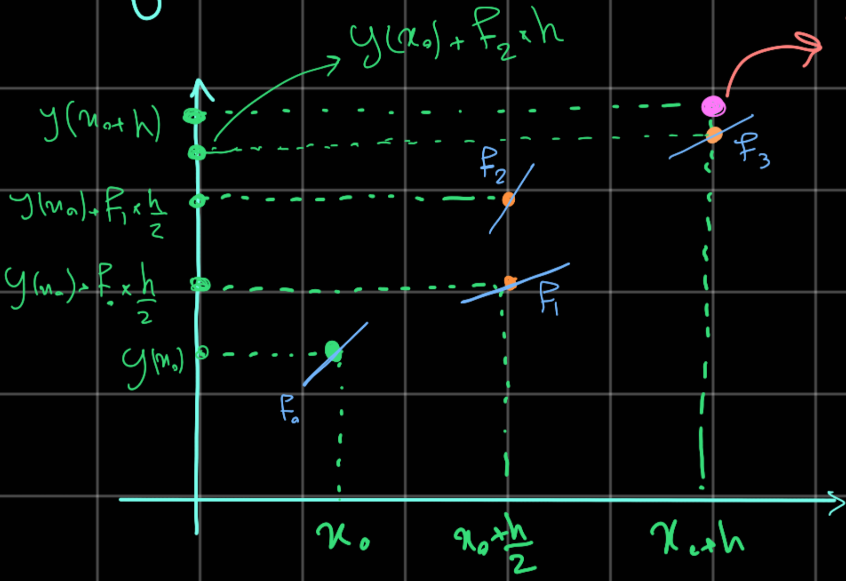
Runge_kutta method uses the mean value of slopes to go from $y(x_0)$ to $y(x_0+h)$

$$\dot{y} = F(x,y)$$



$$y(x_0+h) = y(x_0) + \frac{h}{6}\left(F_0 + 2F_1 + 2F_2 + F_3\right)$$

$$F_0 = F(x_0, y(x_0))$$
$$F_1 = F\left(x_0+\frac{h}{2},\ y(x_0) + \frac{F_0}{2}h\right)$$
$$F_2 = F\left(x_0+\frac{h}{2},\ y(x_0) + \frac{F_1}{2}h\right)$$
$$F_3 = F\left(x_0+h,\ y(x_0) + F_2 h\right)$$

Applying Rung_kutta on "n" order differential equ.

$$y^{(n)} = F\left(x, y, \dot{y}, \ddot{y}, \ldots, y^{(n-1)}\right)$$

We can do the following change of variables:

$$y_0 = y \quad ; \quad y_1 = \dot{y} \quad ; \quad y_2 = \ddot{y} \quad ; \ \ldots \ ; \quad y_{n-1} = y^{(n-1)}$$

So now we can rewrite the original differential equation in form of following system of first order differential equations

$$\begin{cases} \dot{y}_0 = y_1 & = F_0(x, Y_{(x)}) \\ \dot{y}_1 = y_2 & = F_1(x, Y_{(x)}) \\ \dot{y}_2 = y_3 & = F_2(x, Y_{(x)}) \\ \vdots & \\ \dot{y}_{n-1} = y_n = F(x, \underbrace{y_0, y_1, y_2, \dots, y_{n-1}}) & = F_{n-1}(x, Y_{(x)}) \end{cases}$$

$$Y_{(x)} = (y_0(x), y_1(x), \dots, y_{n-1}(x))$$

☆ to solve the system of equations we should have the initial values x_0, $Y(x_0)$

☆ let's rewrite the system of equations in form of vector equations:

$$☆ \quad \boxed{\dot{Y} = F(x, Y_{(x)})} \quad ☆$$

in which we have

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{pmatrix} ; \quad F(x, Y_{(x)}) = \begin{pmatrix} F_0(x, Y_{(x)}) \\ F_1(x, Y_{(x)}) \\ F_2(x, Y_{(x)}) \\ \vdots \\ F_{n-1}(x, Y_{(x)}) \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ F(x, Y_{(x)}) \end{pmatrix}$$

Similar to the single RK method, we can solve the above equation as follows:

$$Y(x_0+h) = Y(x_0) + \frac{h}{6}\left(\tilde{F}_0 + 2\tilde{F}_1 + 2\tilde{F}_2 + \tilde{F}_3\right)$$

$$\tilde{F}_0 = F(x_0, Y(x_0)) \quad ; \quad \tilde{F}_1 = F\left(x_0, Y(x_0) + \frac{h}{2}\tilde{F}_0\right)$$

$$\tilde{F}_2 = F\left(x_0, Y(x_0) + \frac{h}{2}\tilde{F}_1\right) \quad ; \quad \tilde{F}_3 = F\left(x_0, Y(x_0) + \tilde{F}_3 h\right)$$