Using Barometric Pressure Altitude to Verify and Improve GNSS/INS Surface Position Accuracy in Space Vehicle Ascent Guidance

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***Abstract*—In this paper, we detail a method for determining a reduced set of Cartesian coordinates relative to the Earth (or any planetary body with a sufficiently studied atmosphere) given either barometrically-measured surface altitude or ambient temperature, plus a known starting location, velocity, and flight time for an ascending space vehicle. While the accuracy of this method is not yet known, it is worth further investigation to determine the feasibility and efficiency of such a computation and whether or not it is beneficial to the total accuracy of the vehicle's entire navigation system.**

***Index Terms*—barometer, GNSS, INS, space vehicle guidance, navigation error**

In any spacecraft ascent guidance system, a barometric pressure sensor is vitally important for accurately computing the vehicle's altitude above Earth's surface. Tradition (and common sense) dictates that since a barometer can only measure atmospheric pressure, the *only* readings it can be used for are pressure altitude and not location (latitude/longitude or Cartesian coordinates). However, it became apparent that since spacecraft don't always fly straight upwards, there is a difference in total distance traveled and barometric altitude. This difference is illustrated in figure 1 by the equation $\frac{\vec{Y}}{\|\vec{v}\|}t - \Delta y$. Effectively, take the difference between total distance traveled (represented by $\vec{v}t$) and the currently measured barometric altitude (Δy) and compare them along the same axis.

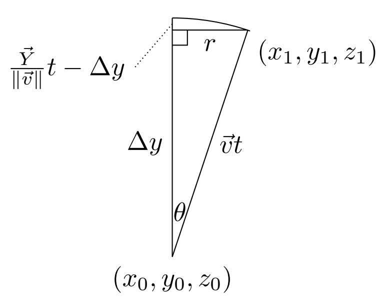

Fig. 1. Difference between distance traveled and barometric altitude

Next, we account for error margins. Assuming an angular size error α in the measurement of θ and an altimeter error margin β defines an ellipse lying in the XY plane. The equation for this ellipse is given in equation 1d. It should be noted that this ellipse is not aligned with the X and Y axis – it is in fact rotated by an angle of $-\theta$ as shown in figure 2.

This ellipse is defined by either setting $E(x, y)$ (as defined in equation 1d) equal to 1 or with parametric equations 1e and 1f in terms of central angle ϕ. Note the semi-axes s and q are derived from the error margins (not shown in any diagrams - α is the angular error in θ and β is the vertical error in Δy).

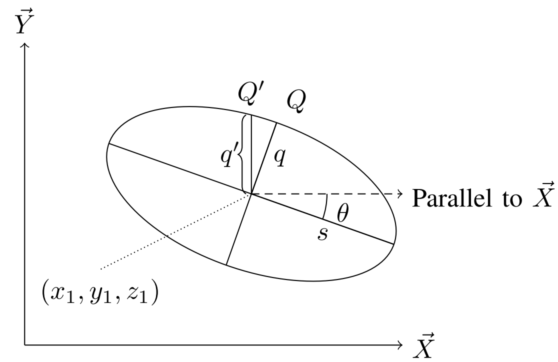

Fig. 2. Rotated ellipse in the XY plane

Now that we have a valid set of points within error bounds (the set of points contained within the ellipse, or where $E(x, y) \leq 1$), we can move into a 3D system. Since we only know distance traveled and height, the actual rocket location can be extrapolated to a ring around the $\vec{Y}$ axis. Including the previously determined error ellipse, we can define a rotated elliptical torus normal to the $\vec{Y}$ axis. This rotation is shown in figure 2.

At this point, we can begin the process of reducing error margins on estimations from other navigation sources. For each of the navigation sources (GNSS, INS, INS postdiction, and any other forms in use), the same steps will be repeated.

First, we determine the angle in the $\vec{X}\vec{Z}$ plane that the point (which for this loop we will call $P2 = (x_2, y_2, z_2)$ lies in and call this angle ψ. The ultimate goal is to determine whether the point lies outside the torus, and if it does, we can bring it to the edge of the torus. Rather than rotate the ellipse around the $\vec{Y}$ axis, which would be unnecessarily complex, we will rotate the point $P2$ around the $\vec{Y}$ axis so that it lies in the $\vec{X}\vec{Y}$ plane. We call this first transformed point (x_{2a}, y_{2a}, z_{2a}). This is done in equations 2a and 2b.

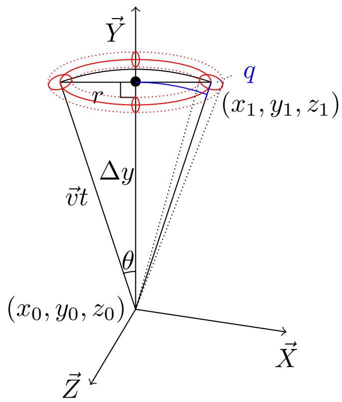

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Fig. 3. Rotated elliptical torus about the Y axis showing error margin in 3D

Next, we need determine whether or not the point (x_{2a}, y_{2a}, z_{2a}) lies within the ellipse previously defined. Since the equation of a rotated ellipse is given in equation 1d and must be equal to 1 in order to form an ellipse, we can simply check whether the function $E(x_{2a}, y_{2a})$ is less than or equal to 1. If it is, the point lies on or inside the ellipse; and if not, the point lies outside.

Now that we know where the point is, we can determine what action must be taken. This is again fairly simple – if the point lies outside the ellipse, we transform it to the boundary of the ellipse. To actually perform this transformation, we first determine the interior angle of the point γ given in equation 3a. Then, we take the appropriate action (scale it to the ellipse) in equation 3b, and finally rotate it back around the $\vec{Y}$ axis to match the original angle in equation 3c.

Now, the point (x_{2c}, y_{2c}, z_{2c}) has been reduced if possible. The steps can now be repeated with another point from another navigation source to further reduce error.

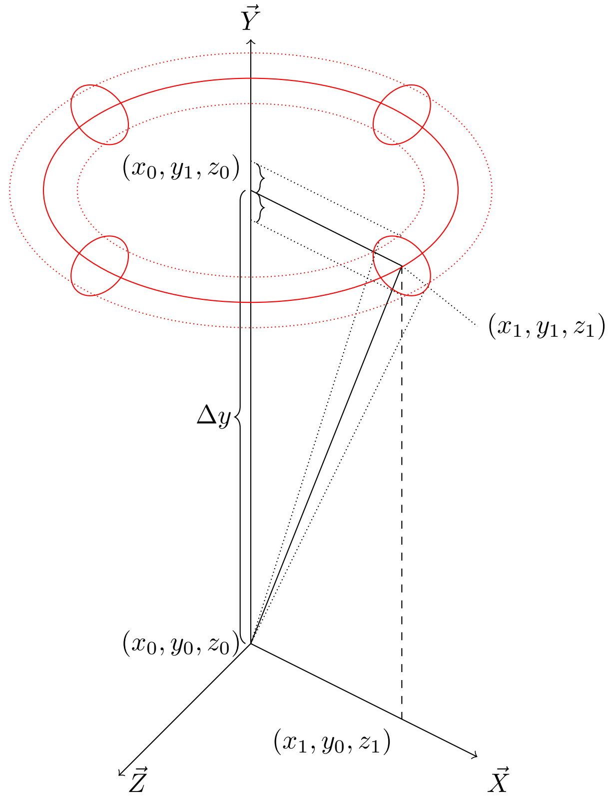

Fig. 4. Blown-up figure depicting all aspects of the problem

$$s = \|\vec{v}\| t \tan(\alpha) \tag{1a}$$

$$q' = \beta \tag{1b}$$

$$q = \csc(\theta)\left(-\left(x_1 + s\cos(\theta)\right)\right), \text{if} \sin(\theta) \neq 0 \tag{1c}$$

$$E(x,y) = \frac{\left((x - x_1)\cos(\theta) + (y - y_1)\sin(\theta)\right)^2}{s^2} + \frac{\left((x - x_1)\cos(\theta) - (y - y_a)\sin(\theta)\right)^2}{q^2} \tag{1d}$$

$$E_x(\phi) = s\cos(\phi)\cos(-\theta) - q\sin(\phi)\sin(-\theta) + x_1 \tag{1e}$$

$$E_y(\phi) = s\cos(\phi)\sin(-\theta) + q\sin(\phi)\cos(-\theta) + y_1 \tag{1f}$$

$$\psi = \text{arctan2}(z_2, x_2) \tag{2a}$$

$$(x_{2a}, y_{2a}, z_{2a}) = (z_2\sin(-\psi) + x_2\cos(-\psi), y_2, z_2\cos(-\psi) - x_2\sin(-\psi)) \tag{2b}$$

$$\gamma = \text{arctan2}(y_{2a} - y_1, x_{2a} - x_1) \tag{3a}$$

$$(x_{2b}, y_{2b}, z_{2b}) = \begin{cases} (x_{2a}, y_{2a}, z_{2a}) & E(x_{2a}, y_{2a}) \leq 1 \\ (E_x(\gamma), E_y(\gamma), z_{2a}) & E(x_{2a}, y_{2a}) > 1 \end{cases} \tag{3b}$$

$$(x_{2c}, y_{2c}, z_{2c}) = (z_{2b}\sin(\psi) + x_{2b}\cos(\psi), y_{2b}, z_{2b}\cos(\psi) - x_{2b}\sin(\psi)) \tag{3c}$$