NAVIGATIONAL ALGORITHMS
Corrections for Sextant Altitude

(F1)

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San Sebastián – Donostia
43º 19'N 002ºW
http://sites.google.com/site/navigationalalgorithms/

Indice

Abstract

Celestial navigation is based on obtaining your position by observing the stars, measuring their altitude over the horizon, or its complement; the zenith distance. This altitude is obtained with an instrument that at sea is the sextant, and on the ground can be a theodolite. It is necessary to apply such a measure obtained, a number of corrections to get a reduced altitude to the centre of the Earth and self-effects such as refraction due to the Earth's atmosphere. All sextant angles need to be corrected for index error and dip to produce the apparent altitude. Calculate the observed altitude by subtracting a correction for refraction. For the Sun, Moon, Venus and Mars a correction for parallax is also applied to H and for the Sun and Moon a further correction for semi-diameter is also required.

Navigational Algorithms
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The formulae for corrections of the measured altitude are described. The values are tabulated in the nautical Almanac.

Variables

Hs	Sextant Altitude measured by the observer.
IE	Instrumental or Index Error.
Dip	Dip of horizon
H	Apparent Altitude
HP	Horizontal Parallax
PA	Parallax in Altitude
SD	Semi-diameter
Ho	Observed Altitude
R	Atmospheric Refraction

The variables are all in degrees.

P	Atmospheric pressure [hPa]
T	Air temperature [ºC]

Corrections for Sextant Altitude

The corrections take into account:

- Intrinsic Errors to the sextant.
- The height of the eye of the observer.
- Adjust of the equivalent reading in the centre of the Earth, and in the centre of the star.
- The refraction due to the terrestrial atmosphere.
- Others.

Dip of horizon

The correction is:

Dip = 0.0293 * SQRT(h) [º]

Where h is the height of the eye above the horizon in meters.

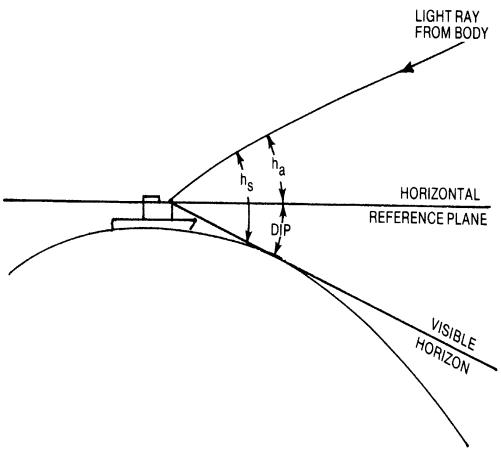


Figure 19–9. Necessity for dip correction illustrated. The sextant altitude hs will always be greater than the apparent altitude ha by the amount of the dip angle.

Apparent Altitude

Is the sextant altitude corrected for index error and dip.

H = Hs + IE - Dip

Refraction

For standard conditions:

- T = 10 ºC
- P = 1010 mb

The correction for refraction is [1]:

Ro = 0.0167 / (tan (H + 7.31) / (H + 4.4)) [º]

If the observation is made under non standard conditions, the correction factor is:

f = 0.28 * P/ (T+ 273)

And the adjusted Refraction is:

R = f * Ro

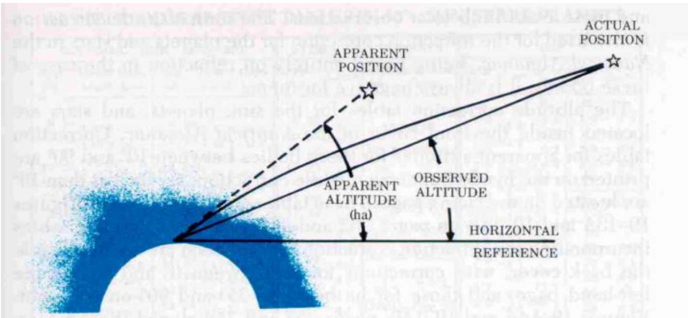


Figure 19–12. Effect of atmospheric refraction. It will always cause the apparent altitude ha to appear to be greater than the observed altitude Ho to the center of the body.

Additional Corrections

For stars: Ho = H - R

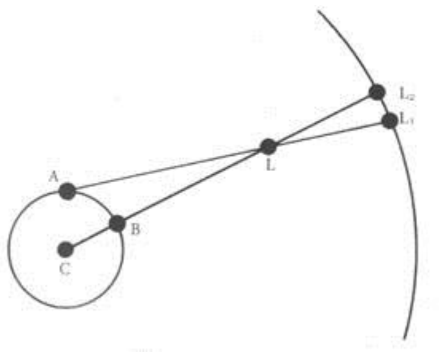
For the Sun, the Moon, and planets:

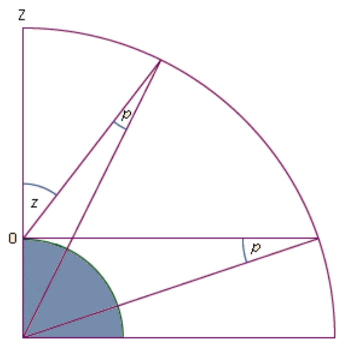
- Parallax (Sun, Moon, Venus, & Mars)
- Semidiameter of the body (Sun, Moon)
- Augmentation (Moon)

Parallax in altitude

Parallax correction is the correction due to the difference between the apparent direction from a point on the surface of the Earth to celestial body and the apparent direction from the centre of the Earth to the same body. Due to geocentric parallax, varying with the body's altitude and distance from the Earth.

The parallax correction setting the equivalent reading in the centre of the Earth.





Calculate the (PA) from the horizontal parallax (HP) and the apparent altitude (H) for the Sun, Moon, Venus and Mars.

Approximated value for the Sun:

HP = 0.0024°

For the Moon, the **Oblateness of the Earth** will be taken into account:

OB = 0.0032*(SIN(2B)*COS(z)*SIN(H)-SQ(SIN(B))*COS(H)) [º]

Where:

- B: latitude of the observer N (+) / S (–)
- z: azimuth of the Moon

Approximate values are sufficient for the calculation.

At mid-latitudes and for altitudes of the Moon below 60º, a simple approximation is made:

OB = - 0.0017 * cos H

The correction for parallax is:

PA = HP * COS(H) + OB

Semidiameter

The semidiameter setting the equivalent reading in the centre of the celestial body.

Because the Sun and Moon appears to be a large disk, we do not attempt to guess where the centre is. We bring the limb of the Sun or Moon to the horizon, thus there is an adjustment factor, the semi-diameter correction, for both. Mainly the semidiameter correction is taken into account for the Sun and the Moon, and maybe for bigger planets: Jupiter and Saturn. Stars appears as very small points and the factor does not apply.

The arithmetic sign is:

- (+) Lower limb
- (-) Upper limb

Approximated values:

- Sun: SD = 16'
- Moon: SD = 0.2724° * HP

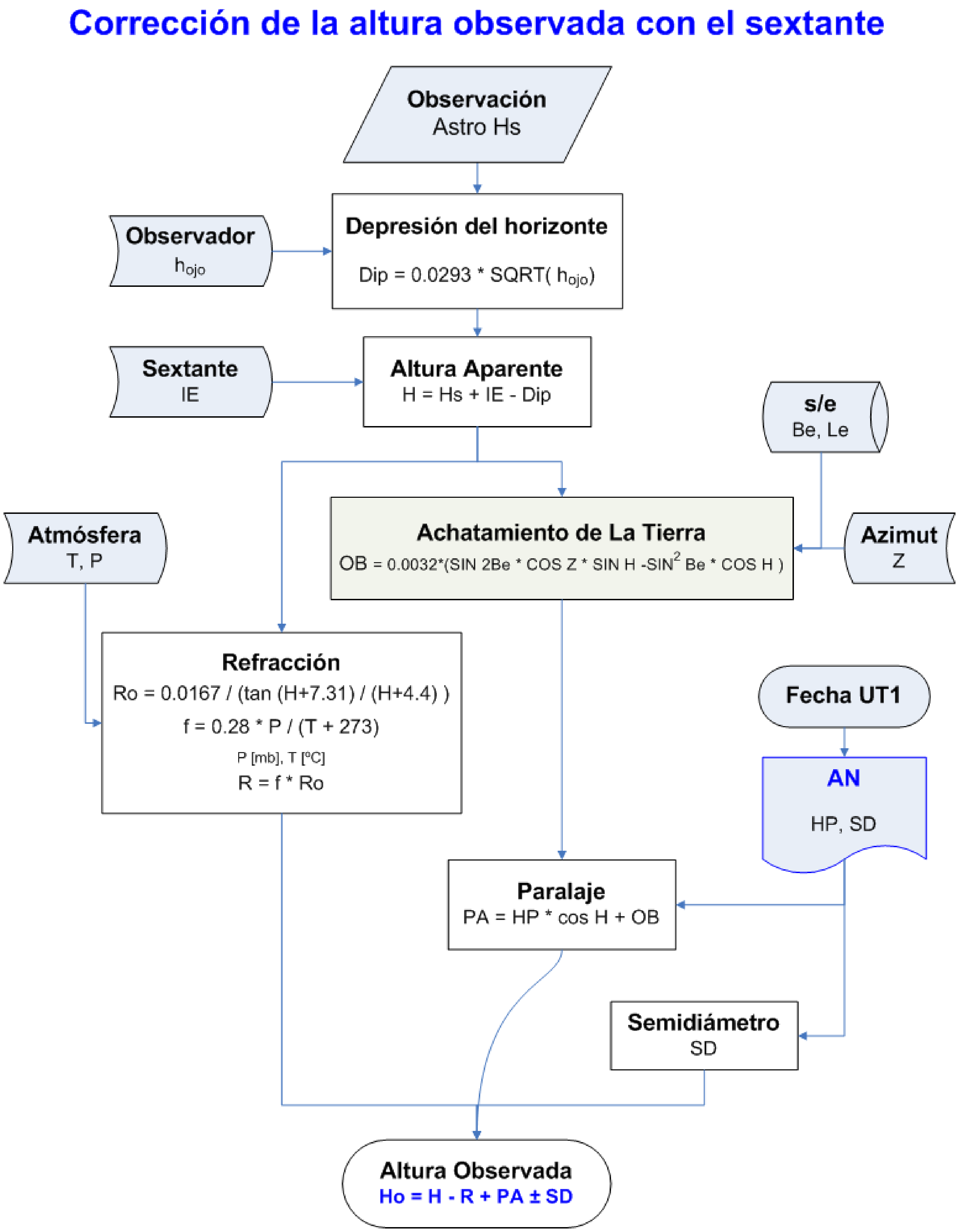
The observed altitude Ho

The Observed Altitude is the apparent altitude corrected for refraction and if appropriate corrected for parallax and semi-diameter.

H = Hs + IE - Dip

Ho = H - R + PA ± SD

A1. Algorithm



A2. Examples

```
Star
Limbo: Centro
altura sextante [°]: Hs = 49.5850
correcion de indice [']: ie = 0.0000
altura del ojo sobre el nivel del mar [m]: heas = 6.0000
temperatura ambiente [°C]: T = 18.0000
presion atmosferica [mb]: P = 1010.0000
en [°]:
altura observada por el sextante: Hos = 49.5850
depresion del horizonte: dip = 0.0719
altura aparente: Ha = 49.5131
correcion por refraccion: R = 0.0134
correcion por achatamiento: OB = 0.0000
Paralaje horizontal: HP = 0.0000
correcion por paralaje: PA = 0.0000
semidiametro: SD  = 0.0000
Aug SD = 0.0000
altura observada corregida: Ho = 49.4997 =  49° 30.0'


04/04/2008              10:12:18
Sun
Limbo: Inferior
altura sextante [°]: Hs = 44.0400
correcion de indice [']: ie = 0.0000
altura del ojo sobre el nivel del mar [m]: heas = 2.5000
temperatura ambiente [°C]: T = 10.0000
presion atmosferica [mb]: P = 1010.0000
en [°]:
altura observada por el sextante: Hos = 44.0400
depresion del horizonte: dip = 0.0464
altura aparente: Ha = 43.9936
correcion por refraccion: R = 0.0168
correcion por achatamiento: OB = 0.0000
Paralaje horizontal: HP = 0.0024
correcion por paralaje: PA = 0.0018
semidiametro: SD  = 0.2665
Aug SD = 0.0000
altura observada corregida: Ho = 44.2451 =  44° 14.7'


21/03/2008              21:22:43
Moon
SD  = 15.125046 '
HP  = 55.508365 '
Limbo: Inferior
altura sextante [°]: Hs = 27.8000
correcion de indice [']: ie = 0.0000
altura del ojo sobre el nivel del mar [m]: heas = 2.5000
temperatura ambiente [°C]: T = 10.0000
presion atmosferica [mb]: P = 1010.0000
en [°]:
altura observada por el sextante: Hos = 27.8000
depresion del horizonte: dip = 0.0464
altura aparente: Ha = 27.7536
correcion por refraccion: R = 0.0308
correcion por achatamiento: OB = -0.0023
Paralaje horizontal: HP = 0.9251
correcion por paralaje: PA = 0.8164
semidiametro: SD  = 0.2521
Aug SD = 0.0019
altura observada corregida: Ho = 28.7933 =  28° 47.6'
```

A3. Software

Available at the Navigational Algorithms web site: Almanaque Nautico.exe

- Set Hs
- Set data in [Parámetros]: Heye, Limb, IE, P, T
- Set estimated position in [s/e], (*Only for the Moon*)
- [Recta de Altura] option calculates Ho.

A4. Source code

An example of how to implement the correction:

```
#include <stdio.h>
#include <math.h>
#include "mathlib.hpp"

/*
  Celestial body:
  [0] Star
  [1] Sun
  [2] Moon
  [3] Venus/Marte
  [4] Jupiter/Saturno

  For Sun & Moon: Limb
  [1] Lower
  [2] Upper

  Units:
  Hs [deg]
  EI [deg]
  hEye [m]
  T [Celsius]
  P [mb]
  sd [deg]
  HP [deg]
*/


double ObservedAltitude( int body, double Hs, double IE, double hEye,
                       double TC = 10, double Pmb = 1010,
                       int limb = 0, double sd = 0,
                       double HP = 0 )
{
  double HO;
  double dip, H, RO, f, R;
  double PA = 0;
  double OB = 0;

  dip = .0293*sqrt( hEye );

  // Aparent Altitude
  H  = Hs + IE- dip;

  // Refraction
  RO = 0.0167/TAN( H+7.31/(H+4.4) );
  f  = 0.28*Pmb/(TC+273);
  R  = f*RO;

  // approximate values
  if( body == 1 ) {
    HP = 0.0024;
    // sd
    }
  else if( body == 2 ) {
    OB = -0.0017*COS( H );
    // HP
    sd = 0.2724*HP;
    }

  // Parallax
  if( body == 1  ||  body == 2 ) {
    PA = HP*COS( H )+ OB;
    if( limb == 2 ) sd = -sd;
    }

  // Observed Altitude
  HO = H - R + PA + sd;

  return( HO ); // [deg]
}
```

A5. References

1. G. G. Bennett, 1982, Journal of the Institute of Navigation, volume 35, page 255.
2. Compact Data for Navigation and Astronomy for the Years 1991-1995, Cambridge University Press ISBN 0-521-38731-0. Yallop B. D. and Hohenkerk C. Y. (1991).
3. Compact Data for Navigation and Astronomy for 1996 to 2000. ISBN 0-11-772467-X. Yallop B. D. and Hohenkerk C. Y. (1995), The Stationery Office.
4. NavPac and Compact Data 2001–2005. ISBN 011-887311-3. HM Nautical Almanac Office. Stationery Office
5. NavPac and Compact Data 2006–2010. ISBN 011-887331-8. HM Nautical Almanac Office. Stationery Office
6. The Nautical Almanac. USNO

(F1) SEXTANTE con pie metálico, siglo XVIII. Imágenes cedidas por el Real Instituto y Observatorio de la Armada en San Fernando, Cádiz, España. (Nº INV. ROA: 0026/I) http://www.roa.es/Biblioteca/exposiciones/exposiciones.html