TESTING A DECENTRALIZED FILTER FOR GPS/INS INTEGRATION

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ABSTRACT

A decentralized Kalman filter strategy is presented and applied to GPS/INS integration. Two Kalman filters are used, one is a local filter, processing GPS data and providing locally best estimates of position and velocity. The second is an INS filter which uses the results from the GPS filter as updates to the estimates obtained from the inertial data. Because of the high short term accuracy of the inertial system, the position results from INS can be used for cycle slip detection and correction. The major advantages of this method, based on decentralized filter theory, are the flexible combination of GPS and INS and the simplicity of the implementation. An extension of this method to more sensors is straightforward. Numerical results will be used to illustrate the salient features of the method.

1. INTRODUCTION

Relative positioning using differential GPS in a dynamic environment is very accurate, especially when using carrier phase measurements. It is plagued, however, by cycle slip problems, by a data rate which is not sufficient for precise trajectory interpolation, and by the lack of attitude. A strapdown inertial system provides differential position, velocity and attitude at high rates and with high accuracy over short time periods. The accuracy degrades quickly, however, as a function of time and results can only be used for precise positioning if the INS is regularly updated. Integration of the INS with differential GPS has therefore many advantages. GPS data can be used to ensure the long term accuracy of the INS performance. The INS will then provide highly accurate positioning results at high rates. GPS benefits from the INS because cycle slip detection and correction is now feasible. Another advantage of GPS/INS integration is precise attitude determination. This paper looks at the integration of measurements from a strapdown inertial system and differential GPS via a decentralized Kalman filtering approach.

The integration of GPS and INS data via a common Kalman filter has been discussed in many papers, see for instance Cox (1980), Wong et al. (1988), Hein et al. (1988), Tazartes et al. (1988). In this case, one dynamic model and thus one Kalman filter is used to model the effects of GPS and INS measurements.

During the past decade, the development of decentralized Kalman filtering methods has received increased attention, see for instance Speyer (1979), Willsky (1982), Hashemipour (1988), Carlson (1988). Based on a parallel processing structure, decentralized filtering is a promising method to deal with a multi-processor system, in which several parallel local filters process data from separate sub-systems, and a master filter combines their results to obtain a optimal global estimate, see for instance Wei and Schwarz (1989). As an alternative to the common Kalman filter for GPS/INS integration, the decentralized filtering technique is discussed and applied in the following.

2. CENTRALIZED AND DECENTRALIZED FILTER

Problem statement

The errors of a dynamic system are usually time dependent and can therefore be described by a state space model of the form

$$\dot{x}(t) = F(t)\, x(t) + G(t)\, w(t) \tag{2-1a}$$

or for discrete measurements

$$x_k = \Phi_{k,k-1}\, x_{k-1} + w_{k-1}, \tag{2-1b}$$

where x_k is the state vector and w_{k-1} is the system noise with covariance matrix Q_{k-1}.

At an instant of time t_k, the measurements z_k, which can be any linear combination of state vector components, are given in the form

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$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \tag{2-2}$$

where $\mathbf{v}_k$ is the measurement noise with covariance matrix $\mathbf{R}_k$.

If measurements are from different independent external systems, the measurements $\mathbf{z}_k$ can be written as

$$\mathbf{z}_k = \{\mathbf{z}_{1k}^T, \cdots, \mathbf{z}_{ik}^T\}^T \tag{2-3}$$

where the subscript i denotes the external system i and $\mathbf{z}_{ik}$ are the measurements from the system i at time t_k.

The corresponding measurement equation for the measurements $\mathbf{z}_{ik}$, linearly related to the components of the state vector $\mathbf{x}_k$, is

$$\mathbf{z}_{ik} = \mathbf{H}_{ik} \mathbf{x}_k + \mathbf{v}_{ik} \tag{2-4}$$

where $\mathbf{v}_{ik}$ is the measurement noise with covariance matrix $\mathbf{R}_{ik}$. Equation (2-4) simplifies to equation (2-2) if only one update system i is considered.

In many cases the update system i can be treated as a local system i which is described by a state space model of the form

$$\mathbf{x}_{ik} = \Phi_{k,k-1}^{i} \mathbf{x}_{i,k-1} + \mathbf{w}_{i,k-1} \tag{2-5}$$

with the corresponding measurement equation

$$\mathbf{z}_{ik} = \mathbf{A}_{ik} \mathbf{x}_{ik} + \mathbf{v}_{ik} \tag{2-6}$$

where $\mathbf{x}_{ik}$ is the state vector of the local system i, $\mathbf{w}_{ik}$ is the local system noise with covariance $\mathbf{Q}_{ik}$, and $\mathbf{v}_{ik}$ is the measurement noise.

Given the models just described, the problem to estimate the state vector $\mathbf{x}_k$, modelled by equation (2-1), based on all measurements $\mathbf{z}_k = \{\mathbf{z}_{1k}, \cdots, \mathbf{z}_{ik}\}$ can be formulated in different ways. The first approach makes an optimal estimate of the state vector $\mathbf{x}_k$ by using standard Kalman filtering which directly uses all measurements $\mathbf{z}_k$ as update measurements to update the state equation (2-1). In the second approach, a decentralized filtering technique which processes data in two stages is used. In a first step, the local systems process their own data to obtain a best local estimate using measurements from the local system only. The results from the local filters are then used to update the master filter, which combines the information from the various local systems to produce the best global estimate of the master system described by equation (2-1).

Centralized Kalman filtering

In contrast to the decentralized filter, the standard Kalman filter, which processes the data from different systems in one step, is called the *centralized Kalman filter*. The Kalman filter prediction and update equations as given in (Gelb, 1974) can be directly applied:

prediction: $$\hat{\mathbf{x}}_k(-) = \Phi_{k,k-1} \hat{\mathbf{x}}_{k-1}(+) \tag{2-7}$$

$$\mathbf{P}_k(-) = \Phi_{k,k-1} \mathbf{P}_{k-1}(+) \Phi_{k,k-1}^T + \mathbf{Q}_{k-1} \tag{2-8}$$

update: $$\hat{\mathbf{x}}_k(+) = \hat{\mathbf{x}}_k(-) + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k(-)) \tag{2-9}$$

$$\mathbf{K}_k = \mathbf{P}_k(-)\mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_k(-)\mathbf{H}_k^T + \mathbf{R}_k]^{-1} \tag{2-10}$$

$$\mathbf{P}_k(+) = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k(-) \tag{2-11}$$

where (-) and (+) denote the estimate before and after update. The standard Kalman filter equations are written without the subscript i.

Assuming that measurements between different local system i are independent, the Kalman filter makes a best estimate of the state vector $\mathbf{x}_k$ in terms of the measurements $\mathbf{z}_{ik}$ of the form:

update: $$\hat{\mathbf{x}}_k(+) = \hat{\mathbf{x}}_k(-) + \sum_i \mathbf{K}_{ik} (\mathbf{z}_{ik} - \mathbf{H}_{ik} \hat{\mathbf{x}}_k(-)) \tag{2-12}$$

$$\mathbf{K}_{ik} = \mathbf{P}_k(+) \mathbf{H}_{ik}^T \mathbf{R}_{ik}^{-1} \tag{2-13}$$

$$\mathbf{P}_k(+) = (\mathbf{I} - \sum_i \mathbf{K}_{ik} \mathbf{H}_{ik}) \mathbf{P}_k(-). \tag{2-14}$$

The update equations (2-9) to (2-11) can be reformulated as

$$\hat{\mathbf{x}}_k(+) = \mathbf{P}_k(+) \mathbf{P}_k^{-1}(-) \hat{\mathbf{x}}_k(-) + \sum_i \mathbf{P}_k(+) \mathbf{H}_{ik}^T \mathbf{R}_{ik}^{-1} \mathbf{z}_{ik} \tag{2-15}$$

and

$$\mathbf{P}_k^{-1}(+) = \mathbf{P}_k^{-1}(-) + \sum_i \mathbf{H}_{ik}^T \mathbf{R}_{ik}^{-1} \mathbf{H}_{ik} \tag{2-16}$$

In equations (2-7) to (2-16), $\mathbf{P}_k(-)$ is the covariance of the predicted state vector $\hat{\mathbf{x}}_k(-)$, while $\mathbf{P}_k(+)$ is the covariance of the updated state vector $\hat{\mathbf{x}}_k(+)$.

Decentralized filtering

Decentralized filtering is a two-stage data processing technique which processes data from multi-data systems. In the first stage, each local processor uses its own data to make a best local estimate. These estimates are obtained in a parallel processing mode. The local estimates are then fused by a master filter to make a best global estimate of the state vector of the master system. Figure 2.1 shows in a block diagram the principle of the decentralized filter for the integration of an INS with other sensors.

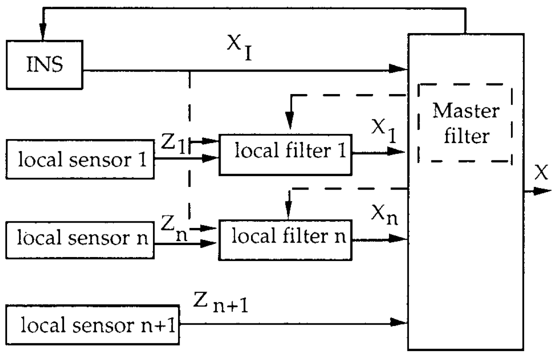

Figure 2.1 Decentralized filtering schema

For the i-th local system, which is described by model (2-5) and (2-6), the best local estimate of the state vector $\mathbf{x}_{ik}$ can be obtained using Kalman filtering based on the local measurements $\mathbf{z}_{ik}$ as follows:

$$\hat{x}_{ik}(+) = \mathbf{P}_{ik}(+)\,\mathbf{P}_{ik}^{-1}(-)\,\hat{x}_{ik}(-) + \mathbf{P}_{ik}(+)\,\mathbf{A}_{ik}^T\,\mathbf{R}_{ik}^{-1}\,\mathbf{z}_{ik} \quad (2\text{-}17)$$

and

$$\mathbf{P}_{ik}^{-1}(+) = \mathbf{P}_{ik}^{-1}(-) + \mathbf{A}_{ik}^T\mathbf{R}_{ik}^{-1}\,\mathbf{A}_{ik} \quad (2\text{-}18)$$

where $\hat{x}_{ik}(+)$ is the best estimate after update with covariance matrix $\mathbf{P}_{ik}(+)$, and $\hat{x}_{ik}(-)$ is the best prediction of the local state vector with corresponding covariance matrix $\mathbf{P}_{ik}(-)$.

In the next step, a best global estimate for the state vector $\mathbf{x}_{ik}$ is obtained in terms of the local estimates. The local estimation results are thus used as measurement updates instead of the measurements themselves.

The transformation from the two stage to the one stage procedure can be done, if a matrix $\mathbf{M}_i$ can be defined, such that

$$\mathbf{H}_{ik} = \mathbf{A}_{ik}\,\mathbf{M}_i. \quad (2\text{-}19)$$

This condition is rather weak and can almost always be satisfied. Particularly, if the local state vector $\mathbf{x}_{ik}$ is part of the state vector $\mathbf{x}_k$, the relationship between $\mathbf{x}_{ik}$ and $\mathbf{x}_k$ can be expressed as

$$\mathbf{x}_{ik} = \mathbf{M}_i\,\mathbf{x}_k. \quad (2\text{-}20)$$

Comparing equation (2-15) with (2-17) and considering the condition (2-19) leads to the best global estimate $\hat{x}_k(+)$

$$\hat{x}_k(+) = \mathbf{P}_k(+)\mathbf{P}_k^{-1}(-)\hat{x}_k(-) + \sum_i \mathbf{P}_k(+)\mathbf{M}_i^T\mathbf{P}_{ik}^{-1}(+)\hat{x}_{ik}(+) - \sum_i \mathbf{P}_k(+)\mathbf{M}_i^T\mathbf{P}_{ik}^{-1}(-)\hat{x}_{ik}(-). \quad (2\text{-}21)$$

The corresponding inverse covariance matrix $\mathbf{P}_k^{-1}(+)$ is obtained from

$$\mathbf{P}_k^{-1}(+) = \mathbf{P}_k^{-1}(-) + \sum_i \mathbf{M}_i^T\,\mathbf{P}_{ik}^{-1}(+)\,\mathbf{M}_i - \sum_i \mathbf{M}_i^T\,\mathbf{P}_{ik}^{-1}(-)\,\mathbf{M}_i. \quad (2\text{-}22)$$

In equations (2-21) and (2-22) the estimate $\hat{x}_{ik}(+)$ with its covariance matrix $\mathbf{P}_{ik}(+)$, and the prediction $\hat{x}_{ik}(-)$ with its covariance matrix $\mathbf{P}_{ik}(-)$ are results of the local Kalman filter i. Equations (2-21) and (2-22) formulate the update equation for the global estimate of the state vector $\mathbf{x}_k$ and its covariance matrix in terms of the local estimates. The prediction of the state vector $\hat{x}_k(-)$ with its covariance matrix $\mathbf{P}_k(-)$ is computed from the prediction equations (2-7) and (2-8).

Equations (2-17) and (2-21) indicate the parallel processing structure and the fusing algorithm of the master filter. Usually, equation (2-21) is reformulated to give formulas similar to the standard Kalman filter. Then,

$$\hat{x}_k(+) = \hat{x}_k(-) + \sum_i \mathbf{K}_{ik}^d(+)(\hat{x}_{ik}(+) - \mathbf{M}_i\hat{x}_k(-)) - \sum_i \mathbf{K}_{ik}^d(-)(\hat{x}_{ik}(-) - \mathbf{M}_i\,\hat{x}_k(-)) \quad (2\text{-}23)$$

with Kalman gain matrices

$$\mathbf{K}_{ik}^d(+) = \mathbf{P}_k(+)\,\mathbf{M}_i^T\,\mathbf{P}_{ik}^{-1}(+) \quad (2\text{-}24)$$

and

$$\mathbf{K}_{ik}^d(-) = \mathbf{P}_k(+)\,\mathbf{M}_i^T\,\mathbf{P}_{ik}^{-1}(-). \quad (2\text{-}25)$$

Using equations (2-21) to (2-25) the master filter gives the best global estimate of the state vector $\mathbf{x}_k$ in terms of the local estimation results.

3. KALMAN FILTERS FOR GPS/INS INTEGRATION

To estimate the errors of a strapdown inertial system, a state vector with 15 error states will be used. It is of the form

$$x_{INS} = (\boldsymbol{\varepsilon}, \delta\mathbf{r}, \delta\mathbf{v}, \mathbf{d}, \mathbf{b})^T \tag{3-1}$$

where

- ε is the vector of attitude errors in the local-level frame;
- $\delta\mathbf{r}$ is the vector of position errors in the local-level frame;
- $\delta\mathbf{v}$ is the vector of velocity errors in the local-level frame;
- $\mathbf{d}$ is the vector of gyro drifts about gyro axes in the body frame;
- $\mathbf{b}$ is the vector of accelerometer biases in the body frame.

If double differenced measurements of GPS are used, the state vector of the GPS filter is

$$x_{GPS} = (\delta\mathbf{r}, \delta\mathbf{v})^T. \tag{3-2}$$

Equation (3-2) is the error state vector of differential GPS if ambiguities have been fixed to their integer values. If cycle slips occur, one can directly correct them by comparing GPS measurements and INS data. One can also augment the state vector to estimate the ambiguities, in which case the ambiguity parameters should be added to the state vector (3-2).

Usually, the INS is considered as the reference system, which provides position, velocity and attitude information. The GPS measurements are used to update the INS solution and estimate the error states.

In order to combine INS and GPS data by using filtering techniques, two strategies, outlined before, will be applied. The first one uses a common Kalman filter for both, the INS and GPS errors. The GPS measurements are directly used as update measurements (Wong et al., 1988). The second one uses the decentralized filtering technique, in which two Kalman filters are formulated. One is the GPS filter, which is used as a local filter and processes GPS data only. The other is the INS filter which is considered as the master filter and estimates position, velocity and attitude along the trajectory. The results from the local GPS filter are used to update the INS master filter at distinct epochs to obtain an optimal global estimate. In general, these filters can work independently and they interact only occasionally. Figure 3.1 shows in a block diagram the principle of the decentralized filter for GPS/INS integration.

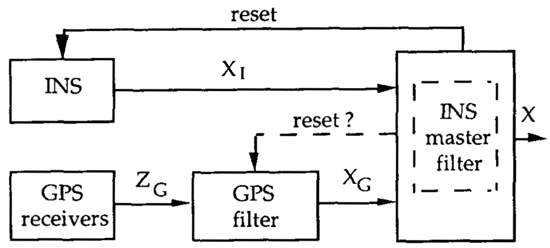

Figure 3.1 Decentralized filter for GPS/INS integration

The decentralized filter can be implemented in a variety of design configurations, which are illustrated in figures 3.2 to 3.4. For each of these designs, the time propagation and measurement update steps are essentially the same. The differences of the designs are the results of fusion and reset between the local GPS filter and the INS master filter.

Figure 3.2 shows a fully decentralized filter design, in which the update from the GPS filter provides the estimated state vector $\mathbf{x}_G(+)$ as well as the predicted state vector $\mathbf{x}_G(-)$ to the master filter. This approach is rigorous and the design corresponds to the update equations (2-21) or (2-23). As an approximation, the third term on the right site of equations (2-21) and (2-23) can be neglected. In that case the predicted state vector $\mathbf{x}_G(-)$ of the GPS filter is not used for the INS filter, as shown in Figure 3.3. This design is called cascaded decentralized filter because this configuration is similar to the cascaded filters in which the output from one filter is used as the measurements input to the next. Despite its theoretical drawback, the algorithm is very practical because of its simplicity. The formulas for the conventional Kalman filter can then directly be used for the INS master filter. Practically, the cascaded filter will often give the same results as the fully decentralized filter because in most cases the error covariance $\mathbf{P}_{ik}(-)$ of the predicted states of the GPS filter is much larger than the covariance $\mathbf{P}_k(+)$. In those cases, the Kalman gain $\mathbf{K}_{ik}^d(-)$ in (2-23) becomes so small that the third term on the right site of equation (2-23) can be neglected. In Figures 3.2 and 3.3 there is no feedback of the fused solution of the INS master filter to the GPS filter. Both filters run in parallel. The predicted position results from the INS are used to detect and correct cycle slips in the GPS phase data. These configurations essentially preserve the integrity of the two data streams. Figure 3.4 shows a design with a feedback of the fused solution from the INS filter to the GPS filter. This design is called sub-decentralized filter.

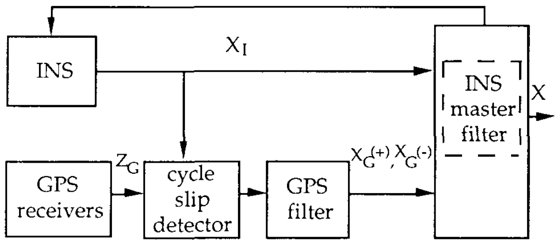

Figure 3.2 Fully decentralized filter

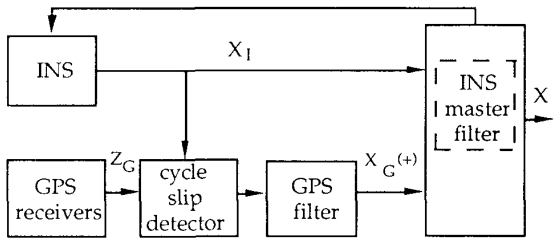

Figure 3.3 Cascaded filter

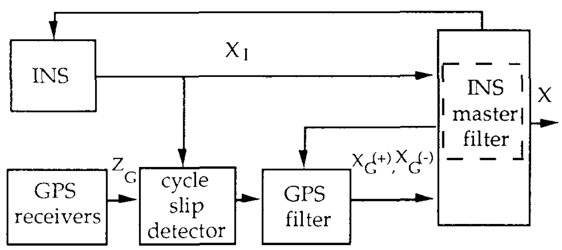

Figure 3.4 Sub-decentralized filter

For our application, the state vector for the INS master filter is given by (3-1) and denoted as $\mathbf{x}_I$. The state vector of the local GPS filter is defined by (3-2) and denoted as $\mathbf{x}_G$. The position or velocity estimated by the local GPS filter will be used to update the INS master filter. The connection matrix $\mathbf{M}_G$ in equation (2-19) is of the form

$$\mathbf{M}_G = \begin{pmatrix} 0 & \mathbf{I} & 0 & 0 & 0 \end{pmatrix} \tag{3-3}$$

for position update, and

$$\mathbf{M}_G = \begin{pmatrix} 0 & 0 & \mathbf{I} & 0 & 0 \end{pmatrix} \tag{3-4}$$

for velocity update. In these equations $\mathbf{I}$ are (3 x 3) unit matrices and $\mathbf{0}$ are (3 x 3) zero matrices.

4. TEST RESULTS

To implement the different filtering designs, software for GPS/INS integration via the centralized or decentralized filter designs has been developed at the University of Calgary. Following the designs discussed previously, the local filter mainly processes GPS data, while the master filter mainly processes INS data.

Test data were collected in the Kananasky Valley west of Calgary, using a road vehicle as the moving base for the INS and the roving GPS receiver. The test started with a ten minutes alignment period, during which data from both the GPS and the INS were collected. The GPS static data were used to determine the initial integer phase ambiguities by way of a least squares adjustment (Cannon and Schwarz, 1989), while the INS data were used to compute the initial attitude of the INS. After this the data from both systems were processed by using the centralized or decentralized Kalman filtering methods shown in Figures 3.2 to 3.4. The principle of cycle slip detection and correction is given in Lapucha et al. (1989).

Data collection in kinematic mode along a well controlled traverse of 6 km length took place on October 12, 1989. Six control points, at which static data for 2-3 minutes were collected, were available to analyse the positioning results, for details of the field test see Lapucha et al. (1989). As a first step the GPS positioning results at the control points were compared to the coordinate of the control points. The comparison is shown in Table 4.1, which indicates that no cycle slips occurred and the results are therefore at the level of a few centimeters .

cont. points	ϕ (m)	λ (m)	h (m)
No. 3	0.031	-0.042	-0.032
No. 4	0.040	-0.025	0.030
No. 6	0.053	-0.084	-0.040
No. 7	0.053	-0.088	-0.077

Table 4.1 Accuracy of kinematic GPS at control points

The results of different filtering algorithms are compared to the GPS results to assess different filter designs. The differences between the GPS results and the coordinates estimated by use of the centralized and the decentralized Kalman filters for GPS/INS integration are shown in Figures 4.1 to 4.3. These figures indicate that the different filter designs give almost the same results, even the cascaded Kalman filter. This is mainly due to the fact that all designs depend strongly on the GPS measurements. Thus, all solutions are correlated between themselves, and also

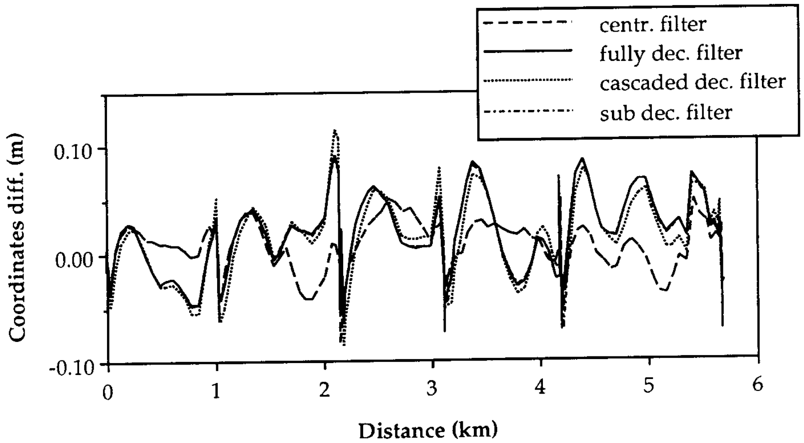

Figure 4.1 Coordinates comparison in latitude

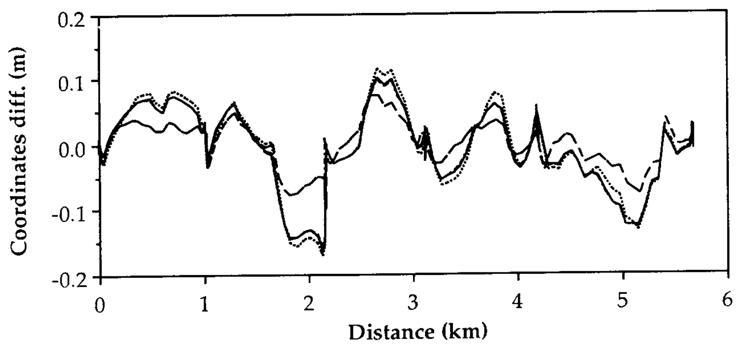

Figure 4.2 Coordinates comparison in longitude

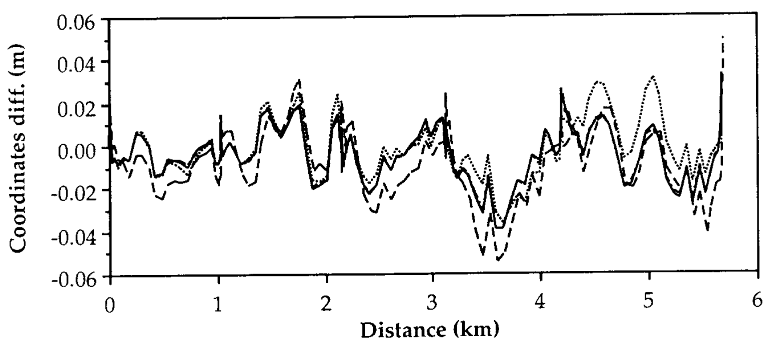

Figure 4.3 Coordinates comparison in height

to the 'control' solution which has been derived using the same data. The coordinates estimated by the centralized Kalman filter are closer to the GPS solution, because this filter directly uses the GPS measurements as updates to the filter.

5. CONCLUSIONS

Centralized and decentralized Kalman filtering for GPS/INS integration is compared in this paper, using different design configurations for the decentralized filter.

Results show that decentralized filtering results in a flexible and simple algorithm for GPS/INS integration, and is suitable for multi-sensor systems of the future. Compared to centralized filtering, the decentralized filter gives globally the same optimal estimation accuracy as the centralized Kalman filter. The accuracy does not deteriorate when a sub-optimal cascaded filter is used which has some advantages in terms of computational efficiency.

In general, a comparison of numerical results from a GPS/INS test run shows that there is very little difference in the accuracy of the results. The decision for the use of one filter over the other should therefore be made on the basis of computational efficiency and operational ease for the intended application.

One limitation of the decentralized filter for GPS/INS integration is that at least four satellites are required to provide acceptable GPS updates for the INS. By using the centralized Kalman filter, information from less than four satellites can be used as updates for the common Kalman filter.

ACKNOWLEDGEMENTS

Financial support for this project was obtained through an NSERC operating grant and a contract with Nortech Survey (Can) Inc. The support of H.E. Martell and D. Lapucha in planning and conducting the test runs is gratefully acknowledged.

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