$$r_1 = \ell_1 (\sin\theta_1, -\cos\theta_1)$$

$$r_2 = \ell_1 (\sin\theta_1, -\cos\theta_1) + \ell_2 (\sin\theta_2, -\cos\theta_2)$$

$$\begin{cases} \dot{r}_1^2 = (\ell_1 \dot{\theta}_1)^2 \\ \dot{r}_2^2 = (\ell_1 \dot{\theta}_1)^2 + (\ell_2 \dot{\theta}_2)^2 + 2\ell_1 \ell_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1) \end{cases}$$

$$\Rightarrow T = \frac{1}{2} m v_1^2 + \frac{1}{2} m_2 v^2$$

$$= \frac{1}{2}(m_1 + m_2)(\ell_1 \dot{\theta}_1)^2 + \frac{1}{2} m_2 (\ell_2 \dot{\theta}_2 + 2\ell_1 \ell_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1))$$

$$V = -mgh_1 - mgh_2 = -(m_1 + m_2) g \ell_1 \cos\theta_1 - m_2 g \ell_2 \cos\theta_2$$

$$\mathcal{L} = T - V$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) = \frac{\partial \mathcal{L}}{\partial \theta_1} \quad ; \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

aftere doing the mathematics we will have:

$$A_1\ddot{\theta}_1 + B_1\ddot{\theta}_2 = C_1 - D_1$$
$$A_2\ddot{\theta}_2 + B_2\ddot{\theta}_1 = C_2 - D_2$$

in which:
$$\begin{cases} A_1 = l_1(m_1+m_2) \\ B_1 = m_2 l_2 \cos(\theta_2-\theta_1) \\ C_1 = m_2 l_2 \dot{\theta}_2^2 \sin(\theta_2-\theta_1) \\ D_1 = (m_1+m_2) g \sin\theta_1 \end{cases}$$

$$\begin{cases} A_2 = l_2 \\ B_2 = l_1 \cos(\theta_2-\theta_1) \\ C_2 = -l_1\dot{\theta}_1 \sin(\theta_2-\theta_1) \\ D_2 = g\sin(\theta_2) \end{cases}$$

$$\ddot{\theta}_1 = \frac{B_2(C_1-D_1) - A_1(C_2-D_2)}{B_1B_2 - A_1A_2} = G_1$$

$$\ddot{\theta}_2 = \frac{A_2(C_1-D_1) - B_1(C_2-D_2)}{A_1A_2 - B_1B_2} = G_2$$

$$\ddot{\Phi} = \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} \Rightarrow \Psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{pmatrix}$$

$$\Rightarrow \dot{\Psi} = \begin{pmatrix} \dot{\psi}_0 \\ \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ G_1 \\ \psi_3 \\ G_2 \end{pmatrix} = F(t, \Psi)$$

$$\Rightarrow \boxed{\dot{\Psi} = F(t, \Psi)}$$

$$F_0 = F(t, \Psi(t))$$

$$F_1 = F\left(t + \frac{h}{2}, \Psi(t) + \frac{F_0 h}{2}\right)$$

$$F_2 = F\left(t + \frac{h}{2}, \Psi(t) + \frac{F_1 h}{2}\right)$$

$$F_3 = F\left(t + h, \Psi(t) + F_2 h\right)$$

$$\boxed{\Psi(t+h) = \Psi(t) + \frac{h}{6}\left(F_0 + 2F_1 + 2F_2 + F_3\right)}$$