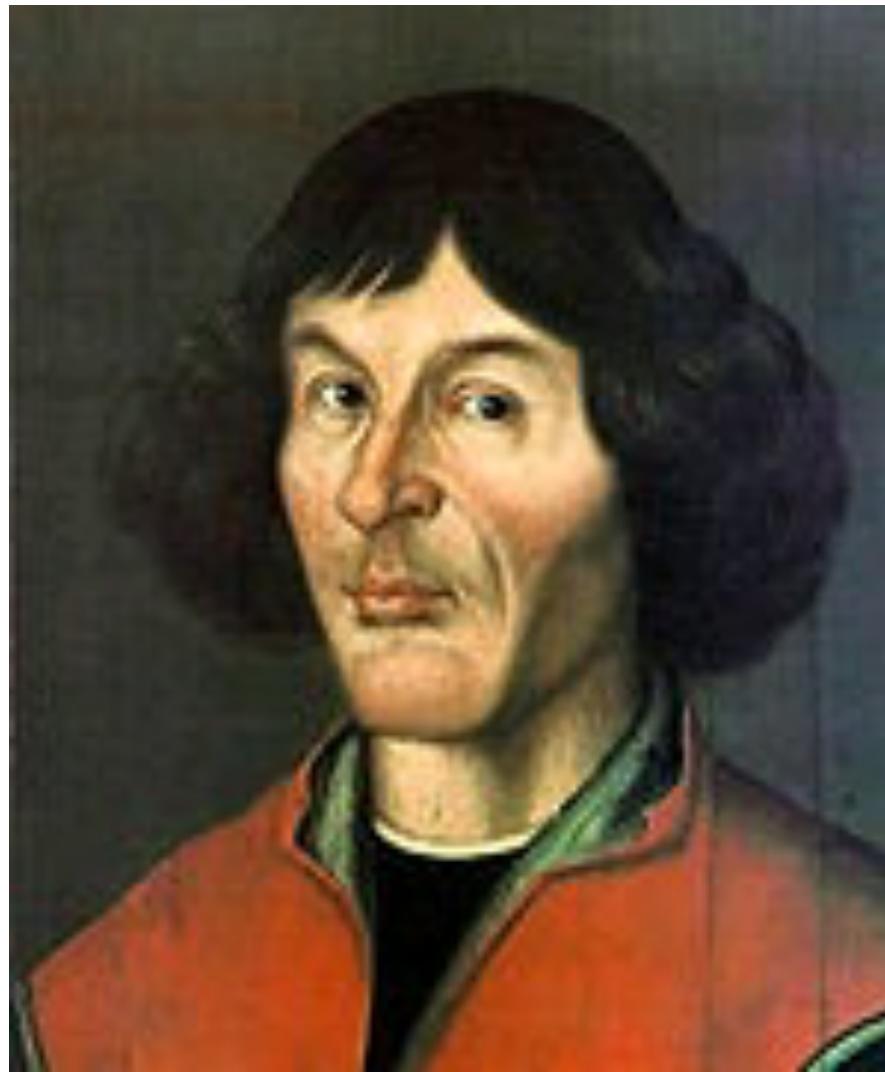


- The *dynamics of a system*: the manner in which the system changes
- Dynamical systems theory gives us a **vocabulary** and **set of tools** for describing dynamics

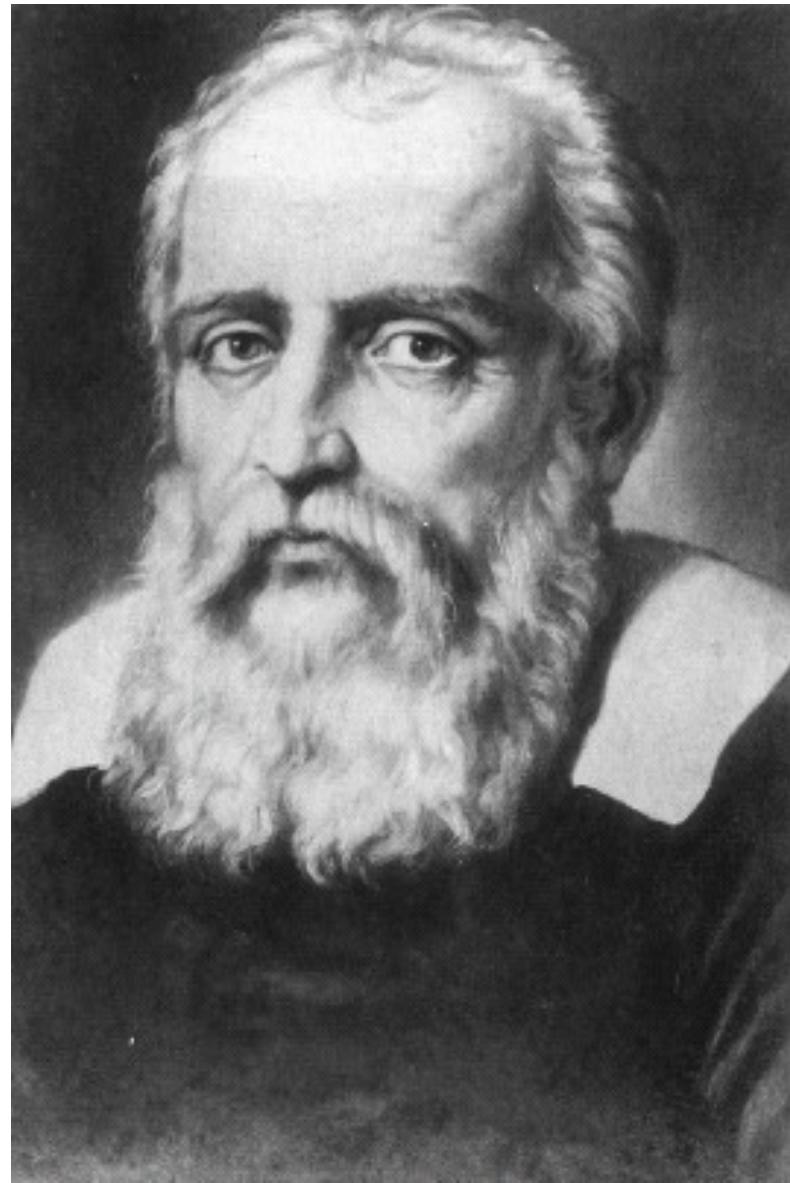
A brief history of the science of dynamics



Aristotle, 384 – 322 BC



Nicolaus Copernicus, 1473 – 1543



Galileo Galilei, 1564 – 1642



Isaac Newton, 1643 – 1727



Pierre-Simon Laplace, 1749 – 1827

“We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.”

—Pierre Simon Laplace, *A Philosophical Essay on Probabilities*



Henri Poincaré, 1854 – 1912

“If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation *approximately*. If that enabled us to predict the succeeding situation with *the same approximation*, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that **small differences in the initial conditions produce very great ones in the final phenomena**. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible...”

“Sensitive dependence on initial conditions”

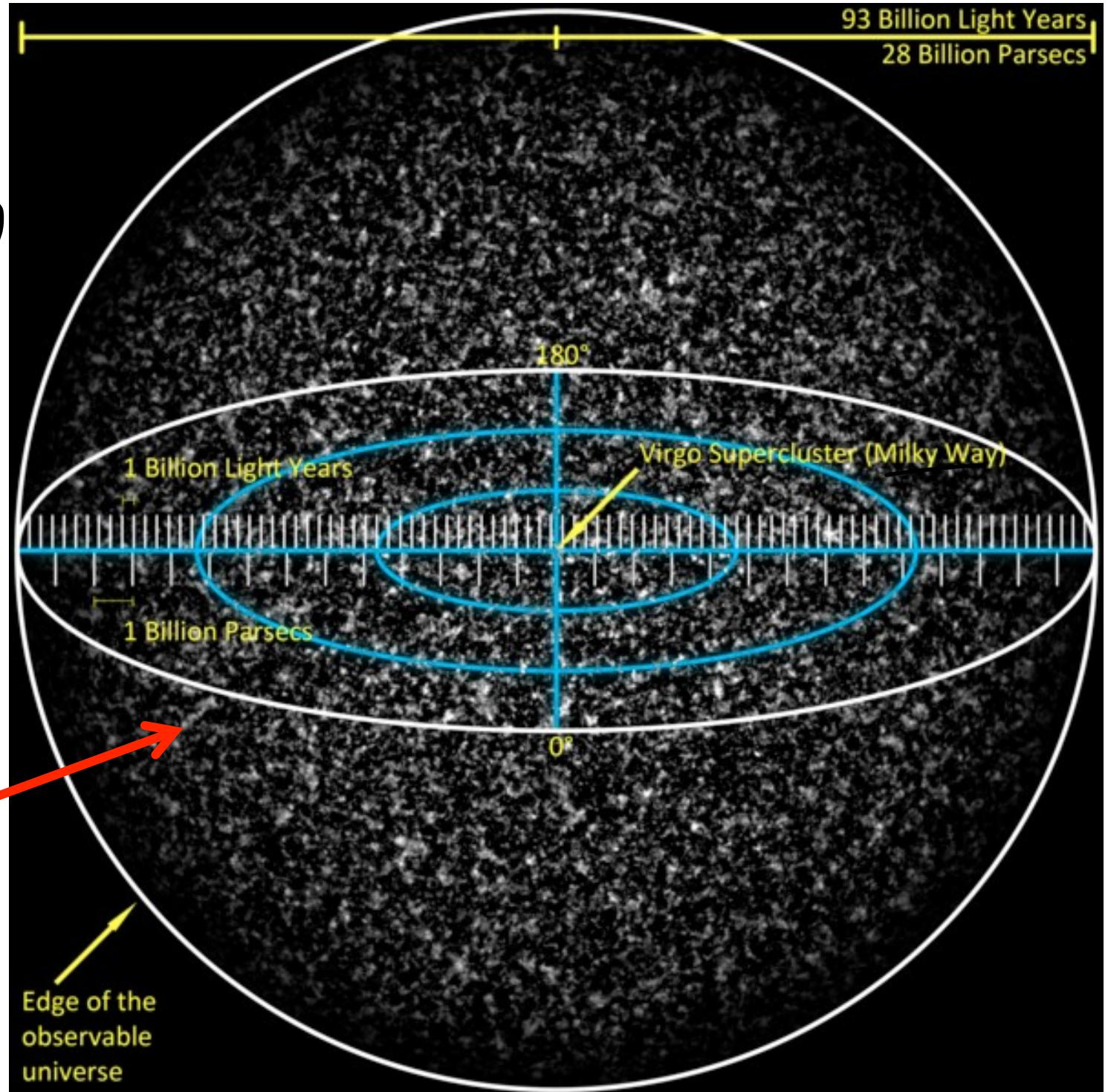
$$F = m a$$

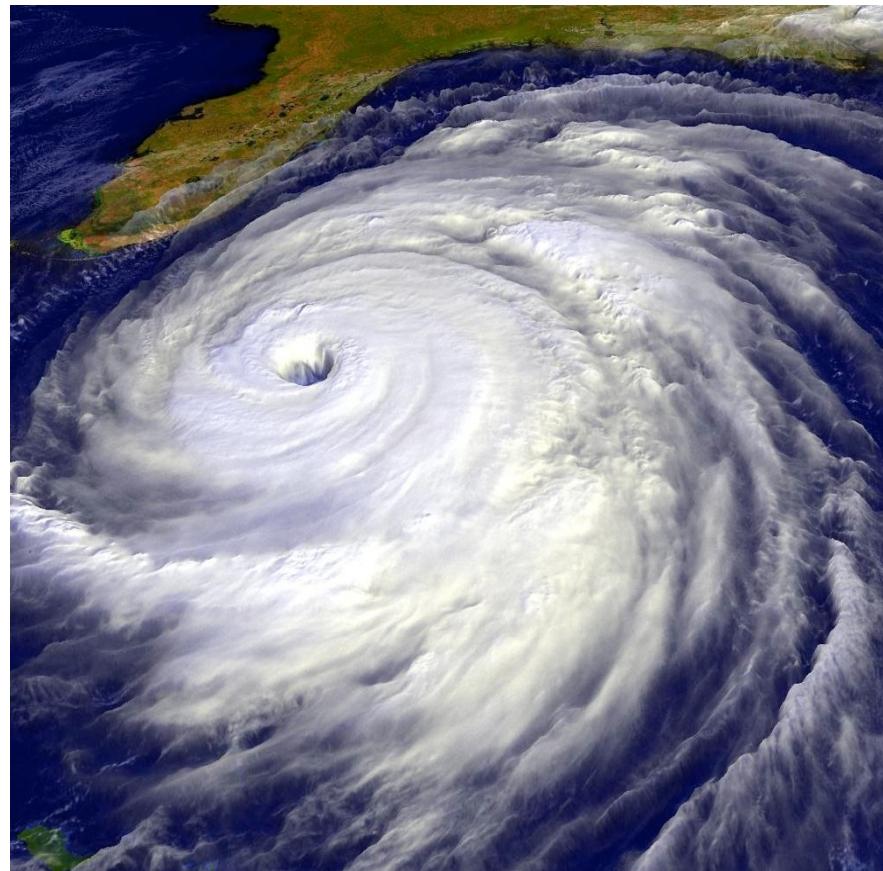
$$F = G m_1 m_2 / d^2$$



Atom's velocity:

3.3430248493930a235958
235238871987324483020
435982972930493093093
....3495092834509809989





[http://www.fws.gov/sacramento/ES_Kids/Mission-Blue-Butterfly/
Images/mission-blue-butterfly_header.jpg](http://www.fws.gov/sacramento/ES_Kids/Mission-Blue-Butterfly/Images/mission-blue-butterfly_header.jpg)

[http://pmm.nasa.gov/sites/default/files/imageGallery/
hurricane_depth.jpg](http://pmm.nasa.gov/sites/default/files/imageGallery/hurricane_depth.jpg)

Chaos:

- One particular type of dynamics of a system
- Defined as “sensitive dependence on initial conditions

Chaos in Nature

- Dripping faucets
- Electrical circuits
- Solar system orbits
- Weather and climate (the “butterfly effect”)
- Brain activity (EEG)

- Heart activity (EKG)
- Computer networks
- Population growth and dynamics
- Financial data

What is the difference between *chaos* and *randomness*?

Notion of “deterministic chaos”



Lord Robert May
b. 1936



Mitchell Feigenbaum
b. 1944

“The fact that the simple and deterministic equation [i.e., the Logistic Map] can possess dynamical trajectories which look like some sort of random noise has disturbing practical implications. It means, for example, that apparently erratic fluctuations in the census data for an animal population need not necessarily betoken either the vagaries of an unpredictable environment or sampling errors; they may simply derive from a rigidly deterministic population growth relationship...Alternatively, it may be observed that in the chaotic regime, arbitrarily close initial conditions can lead to trajectories which, after a sufficiently long time, diverge widely. This means that, even if we have a simple model in which all the parameters are determined exactly, long-term prediction is nevertheless impossible”

— Robert May, 1976

Chaos: Seemingly random behavior with *sensitive dependence on initial conditions*

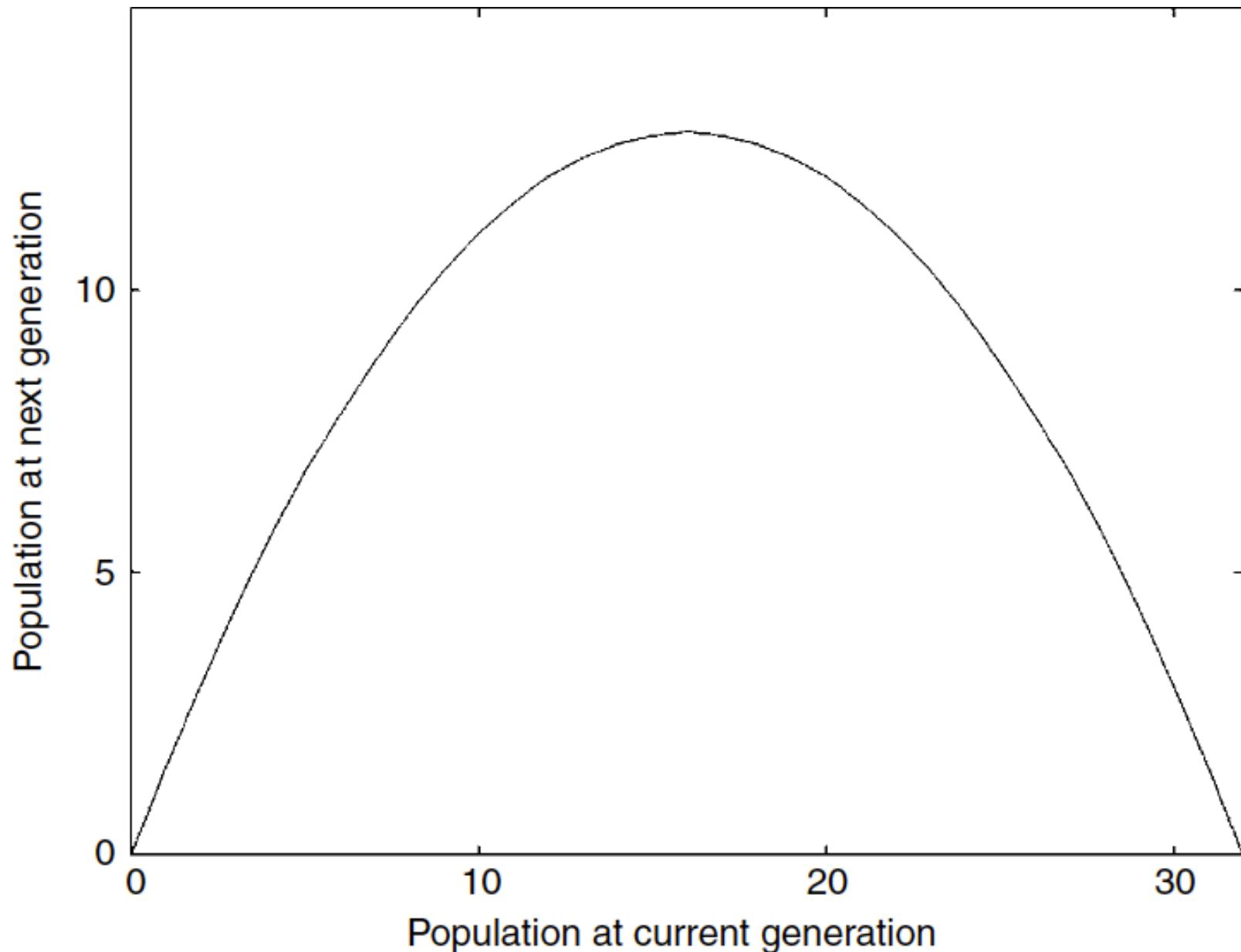
Logistic map: A simple, completely deterministic equation that, when iterated, can display chaos (depending on the value of R).

Deterministic chaos: Perfect prediction, *a la* Laplace's deterministic "clockwork universe", is impossible, even in principle, if we're looking at a chaotic system.

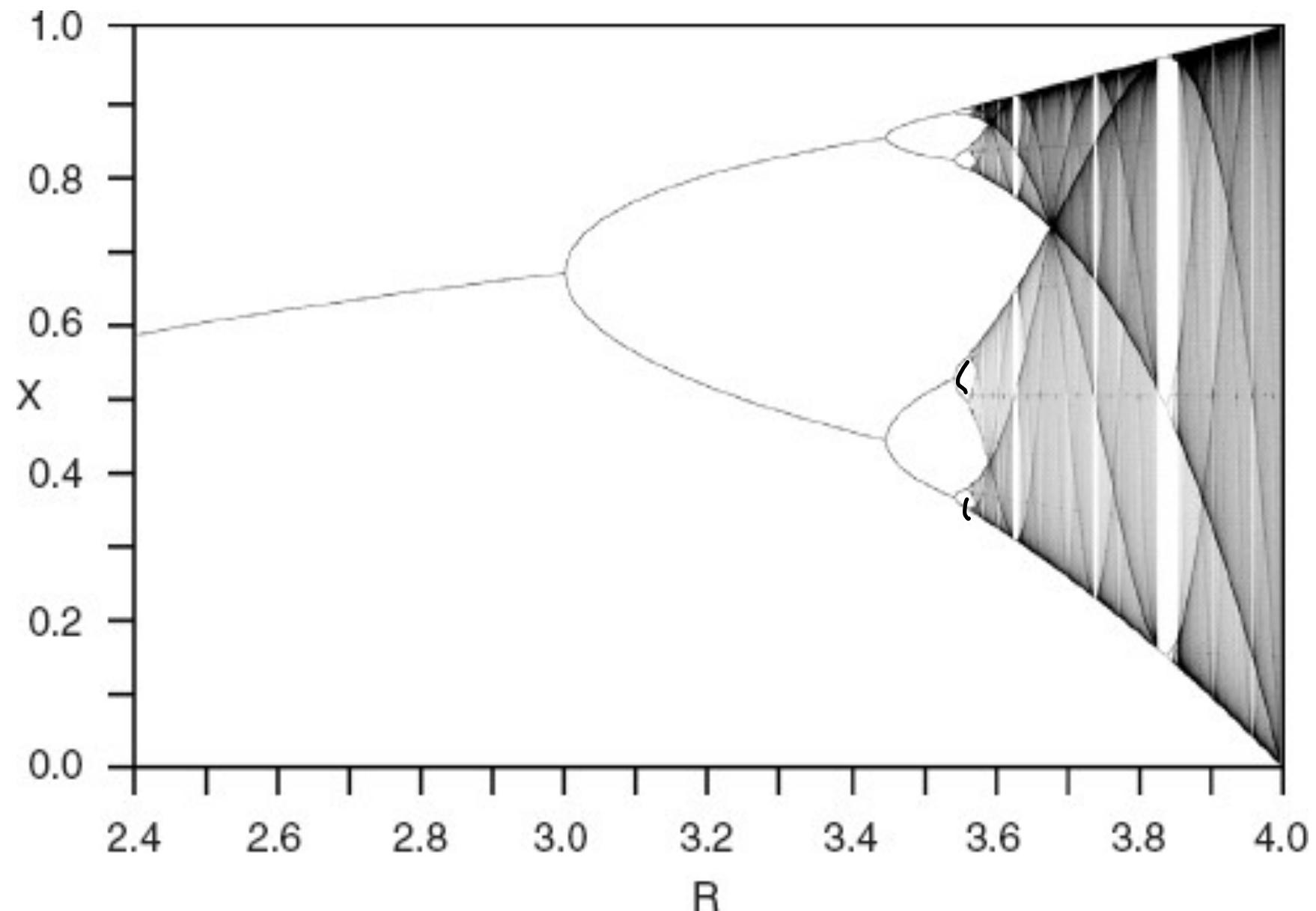
Universality in Chaos

While chaotic systems are not predictable in detail, a wide class of chaotic systems has highly predictable, “universal” properties.

A Unimodal (“one humped”) Map



Logistic Map Bifurcation Diagram



Bifurcations in the Logistic Map

$R_1 \approx 3.0$: period 2

$R_2 \approx 3.44949$ period 4

$R_3 \approx 3.54409$ period 8

$R_4 \approx 3.564407$ period 16

$R_5 \approx 3.568759$ period 32

$R_\infty \approx 3.569946$ period ∞ (onset of chaos)

Bifurcations in the Logistic Map

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$R_5 \approx 3.568759$ period 32

$R_\infty \approx 3.569946$ period ∞
(chaos)

Rate at which distance between bifurcations is shrinking:

$$\frac{R_2 - R_1}{R_3 - R_2} = \frac{3.44949 - 3.0}{3.54409 - 3.44949} = 4.75147992$$

$$\frac{R_3 - R_2}{R_4 - R_3} = \frac{3.54409 - 3.44949}{3.564407 - 3.54409} = 4.65619924$$

$$\frac{R_4 - R_3}{R_5 - R_4} = \frac{3.564407 - 3.54409}{3.568759 - 3.564407} = 4.66842831$$

⋮

$$\lim_{n \rightarrow \infty} \left(\frac{R_{n+1} - R_n}{R_{n+2} - R_{n+1}} \right) \approx 4.6692016....$$

Amazingly, at almost exactly the same time, the same constant was independently discovered (and mathematically derived by) another research team, the French mathematicians Pierre Collet and Charles Tresser.

Summary

Significance of dynamics and chaos for complex systems

- Complex, unpredictable behavior from simple, deterministic rules
- Dynamics gives us a vocabulary for describing complex behavior
- There are fundamental limits to detailed prediction
- At the same time there is universality: “Order in Chaos”