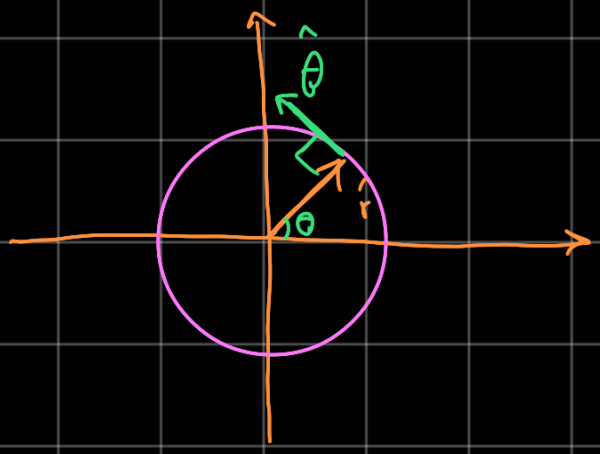
Initial Condition For circular orbit of Stars :

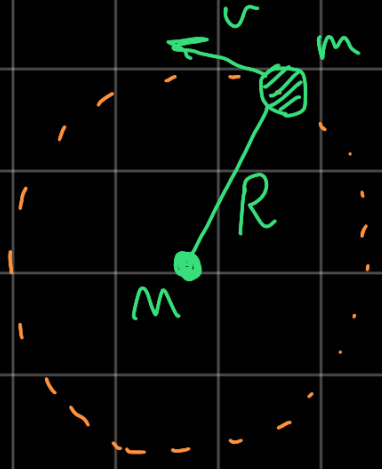
First, let's derive the tangent vector to $\hat{r}$ vector in polar coordinate:



$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

So the initial velocity of stars will be at $\hat{\theta}$ direction but what should be the velocity to maintain a circular motion ?



$$F = \frac{mv^2}{R} = G\frac{mM}{R^2}$$

$$\rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

and since the values of G and M are 1 in my system of units, then we can write :

$$v = \frac{1}{\sqrt{R}}$$