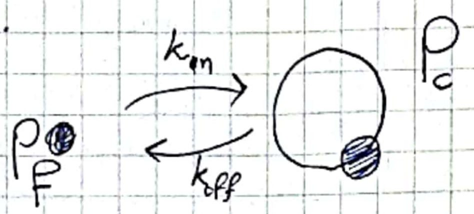
Quasi Steady State Analysis:

Example 1:

Suppose that we have protein P_f (f stands for free) which diffuses in the environment with D_f and decays with rate k:

$$\frac{\partial P_f}{\partial t} = D_f \frac{\partial^2 P_f}{\partial x^2} - k P_f$$

We can change the rate of diffusion and decay by introducing a larger molecule to the environment that P_f can bind and unbind to.



Since P_c (c stands for complex) is large, then its diffusion coefficient will be smaller. Also note that the P_c complex will not decay anymore (do not fit the enzyme). Also we assume that there is unlimited amount of larger molecule.

$$\begin{cases} \dfrac{\partial P_c}{\partial t} = D_c \dfrac{\partial^2 P_c}{\partial x^2} - k_{off} P_c + k_{on} P_f \\[2ex] \dfrac{\partial P_f}{\partial t} = D_f \dfrac{\partial^2 P_f}{\partial x^2} - k P_f + k_{off} P_c - k_{on} P_f \end{cases}$$

By assuming that the binding and unbinding kinetics is very fast, we can assume that the free and binded species equlibrate very quickly.

$$\Rightarrow k_{on} P_f - k_{off} P_c = 0 \Rightarrow \boxed{\frac{P_f}{P_c} = \frac{k_{off}}{k_{on}}} \quad \text{I}$$

Equation I means that since P_f and P_c equlibrate very quickley their ratio can be assumed to be constant at any time.

Now define the total amount of protein (binded or unbinded)

$$\boxed{P = P_c + P_f} \quad \text{II}$$

Using I, II we can write

$$P_f = \frac{k_{off}}{k_{on}} P_c \; ; \; P = P_c + P_f$$

$$\Rightarrow P = \left(\frac{k_{on} + k_{off}}{k_{on}} \right) P_c \Rightarrow \boxed{\begin{aligned} P_c &= \frac{k_{on}}{k_{on} + k_{off}} P \\ P_f &= \frac{k_{off}}{k_{on} + k_{off}} P \end{aligned}} \quad \text{III}$$

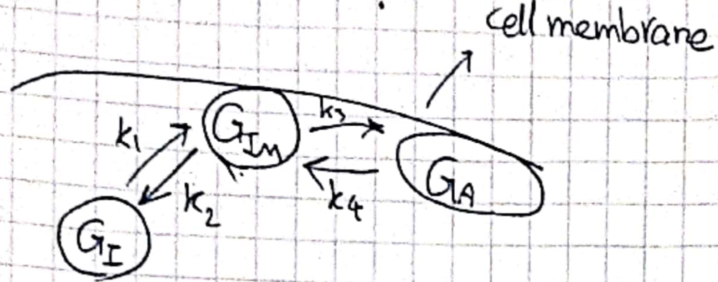
Now by suming the two sides of ODE system and using II, III we will have:

$$\boxed{\frac{\partial P}{\partial t} = \left(D_c \frac{k_{on}}{k_{on} + k_{off}} + D_f \frac{k_{off}}{k_{on} + k_{off}} \right) \frac{\partial^2 P}{\partial x^2} - \frac{k \, k_{off}}{k_{on} + k_{off}} P}$$

$$\Rightarrow \boxed{\begin{aligned} \frac{\partial P}{\partial t} &= \mathcal{D} \frac{\partial^2 P}{\partial x^2} - \bar{k} P \\ \mathcal{D} &= D_c \frac{k_{on}}{k_{on} + k_{off}} + D_f \frac{k_{off}}{k_{on} + k_{off}} \\ \bar{k} &= k \frac{k_{off}}{k_{on} + k_{off}} \end{aligned}}$$

As you can see by playing with k_{on} and k_{off} prameters we can have different behavioural diffusion rate and decay rate

Example II:



Suppose that we have a protein that have two states: Active and Inactive. The inactive form diffuses in the environment with D_I and binds and unbinds to membrane with k_1, k_2. G_{Im} (inactive form binded to membrane) then diffuses and turns into G_A with k_3, k_4:

$$① \quad \frac{\partial G_I}{\partial t} = D_I \frac{\partial^2 G_I}{\partial x^2} - k_1 G_I + k_2 G_{Im}$$

$$② \quad \frac{\partial G_{Im}}{\partial t} = D_{Im} \frac{\partial^2 G_{Im}}{\partial x^2} + k_1 G_I - k_2 G_{Im} - k_3 G_{Im} + k_4 G_A$$

$$③ \quad \frac{\partial G_A}{\partial t} = D_A \frac{\partial^2 G_A}{\partial x^2} + k_3 G_{Im} - k_4 G_A$$

Using the assumption that kinetic between G_I and G_{Im} is very fast and equalibrate very quickley we can write:

$$k_1 G_I - k_2 G_{Im} = 0 \Rightarrow \boxed{\frac{G_I}{G_{Im}} = \frac{k_2}{k_1}} \quad \text{I}$$

let G_i be:

$$\boxed{G_i = G_I + G_{Im}} \quad \text{II}$$

Considering I and II we can write:

$$G_{Im} = \frac{k_1}{k_1 + k_2} G_i$$

$$G_I = \frac{k_2}{k_1 + k_2} G_i$$

And by summing ① and ②:

$$\begin{cases} \dfrac{\partial G_i}{\partial t} = \left(D_I \dfrac{k_2}{k_1 + k_2} + D_{Im} \dfrac{k_1}{k_1 + k_2} \right) \dfrac{\partial^2 G_i}{\partial x^2} - \dfrac{k_1 k_3}{k_1 + k_2} G_i + k_4 G_A \\[2ex] \dfrac{\partial G_A}{\partial t} = D_A \dfrac{\partial^2 G_A}{\partial x^2} + \dfrac{k_1 k_3}{k_1 + k_2} - k_4 G_A \end{cases}$$