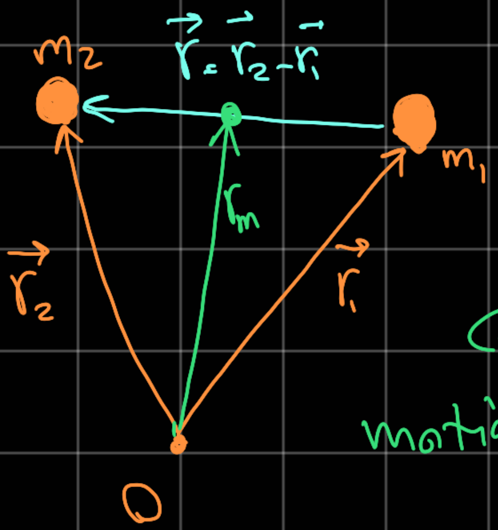
we want to derive initial conditions in which two body at gravitational interaction orbit their center of mass in a circular motion.



let's first write the equations of motion for two particles and convert them into a single equation of motion using reduced mass.

$$m_1 \ddot{r}_1 = F_1 \hat{r} \quad (1) \; ; \quad m_2 \ddot{r}_2 = -F_2 \hat{r} \quad (2)$$

in which we have:
$$\boxed{F_1 = \frac{G m_1 m_2}{r^2} \quad (4)}$$

we difine r_m as:
$$\boxed{r_m = \frac{r_1 m_1 + r_2 m_2}{m_1 + m_2} \quad (3)}$$

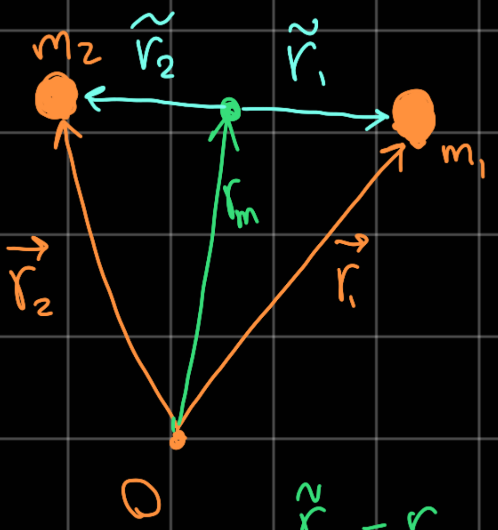
inserting (3) in (1), (2) results in: $\boxed{\ddot{r}_m = 0 \quad (5)}$

$$\rightarrow m_1 m_2 \ddot{r} = m_1 m_2 (\ddot{r}_2 - \ddot{r}_1) = m_1 (m_2 \ddot{r}_2) - m_2 (m_1 \ddot{r}_1)$$
$$= m_1 (-F \hat{r}) - m_2 (F \hat{r})$$
$$= -(m_1 + m_2) F$$

$$\Rightarrow \boxed{\mu \ddot{r} = -f \hat{r} \quad (6) \quad ; \quad \mu = \frac{m_1 m_2}{m_1 + m_2} \quad (7)}$$

So far, we reduced two equations of motion into a single one with reduced mass.

now lets express two original equations of motions in the center of mass coordinates.



$$\boxed{\tilde{r}_1 = r_1 - r_m \quad (8) \quad ; \quad \tilde{r}_2 = r_2 - r_m \quad (9)}$$

inserting (3) in (8) we will have:

$$\tilde{r}_1 = r_1 - r_m = r_1 - \frac{m_1 r_1 + m_2 r_2}{\underbrace{m_1 + m_2}_{m}} = \frac{m_2 r_1 - m_2 r_2}{m}$$

$$= \frac{m_2}{m}(r_1 - r_2)$$

$$\Rightarrow \boxed{\tilde{r}_1 = \frac{m_2}{m} r \quad (10)}$$

using same logic we can derive a similar relation for $\tilde{r}_2$:

$$\boxed{\tilde{r}_2 = \frac{m_1}{m} r \quad (11)}$$

now it is time to derive the inital values for velocities so that the Cores maintain a circular orbit.

using (1), (4), (6) and the assumption that the Cores maintain a Circular motion, we can write:

$$\underbrace{\frac{m_1 m_2}{m_1+m_2}}_{A}\ddot{r} = R\hat{r} = \underbrace{\frac{G m_1 m_2}{R^2}}_{B} = \underbrace{\frac{m_1 v^2}{|\tilde{r}_1|}}_{C} = \underbrace{\frac{m_2 v^2}{\tilde{r}_2^{\,2}}}_{D}$$

$$A, B \Rightarrow \frac{\cancel{G}^{\,1}}{R^2} = \frac{1}{m_1+m_2} = \frac{1}{m} \Rightarrow \frac{1}{m} = \frac{1}{R^2} \Rightarrow \boxed{m = R^2}$$

$$A, C \Rightarrow \frac{m_2}{m} = \frac{v^2}{|\tilde{r}_1|} \Rightarrow v^2 = \frac{|\tilde{r}_1| m_2}{R^2} \Rightarrow \boxed{v_1 = \frac{\sqrt{|\tilde{r}_1| m_2}}{R^2} \quad (12)}$$

using same logic we can derive:

$$\boxed{v_2 = \frac{\sqrt{|\tilde{r}_2| m_1}}{R^2} \quad (13)}$$

So I will use (10), (11), (12), (13) as initial values

$$\vec{r}_1 = \left(\frac{-m_2}{m} r, 0, 0\right) \; ; \; \vec{r}_1 = \left(\frac{m_1}{m} r, 0, 0\right) \; ; \; \vec{v}_1 = (0, -v_1, 0) \; ; \; \vec{v}_2 = (0, v_2, 0)$$