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Adaptive Kalman Filtering for Low-cost INS/GPS

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GPS and low-cost INS sensors are widely used for positioning and attitude determination applications. Low-cost inertial sensors exhibit large errors that can be compensated using position and velocity updates from GPS. Combining both sensors using a Kalman filter provides high-accuracy, real-time navigation. A conventional Kalman filter relies on the correct definition of the measurement and process noise matrices, which are generally defined *a priori* and remain fixed throughout the processing run. Adaptive Kalman filtering techniques use the residual sequences to adapt the stochastic properties of the filter on line to correspond to the temporal dependence of the errors involved. This paper examines the use of three adaptive filtering techniques. These are artificially scaling the predicted Kalman filter covariance, the Adaptive Kalman Filter and Multiple Model Adaptive Estimation. The algorithms are tested with the GPS and inertial data simulation software. A trajectory taken from a real marine trial is used to test the dynamic alignment of the inertial sensor errors. Results show that on line estimation of the stochastic properties of the inertial system can significantly improve the speed of the dynamic alignment and potentially improve the overall navigation accuracy and integrity.

KEY WORDS

1. Integration. 2. GPS. 3. INS.

1. INTRODUCTION.

GPS and Inertial Navigation Systems (INS) are widely used for positioning and attitude determination applications. A range of low-cost Inertial Motion Units (IMU) are available, but these inertial sensors exhibit large errors that must be compensated. The inertial sensor errors can be categorised primarily as first order biases, scale factors and misalignments. Conventional Kalman filtering using GPS position, velocity and, when available, attitude updates can provide estimates of these errors.

To achieve the best performance from the Kalman filter, both the dynamic model and the stochastic information provided to the filter must be accurate. The standard inertial navigation system error model described in texts such as Britting (1971) and Titterton and Weston (1997) is considered to be adequate for this research. However, the stochastic model is recognised to be limited when using *a priori* statistics to model errors that have time varying characteristics (Mohamed and Schwarz, 1999). To achieve the best performance from a Kalman filter, it is therefore necessary to adapt the stochastic model to accommodate for changes in vehicle dynamics and environmental conditions. This paper examines the use of adaptive Kalman filtering algorithms for improving Kalman filter performance.

An example of the time varying nature of the errors involved is highlighted through the initialisation of the inertial sensor error states. When estimating sensor errors, a low process noise variance will result in a precise yet most likely biased estimate. This will result in a long transition to the correct error estimate. Conversely, a larger *a priori* estimate of process noise will result in a quicker transition to the correct error estimate but will result in a less precise estimate. By adapting the process noise matrix in the Kalman filter, both characteristics can be utilized to result in a quick transition to a precise unbiased estimate.

Several methods exist for adapting the measurement and process noise matrices in a Kalman filter. The Adaptive Kalman Filter (AKF) was used in Mohamed and Schwarz (1999) for integrating navigation grade INS measurements with GPS. Mohamed and Schwarz showed that the AKF resulted in a 20% (RMS) improvement in attitude estimation (Mohamed, 1999). Wang *et al.* (1999) used adaptive Kalman filtering to estimate the GPS measurement noise with a tactical grade INS, specifically to improve ambiguity resolution.

Another method used to adapt the weighting between the INS and GPS measurements is to adaptively scale the Kalman filter predicted covariance matrix. Hu *et al.* (2001) described the use of this method for reducing the dynamic modelling errors for filtering DGPS measurements.

The final method considered here is Multiple Model Adaptive Estimation (MMAE) that selects the best state estimate from a bank of simultaneously operating Kalman filters. Previously, MMAE has not been examined due to the computational load caused by running simultaneous filters; however, advances in processor technology now makes this less of a limitation. This paper examines the use of these adaptive filtering techniques with Real Time Kinematic (RTK) GPS and low-cost inertial sensors.

2. ADAPTIVE KALMAN FILTERING ALGORITHMS.

A number of adaptive Kalman filtering techniques exist to achieve the criteria described in adapting the stochastic properties of the Kalman Filter. Most techniques use the innovation sequence as the basis for adapting the measurement noise covariance, R_k, and the process noise covariance, Q_k. These methods use a windowing function on the most recent innovations. Correct identification of the window size also needs to be identified to obtain the correct balance between filter adaptivity and stability. Multiple Model Kalman filtering is also described as a potential method for filter adaptation.

2.1. *Covariance Scaling.* The covariance scaling method was used in Hu *et al.* (2001) for improving the stochastic modelling of differential pseudo-range GPS. The predicted covariance, $P_{k-1}^{(+)}$, is artificially scaled by the factor $S_k > 1$ to apply more weight to the measurements:

$$P_k^{(+)} = S_k(\Phi P_{k-1}^{(-)} \Phi^T + Q_{k-1}). \tag{1}$$

Different techniques can be used in estimating S_k. For example, *a priori* methods can be used in alignment of low-cost IMUs, where it is known that the inertial sensor errors will be larger before the system has been aligned. Another proposed method is presented in Hu *et al.* (2001), which uses an algorithm based on the magnitude of the predicted residuals.

2.2. *Adaptive Kalman Filter*. The principle of the Adaptive Kalman Filter (AKF) is to make the Kalman filter residuals consistent with their theoretical co-variances (Mehra, 1972). An estimate of the covariance of the innovation residual is obtained by averaging the previous residual sequence over a window length N:

$$C_{v_k^{(-)}} = \frac{1}{N} \sum_{j=k-N+1}^{k} v_j^{(-)} (v_j^{(-)})^T, \tag{2}$$

where: $v_j^{(-)} = z_k - H_k x_k^{(-)}$ is the innovation residual from the Kalman filter. The estimated measurement noise is computed by comparing the theoretical covariance $(H_k P_k^{(-)} H_k^T + R_k)$ with the estimated covariance to give:

$$\hat{R}_k = C_{v_k^{(-)}} - H_K P_k^{(-)} H_k^T. \tag{3}$$

The estimated process noise can also be obtained (see Wang *et al.* (1999)) to give:

$$\hat{Q}_k = \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta x_j \Delta x_j^T + P_k^{(+)} - \Phi P_{k-1}^{(+)} \Phi^T, \tag{4}$$

where: $\Delta x_k = x_k^{(-)} - x_k^{(+)}$. This is known as a residual based estimate. Equation (4) can be written in terms of the innovation sequence by making the following substitution for the covariance of the state corrections (Mohamed and Schwarz, 1999),

$$\hat{C}_{\Delta x_k} \approx \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta x_j \Delta x_j^T \approx K_k \hat{C}_{v_k^{(-)}} K_k^T. \tag{5}$$

For a full derivation of these equations using the maximum likelihood method, see Mohamed (1999). These equations result in a full variance/covariance matrix that attempts to model some of the inherent correlations.

2.3. *Multiple Model Adaptive Estimation.* Multiple Model Adaptive Estimation (MMAE) uses multiple Kalman filters that run simultaneously, in this case each using different stochastic properties (Brown and Hwang, 1997). The correct model is identified using the residual probability density function. MMAE has not previously been considered for navigation applications due to the substantial increase in processor load caused by running simultaneous Kalman filters. Recent improvements in processor speed reduce this potential problem.

2.3.1. *MMAE Algorithm.* For each filter, the probability density function, $f_n(z_k)$, for the nth Kalman filter is given by:

$$f_n(z_k) = \frac{1}{\sqrt{(2\pi)^m |C_{v_k^{(-)}}|}} e^{-\frac{1}{2} v_k C_{v_k^{(-)}}^{-1} v_k^T}, \tag{6}$$

where: m is the number of measurements. The probability $p_n(k)$ that the nth model is correct is computed from the recursive formula:

$$p_n(k) = \frac{f_n(z_k) \cdot p_n(k-1)}{\sum_{j=1}^{N} f_j(z_k) \cdot p_j(k-1)}, \tag{7}$$

for N Kalman filters. The optimal state estimate is computed using the weighted combination of states using:

$$\hat{x}_k^{(+)} = \sum_{j=1}^{N} p_j(k)\hat{x}_{k_j}^{(+)}. \tag{8}$$

This results in the identification of a single correct model for which $p_n(k)$ will converge to unity, and the other models to zero. As a result, the filter will ignore new observations; this deficiency can be overcome in a variety of ways. For example, a threshold value can be defined for $p_n(k)$, or the smallest value for $f_n(z_k)$ could be used to identify the best model at each epoch (Welch and Bishop, 2001).

3. SOFTWARE.

The software used to process the measurements was developed at the Institute of Engineering Surveying and Space Geodesy (IESSG) and is called KinPos^i. KinPos^i processes raw GPS and INS measurements using centralised or decentralised Kalman filtering. The software can process dual-frequency pseudo-range, carrier-phase and doppler GPS observations and uses the LAMBDA ambiguity resolution method (De Jonge and Tiberius, 1996). The filter contains nine navigation states and up to 24 inertial error states to estimate sensor bias, scale factor and axis non-orthogonality errors. KinPos^i provides the capability for all the adaptive Kalman filtering methodologies described in the previous section.

4. DATA SIMULATION.

Data simulation is useful in assessing the performance of the algorithms described, as it allows direct control over all the error parameters. Using simulated data for inertial measurements provides truth not only for navigation errors, but also for sensor errors that can be compared to the Kalman filter estimates. The IESSG's Navigation System Simulator (NSS) was developed for the European Space Agency (ESA) Low-Cost Navigator (LCN) project.

NSS creates raw GPS and inertial measurements from a kinematic trajectory file containing position and attitude measurements. The simulator creates L1 and L2 pseudo-range, carrier and doppler measurements using ionospheric, tropospheric and multipath models; satellite and receiver clock errors; satellite orbit errors; and measurement noise. Cycle slips and unhealthy satellites can also be modelled. The simulator uses an SP3 format file to provide ephemeris data for any time period. The inertial measurements are simulated using the models described in Figures 1 and 2. The body frame angular rate and accelerations are computed from the trajectory input, which is interpolated at the high rate of the inertial measurements.

Table 1 describes the inertial errors that are simulated and their respective values, which correspond to a typical low-cost MEMS sensor. The trajectory simulated is taken from a real trajectory obtained using a POS/MV system at a marine trial using a small survey boat in Plymouth, UK. A range of dynamic manoeuvres were performed beginning with figure-of-eight paths for alignment. The 15-minute trajectory used in this paper corresponds to the alignment section of the trajectory. The initial attitude misalignment is 0·6°, −1·6°, 2·0° for roll, pitch and yaw respectively, which corresponds to the level error of the boat on the water, and a heading error which would be obtained from, for example, the boat's on-board compass. The IMU measurements are updated at 100 Hz. The Kalman filter is run in loosely coupled mode with GPS updates at 1 Hz.

Table 1. Inertial Navigation Simulation Specification.

	Acceleration (m/s^{-2})	Gyro (°/s)
Bias	$< \pm 0{\cdot}3$	$< \pm 2{\cdot}0$
Noise	$2{\cdot}8 \times 10^{-2}$	$8{\cdot}5 \times 10^{-2}$
Scale Factor	$< 1{\cdot}0\%$	$< 1{\cdot}0\%$
Saturation	± 2	± 100
Quantisation	$1{\cdot}4 \times 10^{-2}$	$2{\cdot}5 \times 10^{-2}$
Misalignment	$3{\cdot}49 \times 10^{-4}$ rad	$9{\cdot}95 \times 10^{-4}$ rad
Cross Axis Sensitivity	$1{\cdot}0\%$	—

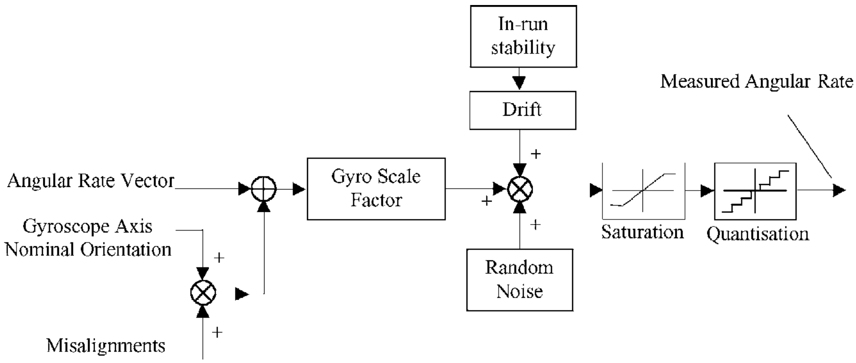


Figure 1. Gyro simulation model.

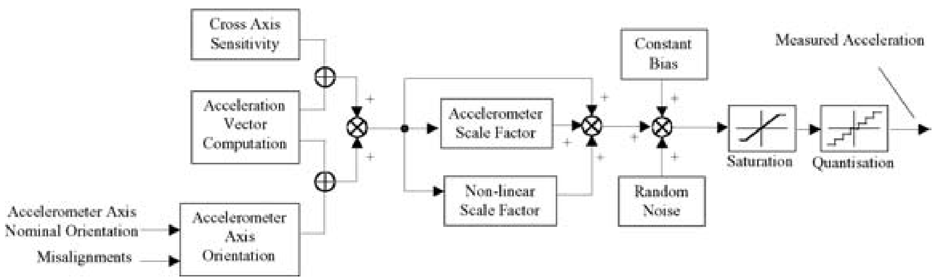


Figure 2. Accelerometer simulation model.

The initial position and velocity of the inertial system is obtained directly from the GPS estimates.

5. CONVENTIONAL KALMAN FILTERING.

For marine applications, it is rarely possible to estimate the initial errors of the inertial sensor using static initialisation techniques. Low-cost inertial sensor errors vary widely from switch-on to switch-on; therefore the errors have to be estimated using a dynamic alignment. Figure 3 shows the Kalman filter x-axis gyro bias estimate during alignment using different gyro bias process noise estimates in the Kalman filter.

It can be seen from Figure 3 that the larger process noise values at the start of the processing run result in a quicker alignment. The smaller process noise values result in the filter ignoring GPS updates resulting in a very slow dynamic alignment, or even

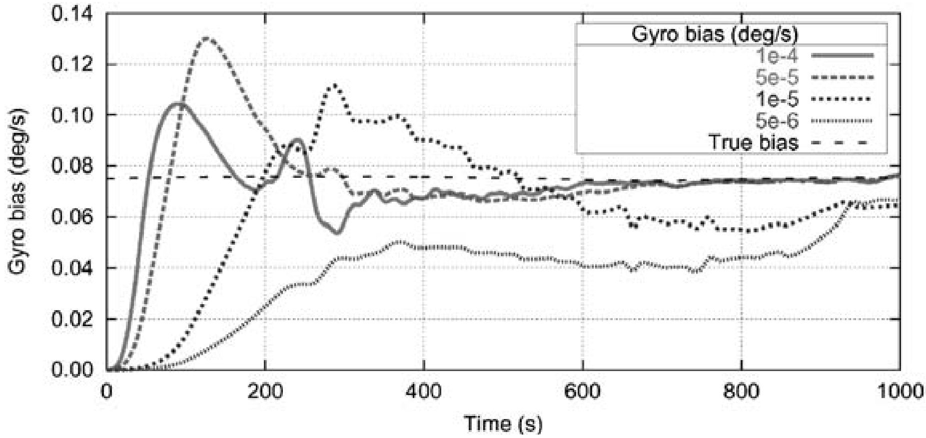


Figure 3. X-axis gyro bias using different gyro process noise.

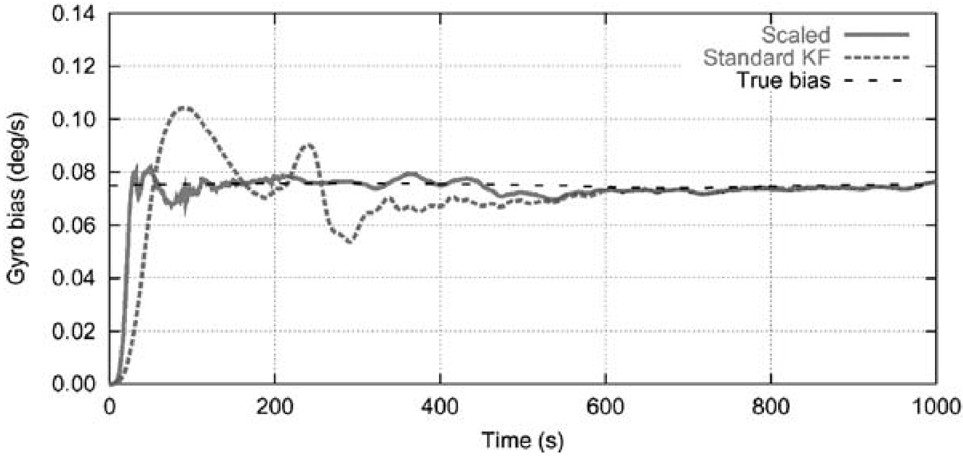
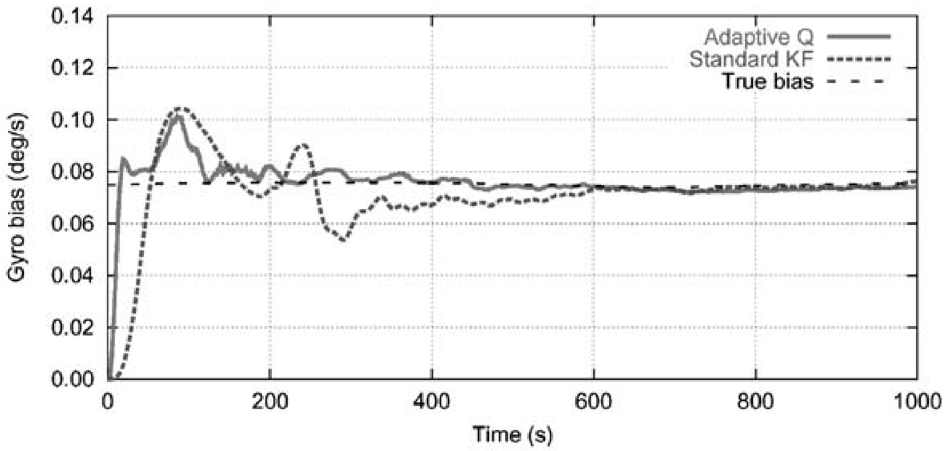


Figure 4. Alignment using scaled predicted covariance.

divergence of the filter, if the inertial errors remain uncompensated. Once the approximate alignment has been obtained, further analysis of the results identifies that the lower process noise estimates result in better performance.

6. COVARIANCE SCALING. For the predicted covariance scaling approach, the Kalman filter is set up using the process noise error estimates used in the conventional Kalman filter. Figure 4 shows the effect of artificially inflating the process noise measurements using *a priori* scale factors. Four empirical scale factors, $S_k = 1{\cdot}2$, 1·15, 1·1 and 1·05 were used with a decrease in scale factor every 50 seconds. After this, conventional Kalman filtering is applied. It can be observed from Figure 4 that the bias estimate is obtained more precisely in a faster time than using the conventional filter. However, the coefficients were obtained empirically with full truth data available for all the errors involved.



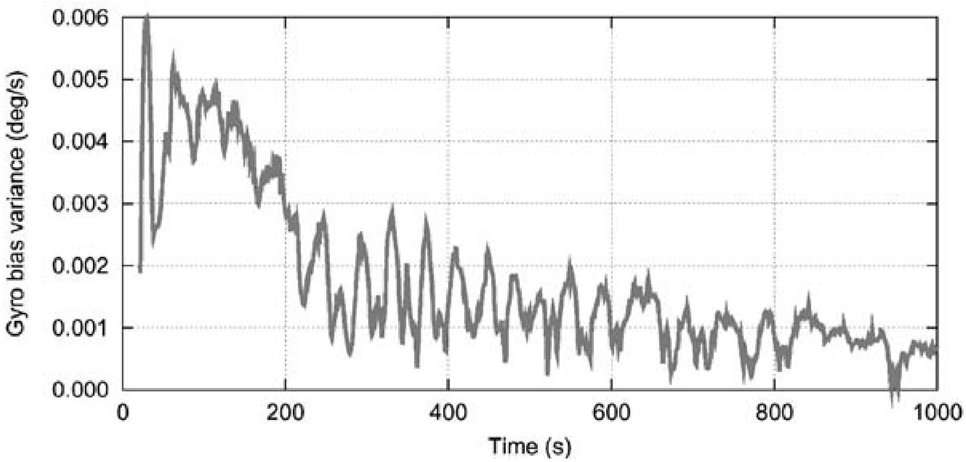


Figure 5. Adaptive Q_k estimate of x-gyro bias and corresponding variance.

7. ADAPTIVE KALMAN FILTERING. The adaptive Kalman filter potentially provides a self-tuning method for adapting the covariance matrices of the filter. Equation (3) provides a method for adapting the measurement noise matrix which, in an integrated system, corresponds to the GPS error. For this paper, the innovation-based process noise estimate (Equation (5) substituted into Equation (4)) was used to estimate the inertial errors. Figure 5 shows the adaptive estimate of the x-axis gyro bias estimate. The gyro estimate is obtained using a window length of 20 epochs, which provides a balance between filter stability and adaptivity. The adaptive filter requires a number of epochs in which to accumulate the innovations used to estimate the covariance of the innovations. Consequently, the first 20 epochs are provided by conventional Kalman filtering with an *a priori* estimate of the gyro bias. After the 20 epochs are accumulated, the adaptive estimate of Q_k is used. Figure 5 demonstrates that the adaptive Kalman filter provides a better estimate than

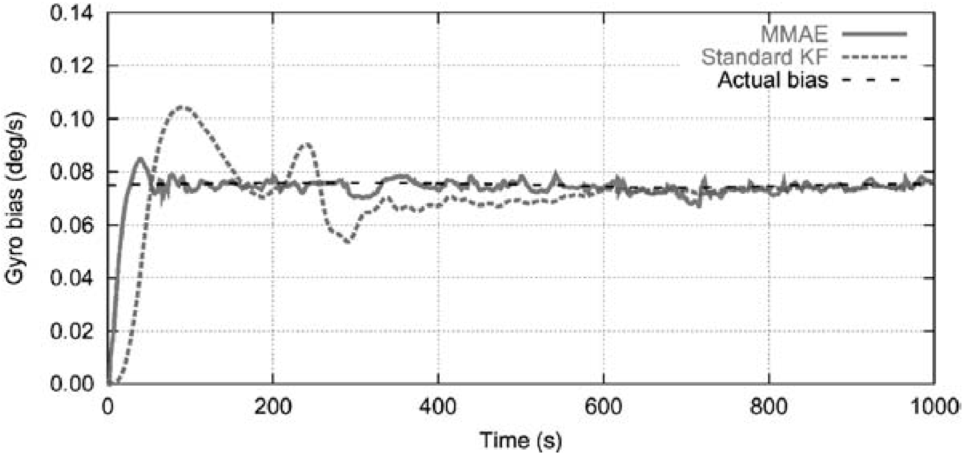


Figure 6. X-axis gyro bias using MMAE.

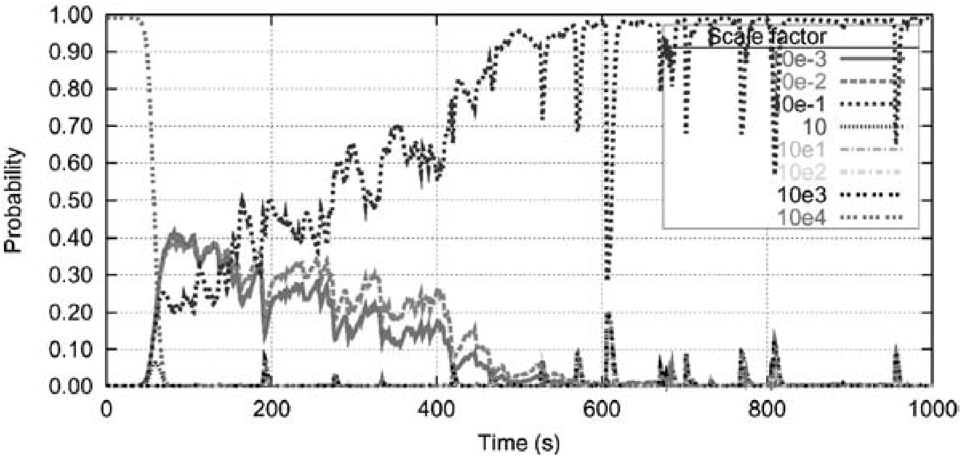


Figure 7. Estimate of $p_n(k)$ during alignment.

the standard Kalman filter. The adaptive filter settles to a precise estimate of the gyro bias in just over 100 seconds. Similar results were obtained for resolution of the other sensor bias states.

Also shown in Figure 5 is the corresponding variance estimate provided by the adaptive filter. It can be observed from the estimated gyro variance that the overall magnitude of the estimate reduces as the system becomes aligned. The fluctuations in the gyro bias variance estimate are caused by the dynamics of the trajectory. This is due to factors such as integration error and uncompensated scale factor error. This shows that the adaptive filter is continually estimating and adapting the stochastic properties of the inertial measurements.

The disadvantages of the adaptive Kalman filter are primarily related to filter stability. Potentially, the estimate of Q_k provides a full variance/covariance matrix. However, this estimate produced a divergent filter estimate. Consequently only the variance estimates were used to form the estimated process noise matrix.

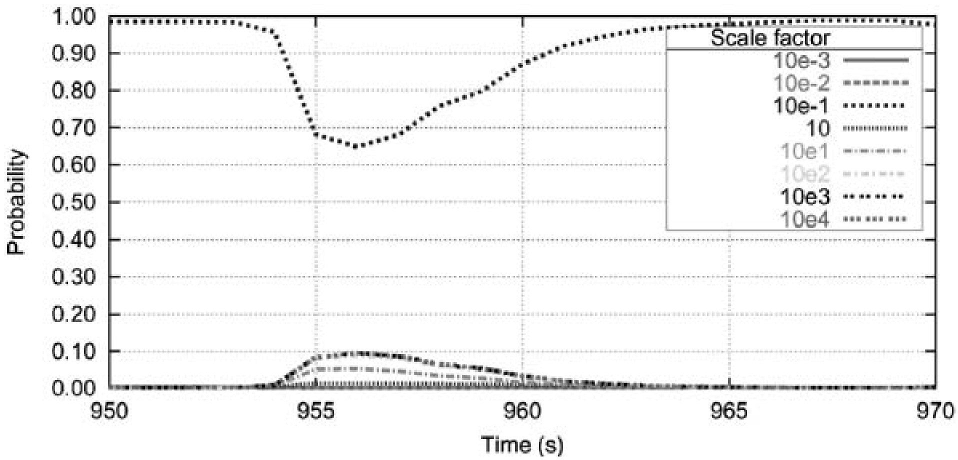


Figure 8. Adaptive nature of MMAE during turn.

While the results obtained have produced stable estimates, further analysis would be required to ensure filter robustness. Furthermore, due to the recursive nature of the filter estimate, the performance of the filter is dependent on the *a priori* estimate of Q_k used in the initialisation of the adaptive filter. This means that the adaptive filter is not entirely self-tuning.

8. MULTIPLE MODEL ADAPTIVE ESTIMATION. To test the MMAE filtering approach, eight parallel Kalman filters were run with each filter using a different process noise matrix. Each process noise matrix was formed from the matrix used in the conventional Kalman filter scaled by a coefficient S_n. The factors used were in powers of ten from 10^{-3} to 10^4. Figure 6 shows the MMAE estimate for the x-gyro bias estimate compared to the conventional filter.

It can be observed from Figure 6 that the x-gyro bias estimate is obtained in approximately 50 seconds to the same accuracy that the conventional filter obtains after 500 seconds. Similar results were obtained for the resolution of all sensor biases. Figure 7 shows the values for the probability coefficients $p_n(k)$ obtained from Equation 7. The filter is initialised with each model having equal probability. To force the filter to remain adaptive, a threshold probability coefficient of $2{\cdot}5 \times 10^{-3}$ was used. At the first epoch, the filter with the largest scale factor is immediately identified as the best estimate with a corresponding probability of 0·99. As the system becomes aligned, the stochastic properties of the filter change, and consequently the probability coefficients adapt to use different models. As the alignment is refined, the scale factor $S_n = 1$ is identified as the best estimate.

It can be observed in Figure 7 that, although $S_n = 1$ is the predominant filter, there are points where weighted measurements from the other filters are being used. Figure 8 shows a close up of one of these points that corresponds to a turn performed in the boat. This shows, in a similar way to the adaptive Kalman filter, that the inertial navigation errors change during manoeuvres due to factors such as integration error and uncompensated errors such as scale factors or cross-axis sensitivity. This demonstrates the MMAE is capable of providing the adaptive characteristics that are necessary to obtain maximum performance from the Kalman filter.

The primary limitation of MMAE is the increased computational burden imposed by the parallel filters. For the loosely coupled integration filter, the standard Kalman Filter running on a 1·4 GHz PC under the Linux operating system takes 0·01 seconds per epoch. For the MMAE filter running eight parallel Kalman filters, processing takes 0·04 seconds per epoch. Such an increase in processing time can be considered negligible for many real-time applications. It has also been noted that the some of the filter estimates are not used in the computation of the state vector, and further analysis of these results can reduce the number of filters used. The MMAE filter provides a method for adapting the stochastic properties that is self-tuning. However, the multiple models have to be estimated initially, and the performance of the filter will be dependent on the definition of these models. MMAE further provides a potentially more robust filter where incorrect error models are effectively ignored.

9. CONCLUSIONS. This paper has shown that there are many different ways in which a Kalman filter can be configured. The innovation and residual sequences provide a useful performance indicator that can be used to adaptively tune the stochastic information used in the filter. This paper has shown the use of adaptive filtering techniques to improve the speed of the dynamic alignment of a MEMS IMU with RTK GPS for a marine application. Initial MEMS sensor errors can be large and require fast resolution for the filter to remain stable. Also demonstrated was that alignment can be obtained in approximately 50–100 seconds using adaptive methods compared to greater than 500 seconds for the conventional Kalman filter.

Work is currently being undertaken at the IESSG to validate the results with data collected at the Plymouth trial, which was used to simulate the data used in this paper. Initial results indicate that the adaptive methods translate well to real data collected with a Crossbow MEMS IMU.

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