Phase space

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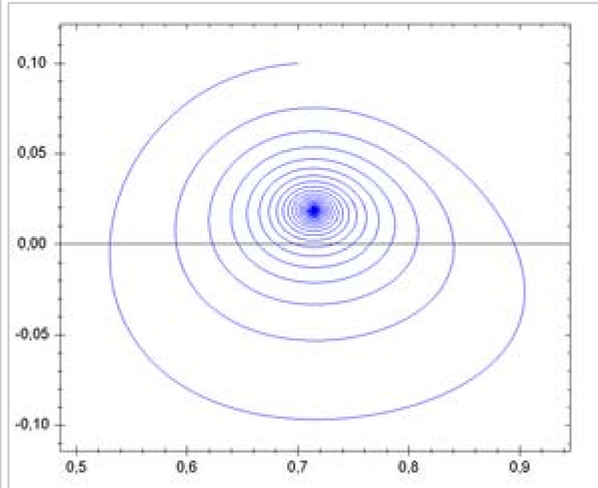
In mathematics and physics, a **phase space**, introduced by Willard Gibbs in 1901, is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space. For mechanical systems, the phase space usually consists of all possible values of position and momentum variables. A plot of position and momentum variables as a function of time is sometimes called a phase plot or a phase diagram. Phase diagram, however, is more usually reserved in the physical sciences for a diagram showing the various regions of stability of the thermodynamic phases of a chemical system, which consists of pressure, temperature, and composition.



Phase space of a dynamical system with focal stability.

In a phase space, every degree of freedom or parameter of the system is represented as an axis of a multidimensional space. For every possible state of the system, or allowed combination of values of the system's parameters, a point is plotted in the multidimensional space. Often this succession of plotted points is analogous to the system's state evolving over time. In the end, the phase diagram represents all that the system can be, and its shape can easily elucidate qualities of the system that might not be obvious otherwise. A phase space may contain very many dimensions. For instance, a gas containing many molecules may require a separate dimension for each particle's *x*, *y* and *z* positions and velocities as well as any number of other properties.

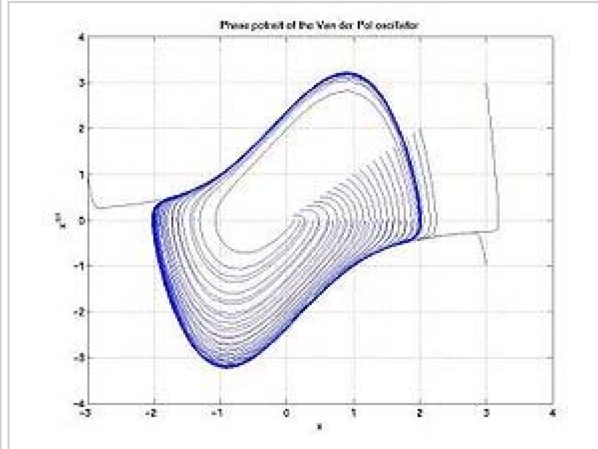
In classical mechanics the phase space co-ordinates are the generalized coordinates q_i and their conjugate generalized momenta p_i. The motion of an ensemble of systems in this space is studied by classical statistical mechanics. The local density of points in such systems obeys Liouville's Theorem, and so can be taken as constant. Within the context of a model system in classical mechanics, the phase space coordinates of the system at any given time are composed of all of the system's dynamical variables. Because of this, it is possible to calculate the state of the system at any given time in the future or the past, through integration of Hamilton's or Lagrange's equations of motion. Furthermore, because each point in phase space lies on exactly one phase trajectory, no two phase trajectories can intersect.

For simple systems, such as a single particle moving in one dimension for example, there may be as few as two degrees of freedom, (typically, position and velocity), and a sketch of the phase portrait may give qualitative information about the dynamics of the system, such as the limit-cycle of the Van der Pol oscillator shown in the diagram.

Here, the horizontal axis gives the position and vertical axis the velocity. As the system evolves, its state follows one of the lines (trajectories) on the phase diagram.

Classic examples of phase diagrams from chaos theory are the Lorenz attractor and Mandelbrot set.

Quantum mechanics



Phase portrait of the Van der Pol oscillator

In quantum mechanics, the coordinates *p* and *q* of phase space become hermitian operators in a Hilbert space, but may alternatively retain their classical interpretation, provided functions of them compose in novel algebraic ways (through Groenewold's 1946 star product). Every quantum mechanical observable corresponds to a unique function or distribution on phase space, and vice versa, as specified by Hermann Weyl (1927) and supplemented by John von Neumann (1931); Eugene Wigner (1932); and, in a grand synthesis, by H J Groenewold (1946). With José Enrique Moyal (1949), these completed the foundations of phase-space quantization, a logically autonomous reformulation of quantum mechanics. Its modern abstractions include deformation quantization and geometric quantization.

Thermodynamics and statistical mechanics

In thermodynamics and statistical mechanics contexts, the term phase space has two meanings: It is used in the same sense as in classical mechanics. If a thermodynamical system consists of *N* particles, then a point in the *6N*-dimensional phase space describes the dynamical state of every particle in that system, as each particle is associated with three position variables and three momentum variables. In this sense, a point in phase space is said to be a microstate of the system. *N* is typically on the order of Avogadro's number, thus describing the system at a microscopic level is often impractical. This leads us to the use of phase space in a different sense.

The phase space can refer to the space that is parametrized by the *macroscopic* states of the system, such as pressure, temperature, etc. For instance, one can view the pressure-volume diagram or entropy-temperature diagrams as describing part of this phase space. A point in this phase space is correspondingly called a macrostate. There may easily be more than one microstate with the same macrostate. For example, for a fixed temperature, the system could have many dynamic configurations at the microscopic level. When used in this sense, a phase is a region of phase space where the system in question is in, for example, the liquid phase, or solid phase, etc.

Since there are many more microstates than macrostates, the phase space in the first sense is usually a manifold of much larger dimensions than the second sense. Clearly, many more parameters are required to register every detail of the system up to the molecular or atomic scale than to simply specify, say, the temperature or the pressure of the system.

See also

- Classical mechanics
- Dynamical system

- Molecular dynamics
- Hamiltonian mechanics
- Lagrangian mechanics
- Cotangent bundle
- Symplectic manifold
- Phase plane
- Phase space method
- Parameter space
- Optical Phase Space
- State space (controls) for information about state space (similar to phase state) in control engineering.
- State space (physics) for information about state space in physics
- State space for information about state space with discrete states in computer science.