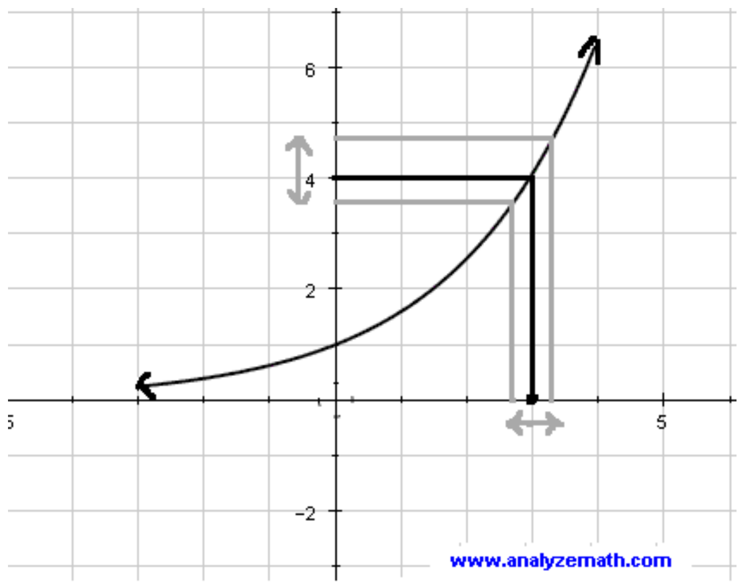
Propagation of Uncertainty

Notation: Δx is the uncertiany in *x*.

<u>Underlying theory:</u>

A function $f(x, y, z)$ which is calculated with variables, each of which has uncertainty $\Delta x, \Delta y, \Delta z$ must also have uncertainty Δf.

The uncertainty in *f* associated with the uncertainty in *x* can be visualized with a graph of *f(x)* (assuming that *y* and *z* are constants). If the best guess for *x* is 4.0 with an uncertainty of +/- 0.5, what is the associated uncertainty in *f*? We can see from the graph.



The uncertainty in *f* due to the uncertainty in *x* can be written as the slope of the *f(x)* graph evaluated at the best guess for *x, y* and *z* times the uncertainty in *x*:

$$\Delta f_x = \left.\frac{\partial f}{\partial x}\right|_{x_0, y_0, z_0} \Delta x$$

$$\Delta f_y = \left.\frac{\partial f}{\partial y}\right|_{x_0, y_0, z_0} \Delta y$$

$$\Delta f_z = \left.\frac{\partial f}{\partial z}\right|_{x_0, y_0, z_0} \Delta z$$

Since there are multiple uncertainties, we need to "add them together."

$$\Delta f = \sqrt{(\Delta f_x)^2 + \left(\Delta f_y\right)^2 + (\Delta f_z)^2}$$

<u>Examples</u>

1) Calculate K, given that m = (2.00 +/- 0.05) kg and v = (5.0 +/- 0.2) m/s.

$$K = \frac{1}{2}mv^2 = 25\,J$$
$$\Delta K_m = \frac{\partial K}{\partial m}\Delta m = \left(\frac{1}{2}v^2\right)\Delta m = 0.625\,J$$
$$\Delta K_v = \frac{\partial K}{\partial v}\Delta v = (mv)\Delta v = 2.0\,J$$
$$\Delta K = \sqrt{(\Delta K_m)^2 + (\Delta K_v)^2} = 2.1\,J$$
$$K = (25 \pm 2)\ \mathrm{J}$$

(We always round the uncertainty to 1 sig. fig. unless it is 1, then we round to 2 sig. fig.)

2) Calculate gravitational force between two masses given m1 = (100 +/- 5) kg and m2 = (200 +/- 7) kg and r = (1.4 +/- 0.1) m. $G = 6.67 \times 10^{-11}$ N m^2/kg^2

$$F = G\frac{m_1 m_2}{r^2} = 681 \times 10^{-9}\ \mathrm{N}$$
$$\Delta F_{m1} = \frac{\partial F}{\partial m_1}\Delta m_1 = \left(G\frac{m_2}{r^2}\right)\Delta m_1 = 34.0 \times 10^{-9}\ \mathrm{N}$$
$$\Delta F_{m2} = \frac{\partial F}{\partial m_2}\Delta m_2 = \left(G\frac{m_1}{r^2}\right)\Delta m_2 = 23.8 \times 10^{-9}\ \mathrm{N}$$
$$\Delta F_r = \frac{\partial F}{\partial r}\Delta r = \left(-2G\frac{m_1 m_2}{r^3}\right)\Delta r = 97.2 \times 10^{-9}\ \mathrm{N}$$
$$\Delta F = \left(\sqrt{(34.0)^2 + (23.8)^2 + (97.2)^2}\right) \times 10^{-9}\ \mathrm{N}$$
$$F = (680 \pm 110) \times 10^{-9}\ \mathrm{N}$$