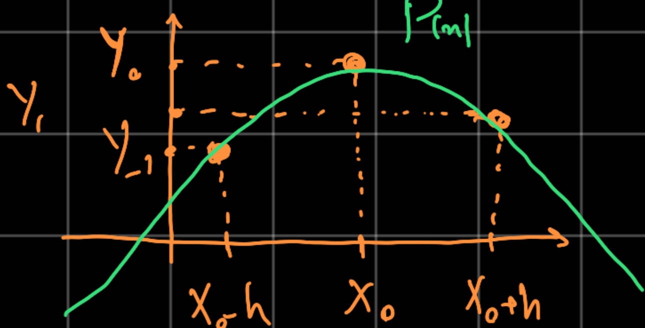
Lagrange interpolation can be used to fit a degree n polynomial to n+1 data points

$$\{(x_i, y_i)\}_{i=1}^{n}$$

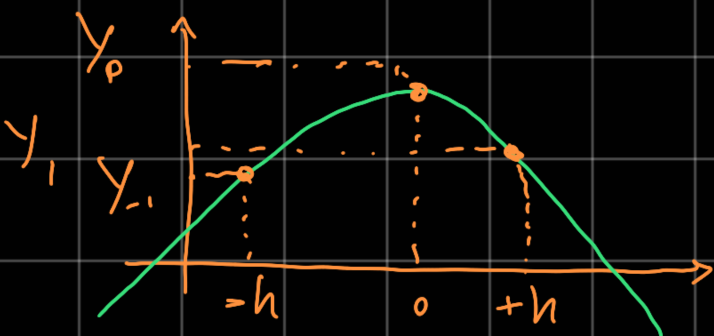
$$P(x) = \sum_{j=1}^{n} y_j L_j(x)$$

$$L_j(x) = \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i}$$

Now suppose that we have $\underline{3}$ data points and we want to fit a 2 degree polynomial to it



Without the loss of generalization, we can write:



$$P(x) = Y_{-1} \underline{L_1(x)} + Y_0 \underline{L_2(x)} + Y_1 \underline{L_3(x)}$$

$$\frac{x-0}{-h-0} \cdot \frac{x-h}{-h-h} \qquad \frac{x+h}{0+h} \cdot \frac{x-h}{0-h} \qquad \frac{x+h}{h+h} \cdot \frac{x-0}{h-0}$$

$$\Rightarrow P(x) = x^2 \left(\frac{Y_{-1}}{2h^2} - \frac{Y_0}{h^2} + \frac{Y_1}{2h^2} \right) + x \left(\frac{-Y_{-1}}{2h} + \frac{Y_1}{2h} \right) + Y_0$$

$$\boxed{\left. \frac{dP(x)}{dx} \right|_{x=0} = \frac{Y_1 - Y_{-1}}{2h}}$$

So in general the derivitive of $P(x)$ will be,

$$\boxed{\frac{dP(x)}{dt} \simeq \frac{P(x+\Delta h) - P(x-\Delta h)}{2\Delta h}}$$