Group delay and phase delay

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(Redirected from Phase delay)

Group delay is a measure of the transit time of a signal through a device under test (DUT), versus frequency. Group delay is a useful measure of phase distortion, and is calculated by differentiating the insertion phase response of the DUT versus frequency. Another way to say this is that group delay is a measure of the slope of the transmission phase response. The linear portion of the phase response is converted to a constant value (representing the average signal-transit time) and deviations from linear phase are transformed into deviations from constant group delay. The variations in group delay cause signal distortion, just as deviations from linear phase cause distortion. Group delay is just another way to look at linear phase distortion.

In LTI system theory, control theory, and in digital or analog signal processing, the relationship between the input signal, $x(t)$, to output signal, $y(t)$, of an LTI system is governed by:

$$y(t) = h(t) * x(t) \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} x(u)h(t-u)\,du$$

Or, in the frequency domain,

$$Y(s) = H(s)X(s)$$

where

$$X(s) = \mathcal{L}\left\{x(t)\right\} \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$Y(s) = \mathcal{L}\left\{y(t)\right\} \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} y(t)e^{-st}\,dt$$

and

$$H(s) = \mathcal{L}\left\{h(t)\right\} \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} h(t)e^{-st}\,dt.$$

Here $h(t)$ is the time domain impulse response of the LTI system and $X(s)$, $Y(s)$, $H(s)$, are the Laplace transforms of $x(t)$, $y(t)$, and $h(t)$, respectively. $H(s)$ is called the transfer function of the LTI system and, as does the impulse response, $h(t)$, *fully* defines the input-output characteristics of the LTI system.

When such a system is driven by a quasi-sinusoidal signal, (a sinusoid with a slowly changing amplitude envelope $A(t)$, relative to the change of phase, ω, of the sinusoid),

$$x(t) = A(t)\cos(\omega t + \theta)$$

the output of such an LTI system is very well approximated as

$$y(t) = |H(i\omega)| A(t - \tau_g) \cos\left(\omega(t - \tau_\phi) + \theta\right)$$

if

$$\frac{d \log\left(A(t)\right)}{dt} \ll \omega$$

and τ_g and τ_ϕ, the **group delay** and **phase delay** respectively, are as shown below and potentially functions of ω. In a linear phase system (with non-inverting gain), both τ_g and τ_ϕ are equal to the same constant delay of the system and the phase shift of the system increases linearly with frequency ω.

It can be shown that for an LTI system with transfer function *H*(*s*) that if such is driven by a complex sinusoid of unit amplitude,

$$x(t) = e^{i\omega t}$$

the output is

$$\begin{aligned} y(t) &= H(i\omega)e^{i\omega t} \\ &= \left(|H(i\omega)|e^{i\phi(\omega)}\right) e^{i\omega t} \\ &= |H(i\omega)|e^{i(\omega t + \phi(\omega))} \end{aligned}$$

where the phase shift ϕ is

$$\phi(\omega) \stackrel{\text{def}}{=} \arg\{H(i\omega)\}$$

Additionally, it can be shown that the group delay, τ_g, and phase delay, τ_ϕ, are related to the phase shift ϕ as

$$\tau_g = -\frac{d\phi(\omega)}{d\omega}$$

$$\tau_\phi = -\frac{\phi(\omega)}{\omega}.$$

In physics, and in particular in optics, the term **group delay** has the following meanings:

1. The rate of change of the total phase shift with respect to angular frequency,

$$\tau_g = -\frac{d\phi}{d\omega}$$

through a device or transmission medium, where ϕ is the total phase shift in radians, and ω is the angular frequency in radians per unit time, equal to $2\pi f$, where f is the frequency (hertz if group delay is measured in seconds).

2. In an optical fiber, the transit time required for optical power, traveling at a given mode's group velocity, to travel a given distance.

Note: For optical fiber dispersion measurement purposes, the quantity of interest is group delay per unit length, which is the reciprocal of the group velocity of a particular mode. The measured group delay of a signal through an optical fiber exhibits a wavelength dependence due to the various dispersion mechanisms present in the fiber.

Source: from Federal Standard 1037C

It is often desirable for the group delay to be constant across all frequencies; otherwise there is temporal smearing of the signal. Because group delay is $\tau_g(\omega) = -\frac{d\phi}{d\omega}$, as defined in (1), it therefore follows that a constant group delay can be achieved if the transfer function of the device or medium has a linear phase response (i.e., $\phi(\omega) = \phi(0) - \tau_g\omega$ where the group delay τ_g is a constant). The degree of nonlinearity of the phase indicates the deviation of the group delay from a constant.

Group delay in the audio field

Group delay has some importance in the audio field and especially in the sound reproduction field. Many components of an audio reproduction chain, notably loudspeakers and multiway loudspeakers crossover networks, introduce group delay in the audio signal. It is therefore important to know the threshold of audibility of group delay with respect to frequency, especially if the audio chain is supposed to provide a high fidelity reproduction. At the time of writing no extensive data is available, and the concept is often treated by "rule of thumb" or based on hunches and received wisdom. The best thresholds of audibility table has been provided by Blauert and Laws:

Frequency	Threshold
500 Hz	3.2 ms
1 kHz	2 ms
2 kHz	1 ms
4 kHz	1.5 ms
8 kHz	2 ms

The table above has been published into the following article:

Blauert, J. and Laws, P "Group Delay Distortions in Electroacoustical Systems", Journal of the Acoustical Society of America, Volume 63, Number 5, pp. 1478–1483 (May 1978)

See also

- Audio system measurements
- Bessel filter

External links

- Discussion of Group Delay in Loudspeakers
- Group Delay Explanations and Applications

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