Correspondence

Iterated Multiplicative Extended Kalman Filter for Attitude Estimation Using Vector Observations

This paper proposes an iterated multiplicative extended Kalman filter (IMEKF) for attitude estimation using vector observations. In each iteration, the vector-measurement model is relinearized based on a new reference quaternion refined by the attitude-error estimate. An implicit reset operation on the attitude error is performed in each iteration to obtain the refined quaternion. With only a little additional computation burden, the IMEKF can much improve on the performance of the MEKF. For large initialization errors, the IMEKF performs even better than the unscented quaternion estimator but with much smaller computational burden. Numerical results are reported to validate its effectiveness and prospect in spacecraft attitude-estimation applications.

I. INTRODUCTION

Spacecraft attitude estimation refers to the process of estimating a spacecraft's orientation from noisy measurement data and known reference observations [1, 2]. Attitude-estimation methods generally fall into two categories: deterministic ones based on solutions to Wahba's problem and recursive stochastic estimators that statistically combine measurements and kinematic models [1–12]. A related advantage of the recursive stochastic estimators is their ability to estimate some parameters other than the attitude, say sensor misalignments and rate-gyro biases. Moreover, recursive estimation methods can generally provide a more accurate attitude estimate than deterministic methods due to their incorporation of memory of past observations, and they have thus been investigated intensely for attitude estimation. Kalman-like filtering estimation methods are in this category.

For spacecraft attitude representation, the quaternion is preferred over other representations due to the bilinear nature of its kinematics and its singularity-free property [2]. However, when applied in attitude-estimation filters, its unity norm constraint can be easily destroyed in the standard filtering process. The most common approach to overcome this shortfall involves using the quaternion for global nonsingular attitude representation and a set of unconstrained parameters for local attitude representation and filtering. By far, the extended Kalman filter (EKF) in its multiplicative form has performed admirably in the vast majority of attitude-estimation applications [4, 5]. Nevertheless, when nonnegligible nonlinear effects and

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large initial condition errors are involved, considerable performance degradation may arise from the explicit linearization in the EKF. Continuous research efforts have been conducted to develop advanced filtering algorithms to circumvent the drawbacks of the EKF for attitude estimation, such as the unscented quaternion estimator (USQUE) [6, 7], cubature Kalman filter [8], sparse Gauss–Hermite quadrature filter [9], and particle filter [10], to name a few. For more details about attitude-estimation algorithms through 2006, the reader can refer to the comprehensive and excellent survey paper [3].

Just as Crassidis et al. argued in [3], although various filtering methods considered to be most promising by their own authors have been developed, the multiplicative EKF (MEKF) remains the workhorse of real-time spacecraft attitude estimation due to its relatively simple and flexible characteristics. Moreover, in the face of filtering performance degradation, the designer can use several strategies within the EKF framework to improve performance [13]. With this consideration, the backwards-smoothing EKF [14] and second-order EKF [5] were developed for spacecraft attitude estimation with efforts to address the drawbacks of the original EKF. However, the computational burdens of these methods have hindered their realistic application in engineering.

Iteration has been widely approved as a promising procedure to improve filtering performance with a moderate addition of computational burden [13, 15]. The performance improvement from iteration is due to relinearizing the measurement model about the a posteriori state estimate, which is considered to be more accurate than the a priori estimate. For spacecraft attitude estimation, the nonlinearity mainly lies in the measurement model, which makes the iteration procedure more attractive. Although the concept of iteration is not new, its extension to the MEKF still must be revisited appropriately, due to the special filtering and propagating structure of MEKF. Specifically, the MEKF actually performs an unconstrained estimation of a three-component attitude error, while the measurement model is a nonlinear function of the constrained quaternion, and the relinearization should be performed about the quaternion. In this respect, iteration cannot be extended to the MEKF directly. To our best knowledge, there has not been investigation of an iterated MEKF (IMEKF) for spacecraft attitude estimation using vector observations. These facts represent the main motivation of this paper, which will focus on investigating the iterated procedure with respect to the MEKF for spacecraft attitude estimation using vector observations. The aim is to propose an IMEKF with performance improvement but moderate addition of computational burden.

The rest of the paper is organized as follows. Section II mathematically formulates the problem of spacecraft attitude estimation. The explicit filtering procedure of the MEKF is presented in Section III, which is used as fundamental material for the following developed IMEKF. Section IV is devoted to the ingenious development of the IMEKF. Section V reports simulation results of the algorithm and compares it with existing methods. Finally, conclusions are drawn in Section VI.

II. PROBLEM FORMULATION

A. Attitude-Kinematics Model

Assume that the spacecraft attitude is represented by a quaternion denoted by $\boldsymbol{q} = [\,\boldsymbol{\rho}^{\mathrm{T}} \;\; \boldsymbol{q}_4\,]^{\mathrm{T}}$, with $\boldsymbol{\rho} = [\,\boldsymbol{q}_1 \;\; \boldsymbol{q}_2 \;\; \boldsymbol{q}_3\,]^{\mathrm{T}}$ being the vector component. The quaternion kinematics equation is given by

$$\dot{\boldsymbol{q}}(t) = \frac{1}{2}\Xi\left[\boldsymbol{q}(t)\right]\boldsymbol{\omega}(t), \tag{1}$$

with

$$\Xi[\boldsymbol{q}] = \begin{bmatrix} \boldsymbol{q}_4 \boldsymbol{I}_{3\times3} + [\boldsymbol{\rho}\times] \\ -\boldsymbol{\rho}^{\mathrm{T}} \end{bmatrix}.$$

In this case, $\boldsymbol{I}_{3\times3}$ is a 3×3 identity matrix and $[\boldsymbol{\rho}\times]$ is the cross-product matrix, which is given by

$$[\boldsymbol{\rho}\times] = \begin{bmatrix} 0 & -\boldsymbol{q}_3 & \boldsymbol{q}_2 \\ \boldsymbol{q}_3 & 0 & -\boldsymbol{q}_1 \\ -\boldsymbol{q}_2 & \boldsymbol{q}_1 & 0 \end{bmatrix}.$$

The corresponding discrete-time kinematic equation of (1) is given by

$$\boldsymbol{q}_k = \Omega(\boldsymbol{\omega}_{k-1})\,\boldsymbol{q}_{k-1}, \tag{2}$$

with

$$\Omega(\boldsymbol{\omega}_{k-1}) = \begin{bmatrix} \boldsymbol{Z}_{k-1} & \boldsymbol{\psi}_{k-1} \\ -\boldsymbol{\psi}_{k-1}^{\mathrm{T}} & \cos\left(0.5\,\|\boldsymbol{\omega}_{k-1}\|\,\Delta t\right) \end{bmatrix}$$

$$\boldsymbol{Z}_{k-1} = \cos\left(0.5\,\|\boldsymbol{\omega}_{k-1}\|\,\Delta t\right)\boldsymbol{I}_{3\times3} - [\boldsymbol{\psi}_{k-1}\times]$$

$$\boldsymbol{\psi}_{k-1} = \sin\left(0.5\,\|\boldsymbol{\omega}_{k-1}\|\,\Delta t\right)\boldsymbol{\omega}_{k-1}\big/\|\boldsymbol{\omega}_{k-1}\|,$$

where Δt is the sampling time interval in the gyro.

In the attitude kinematics model, the angular velocity is measured by rate-integrating gyro. For such a sensor, a widely used model is given by

$$\tilde{\boldsymbol{\omega}}(t) = \boldsymbol{\omega}(t) + \boldsymbol{\beta}(t) + \boldsymbol{\eta}_v(t) \tag{3a}$$

$$\dot{\boldsymbol{\beta}}(t) = \boldsymbol{\eta}_u(t), \tag{3b}$$

where $\tilde{\boldsymbol{\omega}}(t)$ and $\boldsymbol{\omega}(t)$ are the measured angular velocity and the true angular velocity, respectively; $\boldsymbol{\beta}(t)$ is the gyro bias; and $\boldsymbol{\eta}_v(t)$ and $\boldsymbol{\eta}_u(t)$ are zero-mean Gaussian white-noise process noise with covariances $\sigma_v^2\boldsymbol{I}_{3\times3}$ and $\sigma_u^2\boldsymbol{I}_{3\times3}$, respectively.

B. Vector-Observation Model

The vector observations measured by a single sensor are given by

$$\boldsymbol{b}_m = A(\boldsymbol{q})\,\boldsymbol{r}_m + \boldsymbol{v}_m, \tag{4}$$

where $\boldsymbol{b}_m$ is the mth measurement vector, $\boldsymbol{r}_m$ is the mth known reference vector, $\boldsymbol{v}_m$ is zero-mean Gaussian white measurement noise with covariances $\sigma_m^2 \boldsymbol{I}_{3\times 3}$, and $A(\boldsymbol{q})$ is the rotation matrix, given by

$$A(\boldsymbol{q}) = \left[q_4^2 - \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\rho}\right]\boldsymbol{I}_{3\times 3} + 2\boldsymbol{\rho}\boldsymbol{\rho}^{\mathrm{T}} - 2q_4\left[\boldsymbol{\rho}\times\right]. \tag{5}$$

Multiple (n) vector measurements can be concatenated to form

$$\boldsymbol{y}_k = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix}_k = h\left(\boldsymbol{q}_k\right) = \begin{bmatrix} A(\boldsymbol{q})\boldsymbol{r}_1 \\ A(\boldsymbol{q})\boldsymbol{r}_2 \\ \vdots \\ A(\boldsymbol{q})\boldsymbol{r}_n \end{bmatrix}_k + \begin{bmatrix} \boldsymbol{v}_1 \\ \boldsymbol{v}_2 \\ \vdots \\ \boldsymbol{v}_n \end{bmatrix}_k. \tag{6}$$

The corresponding concatenated measurement noise is given by $\boldsymbol{R}_k = \mathrm{diag}([\,\sigma_1^2\boldsymbol{I}_{3\times 3}\ \ \sigma_2^2\boldsymbol{I}_{3\times 3}\ \ \ldots\ \ \sigma_n^2\boldsymbol{I}_{3\times 3}\,])$.

III. MEKF

The MEKF is the extension of the EKF to the quaternion attitude-estimation problem, with a main focus on preserving the quaternion norm constraint. In the MEKF, the quaternion is used for attitude propagation, and the unconstrained three-dimensional parameterization—say, the Euler angle—is used for local attitude-error representation and filtering. The MEKF actually performs an unconstrained estimation of a three-component attitude error, with the quaternion playing the role of a reference to define the corresponding error. The derivation of the MEKF has been reported in many fundamental books [1, 2]; here we present the explicit filtering procedures in discrete-time form directly.

In the time update of the MEKF, the discrete error-state transition matrix is given by

$$\boldsymbol{\Phi}_{k-1} = \begin{bmatrix} \boldsymbol{\Phi}_{k-1,11} & \boldsymbol{\Phi}_{k-1,12} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{I}_{3\times 3} \end{bmatrix},$$

with

$$\begin{aligned}\boldsymbol{\Phi}_{k-1,11} = \boldsymbol{I}_{3\times 3} &- [\hat{\boldsymbol{\omega}}_{k-1}\times]\sin(\|\hat{\boldsymbol{\omega}}_{k-1}\Delta t\|)/\|\hat{\boldsymbol{\omega}}_{k-1}\| \\ &+ [\hat{\boldsymbol{\omega}}_{k-1}\times]^2(1-\cos(\|\hat{\boldsymbol{\omega}}_{k-1}\Delta t\|))/\|\hat{\boldsymbol{\omega}}_{k-1}\|^2\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Phi}_{k-1,12} = [\hat{\boldsymbol{\omega}}_{k-1}\times]&(1-\cos(\|\hat{\boldsymbol{\omega}}_{k-1}\Delta t\|))/\|\hat{\boldsymbol{\omega}}_{k-1}\|^2 \\ &- \boldsymbol{I}_{3\times 3}\Delta t + [\hat{\boldsymbol{\omega}}_{k-1}\times]^2(\|\hat{\boldsymbol{\omega}}_{k-1}\|\,\Delta t \\ &- \sin(\|\hat{\boldsymbol{\omega}}_{k-1}\Delta t\|))/\|\hat{\boldsymbol{\omega}}_{k-1}\|^3.\end{aligned}$$

The discrete process-noise covariance is given by

$$\boldsymbol{Q}_{k-1} = \begin{bmatrix} \left(\sigma_v^2\Delta t + \sigma_u^2\Delta t^3/3\right)\boldsymbol{I}_{3\times 3} & -\left(\sigma_u^2\Delta t^2/2\right)\boldsymbol{I}_{3\times 3} \\ -\left(\sigma_u^2\Delta t^2/2\right)\boldsymbol{I}_{3\times 3} & \left(\sigma_u^2\Delta t\right)\boldsymbol{I}_{3\times 3} \end{bmatrix}.$$

The corresponding process-noise transition matrix is given by

$$\boldsymbol{G}_{k-1} = \begin{bmatrix} -\boldsymbol{I}_{3\times 3} & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & \boldsymbol{I}_{3\times 3} \end{bmatrix}.$$

REMARK 1 It is shown in Table I that the estimated value in each filtering recursion of the MEKF is actually the unconstrained attitude error, while the quaternion is used to globally propagate the attitude. The fact that the dynamic model is the function of the attitude quaternion, while the filtering state is the unconstrained attitude error, is precisely the reason why the traditional iterated procedure cannot be applied in the MEKF directly.

TABLE I
MEKF

Initialization set values $\hat{\boldsymbol{q}}_0$, $\hat{\boldsymbol{\beta}}_0$, and $\boldsymbol{P}_0$
Recursive update for $k = 1 : K$ (K, total time steps)
Time update
$\hat{\boldsymbol{q}}_{k|k-1} = \Omega(\hat{\boldsymbol{\omega}}_{k-1})\hat{\boldsymbol{q}}_{k-1}$
$\hat{\boldsymbol{\beta}}_{k|k-1} = \hat{\boldsymbol{\beta}}_{k-1}$
$\hat{\boldsymbol{\omega}}_{k-1} = \tilde{\boldsymbol{\omega}}_{k-1} - \hat{\boldsymbol{\beta}}_{k-1}$
$\boldsymbol{P}_{k|k-1} = \boldsymbol{\Phi}_{k-1}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_{k-1}^{\mathrm{T}} + \boldsymbol{G}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{G}_{k-1}^{\mathrm{T}}$
Measurement update
$$h_k(\hat{\boldsymbol{x}}_{k|k-1}) = \begin{bmatrix} A(\hat{\boldsymbol{q}}_{k|k-1})\boldsymbol{r}_1 \\ \vdots \\ A(\hat{\boldsymbol{q}}_{k|k-1})\boldsymbol{r}_n \end{bmatrix},\ \boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k|k-1}) \begin{bmatrix} A(\hat{\boldsymbol{q}}_{k|k-1})\boldsymbol{r}_1 & \boldsymbol{0}_{3\times 3} \\ \vdots & \vdots \\ A(\hat{\boldsymbol{q}}_{k|k-1})\boldsymbol{r}_n & \boldsymbol{0}_{3\times 3} \end{bmatrix}$$
$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T(\hat{\boldsymbol{x}}_{k|k-1})[\boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k|k-1})\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T(\hat{\boldsymbol{x}}_{k|k-1}) + \boldsymbol{R}_k]^{-1}$
$\boldsymbol{P}_k = [\boldsymbol{I}_{6\times 6} - \boldsymbol{K}_k\boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k|k-1})]\boldsymbol{P}_{k|k-1}$
$\Delta\hat{\boldsymbol{x}}_k = \boldsymbol{K}_k[\boldsymbol{y}_k - h_k(\hat{\boldsymbol{x}}_{k|k-1})]$
Attitude update
$\Delta\hat{\boldsymbol{x}}_k = \begin{bmatrix}\Delta\hat{\boldsymbol{a}}_k^T & \Delta\hat{\boldsymbol{\beta}}_k^T\end{bmatrix}$
$\hat{\boldsymbol{q}}_k = \hat{\boldsymbol{q}}_{k|k-1} + 0.5\Xi(\hat{\boldsymbol{q}}_{k|k-1})\Delta\hat{\boldsymbol{a}}_k$
$\hat{\boldsymbol{\beta}}_k = \hat{\boldsymbol{\beta}}_{k|k-1} + \Delta\hat{\boldsymbol{\beta}}_k$

REMARK 2 There is an implicit reset for the error state occurring after the attitude update, which is used to move the update information from the error state to the full state estimate—i.e., the attitude quaternion. Meanwhile, the a priori error state remains the zero vector during the time update, which is due to the fact that the full state estimate and the error-state estimate are propagated using the same process. The reset operation reflects that the attitude error in the next filtering recursion is calculated about a new and more accurate quaternion. This means that the MEKF is actually a closed-loop corrector for the global attitude quaternion. The reset operation also plays a crucial role in our developed IMEKF that will be presented explicitly in the next section.

IV. IMEKF

In the EKF, the nonlinear measurement model is expanded and truncated around the a priori state estimate, since this is the best estimate before the measurement in this filtering recursion is taken into account. But when the measurement update of the EKF has been implemented, the a posteriori estimate—which is more accurate than the a priori one—is obtained and can be used to relinearize the measurement model. This is the basic idea of the iterated filtering algorithms. For the spacecraft attitude estimation studied in this paper, the measurement model is actually a function of the quaternion. In this respect, the core of developing the IMEKF is to obtain an updated quaternion and reformulate the Taylor-series expansion of the measurement model around the new quaternion.

According to the principle of successive transformations by quaternion, we have

$$A\left(\delta q\left(\Delta\boldsymbol{\alpha}\right)\right)A\left(\boldsymbol{q}\right)=A\left(\delta q\left(\Delta\boldsymbol{\alpha}\right)\otimes\boldsymbol{q}\right),\tag{7}$$

where $\delta q(\Delta\boldsymbol{\alpha})$ denotes the error quaternion corresponding to the unconstrained attitude error $\Delta\boldsymbol{\alpha}$. The attitude-error matrix in terms of the small-angle approximation $\Delta\boldsymbol{\alpha}$ is given by

$$A\left(\delta q\left(\Delta\boldsymbol{\alpha}\right)\right)\approx\boldsymbol{I}_{3\times3}-\left[\Delta\boldsymbol{\alpha}\times\right]-\left(\|\Delta\boldsymbol{\alpha}\|^{2}-\Delta\boldsymbol{\alpha}\Delta\boldsymbol{\alpha}^{\mathrm{T}}\right)\big/2.\tag{8}$$

According to (7) and (8), the vector-measurement model in (4) can be rewritten as

$$\begin{aligned}\boldsymbol{b}_{m}&=A\left(\boldsymbol{q}\right)\boldsymbol{r}_{m}+\boldsymbol{v}_{m}=A\left(\delta q\left(\Delta\boldsymbol{\alpha}\right)\right)A\left(\boldsymbol{q}_{\mathrm{ref}}\right)\boldsymbol{r}_{m}\\&\approx\left(\boldsymbol{I}_{3\times3}-\left[\Delta\boldsymbol{\alpha}\times\right]-\left(\|\Delta\boldsymbol{\alpha}\|^{2}-\Delta\boldsymbol{\alpha}\Delta\boldsymbol{\alpha}^{\mathrm{T}}\right)\big/2\right)\\&\quad\times A\left(\boldsymbol{q}_{\mathrm{ref}}\right)\boldsymbol{r}_{m},\end{aligned}\tag{9}$$

where $\boldsymbol{q}_{\mathrm{ref}}$ is some chosen reference unit quaternion and $\delta q(\Delta\boldsymbol{\alpha})$ is the difference between the reference quaternion and the true quaternion.

Denote $h(\boldsymbol{b}_m)=A(\boldsymbol{q})\boldsymbol{r}_m$ and $\hat{\boldsymbol{b}}_m=A(\boldsymbol{q}_{\mathrm{ref}})\boldsymbol{r}_m$. The vector-measurement model (9) can then be expanded about the reference vector and truncated as

$$\begin{aligned}h\left(\boldsymbol{b}_{m}\right)&\approx\hat{\boldsymbol{b}}_{m}-\left.\frac{\partial h\left(\boldsymbol{b}_{m}\right)}{\partial\boldsymbol{b}_{m}}\right|_{\hat{b}_{m}}\left[\Delta\boldsymbol{\alpha}\times\right]\hat{\boldsymbol{b}}_{m}\\&=A\left(\boldsymbol{q}_{\mathrm{ref}}\right)\boldsymbol{r}_{m}+\left.\frac{\partial h\left(\boldsymbol{b}_{m}\right)}{\partial\boldsymbol{b}_{m}}\right|_{\hat{b}_{m}}\left[\hat{\boldsymbol{b}}_{m}\times\right]\Delta\boldsymbol{\alpha}.\end{aligned}\tag{10}$$

For the MEKF, the reference quaternion is selected as the a priori quaternion estimate—that is, $\hat{\boldsymbol{q}}_{k|k-1}$. Meanwhile, the error state is the zero vector due to the reset operation—that is, $\Delta\boldsymbol{\alpha}=0$. In this respect, the predicted measurement is actually

$$h\left(\boldsymbol{b}_{m}\right)=A\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\boldsymbol{r}_{m}.\tag{11}$$

When the iterated procedure has been implemented, assume that the a posteriori quaternion estimate $\hat{\boldsymbol{q}}_{k,i}$ is obtained after the ith iteration. Then the vector-measurement model can be re-expanded as

$$\begin{aligned}h\left(\boldsymbol{b}_{m}\right)&\approx A\left(\hat{\boldsymbol{q}}_{k,i}\right)\boldsymbol{r}_{m}+\left.\frac{\partial h\left(\boldsymbol{b}_{m}\right)}{\partial\boldsymbol{b}_{m}}\right|_{A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_{m}}\\&\quad\times\left[A\left(\hat{\boldsymbol{q}}_{k,i}\right)\boldsymbol{r}_{m}\times\right]\Delta\boldsymbol{\alpha}.\end{aligned}\tag{12}$$

As has been discussed, in order to reduce the linearization error, the a posteriori quaternion estimate is required, which necessitates the reset operation in each iteration. In this respect, the error state in (12) is also the zero vector. That is to say, the predicted measurement in iteration $i+1$ is

$$h\left(\boldsymbol{b}_{m}\right)=A\left(\hat{\boldsymbol{q}}_{k,i}\right)\boldsymbol{r}_{m}.\tag{13}$$

Based on the aforementioned discussion, the explicit procedure of the developed IMEKF can now be summarized as in Table II.

REMARK 3 It should be noted that the attitude quaternion update equation is not a Kalman estimator, although it seems to be. It is actually an approximation of quaternion multiplication—that is,

$$\begin{aligned}\hat{\boldsymbol{q}}_{k,i+1}&=\delta\hat{\boldsymbol{q}}_{k,i}\otimes\hat{\boldsymbol{q}}_{k,i}\approx\begin{bmatrix}0.5\Delta\hat{\boldsymbol{\alpha}}_{k,i}\\1\end{bmatrix}\otimes\hat{\boldsymbol{q}}_{k,i}\\&=\hat{\boldsymbol{q}}_{k,i}+0.5\,\Xi\left(\hat{\boldsymbol{q}}_{k,i}\right)\Delta\hat{\boldsymbol{\alpha}}_{k,i}.\end{aligned}$$

The attitude-error estimate $\Delta\hat{\boldsymbol{\alpha}}_{k,i}$ is actually used to correct its corresponding reference attitude, about which the vector-measurement model has been linearized. In this respect, the attitude update should not be processed as

$$\hat{\boldsymbol{q}}_{k,i+1}=\hat{\boldsymbol{q}}_{k|k-1}+0.5\,\Xi\left(\hat{\boldsymbol{q}}_{k|k-1}\right)\Delta\hat{\boldsymbol{\alpha}}_{k,i},$$

although it looks more like the original form of the iterated EKF (IEKF).

REMARK 4 It is shown that the reset operation for the error state is implicitly performed after the attitude update in each iteration, which is used to obtain the more accurate attitude quaternion. The reset operation in each iteration is precisely the reason why the innovation used in the IMEKF exhibits a different form from that in the original IEKF. The innovation used in the original IEKF has the form

$$\boldsymbol{y}_{k}-h\left(\hat{\boldsymbol{x}}_{k,i}\right)-\boldsymbol{H}_{k,i}\left(\hat{\boldsymbol{x}}_{k|k-1}-\hat{\boldsymbol{x}}_{k,i}\right).$$

According to (12), the Jacobian matrix $\boldsymbol{H}_{k,i}$ in the developed IMEKF is given by

$$\boldsymbol{H}_{k,i}=\left.\frac{\partial h\left(\boldsymbol{b}_{m}\right)}{\partial\boldsymbol{b}_{m}}\right|_{A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_{m}}\left[A\left(\hat{\boldsymbol{q}}_{k,i}\right)\boldsymbol{r}_{m}\times\right].$$

Due to the reset operation, the term $\boldsymbol{H}_{k,i}\Delta\hat{\boldsymbol{\alpha}}_{k,i}$ is actually zero.

TABLE II
IMEKF

Initialization set values $\hat{\boldsymbol{q}}_0$, $\hat{\boldsymbol{\beta}}_0$, and $\boldsymbol{P}_0$
Recursive update for $k=1:K$ (K, total time steps)
Time update
$\hat{\boldsymbol{q}}_{k|k-1}=\Omega(\hat{\boldsymbol{\omega}}_{k-1})\hat{\boldsymbol{q}}_{k-1}$
$\hat{\boldsymbol{\beta}}_{k|k-1}=\hat{\boldsymbol{\beta}}_{k-1}$
$\hat{\boldsymbol{\omega}}_{k-1}=\tilde{\boldsymbol{\omega}}_{k-1}-\hat{\boldsymbol{\beta}}_{k-1}$
$\boldsymbol{P}_{k|k-1}=\boldsymbol{\Phi}_{k-1}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_{k-1}^{\mathrm{T}}+\boldsymbol{G}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{G}_{k-1}^{\mathrm{T}}$
Measurement update
Set $\hat{\boldsymbol{q}}_{k,0}=\hat{\boldsymbol{q}}_{k|k-1}$,$\hat{\boldsymbol{\beta}}_{k,0}=\hat{\boldsymbol{\beta}}_{k|k-1}$
For $i=0,1,\ldots,N$, evaluate the following equations (where N is the desired number of measurement-update iterations):

$$h_k(\hat{\boldsymbol{x}}_{k,i})=\begin{bmatrix}A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_1\\\vdots\\A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_n\end{bmatrix},\ \boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k,i})=\begin{bmatrix}A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_1&\boldsymbol{0}_{3\times3}\\\vdots&\vdots\\A(\hat{\boldsymbol{q}}_{k,i})\boldsymbol{r}_n&\boldsymbol{0}_{3\times3}\end{bmatrix}$$

$\boldsymbol{K}_{k,i}=\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}}(\hat{\boldsymbol{x}}_{k,i})\left[\boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k,i})\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^{\mathrm{T}}(\hat{\mathbf{x}}_{k,i})+\boldsymbol{R}_k\right]^{-1}$
$\Delta\hat{\boldsymbol{x}}_{k,i}=\boldsymbol{K}_{k,i}[\boldsymbol{y}_k-h_k(\hat{\boldsymbol{x}}_{k,i})]$
Attitude update
$\Delta\hat{\boldsymbol{x}}_{k,i}=\left[\ \Delta\hat{\boldsymbol{\alpha}}_{k,i}^{\mathrm{T}}\ \ \Delta\hat{\boldsymbol{\beta}}_{k,i}^{\mathrm{T}}\ \right]$
$\hat{\boldsymbol{q}}_{k,i+1}=\hat{\boldsymbol{q}}_{k,i}+0.5\Xi(\hat{\boldsymbol{q}}_{k,i})\Delta\hat{\boldsymbol{\alpha}}_{k,i}$
$\hat{\boldsymbol{\beta}}_{k,i+1}=\hat{\boldsymbol{\beta}}_{k,i}+\Delta\hat{\boldsymbol{\beta}}_{k,i}$
End for
$\hat{\boldsymbol{q}}_k=\hat{\boldsymbol{q}}_{k,N+1}$, $\hat{\boldsymbol{\beta}}_k=\hat{\boldsymbol{\beta}}_{k,N+1}$
$\boldsymbol{P}_k=[\boldsymbol{I}_{6\times6}-\boldsymbol{K}_{k,N}\boldsymbol{H}_k(\hat{\boldsymbol{x}}_{k,N})]\boldsymbol{P}_{k|k-1}$

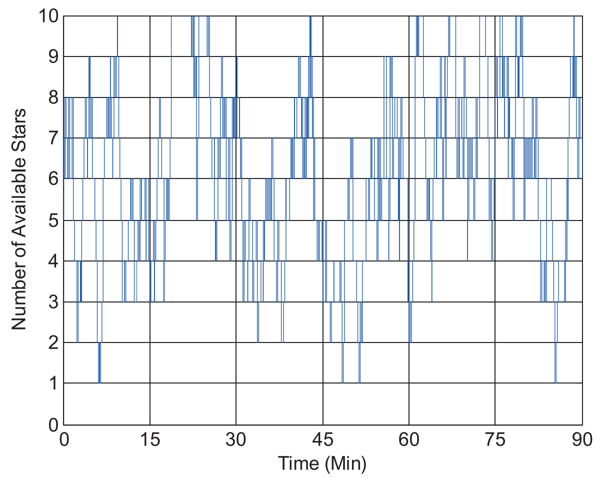

Fig. 1. Number of available stars.

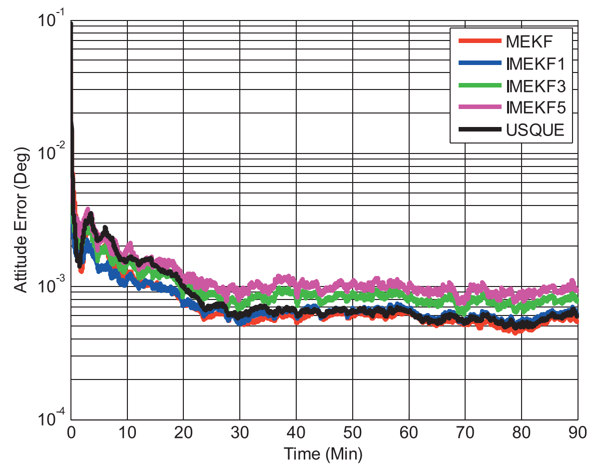

Fig. 2. Norm of attitude-estimation errors, Case 1.

REMARK 5 In the IMEKF, the covariance update exhibits the same form as in the original IEKF, which indicates that the reset operation does not modify the covariance. The corresponding reason has been discussed in previous work—that is, the reset operation merely moves the update information from the error state to the full state estimate and therefore neither increases nor decreases the total information content of the estimate [2, 5].

V. SIMULATION RESULTS AND ANALYSIS

In this section, several test cases are simulated to evaluate the performance of the developed IMEKF against that of the well-known nonlinear filtering techniques—that is, the MEKF and USQUE. In the simulation, a typical star tracker is used to determine the attitude of a spacecraft in a 90-min low-Earth orbit. The star tracker can sense up to 10 stars in a $6^\circ \times 6^\circ$ field of view. The catalog contains stars that can be sensed up to a magnitude of 6.0 (larger magnitudes indicate dimmer stars) [1, 2]. The gyro measurements and star images are both taken at 1-s intervals. The noise of the star-tracker measurement model is zero-mean white Gaussian noise with a standard deviation of $\sigma_m = (0.005/3)^\circ$, $m = 1, 2, \ldots, n$. The gyro noise is also assumed to have white Gaussian distribution with a zero mean and standard deviations of $\sigma_u = \sqrt{10} \times 10^{-10}$ rad/s$^{3/2}$ and $\sigma_v = \sqrt{10} \times 10^{-7}$ rad/s$^{1/2}$. The initial bias for each axis is given by $0.1^\circ/h$. The number of available stars during the simulation is shown in Fig. 1. The following four simulation cases, with different initial attitude-estimation errors, are studied for comparison of the developed IMEKF with the MEKF and USQUE:

Case 1: Initial attitude-estimation error = $[\,1^\circ\ 1^\circ\ 1^\circ\,]^{\mathrm{T}}$.

Case 2: Initial attitude-estimation error = $[\,10^\circ\ 10^\circ\ 30^\circ\,]^{\mathrm{T}}$.

Case 3: Initial attitude-estimation error = $[\,30^\circ\ 30^\circ\ 30^\circ\,]^{\mathrm{T}}$.

Case 4: Initial attitude-estimation error = $[\,50^\circ\ -50^\circ\ 160^\circ\,]^{\mathrm{T}}$.

Meanwhile, for the developed IMEKF, different iterations have also been studied. Specifically, IMEKF1 denotes the developed IMEKF with one iteration, IMEKF3 three iterations, and IMEKF5 five iterations. The following simulation results of the norm of total attitude-estimation error are all averaged values of 100 Monte Carlo runs. Specifically, the averaged norm error ANE is defined as

$$\mathrm{ANE}(k) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{norm}\left(q2\mathrm{att}\left(\hat{\boldsymbol{q}}_k^i \otimes \left(\boldsymbol{q}_{\mathrm{ref},k}\right)^{-1}\right)\right),$$

where $\boldsymbol{q}_{\mathrm{ref},k}$ represents the true and reference attitude quaternion; $\hat{\boldsymbol{q}}_k^i$ represents the quaternion estimate of the ith Monte Carlo run; $q2\mathrm{att}(\cdot)$ denotes the transition function from the quaternion to the Euler angle; norm$(\cdot)$ denotes the norm operation; and N is the number of Monte Carlo runs (100 in our study).

In Case 1, the initial attitude covariance is set to $(1^\circ)^2$ and the bias covariance is set to $(0.2^\circ/h)^2$. These quantities are converted to radians and seconds, respectively, for initial attitude and gyro-drift covariances in the simulation study. We use the same initial covariance for the MEKF, USQUE, and IMEKF. The norm of total attitude-estimation error for this case is shown in Fig. 2. It is shown that the simulation exhibits very similar performance for all the evaluated filters in this case, and the USQUE and IMEKF do not reveal any superior accuracy over the MEKF. This indicates that the linearization procedure in the MEKF is valid when a good a priori estimate of the state can be provided. In this case, the MEKF remains the method of choice for attitude estimation due to its relatively simple and flexible characteristic.

In Case 2, the initial bias estimate is still set to zeros. The initial attitude covariance is set to $(10^\circ)^2$ and the bias covariance is set to $(0.2^\circ/h)^2$. The norm of total attitude-estimation error for this case is shown in Fig. 3. It can be seen from Figs. 2 and 3 that the performance of the MEKF is much degraded when the initial attitude-estimate

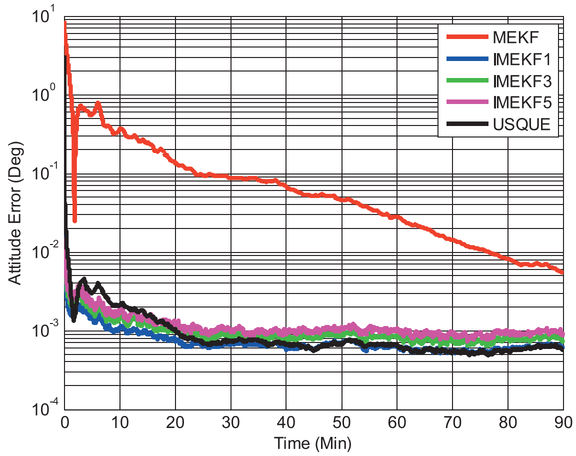

Fig. 3. Norm of attitude-estimation errors, Case 2.

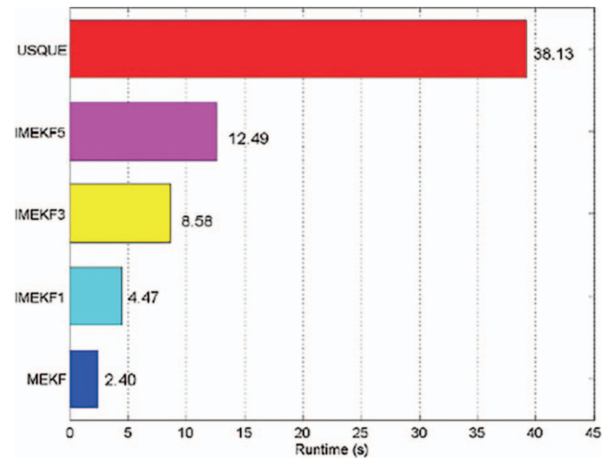

Fig. 5. Averaged estimator run time.

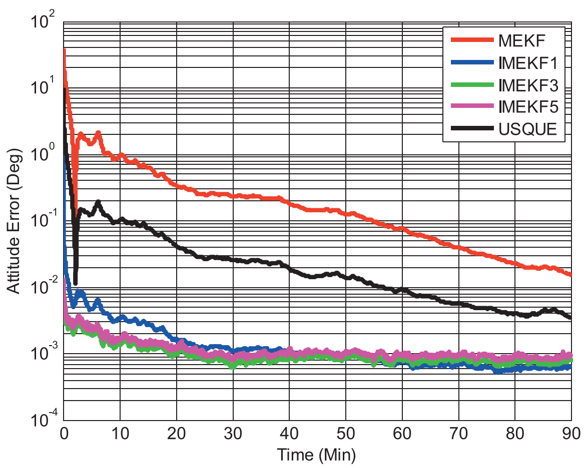

Fig. 4. Norm of attitude-estimation errors, Case 3.

error increases, which indicates that the MEKF can fail in cases that lack a good a priori estimate of the state. In contrast, the USQUE and the developed IMEKF can still perform satisfactorily. Meanwhile, the IMEKF with different iterations has a comparable performance to that of the USQUE. The superiority of the USQUE over the MEKF, due to nonnegligible nonlinear effects, has been widely recognized by researchers. The reason for the superiority of the IMEKF over the MEKF is the same as that for the IEKF over the EKF.

In Case 3, the initial bias estimate is still set to zeros. The initial attitude covariance is set to $(30°)^2$ and the bias covariance is still set to $(0.2°/h)^2$. The norm of total attitude-estimation error for this case is shown in Fig. 4. It is shown that the performance of both the MEKF and the USQUE is degraded in terms of convergent speed and accuracy. This indicates that the nonnegligible nonlinear effect caused by the lack of a good a priori estimate also has a negative effect on the USQUE. In contrast, the performance of the IMEKF does not degrade so much compared with the previous two cases, and its superiority over the USQUE and MEKF is more obvious. This implies that the IMEKF is able to capture large uncertainties more efficiently than the USQUE.

It can be seen from Figs. 2–4 that the majority of the possible filtering-performance improvement is obtained by only one iteration, and further iterations do not further improve performance for the studied problem. The corresponding finding is consistent with the argument in [13]. A comparison of the averaged computation times of 100 Monte Carlo runs is shown in Fig. 5 for simulations implemented on a computer with a 2.66-Gb CPU, 2.0 Gb of memory, and the Windows 7 operating system using MATLAB. Note that, as expected, the computation time for the USQUE is much higher than for the MEKF-type filters. That is, the accuracy improvement of the USQUE over the MEKF is at the cost of a notable increase in computation burden. Therefore, the performance improvement of the USQUE over the MEKF is compromised, especially for a case with a large initial estimate error, as shown in Fig. 4. In contrast, the developed IMEKF can much improve the performance of the MEKF with only a little addition of computation time, especially for the IMEKF with one iteration. In this respect, the developed IMEKF will be more celebrated in real-time application. Taking both accuracy and computational efficiency into account, the IMEKF outperforms the USQUE, as the IMEKF can always achieve comparable or even better accuracy than the USQUE but at much less computational cost.

The attitude-estimation errors and their respective 3σ bounds for the USQUE and IMEKF1 in Case 3 are plotted in Figs. 6 and 7, respectively, for a single simulation. It is shown that the USQUE attitude errors converge to within their respective 3σ error bounds until well after 1 h for the horizontal angles. For the yaw angle, consistency cannot even be obtained after 1.5 h. In contrast, as shown in Fig. 7, the IMEKF1 attitude errors converge to within their respective 3σ bounds well within 5 min, which indicates that the IMEKF1 is performing in a near-optimal fashion.

In Case 4, the initial bias estimate is still set to zeros. The initial attitude covariance is set to $(50°)^2$ and the

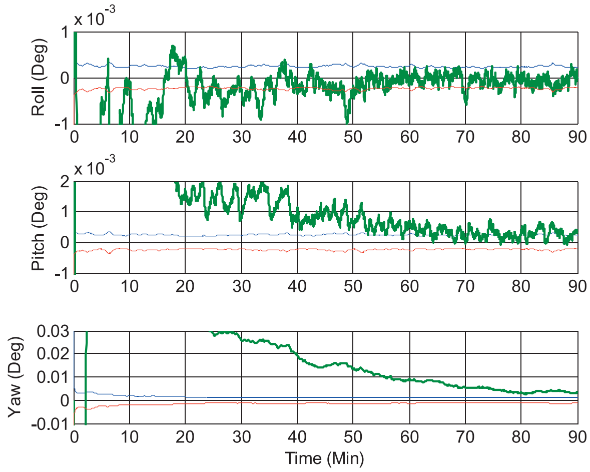

Fig. 6. Attitude errors in USQUE with 3σ error bounds.

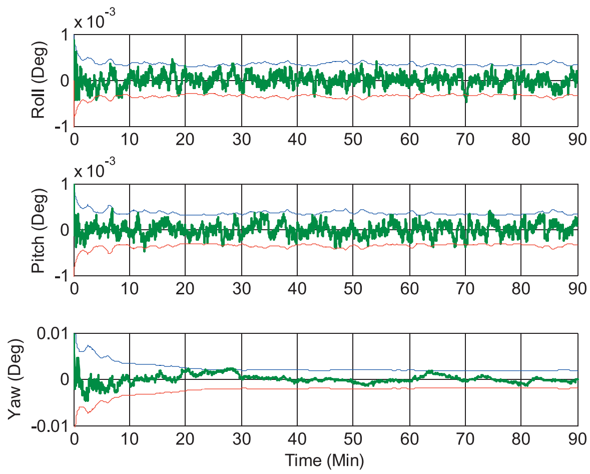

Fig. 7. Attitude errors in IMEKF1 with 3σ error bounds.

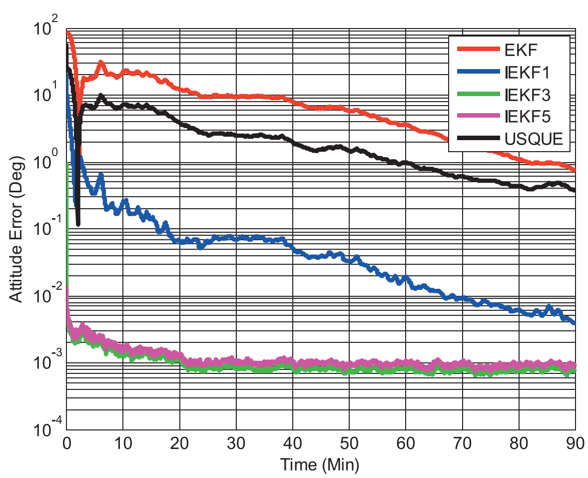

Fig. 8. Norm of attitude-estimation errors, Case 4.

initial bias covariance is unchanged. This setting is similar to that in [6], which is a very dramatic situation, as the initial attitude errors are extremely large. The norm of total attitude-estimation error for this case is shown in Fig. 8.

It can be seen that the proposed IMEKF can still perform sufficiently well in terms of speed of convergence and economy of processing, when compared to the more complex USQUE. Compared with other simulation cases, it can be concluded that the IMEKF requires more iterations for convergence with very large initial attitude-estimation error. Since we cannot know how large the initial attitude-estimation error is in practical application, we can perform more iterations, say three, to guarantee filtering accuracy and convergence speed. This is a conservative approach that incurs greater computational cost; however, the corresponding computational cost is still moderate when compared to that of the more complex USQUE, as shown in Fig. 5.

From the four simulation cases, it can be concluded that the IMEKF can always achieve comparable or even better accuracy than the USQUE and at much less computational cost. In light of this, we can conclude that the proposed filter is better than the USQUE in a wide range of original conditions when accuracy and computational efficiency are both considered. Hopefully, the proposed IMEKF can be widely approved as a promising substitute for the USQUE when a good a priori estimate of the state is unavailable.

It should be noted that iteration can also be extended to the USQUE, which has not been studied in this paper. The reasons are as follows: First, the trade-off between performance and computational time for the iterated USQUE would make the development more or less inappreciable. As is shown in Fig. 5, the computational time of the USQUE is already so great, and the iteration procedure can further increase the computational burden manyfold. Second, the USQUE—which uses some deterministically selected samples to capture the state probability distribution—has a quite different filtering principle from that of the MEKF. The iteration procedure developed in this paper may not be extended to the USQUE directly. In any case, the investigation of the iterated USQUE is open to further research.

VI. CONCLUSION

In this paper, the IMEKF for spacecraft attitude estimation is proposed by incorporating an iteration scheme into the MEKF. In each iteration of the IMEKF, the attitude-error estimate is used to refine the attitude quaternion, which is then used to relinearize the vector-measurement model in the next iteration. Due to the reset operation on the attitude-error estimate, the developed IMEKF exhibits a different form from that of the traditional IEKF. However, the performance improvement can be obtained by the IMEKF for similar reasons as the IEKF. Simulation results indicate that the performance of the developed IMEKF exceeds that of the MEKF and USQUE for large initialization errors. The computational burden of the IMEKF with one iteration is only a little larger than that of the MEKF, and much smaller than that of the USQUE.

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LUBIN CHANG, Member, IEEE
BAIQING HU
KAILONG LI
Department of Navigation Engineering
Naval University of Engineering
Jiefang Rd. 717, Qiaokou District, Wuhan, 430033
China
E-mail: (changlubin@163.com)

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