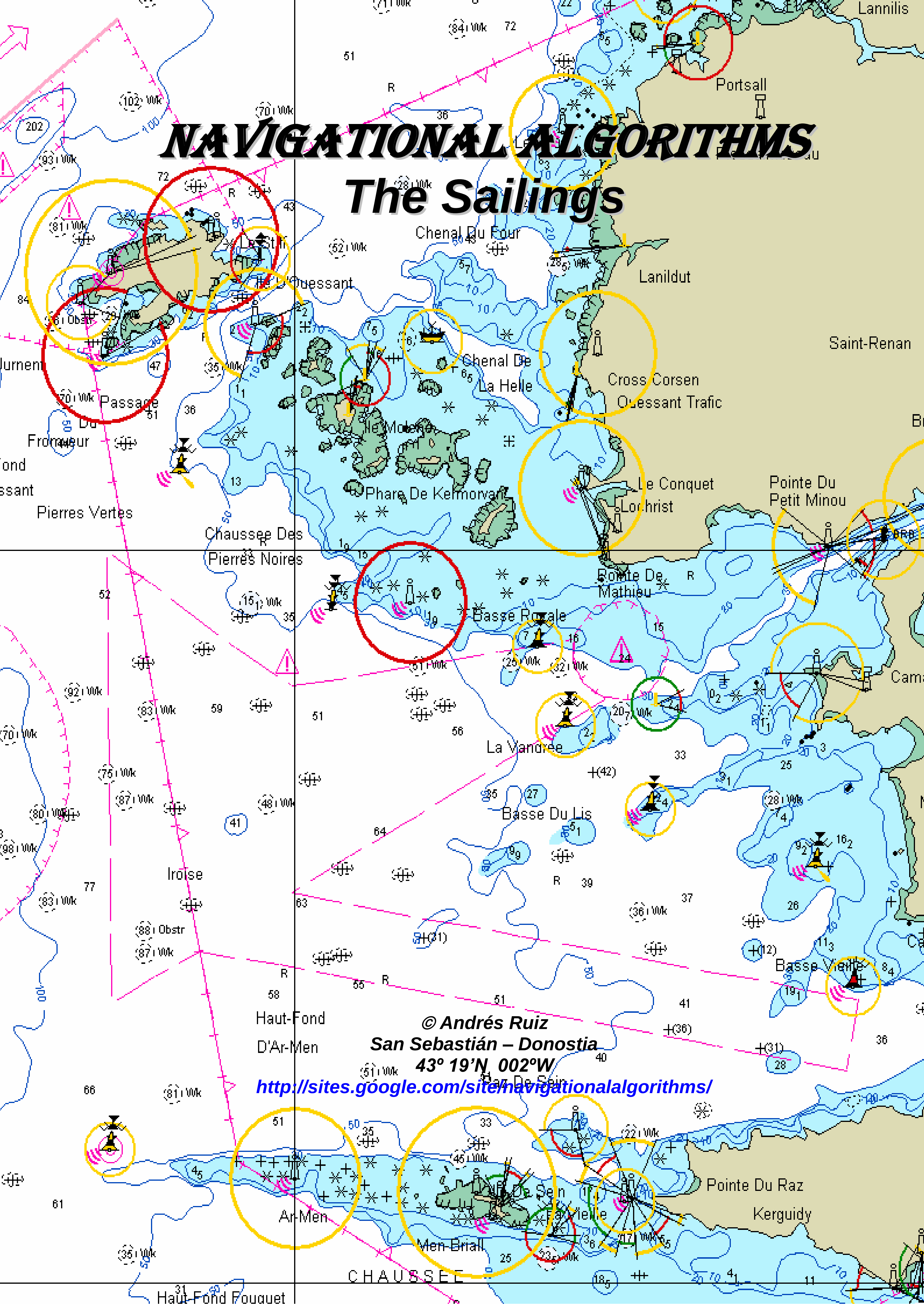
# NAVIGATIONAL ALGORITHMS

## The Sailings






**San Sebastián – Donostia**
**43º 19'N 002ºW**
**http://sites.google.com/site/navigationalalgorithms/**

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### *Abstract*

Dead reckoning involves the determination of one's present or future position by projecting the ship's course and distance run from a known position. A closely related problem is that of finding the course and distance from one known point to another. For short distances, these problems are easily solved directly on charts, but for trans-oceanic distances, a purely mathematical solution is often a better method. Collectively, these methods are called The Sailings [3].


*Navigational Algorithms*
*San Sebastián – Donostia*
*43º 19'N 002ºW*
Current version: 200801

## The Sailings

### *Variables*

| | |
|---|---|
| **B1** | Latitude departure |
| **L1** | Longitude departure |
| **B2** | Latitude destination |
| **L2** | Longitude destination |
| **d** | Distance |
| **R** | Rhumb |
| **ΔB** | Difference in latitude |
| **ΔL** | Difference in longitude |

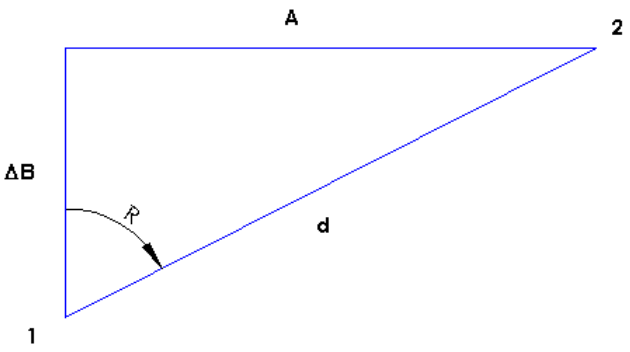
| | Intervals |
|---|---|
| B | -90º (S) <= B <= +90º (N) |
| L | -180º (W) <= L <= +180º (E) |
| R | 0º <= R <= 360º |
| d | d > 0 |

## Middle-Latitude Sailing

### *Variables*

| | |
|---|---|
| **A** | Departure |
| **Bm** | Middle latitude |

- $B_m < 60º$
- d < 200 nm
- $\Delta B < 5º$




Difference in latitude and departure.

- $\Delta B = d \cos R$
- $A = d \sin R$
- $\Delta L = A/ \cos B_m$
- $B_m = (B1+ B2)/2$

### *Position*

ΔB = d cos R
A = d sin R
ΔL = A/ cos Bm

B2 = B1 + d cos R
L2 = L1 + d sin R/ cos Bm

### *Course & Distance*

ΔB = B2 - B1
ΔL = L2 - L1
A = ΔL cos Bm

$$d = \sqrt{\Delta B^2 + A^2}$$

R = arctan( A/ΔB )

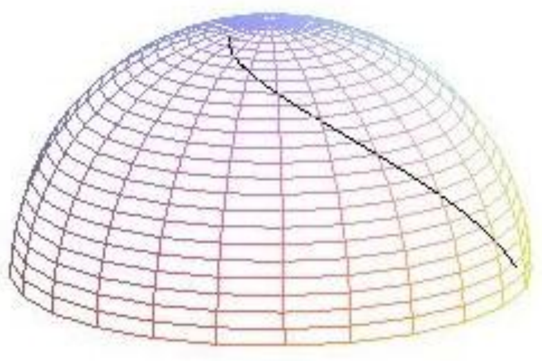
d = d/60 [nm]
if( R<0 ) R=R+360º

### *Example*

(B1, L1 ) = ( 43º 40.5'N 02º 0.00'W )
(B2, L2 ) = ( 45º 36.2'N 03º 15.5'W )

d = 127.56 millas náuticas
R = 335.09º

## Loxodromic. Rhumb Line. Mercator Sailing



Loxodromic on the sphere.

Loxodromic in a Mercator chart.

$$m = \Delta M$$

$$\tan R = \Delta L / m$$

$$L2 = L1 + m \tan R$$

$$d = \Delta B / \cos R$$

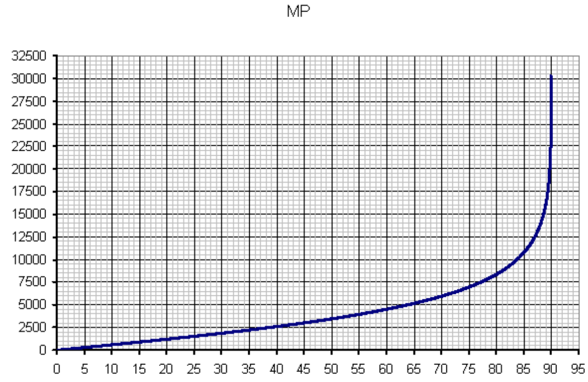
### *Meridional Parts*

$$M = a \log_e 10 \log \tan\left(45 + \frac{L}{2}\right) - a\left(e^2 \sin L + \frac{e^4}{3} \sin^3 L + \frac{e^6}{5} \sin^5 L + \ldots\right),$$

$$M(B) = a \cdot \mathrm{Ln}(\ \tan(45 + B/2) \left(\frac{1 - e \cdot \sin B}{1 + e \cdot \sin B}\right)^{e/2})$$

For ***WGS84***, (World Geodetic System):

```
a = 6378137/1852
f = 1.0/298.257223563
e = sqrt(2*f-SQ(f))
M1 = log(10)*log10(tan( 45+B/2) )
M2 = SQ( eoe )*sin( B )
M3 = pow( eoe, 4 )/3*pow( sin( B ), 3)
M4 = pow( eoe, 6 )/5*pow( sin( B ), 5)
M = a*(M1-M2-M3-M4)
```




M(B)

### *Sphecical Earth*:

Axes: $a = b = R_T = 360*60/(2\pi)$ [nm]

Flattening: $f = 1 - b/a = 0$

eccentricity $e = 0$

$$M = a \int_0^B \sec B \cdot dB$$

**M = 21600/(2π)*Ln( tan(45+B/2) )**

Using hyperbolic functions:

M = 21600/(2π)*arctanh( sin B )

M = 21600/(2π)* arcsinh( tan B )

### *Singular cases*

| | R [º] | sin R | cos R | tan R | ΔB | ΔL |
|---|---|---|---|---|---|---|
| **N** | 0 | 0 | 1 | 0 | d | 0 |
| **E** | 90 | 1 | 0 | ∞ | 0 | d/cos B |
| **S** | 180 | 0 | -1 | 0 | -d | 0 |
| **W** | 270 | -1 | 0 | ∞ | 0 | -d/cos B |

### *Position*

#### Latitude:

```
ΔB = d/60 * COS( R )
B2 = B1 + ΔB
```

## Longitude:

```
if( R == 90 || R == 270 )
  ΔL = d/60 * sin R/cos B
else {
  m = (M(B2)-M(B1))/60
  ΔL = m * tan R
  }
L2 = L1 + ΔL
```

### *Course & Distance*

```
ΔB = B2 - B1
ΔL = L2 - L1
m = (M(B2)-M(B1))/60
```

## Course:

```
if( abs( m ) > 0 ) {
  R = atan( ΔL/m )
  if( m >= 0 AND ΔL >= 0 )
    R = R
  else if( m <= 0 AND ΔL >= 0 )
    R = R + 180°
  else if( m <= 0 AND ΔL <= 0 )
    R = R + 180°
  else if( m >= 0 AND ΔL <= 0 )
    R = R + 360°
  }
// ΔB = 0
else if( ΔL > 0 )
  R = 90°
else if( ΔL < 0 )
  R = 270°
```

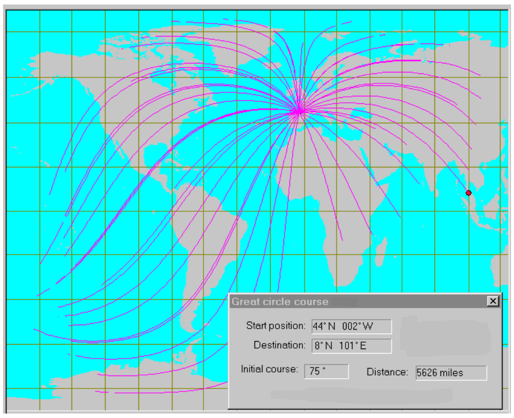
## Distance (nm):

```
if( R == 90 OR R == 270 )
  // cos R = 0
  d = |ΔL*cos B1|
else
  d = ΔB / cos R
d = d * 60
```

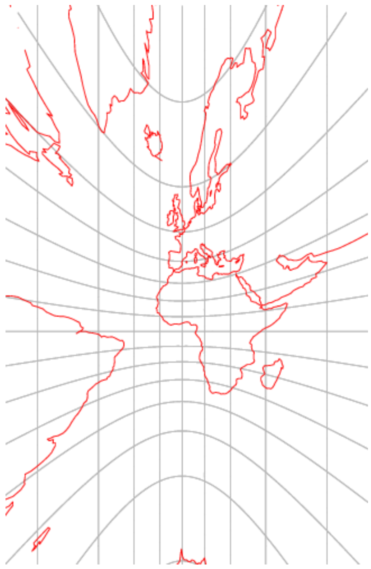
# Great Circle Sailing

### *Variables*

**D** Distancia Ortodrómica entre 1 y 2
**Ri** Rumbo inicial




GC from one departure point to some destinations.



Gnomonic Chart.

**Great circle sailing** involves the solution of courses, distances, and points along a great circle between two points.

### *Distance & Initial GC course*

$\Delta L = L2 - L1$
$-180^\circ <= \Delta L <= 180^\circ$

$$D = \mathrm{acos}(\sin B1 \sin B2 + \cos B1 \cos B2 \cos \Delta L)$$

D = 60*D [millas náuticas]

$$Ri = \mathrm{acos}\left(\frac{\sin B2 - \cos D \sin B1}{\sin D \cos B1}\right)$$

If( $\Delta L < 0$ ) Ri = 360- Ri

***(B1,L1, D, Ri) ⇨ (B2,L2)***

cD = 90-D/60

B2 = 90º - acos( sin B1 sin cD
+ cos B1 cos cD cos Ri )

$$\Delta L = a\cos\left(\frac{\sin cD - \cos(90^{o}\text{-B2})\sin B1}{\sin(90^{o}\text{-B2})\cos B1}\right)$$

L2 = L1 + ΔL

### *Latitude Equation of the Mid-longitude*

Consider a great-circle route, from WP1(B1, L1) to WP2(B2, L2).

The latitude, at the mid-longitude point, where the longitude is Lm, It's given by the expression [10]:

Lm = (L1 + L2) / 2

$$\tan B_m = \frac{\tan B_1 + \tan B_2}{2\cos(\frac{L_2 - L_1}{2})}$$

That is, you average the tangents of the latitudes at both ends, divide by the cos of half the longitude difference, that's the tan of the latitude you are after.

Having the coordinates of that middle-point of the route, you can then easily split each half further, and so on, using the same method, until your point-to-point legs are short enough to treat each one as a rhumb-line.

### *Example*

(B1, L1 ) = ( 43º 40.5'N 02º 0.00'W )
(B2, L2 ) = ( 45º 36.2'N 03º 15.5'W )

D = 127.56 nm
Ri = 335.09º

## Composite sailing

**Composite sailing** is a modification of great-circle sailing to limit the maximum latitude, generally to avoid ice or severe weather near the poles.

## Vector GC

Under construction

### *Vector Equation*

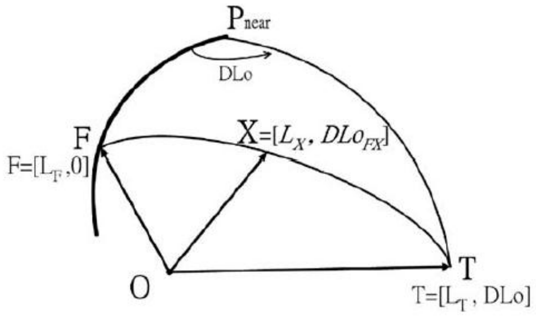



Figure 4. An illustration for three co-planar position vectors on the great circle track.

$$\vec{F} = [\cos L_F, 0, \sin L_F],$$

$$\vec{T} = [\cos L_T \cdot \cos DLo, \cos L_T \cdot \sin DLo, \sin L_T],$$

$$\vec{X} = [\cos L_X \cdot \cos DLo_{FX}, \cos L_X \cdot \sin DLo_{FX}, \sin L_X],$$

$$\vec{X} \bullet (\vec{P}_1 \wedge \vec{P}_2) = 0$$

### *Distance & Initial GC course*

$$D = R_e \mathrm{arcos}[\mathbf{V_1} \cdot \mathbf{V_2}].$$

$$\mathrm{Cos}\ \gamma = \left(\frac{(\mathbf{V_1 x V_p})}{|\mathbf{V_1 x V_p}|} \cdot \frac{(\mathbf{V_1 x V_2})}{|\mathbf{V_1 x V_2}|}\right)$$

### *Vertices*

$$\frac{d\phi}{d\theta} = \frac{\lambda\sin\theta - \mu\cos\theta}{\sec^2\phi} = 0$$

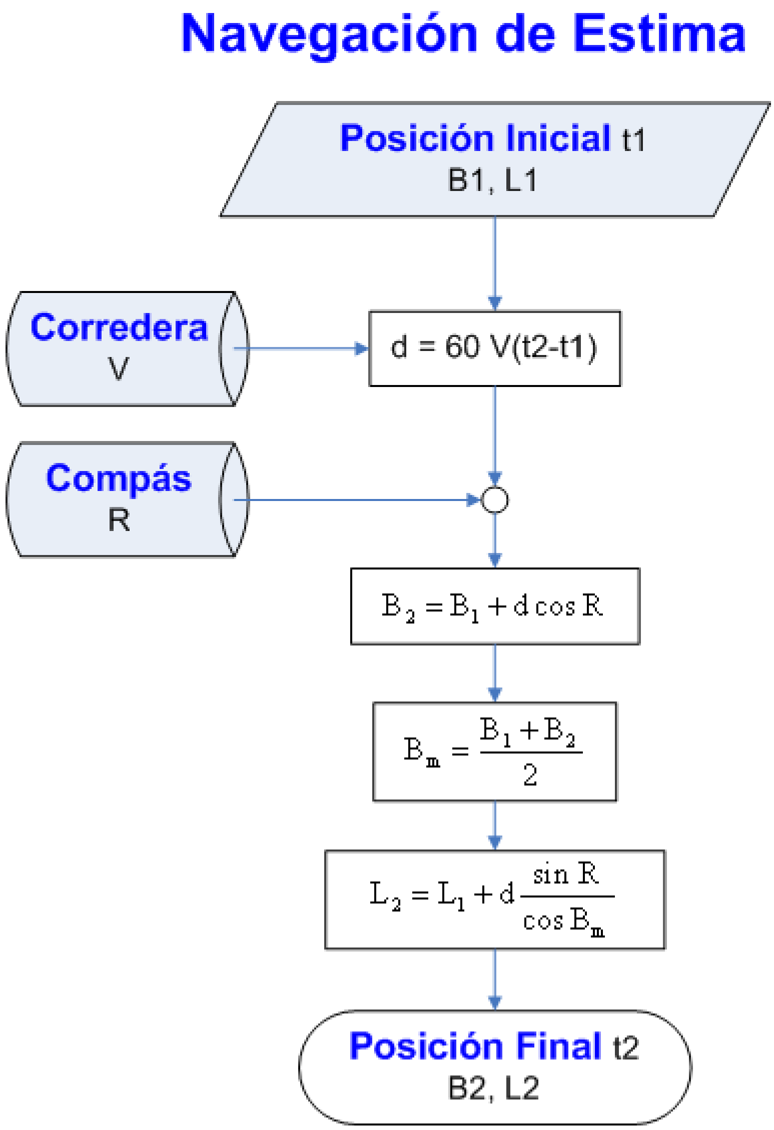
$$\theta_v = \arctan\left(\frac{\mu}{\lambda}\right)$$

$$\phi_v = \pm\arctan\sqrt{\lambda^2 + \mu^2}$$

### *Nodes*

$$\theta_o = \theta_v \pm \pi/2.$$

## A1. Algorithms. Dead Reckoning




## A2. Examples

Example 1 pg 368 Bowditch

| | | R | d |
|---|---|---|---|
| B1 = 32.245 º | | | |
| L1 = -66.4817 º | Latitud Media | 301.9501 | 536.6754 |
| B2 = 36.9783 º | Loxodrómica | 301.8474 | 538.2231 |
| L2 = -75.7033 º | Ortodrómica | 304.5122 | 536.2734 |

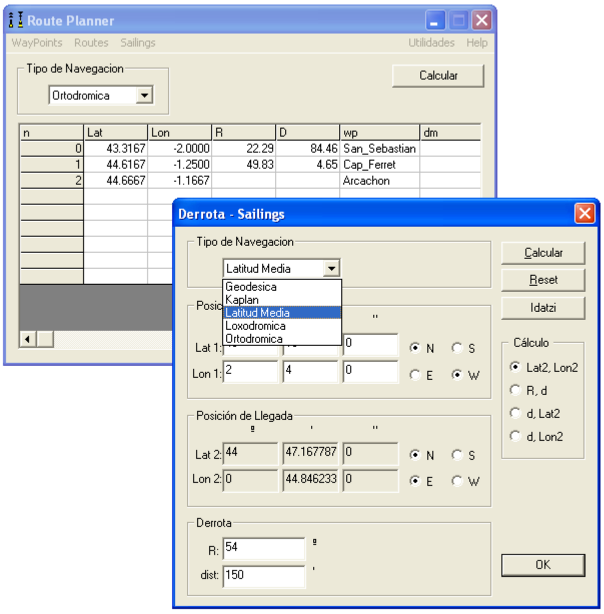
Example 2 pg 368 Bowditch

| | | B2 | L2 |
|---|---|---|---|
| B1 = 75.5283 º | | | |
| L1 = -79.145 º | Latitud Media | 71.5481 | -72.5954 |
| R = 155 º | Loxodrómica | 71.5481 | -72.5672 |
| d = 263.5 mn | Ortodrómica | 71.457 | -73.3044 |

## A3. Software

RoutePlanner

Available at the author's web site.




## A4. Source code

## A5. Bibliography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2. Navigation and Piloting. **Dutton**. 14th ed. ISBN: 0-87021-157-9
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6. A Vector Solution for Navigation on a Great Ellipse. Michael A. Earle. Journal of Navigation, Volume 53, Issue 03, September 2000, pp 473-481
7. A Novel Approach to Great Circle Sailings: The Great Circle Equation. Chih-Li Chen, Tien-Pen Hsu and Jiang-Ren Chang. Journal of Navigation, Volume 57, Issue 02, May 2004, pp 311-320
8. Vector Solutions for Great Circle Navigation. Michael A. Earle. Journal of Navigation, Volume 58, Issue 03, September 2005, pp 451-457
9. The Vector Function for Distance Travelled in Great Circle Navigation. Wei-Kuo Tseng and Hsuan-Shih Lee. Journal of Navigation, Volume 60, Issue 01, January 2007, pp 158-164
10. Building the Latitude Equation of the Mid-longitude. Wei-Kuo Tseng and Hsuan-Shih Lee. Journal of Navigation, Volume 60, Issue 01, January 2007, pp 164-170


### Meridional Parts tables

**Bowditch:**

http://www.nga.mil/MSISiteContent/StaticFiles/NAV_PUBS/APN/Tables/T-06-A.pdf
*Estas tablas mezclan conceptos del modelo esférico con el elipsoide WGS72, ver:*
*"A Comment on Navigation Instruction. Michael A. Earle. Journal of Navigation, Volume 58, Issue 02, pp 337-340"*

**Marine navigation - Navigational Algorithms:**

http://sites.google.com/site/navigationalalgorithms/downloads/mp.txt