The Earth's Magnetic Field

Our planet is surrounded by a magnetic field (Fig. 1.1). In our experience this phenomenon is revealed for example by a compass needle that points approximately to the north. According to modern geophysical ideas when, at a given point and at a certain time, a measurement of the Earth's magnetic field is carried out, the measured value is the result of the superimposition of contributions having different origins. These contributions can be, at a first glance, considered separately, each of them corresponding to a different source:

a The main field, generated in the Earth's fluid core by a *geodynamo* mechanism;
b The crustal field, generated by magnetized rocks in the Earth's crust;
c The external field, produced by electric currents flowing in the ionosphere and in the magnetosphere, owing to the interaction of the solar electromagnetic radiation and the solar wind with the Earth's magnetic field;
d The magnetic field resulting from an electromagnetic induction process generated by electric currents induced in the crust and the upper mantle by the external magnetic field time variations.

In order to analyze the various contributions, we will start here with the spatial analysis of the most stable part of the Earth's magnetic field (parts a and b), following in particular the procedure used by Gauss who was the first to introduce the analysis of the Earth's magnetic field potential. After this we will describe the Earth's magnetic

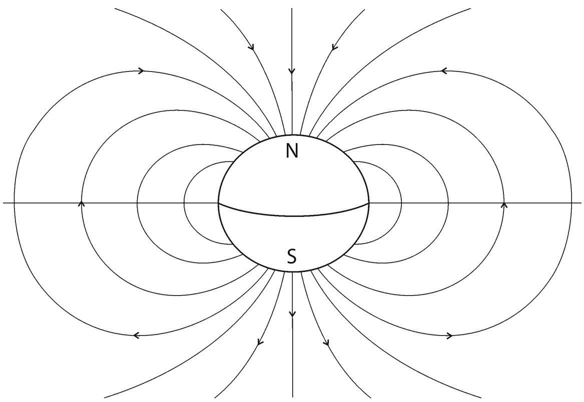


Fig. 1.1. Idealized view of the Earth's magnetic field lines of force with Earth represented as a sphere. N and S are the ideal location of the two magnetic poles

field time variations. In fact the Earth's magnetic field not only shows a peculiar spatial structure, mainly determined by 'a' and 'b' contributions, but is also subject to continuous time variations. These variations, which can have different origins, can be subdivided into two broad classes: long-term and short-term time variations. The former, generally denoted by the name *secular variation*, can be detected when at least 5–10 years, or more, magnetic data from a certain area are examined; this variation is due to the evolution of the deep sources within the Earth, the same sources that also generate the main field. The short-term variations are of external origin to the Earth and are detected over shorter time windows, that can go from fractions of a second generally to no more than a few years (they are essentially included in contribution 'c' above). Finally a magnetic field results from the electromagnetic induction process that is generated by electric currents induced in the crust and the upper mantle, by the external magnetic field time variations. This happens because the Earth is partially an electric conductor and electrical currents can be induced in its conducting parts by external time variations. The secondary magnetic field generated in this way, adds to the other sources.

Only after the results from global analyses of the Earth's magnetic field will be shown, we will give a description of the most important time variations and give an overview of the geodynamo theory. In other chapters the magnetic field of crustal origin and its applications will be discussed.

1.1 Observations and Geomagnetic Measurements

1.1.1 The Magnetic Dipole

The fundamental entity in the study of magnetism is the dipole, that is a system consisting of two magnetic charges, or magnetic masses, of equal intensity and opposite signs. In practice any magnetic bar can be considered a dipole. In some elementary physics books, the term *magnetic mass* is still associated with each end of the dipole. This concept was historically introduced because the magnetic actions exerted by the dipole appear as produced by sources concentrated at its ends, as similarly happens in the case of the electric dipole. However it is well known that if we break a magnetic bar, we do not obtain two separate magnetic charges but two new dipoles. The magnetic bar acts as it consisted of two magnetic masses of equal and opposite signs but this is only a schematic approach. The physical origin of magnetism lies in the electrical properties of matter, an electron in its orbit generates an electric current that in turn generates a magnetic field equivalent to that of a magnetic bar. Therefore we will not deal here with the concept of magnetic mass but we will consider the dipole to be the elementary magnetic structure.

It is simple to show that the magnetic potential V, produced by a magnetic dipole (Fig. 1.2) at a point P, with coordinates (r, θ) in a plane whose polar axis coincides, in direction and versus, with the moment $\boldsymbol{M}$ of the dipole and the origin with its center, is given by

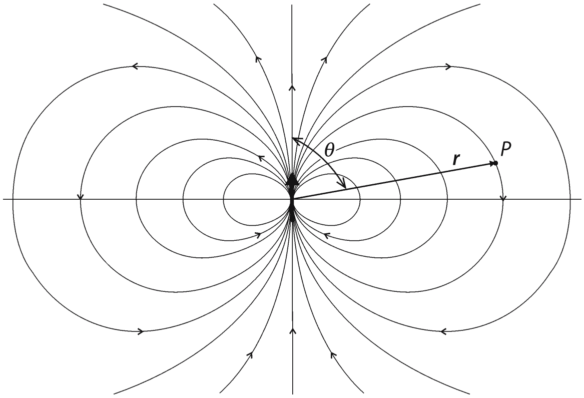


Fig. 1.2. Magnetic dipole field lines of force. The *arrow* indicates the magnetic dipole, $\boldsymbol{r}$ is the vector distance and θ colatitude, as referred to a point P in polar coordinates

$$V = \frac{\mu_0}{4\pi}\frac{\boldsymbol{M}\cdot\boldsymbol{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2} \tag{1.1}$$

or similarly, using $\boldsymbol{r}$ as a radius vector, the gradient formulation can also be used:

$$V = \frac{\mu_0}{4\pi}\boldsymbol{M}\, grad\left(\frac{1}{\boldsymbol{r}}\right) = \frac{\mu_0}{4\pi}\boldsymbol{M}\nabla\left(\frac{1}{\boldsymbol{r}}\right) \tag{1.2}$$

For symmetry reasons this relationship is valid in all planes passing through the polar axis of the dipole. Therefore in each of these planes we can split the magnetic vector field[1] that we will call $\boldsymbol{F}$ into two components; starting from the following relationship

$$\boldsymbol{F} = -grad V = -\nabla V \tag{1.3}$$

Taking into account the polar coordinate system described above and referring to F_t as the component transverse to the radius vector (positively oriented towards increasing θ, which is called *colatitude*), and to F_r the component directed along $\boldsymbol{r}$ (positively oriented outward), we will obtain

$$F_r = -\frac{\partial V}{\partial r}; \qquad F_t = -\frac{1}{r}\frac{\partial V}{\partial \theta} \tag{1.4}$$

1 $\mu_0 = 4\pi \times 10^{-7}$ Henry m^{-1}, is the magnetic permeability of vacuum. IAGA (International Association of Geomagnetism and Aeronomy) has recommended, as a regular procedure, to use $\boldsymbol{B}$, magnetic induction, for measurements of Earth's magnetic field instead of $\boldsymbol{H}$, magnetic field strength.

$$F_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}; \qquad F_t = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3} \tag{1.5}$$

Along the dipole axis, for $\theta = 0$ or $\theta = \pi$, and orthogonally to this axis, for $\theta = \pi/2$, we will have respectively two polar positions and, in an immediate analogy with the Earth's case, an equatorial position. In these cases the defined components have the following values

Polar position $\quad F_t = 0; \quad F_r = \pm\frac{\mu_0}{4\pi}\frac{2M}{r^3}; \quad \theta = 0(\pi)$

Equatorial position $\quad F_t = \frac{\mu_0}{4\pi}\frac{M}{r^3}; \quad F_r = 0; \quad \theta = \frac{\pi}{2}$

While for any given value of θ, we have:

$$F = \left(F_t^2 + F_r^2\right)^{\frac{1}{2}} \tag{1.6}$$

$$F = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{3\cos^2\theta + 1} \tag{1.7}$$

This last relation constitutes the equation of a generic line of force of the dipole magnetic field in polar coordinates.

If the magnetic dipole is immersed in an external magnetic field, as in the case of a magnetic needle or a compass in the Earth's magnetic field, and we let it be free to rotate, both in the horizontal and vertical plane, we can note that it aligns along a particular direction, whatever its original direction was. This is because the needle tends to minimize its interaction energy with the magnetic field in which it is immersed. It is possible to note that to make the interaction energy with an external magnetic field a minimum, a dipole tends to be parallel to a line of force of the external field. If we indicate with $\boldsymbol{F}$ the external magnetic field and with $\boldsymbol{M}$ the dipole (magnetic needle) magnetic moment, the interaction energy E can be expressed as

$$E = -\boldsymbol{M} \cdot \boldsymbol{F} \tag{1.8}$$

while the mechanical couple, Γ, acting on the dipole is

$$\Gamma = \boldsymbol{M} \times \boldsymbol{F} \tag{1.9}$$

The above formulas use the magnetic field $\boldsymbol{F}$ dimensionally as a magnetic induction; we will see that this is considered a standard approach for the Earth's magnetic field. In geomagnetism most of the theoretical studies and data analyses have been devoted to the reconstruction of the configuration of the lines of force of the Earth's magnetic field.

A noticeable analogy can be made between a simple dipole and the source of the Earth's magnetic field. In fact the first analyses carried out by Gauss, already in the first half of 19th century, confirmed the early Gilbert statement that the Earth's magnetic field, in first approximation, appears as generated by a huge magnetic dipole. This dipole is located, inside the Earth, at its center, and has its axis almost parallel to the axis of Earth's rotation. In order to match the orientation of a magnetic needle with its magnetic north pointing to geographic north, the Earth's dipole moment must be oriented in the opposite direction with respect to the Earth's rotation axis (see Fig. 1.2).

1.1.2 Elements of the Earth's Magnetic Field

From now on we will denote by $\boldsymbol{F}$ the Earth's magnetic field vector and, even though currently called a magnetic field, it is intended as a magnetic induction field, which in common physics text books is referred to as $\boldsymbol{B}$. It can be decomposed on the Earth's surface, along three directions. Considering the point of measurement as the origin of a Cartesian system of reference, the x-axis is in the geographic meridian directed to the north, y-axis in the geographic parallel directed to the east and z-axis parallel to the vertical at the point and positive downwards. The three components of the Earth's magnetic field along such axes are called X, Y and Z (Fig. 1.3). We will then have

$$\sqrt{X^2 + Y^2 + Z^2} = F\,; \qquad \sqrt{X^2 + Y^2} = H \tag{1.10}$$

where we have also included H as the horizontal component. In order to describe the field, in addition to the intensive components, we can also use angular elements. They

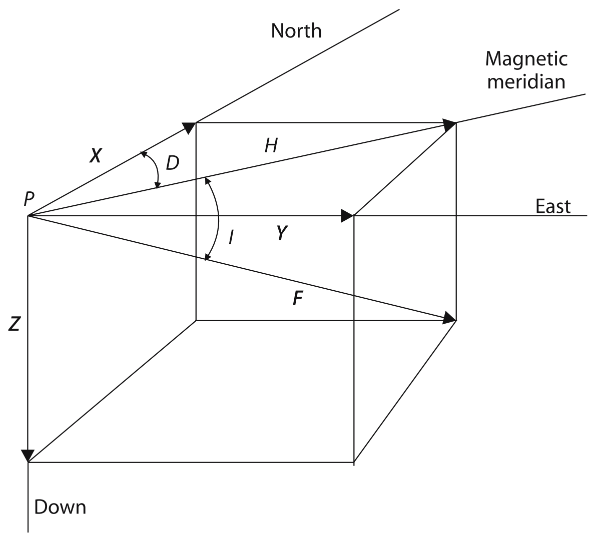


Fig. 1.3. Elements of the Earth's magnetic field. At point P, on the Earth, three axes point respectively to north geographic (x), east geographic (y), and along the vertical downwards (z). The Earth's magnetic field vector $\boldsymbol{F}$ can be projected along the three axes and three magnetic components are obtained X, Y and Z. $\boldsymbol{F}$ also forms an angle I, inclination, with the horizontal plane; H is the horizontal projection of $\boldsymbol{F}$ and angle D, declination, is the angle between H and X

are obtained by introducing two angles, that is I, the inclination of vector $\boldsymbol{F}$ with respect to the horizontal plane, and D declination, the angle between H, the horizontal component of $\boldsymbol{F}$, and the X component, along the geographic meridian. The relationships among these quantities now defined, are

$$H = F\cos I; \quad Z = F\sin I; \quad Z = H\tan I; \quad X = H\cos D; \quad Y = H\sin D \tag{1.11}$$

Three of these quantities (provided they are independent of each other) are completely sufficient to determine the Earth's magnetic field. Note that H is in geomagnetism the horizontal component of $\boldsymbol{F}$ and must not be confused with the generally agreed use of $\boldsymbol{H}$ in physics where the magnetic field strength is in general intended.

Representing the Earth as a sphere and assuming in first approximation that the field is generated by a dipole placed at its center and pointing towards a given direction, we can visualize a new geometry. The dipole axis through the Earth's center, can be called a *geomagnetic axis* and we obtain that, at a point P on the Earth's surface, what in the magnetic dipole geometry previously were indicated by F_t and F_r, now are equivalent to the horizontal and vertical components of the geomagnetic dipole field:

$$H = F_t; \quad Z = -F_r; \quad \sqrt{H^2 + Z^2} = F^2 \tag{1.12}$$

In such a dipole field the geometry of the lines of force, which will be denoted by the function $r = r(\theta)$, can be derived from

$$\frac{Z}{H} = \frac{\mathrm{d}r}{r\mathrm{d}\theta} = 2\frac{M\cos\theta}{M\sin\theta} = 2\cot\theta$$

and moreover

$$\frac{\mathrm{d}r}{r\mathrm{d}\theta} = 2\frac{\cos\theta}{\sin\theta}; \qquad \frac{\mathrm{d}r}{r} = \frac{2\cos\theta\mathrm{d}\theta}{\sin\theta} = 2\frac{\mathrm{d}(\sin\theta)}{\sin\theta}$$

$$\ln r = 2\ln(\sin\theta) + C \quad \rightarrow \quad r = r_e\sin^2\theta; \quad r_e = C$$

where r_e is the Earth's equatorial radius. This analytical representation of the lines of force of the Earth's magnetic field is very useful in the representation of the magnetic field outside the Earth in the so-called magnetosphere (Sect. 1.3.4.1).

As mentioned above, by international agreement, the measurement unit for the Earth's magnetic field is usually expressed in terms of the induction vector $\boldsymbol{B}$. The SI unit of $\boldsymbol{B}$ is the Tesla, but in practice in geophysics its submultiple, the nanoTesla, nT (10^{-9} T) is currently used. The Gauss is instead the fundamental unit of measurement for magnetic field induction in the cgs-emu system (Appendix). On the Earth's surface the Earth's magnetic field varies in magnitude mainly with latitude; to grab an idea, the field varies from about 20 000 nT to about 68 000 nT from the equator to the poles. In

Figs. 1.4, 1.5 and 1.6, the horizontal, vertical and total magnetic field isodynamic charts showing the spatial variations of the given element on the Earth's surface for the year 2005, are reported; in Fig. 1.7 the isogonic map for declination at year 2005 is reported.

1.1.3 Early Measurements of the Earth's Magnetic Field

The object of geomagnetic measurements is the quantitative determination of the Earth's magnetic field elements; this is done using magnetic instruments, called *magnetometers*. Over the years many kind of magnetometers have been designed by scholars and specialists in order to improve the quality of the measurement or to reach a better portability, efficiency, or ease of use. We will not go all the way through the long history of magnetic instruments here, we will however start with a brief introduction describing classical mechanical magnetometers and then we will directly proceed with the more modern and widely used instruments, based on electromagnetic or nuclear phenomena, which make a large use of modern electronics.

Gauss was the first to construct a complete set for the absolute determination of the geomagnetic field elements in the early years of the 19th century. Being the geomagnetic field a vector it is in fact self evident that its complete determination needs the quantification of all elements of this vectorial quantity. The magnetic compass was already used in the middle ages employing magnetic needles to point the magnetic north. The almost faithful north indication made the compass a very useful instrument for north bearing, especially for ships. Around the 15th century it became clear that the compass was not pointing precisely to the geographic north but that an angle, later on called declination, was separating magnetic north indication from geographic north indication. So by an independent measurement of the geographic north, a magnetic needle mounted on a horizontal circle allowed the determination of the declination angle in the horizontal plane. The inclinometer, probably introduced during the 16th century, gives the magnetic field ***F*** inclination with respect to the horizontal plane. Inclinometers also used magnetic needles but the needle was pivoted around a horizontal axis;

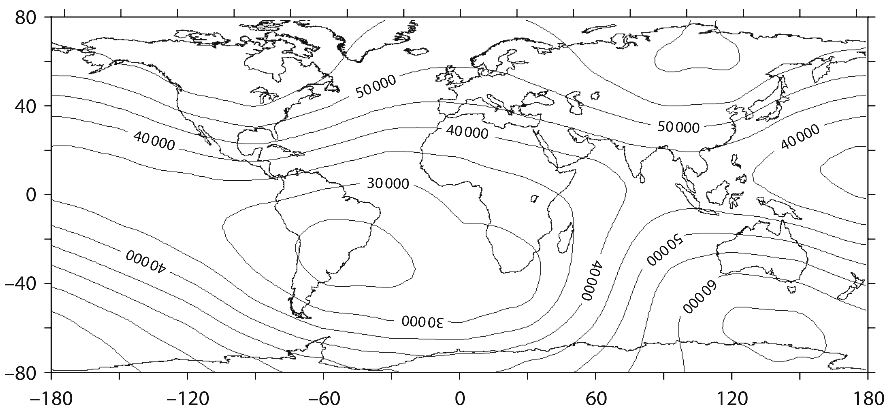


Fig. 1.4. Isodynamic world chart for Earth's magnetic total field *F*. Contour lines in nT, for the year 2005 from IGRF 10th generation model

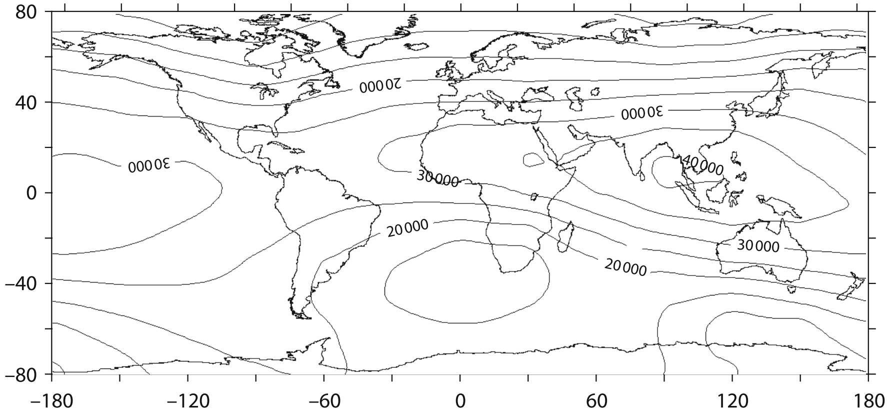


Fig. 1.5. Isodynamic world chart for Earth's magnetic field horizontal intensity *H*. Contour lines in nT, for the year 2005 from IGRF 10th generation model

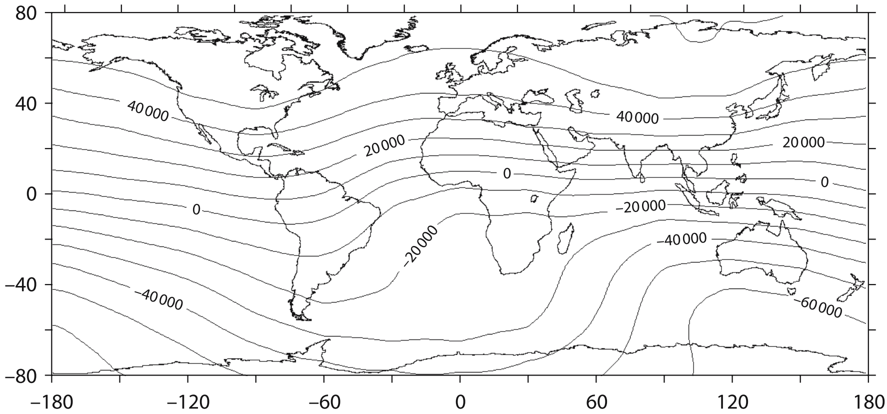


Fig. 1.6. Isodynamic world chart for Earth's magnetic field vertical intensity *Z*. Contour lines in nT, for the year 2005 from IGRF 10th generation model

the inclination angle being measured on a vertical circle. The vertical circle was first carefully oriented in the magnetic meridian plane, then the angle the needle formed with respect to the horizontal, that is the Earth's magnetic field inclination, was measured.

Neither of these angular measurements were sufficiently precise for scientific procedures. One step forward in the measurement of declination, improving its accuracy, was made with the introduction of suspended needles, kept horizontal by means of a special supporting equipment, the equipment in turn suspended by means of a thread. In this way the effect of friction on the pivot was eliminated. A more accurate reading became possible by the use of an optical telescope.

A full knowledge of the Earth's magnetic field vector ***F*** needs at least the measurement of one of its intensive components. The well known explorer Von Humboldt used

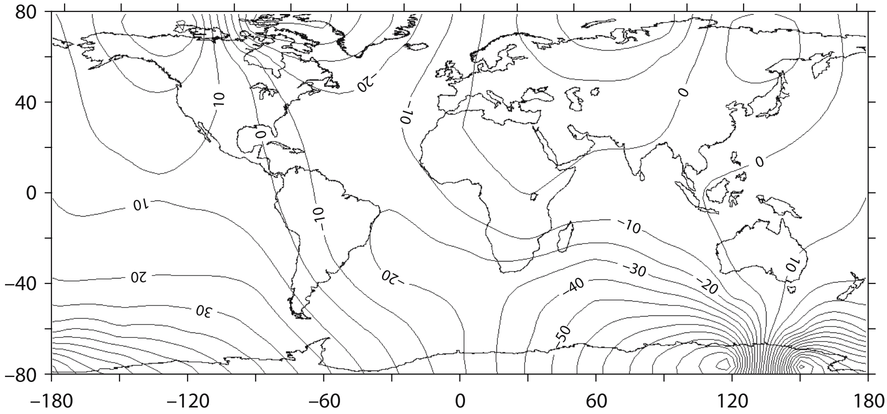


Fig. 1.7. Isogonic world chart for Earth's magnetic field declination *D*. Contour lines in degrees (°), for the year 2005 from IGRF 10th generation model

the observation of the time of oscillation of a compass needle in the horizontal plane to determine relative measurements of horizontal intensity using the relation

$$T = 2\pi\,(I\,/\,MH)^{1/2} \tag{1.13}$$

which connects, for small amplitude oscillations, the period of oscillation T of a magnet with its moment of inertia I and magnetic moment M in a horizontal magnetic field H. This very simple method reduced relative H measurements to the measurement of the oscillation period of a magnet. The procedure was generally adopted by several observers in scientific journeys allowing to obtain a first order approximation knowledge on magnetic field magnitude variation around the globe. Unfortunately in order to establish the absolute magnitude of the magnetic field H, the determination of the needle magnetic moment M and moment of inertia I was necessary.

In 1832 Gauss was the first to realize that it was possible to devise a procedure for the correct absolute determination of the Earth's magnetic field horizontal intensity. This method, modified later by Lamont, consists in the comparison of two mechanical couples acting on a horizontal suspended magnetic needle. One couple is the Earth's magnetic field couple, while the second is artificially acted by a magnet located at a fixed distance from the oscillating needle. In a first phase of the measurements the magnetic needle is accurately oriented along the Earth's magnetic field; in a second phase a deflecting magnet is put in operation at a distance r, laterally at a right angle to the central needle. Calling $\boldsymbol{M}$ the deflecting magnet magnetic moment, the central needle will experience not only the Earth's magnetic field horizontal intensity, H, but also a second field, whose intensity we can call H_1, generated by the deflecting magnet $\boldsymbol{M}$:

$$H_1 = 2\boldsymbol{M}\,/\,r^3 \tag{1.14}$$

As a result the central needle (Fig. 1.8) will be under the influence of the two couples which will move it to a new position, forming an angle α with the initial direction.

The equilibrium position will now be given by

$$H / M = 2 / r^3 \tan\alpha \tag{1.15}$$

Both quantities r and α can easily be measured by a centimeter scale and an optical telescope. In the Lamont variant, all the procedure is such that at the end of the measurement the deflecting magnet M and the central needle are mutually perpendicular so that the final formulation simplifies to

$$H / M = 2 / r^3 \tag{1.16}$$

If the deflecting magnet is the same magnet used in the first part of the experiment with the two equations (Eqs. 1.13 and 1.15), the first, as mentioned above already known by Von Humboldt, and the second found in his experiment, Gauss was able to determine for the first time the magnetic field horizontal absolute intensity H. In this manner the Earth's magnetic field became the first non-mechanical quantity expressed in

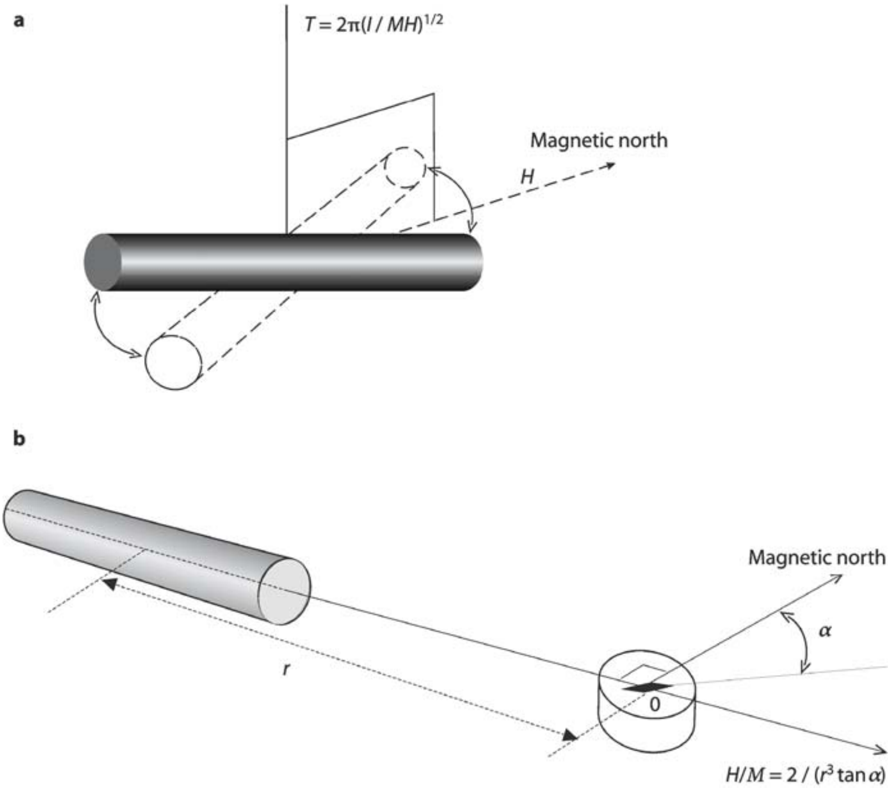


Fig. 1.8. Gauss Lamont magnetometer; **a** a magnetic bar oscillates with period T in the Earth's magnetic field; **b** the magnetic bar is now used to deflect a magnetic needle that rotates freely to an equilibrium position in the magnetic bar and the Earth's magnetic fields

terms of the three fundamental mechanical quantities: mass, length and time. This result was reported in the Gauss's memoir *Intensitas vis Magneticae Terrestris ad Mensuram Absolutam Revocata* in 1833, the last great scientific memoir written in Latin. The complete instrument used in his procedure was for the first time called a magnetometer.

1.1.4 Modern Magnetic Measurements

Since the Earth's magnetic field is a vector quantity, the field magnitude is absolute if expressed in terms of the fundamental quantities (for example mass, length, time and electrical current intensity), while the vector spatial orientation can be expressed for example in terms of D and I, angular dimensionless quantities. From the total field F magnitude and the angular quantities, the geomagnetic field components H, Z and also X, Y can be computed. Sometimes magnetic instruments give as outputs directly the geomagnetic components; it is self evident that once three independent elements are determined, the magnetic field measurement is considered complete.

Nowadays magnetic instruments that utilize magnets for their operation are only very seldom used in magnetic observatories. Moreover the measurement of declination and inclination angles is a procedure employed mainly for absolute magnetic measurements in magnetic observatories or at repeat magnetic stations. An instrument is called absolute when it gives the value of the measured quantity in terms of one or more of the absolute basic fundamental quantities of physics. For this reason in geomagnetism the term *absolute measurement* is still often used to indicate a procedure for the complete absolute determination of the magnetic field elements. An instrument is called relative when it measures the value of one element of the Earth's field as a deviation from a certain initial value not necessarily known. Many of these instruments require a reference initial value that must be determined independently, for example by means of an absolute instrument. The use of relative instruments can of course be very convenient especially in some field operations, for example when only the spatial variation of the magnetic field in an investigated area is required. A second case is when, at a given place, a time variation of the Earth's magnetic field needs to be recorded.

Instruments are delivered with information and data sheets that provide the values of the parameters necessary to evaluate their measurement capability. The most frequently used parameters are reported in what follows.

- *Accuracy:* indicates how an instrument is accurate, that is the maximum difference between measured values and true values.
- *Precision:* is related to the scatter of the measured values and refers to the ability of the instrument of repeating the same value when measuring the same quantity.
- *Resolution:* represents the smallest change of the measured quantity that is detectable by the instrument.
- *Range:* refers to the upper and lower (extreme) limits that can be measured with the instrument. The dynamic range is the ratio between the maximum measurable quantity and the resolution, normally expressed in dB, i.e. $20\log(A_{max}/A_{min})$.

- *Sensitivity:* indicates how many scale units of the instrument correspond to one unit of the measured physical unit.
- *Scale value:* is the reciprocal of sensitivity.

Magnetic instruments are nowadays not only devoted to magnetic measurement, they are also frequently equipped with electronic cards, able to memorize measured data and to interface to PCs for real-time or off-line data communication.

1.1.4.1
Absolute Instruments

Proton Precession Magnetometers and Overhauser Magnetometers
These instruments are based upon the nuclear paramagnetism, i.e. the circumstance that atomic nuclei posses a magnetic spin that naturally tends to orient itself along an external magnetic field. In these magnetometers the sensor is made up of a small bottle full of a hydrogenated liquid (such as propane, decane or other that can operate as liquid in a reasonable temperature range) around which a two coil system is wounded. A direct electrical current is applied to the first winding (polarization coil) by means of an external power supply and consequently generates a magnetic field inside the bottle. Protons in the bottle are then forced to align their spin along this magnetic field starting to precess at a frequency rate depending on the magnetic field magnitude. If the external current is interrupted, the artificial magnetic field is removed and then protons in the bottle will start precessing around the Earth's magnetic field direction at a frequency f given by

$$f = (\gamma / 2\pi)F \tag{1.17}$$

where γ is the so-called magneto-mechanical proton ratio (gyromagnetic ratio) a fundamental quantity, very precisely known in atomic physics (2.6751525×10^8 rad s^{-1} T^{-1}) and F is the external Earth's magnetic field. The proton precession generates at the ends of the second winding (pick-up coil) a time varying electromotive force (e.m.f.) with the same frequency, which can easily be measured to obtain the absolute total field F magnitude. In the average Earth's magnetic field (for example 45 000 nT) the frequency is very close to 2 kHz (1 916 Hz) (Fig. 1.9).

The loss of coherence inside the bottle allows only a small time window (about 2–3 s) for the detection of the e.m.f. frequency. This time is however now more than sufficient for modern electronic frequency meters to give the precession frequency. In fact due to progress in electronic technology, the measurement of frequency is in contemporary physics one of the most accurate techniques. Since it is only dependent on the measurement of a frequency, the measurement of the Earth's magnetic field by means of a proton precession magnetometer is both very precise and absolute: resolution reaches now easily 0.1 to 0.01 nT.

One disadvantage of proton precession magnetometers is the limitation due to the fact that the polarization current needs to be switched off in order to make a measurement. The operation is therefore discontinuous with a time interval of a few seconds between measurements. A continuous proton precession signal can however be obtained by taking advantage for example of the so-called Overhauser effect. The ad-

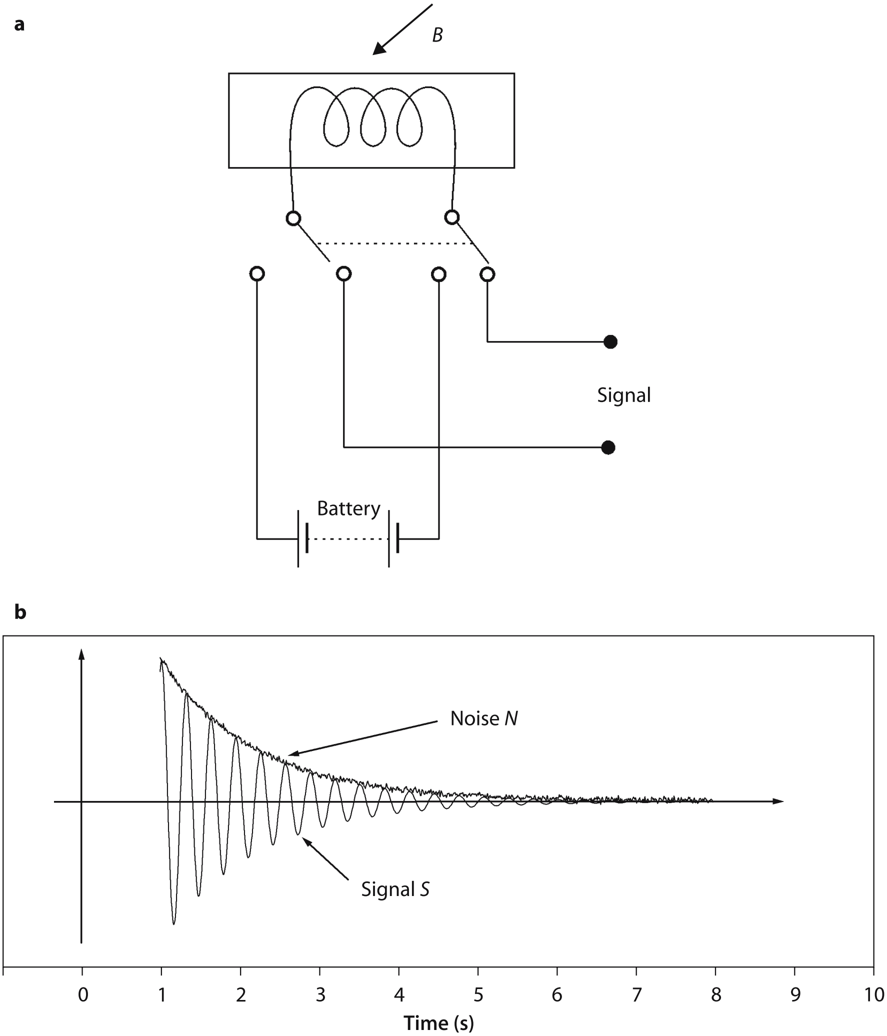


Fig. 1.9. Proton precession magnetometer; **a** electric circuitry schematics for measurement of field B. The measurement is performed in two steps: (1) generation of free proton precession by power injection; (2) signal detection after switching; **b** typical detected signal amplitude decrease. Signal to noise ratio is optimum for only a few seconds after polarization is turned off (from Jankowsky et al. 1996)

dition of free electrons into the liquid in the bottle and the application of a suitable radio frequency, can in fact increase the magnetization of the liquid sample. Without going into details, we will just remember here that as an alternative to applying a strong polarizing field, in Overhauser magnetometers the magnetization is increased by applying a suitable radio frequency electromagnetic field to put the free electrons into resonance. This electron resonant frequency that exceeds by 658 times the proton resonant frequency, has the role of increasing the proton level saturation making the proton precession process signal output in Overhauser magnetometers continuous rather than discrete.

Optically Pumped Magnetometer

This magnetometer is based on the Zeeman effect and the so-called stimulated emission of radiation in certain substances, as in the Maser effect. The instrument consists of a bottle containing a gas such as helium, rubidium or cesium vapors and some sophisticated light detectors. The Zeeman effect deals with the splitting of electron sublevels separation in energy levels under the influence of a magnetic field. Since the energy differences between levels of hyperfine splitting are very small, a specific technique, called optical pumping, is used. The term optical pumping refers to the process of increasing the population of one of the sublevels in the gas that, in the measuring procedure is initially underpopulated, by means of a circularly polarized external radiation at the spectral line frequency corresponding to the level separation in the Earth's magnetic field. Once the overpopulation is obtained, an electromagnetic discharge takes place at the frequency $f = \Delta E / \mathrm{h}$ where ΔE is the transition energy between Zeeman sublevels and h is Planck's constant (6.62×10^{-34} joule–second). ΔE is proportional to the Earth's magnetic field magnitude, which can be calculated from an equation similar to Eq. 1.17: $f = (\gamma_e / 2\pi)F$, where γ_e is the gyromagnetic ratio of the electron. Without going in details, for example in the rubidium vapor magnetometer, the rubidium light passes through an interference filter, a circular polarizer and an absorbing cell, which is filled with rubidium vapor. The light is pumping atoms and the cell becomes transparent to the resonant light. The light intensity is measured by means of a photocell; the radio frequency from the oscillator follows the resonance frequency due to negative feedback from the photocell.

The frequency f is of the order of 200 kHz and γ_e is known to a precision of about $1/10^7$. Accordingly, optically pumped magnetometers have a very high resolution of 0.01–0.001 nT and are some of the most sensitive instruments for magnetic measurements. Their performance can be exploited almost continuously in time, making this instrument very useful for rapid data acquisition at a very high resolution. For this reason they are very common in space magnetometry as well as in aeromagnetism and also in some magnetic prospecting on the ground.

1.1.4.2
Relative Instruments

Fluxgate Magnetometers

Fluxgate magnetometers are electromagnetic instruments that can give direct magnetic measurements along a built-in direction. By orienting this direction along the Earth's magnetic field elements such as F, Z or H, these elements may be measured. The orienting device may vary according to requirements. In some fluxgate magnetometers this built-in direction is along a straight cylinder, while in others the direction is taken along the plane of a ring shaped sensor.

In one class of fluxgate magnetometers the sensor unit is constituted by a cylindrical core with very high magnetic permeability (for example made of permalloy, mumetal or ferrite) placed inside two windings. In the first winding a 1 000 Hz excitation current flows and generates an alternate magnetic field, large enough to saturate the core. In the absence of an external steady magnetic field acting on the sensor, such as a component of the Earth's field, the alternating field collected by the second winding (pick-up coil) contains only the odd harmonics of the excitation current. If a steady magnetic field acts along the core axis, then this field sums to the alternating one in such a way that one semiwave of the e.m.f. in the pick-up coil is now larger in amplitude than the one generated in the opposite direction. In this case in fact the core is brought to saturation faster in the direction parallel to the Earth's field than in the opposite direction. In the pick-up coil a double frequency current will now appear; the amplitude of this current is linearly proportional to the magnitude of the external field acting along the core direction (Fig. 1.10).

In other fluxgate magnetometer models, the central core is substituted by two parallel ferromagnetic cores arranged in such a way that the alternating current acting on the cores produces at the excitation winding terminals two equal and out of phase e.m.f. exactly balanced, which thus sum up to zero. When an external magnetic field acts on the cores, this symmetry is broken and the varying e.m.f. induced in the pick-up coil is linearly proportional to the magnitude of the external field.

In actual fluxgate magnetometers the sensor excitation is produced by means of an electronic oscillator, the signal from the pick-up coil is fed into a tuned amplifier and the output is fed to a phase sensitive detector referenced to the second harmonic of the excitation frequency. The fluxgate magnetometer is a zero field instrument. This means that in order to measure the full intensity of the geomagnetic field along one of its components it also needs an auxiliary compensation system. One serious problem in fluxgate magnetometers, is the temperature variation; in fact the bias coils need a stabilization. To obtain this stabilization the coils have to be wound around quartz tubes or other thermally stable material frames. The fluxgates have a reasonable 0.1 nT resolution and are non absolute instruments frequently used for recording magnetic time variations.

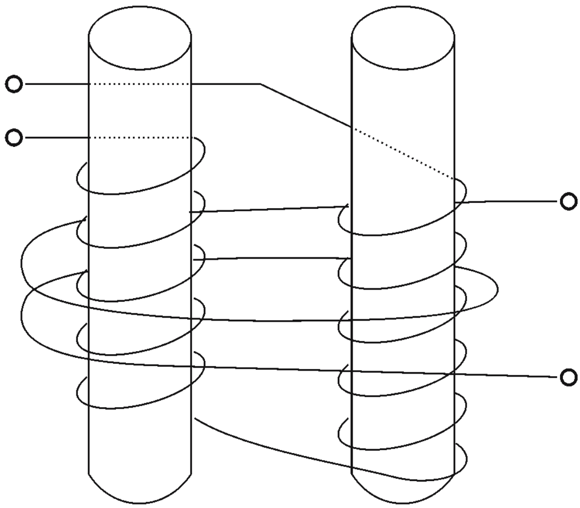


Fig. 1.10a. Fluxgate magnetometer; winding schematics in the case of a two core fluxgate instrument

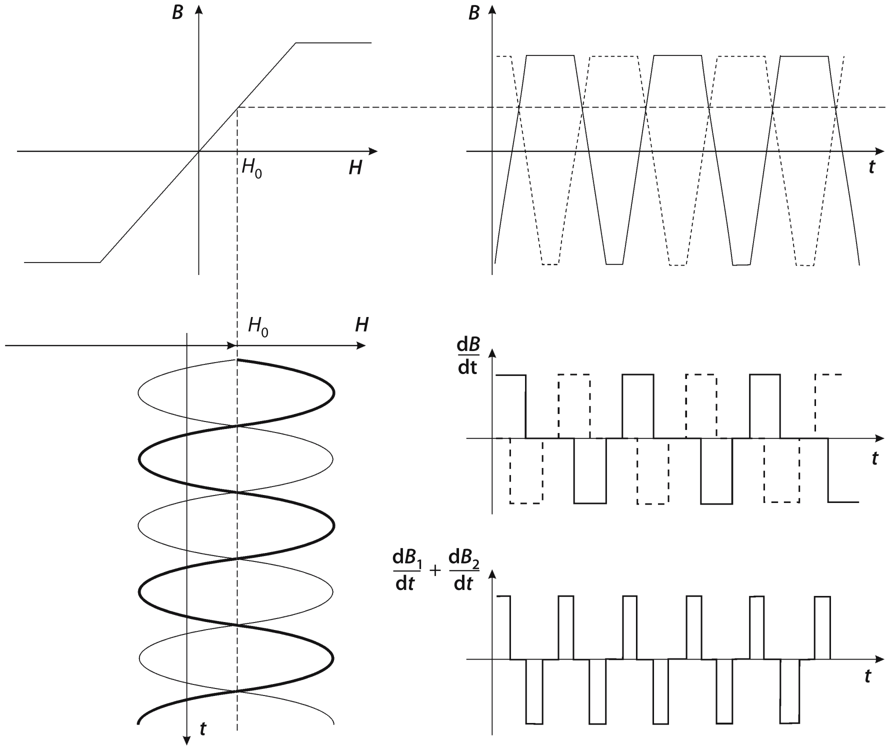


Fig. 1.10b. Fluxgate magnetometer; ***B*** field waveforms at the output signal of a two core fluxgate instrument (from Lowrie 1997)

1.2 Mathematical Description

1.2.1 Spherical Harmonic Analysis

In order to prove analytically that the magnetic field on the Earth's surface is approximately similar to that generated by a dipole placed within the Earth, and to understand this aspect, it is necessary to look at the governing equations for magnetism and introduce the so-called spherical harmonic analysis technique. From Maxwell's equations, for the magnetic induction $\boldsymbol{B}$, we have

$$\mathrm{div}\boldsymbol{B} = \nabla \bullet \boldsymbol{B} = 0\,; \qquad \mathrm{curl}\boldsymbol{B} = \nabla \times \boldsymbol{B} = \mu(\boldsymbol{I} + \frac{\partial \boldsymbol{D}}{\partial t}) \tag{1.18}$$

where $\boldsymbol{I}$ denotes the electric current density, $\boldsymbol{D}$ is the dielectric induction and μ is the magnetic permeability. In a space where there are no discontinuity surfaces and no electric currents, it can be assumed that $\boldsymbol{B}$ can be derived from a magnetic potential V

$$\begin{aligned}
&\operatorname{curl}\boldsymbol{B}=0;\quad \boldsymbol{B}=-\operatorname{grad}V\\
&\operatorname{div}(-\operatorname{grad}V)=\nabla^2 V=0\\
&\Delta V=0
\end{aligned}$$

where Δ is the so-called Laplacian operator equivalent to ∇^2. The last equation for V, known as Laplace's equation, in orthogonal Cartesian coordinates, becomes

$$\Delta V=\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0 \tag{1.19}$$

and can be written in a spherical coordinate system, with the origin at the Earth's center, as

$$\Delta V=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial V}{\partial r})+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial V}{\partial\theta})+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\lambda^2}=0 \tag{1.20}$$

Each function $V=V(r,\theta,\lambda)$ that satisfies this equation is called a harmonic function; r and θ are defined as in Sect. 1.1.1 where the dipole geometry was described in a given plane and let now λ be the geographic longitude, we can assume the Earth to be a sphere of radius a (Fig. 1.11).

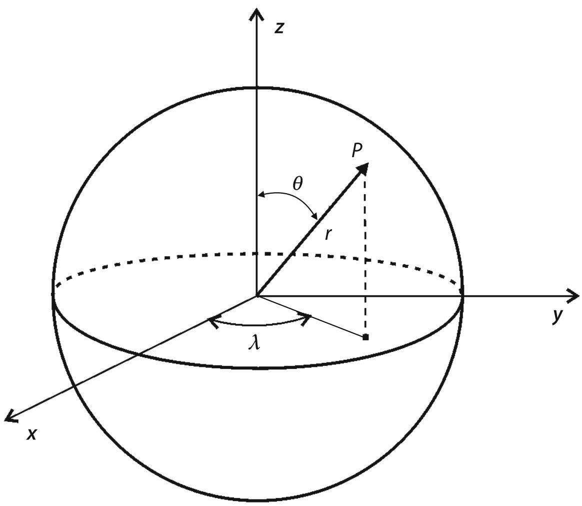


Fig. 1.11. Earth-centered coordinates x, y and z with origin at the Earth's center. Spherical coordinates: for point P on the Earth's surface, r is the distance from Earth's center, θ colatitude and λ longitude

The general solution for a potential in Laplace's equation can be obtained (as similarly occurs in the Earth's gravity case), by means of a technique called spherical harmonic analysis (SHA). The determination of three orthogonal functions, expressed in terms of only one variable each, is needed. In the search for these functions we will take into account the characteristics of the field as considered in spherical coordinates.

Starting from the variable r those functions that take into account the two possible origins of the field, internal or external to the Earth respectively, are considered. As regards λ its definition demands a periodic behavior from 0 to 2π, suggesting the use of periodic functions, as in case of a Fourier series in λ. For what concerns θ, in geomagnetism, the Schmidt quasi-normalized functions are used; similarly to the case of gravity potential solutions, these are equivalent to Legendre functions $P_{n,m}(\theta)$, but with a different normalizing factor. Schmidt functions are in fact normalized to be of the same order of magnitude as the zonal Legendre functions of the same degree. Let us first refer to usual Legendre functions

$$P_{n,m}(\theta) = \sin^m \theta \frac{d^m P_n(\theta)}{d(\cos\theta)^m} \tag{1.21}$$

where n denotes the degree and m is the order, and, for $m = 0$, these reduce to the standard zonal functions

$$P_n(\theta) = \frac{1}{n!2^n} \frac{d^n}{d(\cos\theta)^n} (\cos^2\theta - 1)^n \tag{1.22}$$

Only as an example we recall the first few Legendre zonal functions $P_n(\theta)$.

$$\begin{aligned}
P_0 &= 1 \\
P_1 &= \cos\theta \\
P_2 &= \frac{1}{2}(3\cos^2\theta - 1) \\
P_3 &= \frac{1}{2}(5\cos^3\theta - 3\cos\theta) \\
P_4 &= \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)
\end{aligned}$$

These functions represent the latitudinal magnetic field variations. Some of the first associated Legendre functions (also called spherical functions), drawn from Legendre (zonal) functions using the above formulation, are also shown here as an example (Fig. 1.12).

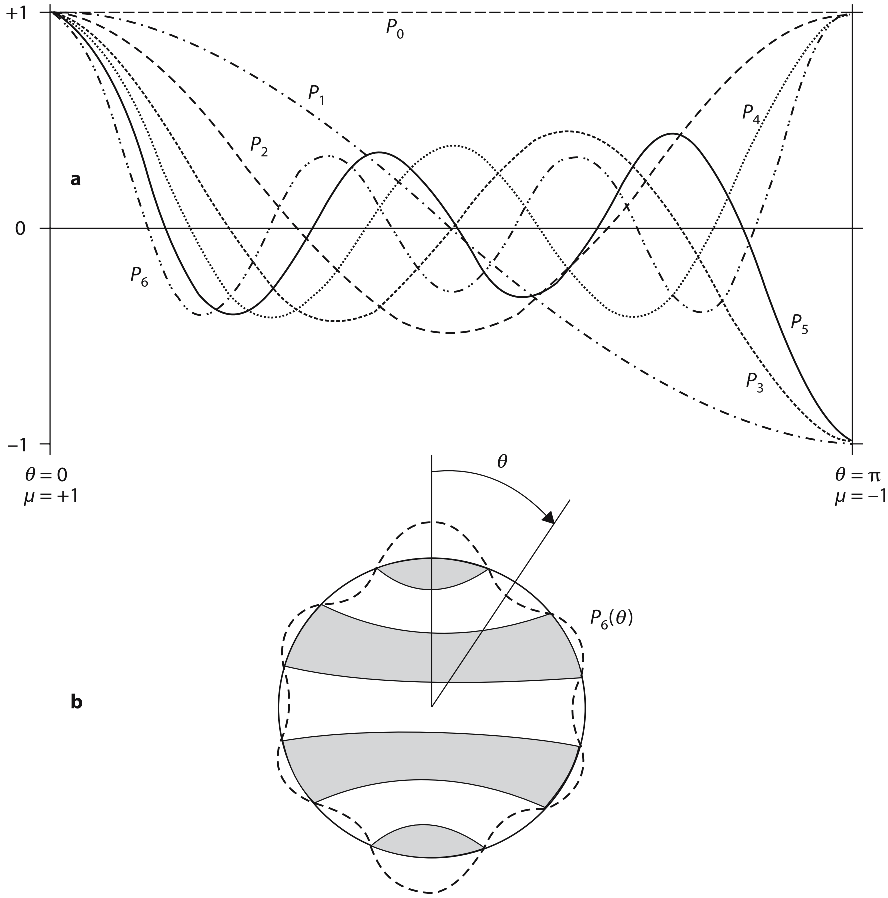


Fig. 1.12. A few low-degree (P_0–P_6) Legendre zonal harmonics on the Earth surface shown for $0 < \theta < \pi$. In the *lower part* function P_6 is shown along the circumference of a circle with gray and white zones to indicate negative and positive values for P_6 on the spherical surface (from Ahern, Copyright 2004)

$$P_1^1 = \sin\theta$$
$$P_2^1 = 3\cos\theta\sin\theta$$
$$P_2^2 = 3\sin^2\theta$$
$$P_3^3 = 15\sin^3\theta$$
$$P_4^4 = 105\sin^4\theta$$

The Schmidt functions used in geomagnetism, indicated by $P_n^m(\theta)$, are partially normalized Legendre functions, differing from Legendre functions only by a normalizing factor. They are defined as follows:

$$P_n^m(\theta)=P_{n,m}(\theta) \quad \text{for } m=0; \quad P_n^m(\theta)=\left(2\frac{(n-m)!}{(n+m)!}\right)^{\frac{1}{2}} P_{n,m}(\theta) \quad \text{for } m>0 \tag{1.23}$$

On the Earth's surface a solution of the Laplace equation should include, in principle, the two possible origins of the Earth's magnetic field, and consequently of the potential that generates the field. The field can in fact have an internal and an external origin. For this reason the coefficients of the selected functions that will result from the analysis, will be denoted by the i and e indices indicating internal and external contributions to the potential. This implies that a field of external origin adds to a field of internal origin to give the value measured on the Earth's surface. In order to satisfy the boundary conditions in Laplace's equation, a field of external origin would have to be generated at great distances from Earth, moreover its magnitude must, within the Earth's sphere, decrease from the surface to the Earth's center, going to zero at its center: a radial variability of the form of $(r/a)^n$ will satisfy this condition. On the contrary a magnetic field of internal origin will depend on r as $(a/r)^{n+1}$ since it must be valid in the space external to the Earth's sphere and decrease its intensity gradually moving outwards, going to zero at infinity.

The general expression of the magnetic potential can then be written as follows

$$V=a\sum_{n=1}^{\infty}\left(\left(\frac{r}{a}\right)^n T_n^{\mathrm{e}}+\left(\frac{a}{r}\right)^{n+1} T_n^{\mathrm{i}}\right) \tag{1.24}$$

where each function T_n with its indices i for internal and e for external, will be represented by the product of the two angular functions to represent the dependence on latitude and longitude, as described above. In general we have for each so-called spherical harmonic function $T_n(\theta,\lambda)$

$$T_n=\sum_{m=0}^{n}(g_n^m\cos m\lambda+h_n^m\sin m\lambda)P_n^m(\theta) \tag{1.25}$$

where g_n^m and h_n^m are the expansion coefficients for the magnetic potential, traditionally called in geomagnetism the Gauss coefficients.

Since the quantities on which an actual data analysis can be undertaken are the magnetic field components, the mathematical form of these quantities, that is the components of the vector field, must be considered. They are the space derivatives of the potential function V defined above:

$$X=\frac{\partial V}{r\partial\theta}=\frac{a}{r}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\frac{\mathrm{d}P_n^m}{\mathrm{d}\theta}\left[\begin{array}{l}\left\{g_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^n+g_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+1}\right\}\cos m\lambda\\ +\left\{h_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^n+h_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+1}\right\}\sin m\lambda\end{array}\right]$$

$$Y = \frac{-\partial V}{r\sin\theta\partial\lambda} = \frac{a}{r\sin\theta}\sum_{n=1}^{\infty}\sum_{m=0}^{n} P_n^m \begin{bmatrix} m\left\{g_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^n + g_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+1}\right\}\sin m\lambda \\ -m\left\{h_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^n + h_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+1}\right\}\cos m\lambda \end{bmatrix}$$

$$Z = \frac{\partial V}{\partial r} = \sum_{n=1}^{\infty}\sum_{m=0}^{n} P_n^m \begin{bmatrix} \left\{n g_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^{n-1} - (n+1) g_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+2}\right\}\cos m\lambda \\ +\left\{n h_n^{m\mathrm{e}}\left(\frac{r}{a}\right)^{n-1} - (n+1) h_n^{m\mathrm{i}}\left(\frac{a}{r}\right)^{n+2}\right\}\sin m\lambda \end{bmatrix}$$

On the Earth's surface, for $r = a$, these equations can be simplified and become

$$X = \sum_{n=1}^{\infty}\sum_{m=0}^{n} \frac{\mathrm{d}P_n^m}{\mathrm{d}\theta}\left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right) \tag{1.26}$$

$$Y = \frac{1}{\sin\theta}\sum_{n=1}^{\infty}\sum_{m=0}^{n} P_n^m\left(m g_n^m \sin m\lambda - m h_n^m \cos m\lambda\right) \tag{1.27}$$

$$Z = \sum_{n=1}^{\infty}\sum_{m=0}^{n} P_n^m\left[\left(n g_n^{m\mathrm{e}} - (n+1) g_n^{m\mathrm{i}}\right)\cos m\lambda + \left(n h_n^{m\mathrm{e}} - (n+1) h_n^{m\mathrm{i}}\right)\sin m\lambda\right] \tag{1.28}$$

We immediately note that in the equations for X and Y, the terms of the Gauss coefficients are now expressed as $g_n^m = g_n^{m\mathrm{e}} + g_n^{m\mathrm{i}}$ and $h_n^m = h_n^{m\mathrm{e}} + h_n^{m\mathrm{i}}$. They appear only as the sum of the internal and external contributions making in this way the separation of the two contributions impossible. On the contrary in the equation for Z these terms are still separated. This consideration will let us estimate such contributions separately. It is important to note, however, that from the distribution of measurements of the X and Y components on the Earth's surface, we can obtain two independent evaluations of g_n^m and h_n^m.

1.2.2 Methods for g_n^m and h_n^m Computation

In the last twenty years or so new methods of analysis have brought to sophisticated procedures for the computation of Gauss coefficients from magnetic measurements. For many years however a very simple and intuitive procedure was used.

Firstly isodynamic maps of the X and Y components were drawn for the whole Earth, thanks to a series of values of the field measured at an irregular distribution of points: observatories, field stations and also ship logs. Then from the obtained maps, values of the magnetic field components on a regular net of points, for instance at the crossing of the meridians and the parallels every 10° of latitude and longitude, could be obtained by interpolation. Fixing our attention on a particular fixed value of colatitude $\theta=\theta_0$ (parallel) and considering all values of the X component on this parallel, a simple procedure enabled the determination of Gauss coefficients. In fact let us start, for example, from the expansion in spherical harmonics for X (Eq. 1.26). On the array of measured points, on that parallel, let us apply a Fourier expansion (denoted by f) to the real data obtained for the component under consideration

$$f(\theta)=\sum_{m=0}^{n}(a_m\cos m\lambda+b_m\sin m\lambda) \tag{1.29}$$

whose coefficients for a total length of the parallel L, are generally denoted by

$$a_m=\frac{1}{L}\int_{-L}^{L} f(\theta)\cos\frac{m\pi\theta}{L}\,\mathrm{d}\theta$$

$$b_m=\frac{1}{L}\int_{-L}^{L} f(\theta)\sin\frac{m\pi\theta}{L}\,\mathrm{d}\theta$$

In order to make these two expansions identical, i.e. $f(\theta)$ were equal to $X(\theta_0)$, at $\theta=\theta_0$, the coefficients of the two respective series expansions must be equal, in detail:

$$X=\frac{\mathrm{d}P_1^0}{\mathrm{d}\theta}g_1^0+\frac{\mathrm{d}P_1^1}{\mathrm{d}\theta}g_1^1\cos\lambda+\frac{\mathrm{d}P_1^1}{\mathrm{d}\theta}h_1^1\sin\lambda+\frac{\mathrm{d}P_2^0}{\mathrm{d}\theta}g_2^0+\frac{\mathrm{d}P_2^1}{\mathrm{d}\theta}g_2^1\cos\lambda+\frac{\mathrm{d}P_2^1}{\mathrm{d}\theta}h_2^1\sin\lambda$$
$$+\frac{\mathrm{d}P_3^0}{\mathrm{d}\theta}g_3^0+\ldots$$

$$f(\theta)=a_0+a_1\cos\lambda+b_1\sin\lambda+\ldots$$

Once the Fourier expansion coefficients are known, this allows us to determine the Gauss coefficients for that component (on the given parallel). The above procedure, referred to the X component, can be similarly applied to the Y component. According to this analysis we have that, if the field is derivable from a potential, the coefficients g_n^m and h_n^m obtained by the two component analysis must be equal within measurement errors.

1.2.3 Results of Spherical Harmonic Analysis

From all data analyses made up to now, it has resulted that the differences between the two sets of coefficients g_n^m and h_n^m obtained independently by the analysis of X and Y components separately, are very small and can be only attributed to measurement errors. The spherical harmonic analysis of the Earth's magnetic field confirms that the assumptions underlying the derivation of Laplace's equation are correct and by consequence the assumption that under certain limitations the Earth's magnetic field is conservative, is valid.

In order to determine separately the contributions of the fields of external and internal origin to the Earth, it is necessary to analyze also the distribution of the vertical component Z. Remembering the formula (Eq. 1.28) obtained for Z from Laplace's equation solution

$$Z = \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\theta)\left[\left(ng_n^{me} - (n+1)g_n^{mi}\right)\cos m\lambda + \left(nh_n^{me} - (n+1)h_n^{mi}\right)\sin m\lambda\right]$$

We can define now

$$C_n^m A_n^m = g_n^{me}\ ;\quad (1-C_n^m)A_n^m = g_n^{mi}\ ;\quad S_n^m B_n^m = h_n^{me}\ ;\quad (1-S_n^m)B_n^m = h_n^{mi}$$

from which $g_n^{me} + g_n^{mi} = g_n^m = A_n^m$; $h_n^{me} + h_n^{mi} = h_n^m = B_n^m$.

The expression for Z becomes

$$Z = \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\theta) \begin{bmatrix} \left(nC_n^m - (n+1)(1-C_n^m)\right) g_n^m \cos m\lambda \\ +\left(nS_n^m - (n+1)(1-S_n^m)\right) h_n^m \sin m\lambda \end{bmatrix} \tag{1.30}$$

As done for X and Y components, by means of a series of measurements on a regular network, we can approximate the measurements at a given colatitude θ by a periodic trigonometric function. A similar procedure can now be obtained by using the Fourier expansion

$$Z = \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\theta)(\alpha_n^m \cos m\lambda + \beta_n^m \sin m\lambda) \tag{1.31}$$

but in this case we must equate the new coefficients α_n^m and β_n^m, deduced in the same way it was done for X and Y components:

$$\alpha_n^m = \left[nC_n^m - (n+1)(1-C_n^m)\right]g_n^m$$
$$\beta_n^m = \left[nS_n^m - (n+1)(1-S_n^m)\right]h_n^m$$

From these equations it is possible to obtain (by the same least mean squares method) α_n^m and β_n^m. Since g_n^m and h_n^m are already known from the horizontal components, we can get the fractions of the harmonic terms respectively of external and internal origin, i.e. C_n^m and S_n^m.

Starting from the analyses carried out since the 1830s from Gauss until today, it results that the terms of external origin show an amplitude far lower, almost negligible, than those of internal origin. In particular as regards the g_1^0 coefficient, the part of external origin is about 0.2% of the internal one, while for g_1^1 and g_1^1 they are about 2%. As it was noted in the case of the comparison between g_n^m and h_n^m, obtained from X and Y (separately computed), we can assume, at this time, that the contribution of external origin is not exactly equal to zero essentially for the following reasons: experimental errors, inability to draw exactly the isodynamic maps and the difficulty to compute the real "mean" magnetic field values within a certain time interval.

In conclusion we can say that the potential, and therefore the Earth's magnetic field, is of internal origin. The potential function of the geomagnetic field can be completely formulated taking only into account the terms of internal origin, denoting from now on Gauss coefficients without the index i

$$V = a\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}P_n^m(\theta)(g_n^m\cos m\lambda + h_n^m\sin m\lambda) \tag{1.32}$$

A full set of Gauss coefficients for the years 2000 and 2005, up to degree and order $n = m = 6$, is given in Table 1.1.

1.2.4
A Predominantly Dipolar Field

Taking only into account the terms of the potential expansion up to $n = 1$, the expression for V becomes at any point on the Earth's surface $P = P(\theta, \lambda)$

$$V_1 = \frac{a^3}{r^2}(g_1^0P_1^0 + g_1^1P_1^1\cos\lambda + h_1^1P_1^1\sin\lambda) \tag{1.33}$$

since $P_1^0 = \cos\theta$ and $P_1^0 = \sin\theta$

$$V_1 = \frac{a^3}{r^2}(g_1^0\cos\theta + g_1^1\sin\theta\cos\lambda + h_1^1\sin\theta\sin\lambda) \tag{1.34}$$

Table 1.1. Gauss coefficients g and h in nT for $n = 1$ to 6, for the years 2000 and 2005 with secular variation coefficients for 2005–2010 in the 10th generation IGRF

g/h	n	m	2000.0	2005.0	SV
g	1	0	−29 619.4	−29 556.8	8.8
g	1	1	−1 728.2	−1 671.8	10.8
h	1	1	5 186.1	5 080.0	−21.3
g	2	0	−2 267.7	−2 340.5	−15.0
g	2	1	3 068.4	3 047.0	−6.9
h	2	1	−2 481.6	−2 594.9	−23.3
g	2	2	1 670.9	1 656.9	−1.0
h	2	2	−458.0	−516.7	−14.0
g	3	0	1 339.6	1 335.7	−0.3
g	3	1	−2 288.0	−2 305.3	−3.1
h	3	1	−227.6	−200.4	5.4
g	3	2	1 252.1	1 246.8	−0.9
h	3	2	293.4	269.3	−6.5
g	3	3	714.5	674.4	−6.8
h	3	3	−491.1	−524.5	−2.0
g	4	0	932.3	919.8	−2.5
g	4	1	786.8	798.2	2.8
h	4	1	272.6	281.4	2.0
g	4	2	250.0	211.5	−7.1
h	4	2	−231.9	−225.8	1.8
g	4	3	−403.0	−379.5	5.9
h	4	3	119.8	145.7	5.6
g	4	4	111.3	100.2	−3.2
h	4	4	−303.8	−304.7	0.0
g	5	0	−218.8	−227.6	−2.6
g	5	1	351.4	354.4	0.4
h	5	1	43.8	42.7	0.1
g	5	2	222.3	208.8	−3.0
h	5	2	171.9	179.8	1.8
g	5	3	−130.4	−136.6	−1.2
h	5	3	−133.1	−123.0	2.0
g	5	4	−168.6	−168.3	0.2
h	5	4	−39.3	−19.5	4.5
g	5	5	−12.9	−14.1	−0.6
h	5	5	106.3	103.6	−1.0
g	6	0	72.3	72.9	−0.8
g	6	1	68.2	69.6	0.2
h	6	1	−17.4	−20.2	−0.4
g	6	2	74.2	76.6	−0.2
h	6	2	63.7	54.7	−1.9
g	6	3	−160.9	−151.1	2.1
h	6	3	65.1	63.7	−0.4
g	6	4	−5.9	−15.0	−2.1
h	6	4	−61.2	−63.4	−0.4
g	6	5	16.9	14.7	−0.4
h	6	5	0.7	0.0	−0.2
g	6	6	−90.4	−86.4	1.3
h	6	6	43.8	50.3	0.9

Let us introduce a new point (θ_0, λ_0) and the corresponding value of horizontal intensity H_0, whose meaning will be made clear in what follows, and assume the following identities

$$g_1^0 = H_0 \cos\theta_0 \,; \quad \frac{h_1^1}{g_1^1} = \tan\lambda_0 = \frac{k \sin\lambda_0}{k \cos\lambda_0}$$

$$g_1^{0^2} + h_1^{1^2} + g_1^{1^2} = H_0^2$$

Then we have

$$H_0^2 = (H_0^2 \cos^2\theta_0 + k^2 \cos^2\lambda_0 + k^2 \sin^2\lambda_0)$$

$$H_0^2 = (H_0^2 \cos^2\theta_0 + k^2)$$

$$H_0^2 (1 - \cos^2\theta_0{}^2) = k^2 = H_0^2 \sin^2\vartheta_0$$

$$k = H_0 \sin\theta_0$$

therefore

$$g_1^1 = H_0 \sin\theta_0 \cos\lambda_0 \,; \quad h_1^1 = H_0 \sin\theta_0 \sin\lambda_0 \,; \quad g_1^0 = H_0 \cos\theta_0$$

We can now obtain for the magnetic potential V_1 the following expression obtained by the insertion of the Gauss coefficients new formulation

$$V_1 = \frac{a^3}{r^2}\left[H_0\left(\cos\theta_0 \cos\theta + \sin\theta_0 \sin\theta \cos\lambda_0 \cos\lambda + \sin\theta_0 \sin\theta \sin\lambda_0 \sin\lambda\right)\right]$$

$$V_1 = \frac{a^3}{r^2}\left[H_0\left\{\cos\theta_0 \cos\theta + \sin\theta_0 \sin\theta\left(\cos\lambda_0 \cos\lambda + \sin\lambda_0 \sin\lambda\right)\right\}\right]$$

$$V_1 = \frac{a^3}{r^2}\left[H_0\left\{\cos\theta_0 \cos\theta + \sin\theta_0 \sin\theta \cos\left(\lambda - \lambda_0\right)\right\}\right] \tag{1.35}$$

where the values of H_0, θ_0 and λ_0 can be obtained from the Gauss coefficients using the relationships previously introduced. Once g_0^1; g_1^1; h_1^1 coefficients are known the relation for V_1 can be written as follows

$$V_1 = \frac{a^3 H_0}{r^2} \cos\Theta = \frac{M}{r^2} \cos\Theta \tag{1.36}$$

where the new angle Θ, can be obtained from the spherical trigonometry cosine theorem[2]. This is the angle between a new pole on the Earth's surface, whose coordinates are (θ_0, λ_0) and that we will name *geomagnetic pole*, and the point of observation $P = P(\theta, \lambda)$.

The expression obtained for V_1 is that of the potential of a magnetic dipole placed at the center of the Earth, whose axis intersects the Earth's surface at the point of coordinates θ_0 and λ_0 so that the colatitude from point $P(\theta, \lambda)$ referred to the new dipole axis, will be given by Θ. H_0 represents the horizontal component of the magnetic dipole field on the Earth's surface in the dipole equatorial plane (Fig. 1.13). On the Earth, being g_1^0 negative the geomagnetic north pole in the northern hemisphere corresponds to a magnetic south pole for the dipole at the Earth's center. Denoting by ϕ_0 the latitude $(90° - \theta_0)$, we can obtain the location of the geomagnetic poles

$$\phi_0 = \arctan\left(g_1^0 / \sqrt{g_1^{1^2} + h_1^{1^2}}\right); \quad \lambda_0 = \arctan(h_1^1 / g_1^1) \tag{1.37}$$

In the year 2000 the geomagnetic North Pole was located at $\phi_0 = 79.542°$ N, $\lambda_0 = 71.572°$ W.

The potential V expansion can be extended to include also all terms for $n = 2$. These terms are called quadrupole terms. Kelvin showed that the sum of all terms for $n = 1$ to $n = 2$ included, gives rise to a magnetic field similar to that generated by a magnetic dipole parallel to the centered dipole, but displaced with respect to the Earth's center by about 500 km in the direction of the West Pacific Ocean plus a term of minor importance in P_2^2. This new dipolar representation of the Earth's magnetic field is that obtained with an *eccentric dipole* and of course it gives a better fit to the global representation of the Earth's magnetic field than with the *central dipole* obtained only by using terms for $n = 1$.

1.2.5 Geomagnetic Coordinates

Geomagnetic coordinates are defined using colatitudes and longitudes in the frame of the geomagnetic dipole, obtained for $n = 1$. Colatitudes Θ, are angles defined with respect to the axis of the geomagnetic dipole, instead of to the usual geographic representation that refers colatitudes to the Earth's rotation axis. Similarly a geomagnetic longitude can be defined with respect to a new zero meridian line, that will be defined in what follows. As in the case of geographic coordinates, the geomagnetic coordinates enable the identification of the position of points on the Earth's surface with respect to a geomagnetic frame of reference. As mentioned above in this new frame it is moreover possible to identify north and south geomagnetic poles as the points, on the Earth's

[2] In spherical trigonometry the cosine theorem states that: The cosine of an angle at the center is given by the product of the cosines of the other two angles at the center plus the product of their sines times the cosine of the angle at the surface opposite to the angle at the center.

surface, where the axis of ideal central dipole intersects the surface. Similarly it is possible to define an ideal line on the Earth's surface representing the intersection of the plane passing through the Earth's center orthogonal to the central dipole. This line is called by analogy, the geomagnetic equator.

From the geographic coordinates θ, λ we can get the geomagnetic colatitude Θ of any given point on the Earth surface, as follows

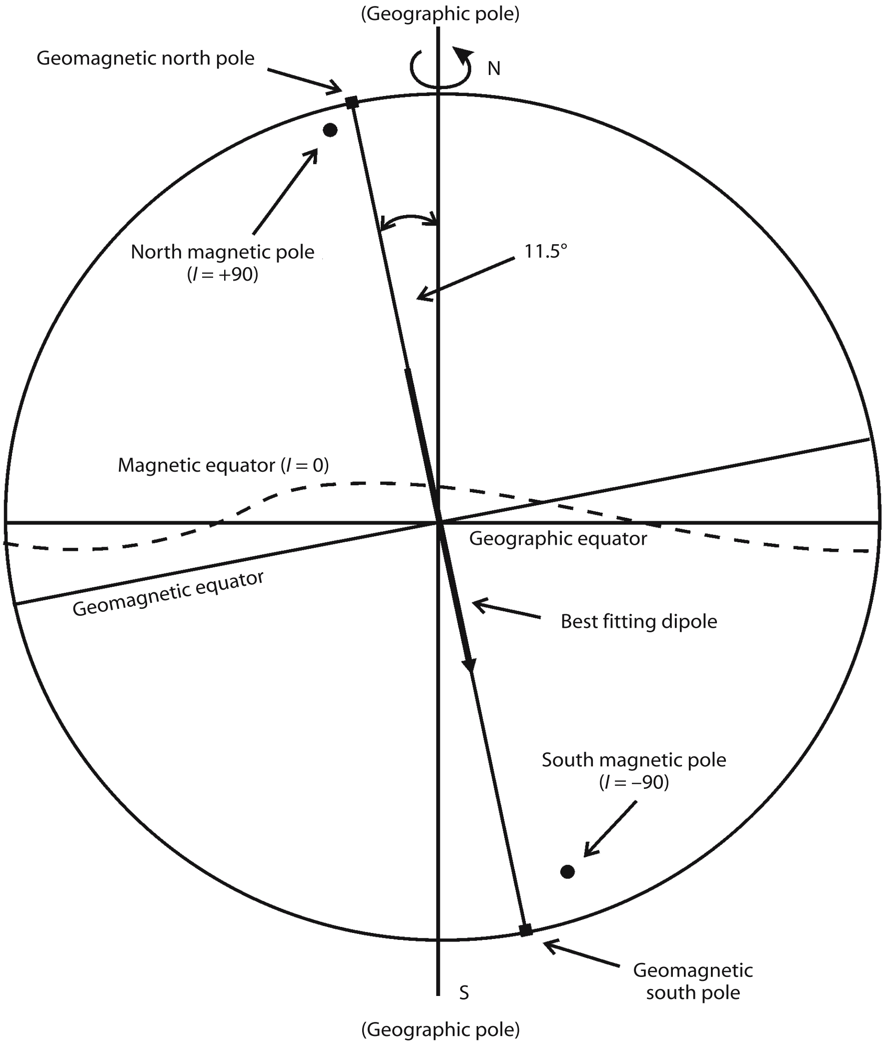


Fig. 1.13. Centered magnetic dipole at the center of a circle representing the Earth with corresponding geographic and geomagnetic poles. On the Earth's surface the geomagnetic poles are only an ideal entity corresponding to the best fitting dipole ($n = 1$). The north and south magnetic poles are measurable points (areas) where dip is $I = \pm 90°$. In analogy with the geographic equator a geomagnetic equator can be drawn for geomagnetic latitude 0°

$$\cos\Theta = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\lambda-\lambda_0) \tag{1.38}$$

which follows directly from the cosine theorem in spherical trigonometry. To obtain the geomagnetic longitude Λ, we firstly define the new zero meridian as the great circle that joins the two poles, geographic and geomagnetic, and then apply the cosine theorem to the θ angle (Fig. 1.14) (geomagnetic longitudes are positive eastward):

$$\cos\theta = \cos\Theta\cos\theta_0 + \sin\Theta\sin\theta_0\cos(\pi-\Lambda) = \cos\Theta\cos\theta_0 - \sin\Theta\sin\theta_0\cos\Lambda$$

$$\cos\Lambda = \frac{\cos\Theta\cos\theta_0 - \cos\theta}{\sin\theta_0\sin\Theta} \tag{1.39}$$

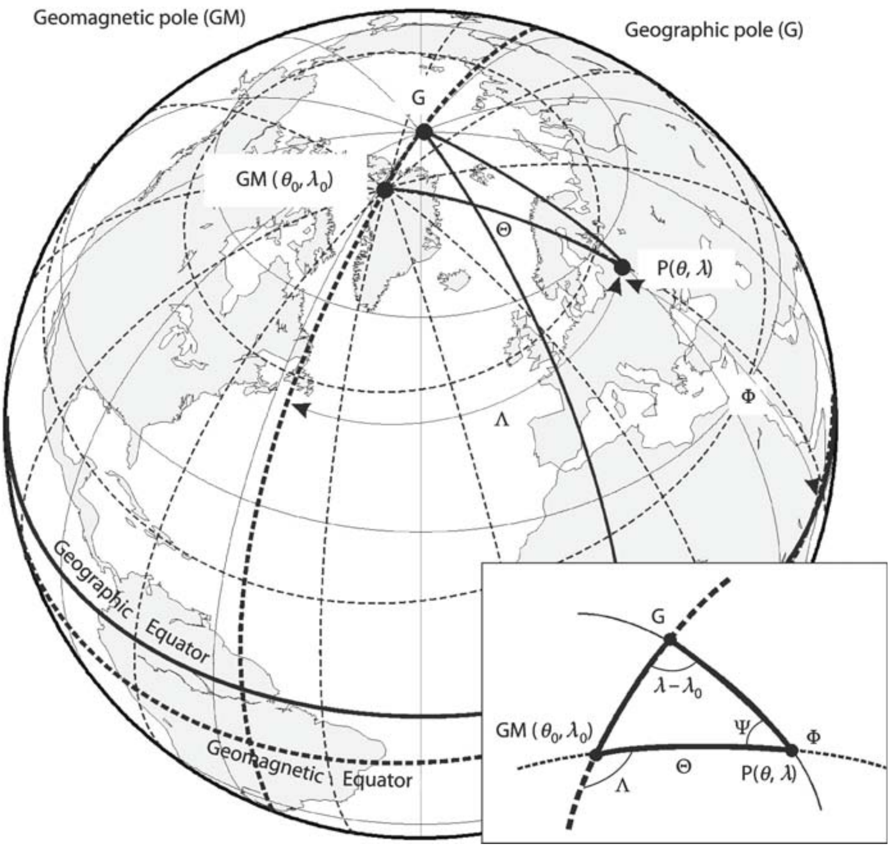


Fig. 1.14. Geographic and geomagnetic coordinates reported on the Earth's surface. *G* geographic north pole; *GM* geomagnetic north pole; *P* is a generic point on the Earth's surface; its geomagnetic coordinates are: Θ = geomagnetic colatitude; Φ = geomagnetic latitude; Λ = geomagnetic longitude

By the sine theorem we can complete the information by obtaining the quadrant in which Λ falls:

$$\frac{\sin(\pi-\Lambda)}{\sin\theta}=\frac{\sin(\lambda-\lambda_0)}{\sin\Theta}; \quad \sin\Lambda=\sin(\lambda-\lambda_0)\frac{\sin\theta}{\sin\Theta}$$

At all points on the Earth's surface, we can also deduce a geomagnetic declination Ψ, as the angle in the new geomagnetic system between the two directions (a) towards the geographic north and (b) towards the geomagnetic north:

$$\frac{\sin\Psi}{\sin\theta_0}=\frac{\sin(\lambda-\lambda_0)}{\sin\Theta}; \quad \sin\Psi=\frac{\sin(\lambda-\lambda_0)\sin\theta_0}{\sin\Theta} \tag{1.40}$$

We will attribute to the Ψ angle the same sign of magnetic element D previously introduced.

The dipolar field at any point P can be computed from the first three Gauss coefficients; this field depends only on the geomagnetic latitude defined as

$$\Phi=\frac{\pi}{2}-\Theta$$

$$H=\frac{\mu_0}{4\pi}\frac{M\cos\Phi}{a^3}; \quad Z=\frac{\mu_0}{4\pi}\frac{2M\sin\Phi}{a^3} \tag{1.41}$$

as we obtain immediately from the dipole equation.

Being the magnetic central dipole only an approximation, the measured field on the Earth's surface at any point can differ also significantly from the theoretical dipolar field. A theoretical geomagnetic pole can be defined from the field measured at a given point as the geomagnetic pole that would be obtained considering the field at that point, as it was simply a dipolar field. This briefly corresponds to the pole that we would obtain by considering the measured declination and the inclination, as if they were purely dipolar. This new pole is called in this case a *virtual pole*. And we obtain

$$Z=H\tan I; \quad \tan I=\frac{Z}{H}=\frac{2M\sin\Phi}{a^3}\frac{a^3}{M\cos\Phi}=2\tan\Phi \tag{1.42}$$

therefore the tangent of the measured inclination in any point is twice the tangent of the virtual geomagnetic latitude at the point Φ_v.

We must remember that calling $\varphi_{0\nu}$ and $\lambda_{0\nu}$ the virtual geomagnetic pole coordinates:

$$\cos\theta_{0\nu} = \cos\Theta_\nu \cos\theta + \sin\Theta_\nu \sin\theta \cos D \tag{1.43}$$

And considering the complementary angles, we will have

$$\sin\varphi_{0\nu} = \sin\Phi_\nu \sin\varphi + \cos\Phi_\nu \cos\varphi \cos D \tag{1.44}$$

$$\sin(\lambda_{0\nu} - \lambda) = \frac{\cos\Phi_\nu \sin D}{\cos\varphi_{0\nu}} \tag{1.45}$$

Equation 1.44 follows directly from the cosine theorem, Eq. 1.45 follows directly from the sine theorem. Hence it is possible to obtain the geomagnetic pole coordinates $\varphi_{0\nu}$ and $\lambda_{0\nu}$.

1.2.6 Harmonic Power Spectra of the Geomagnetic Field

The classical technique of SHA depicts the spatial structure of the Earth's magnetic field potential in terms of spherical harmonics. The solution is represented by mathematical functions of:

a radial distance from the center of a spherical Earth (considered the origin of a spherical reference system);
b Legendre functions in (cosine of) colatitude;
c Fourier series in longitude.

As a consequence of this operation the resulting basis functions, i.e. the spherical harmonics, are orthogonal over the sphere, very helpful in the practical application of SHA to experimental data. As we have seen before, an important aspect of SHA is that it allows the separation of internal and external contributions to the Earth's magnetic field, although it cannot give the exact location of magnetic sources. These can be imagined as sources together ideally grouped at the origin, i.e. at the Earth's center, for internal sources, and at infinite distance, for external sources. Sources are then given in the form of so-called multipoles, each characterized by a given spherical harmonic degree n.

In the actual case of only internal contributions for degree n and order m up to a given value N_{max}, a global model in ordinary spherical harmonics, can represent details of the field with minimum wavelengths given by

$$\lambda_{min} = \frac{4\pi r}{N_{max}} \tag{1.46}$$

The possibility of accurate determination of Gauss coefficients for any global magnetic data set is inversely related to N_{max}, therefore SHA is most suitable to model the longest-wavelength part of the geomagnetic field. This is confirmed also by the introduction of the so-called IGRF (International Geomagnetic Reference Field), which is the accepted global model of the geomagnetic field based on spherical harmonics. It is believed to contain all, or most of, the core field, i.e. the largest part of the field observed at the Earth's surface (for this reason also called the main field). The IGRF is given in the form of sets of spherical harmonic coefficients up to degree and order 10. It also includes a separate set of so-called predictive secular-variation model coefficients which extends to degree and order 8. The current 10th generation was updated in the year 2005 and is composed of sets of main-field models, ranging from 1900 to 2005, at 5-year intervals designated as definitive from 1945 to 2000, inclusive. At this time a provisional main-field set for 2005, and a secular-variation predictive model for the interval 2005 to 2010 are the latest available coefficients.

When only internal sources are considered all contributions to the geomagnetic field, i.e. the sum of all Gauss coefficients up to a given degree n, can be shown on in a semi-logarithmic scale plot as a function of the order number n. In Fig. 1.15 the so-called geomagnetic field power spectrum showing R_n vs. n is reported. R_n is expressed in terms of the Gauss coefficients as follows

$$R_n = (n+1) \sum_{m=0}^{n} \left[(g_n^m)^2 + (h_n^m)^2 \right] \tag{1.47}$$

This expression represents the energy contribution brought by every degree n term in the expansion from $n = 1$ to $n = m = 23$. As is clearly observable, and understandable from the Gauss coefficient values, the contribution to the power decreases as the n order number increases. An isolated point is obtained in the plot for $n = 1$, this corresponds to the centered Earth dipole as shown by the meaning of the $n = 1$ spherical

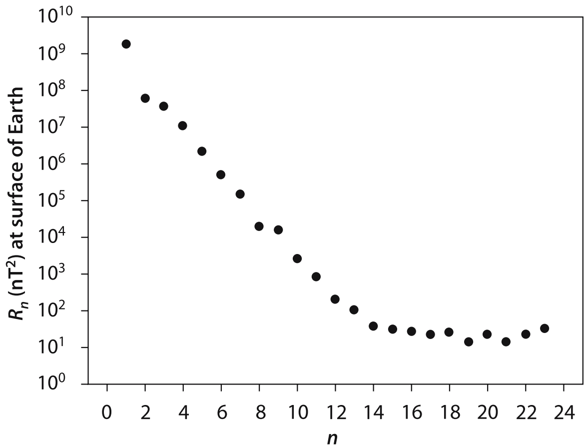


Fig. 1.15. Semi-logarithmic geomagnetic field power spectrum R (nT^2) at Earth's surface for degree $n = 1$ to 23

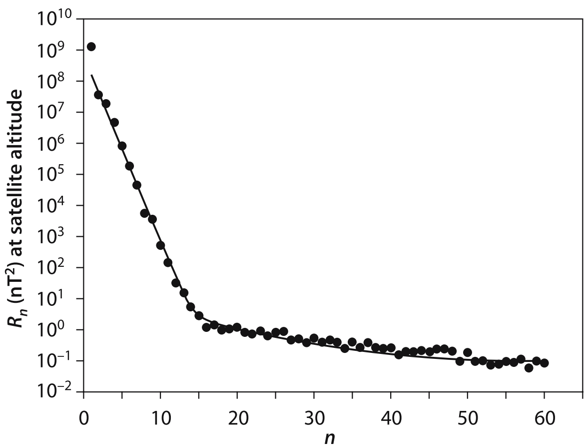
harmonic analysis. Using the general rule given above, Eq. 1.46, we should attribute a maximum wavelength to the magnetic dipole source. All other points in the plot can be easily interpolated by two well separated straight lines. The contributions from $n = 2$ to $n = 12, 13$ decrease to an almost white noise level. From $n = 13, 14$ onwards, a second almost horizontal line can be fitted. All points lying on the first straight line correspond to the maximum intensity values of the power spectrum (excluding the dipole point) and are related to the contribution of the deepest sources, reasonably located in the fluid core up to the core-mantle boundary. The second straight line is almost horizontal instead and corresponds, in the first approximation, to the crustal sources (possibly with a core contamination) with largest n values almost corresponding to a white noise. For this reason this power spectrum plot cannot express the crustal contribution separately from the white noise.

Magnetic satellite data can come in the picture to give better models of core sources and also help to discriminate the largest harmonic degree n values from the white noise. In fact not only Earth's surface data but also satellite data can be used to compute Gauss coefficients. As will be shown in other parts of the text only a few satellites were in fact used to investigate the Earth's magnetic field over about the last thirty years. In Fig. 1.16 a power spectrum plot from satellite data is shown. In this case a possible crustal field contribution can be inferred for the part of the spectrum that goes from $n = 14$ onwards, and that shows up clearly as a different line with a different slope from that for $n < 14$.

1.3 Time Variations

Time variations of the Earth's magnetic field can be divided into two classes: those having an internal origin and those having an external origin with respect to the Earth's surface. Although it is not precisely possible to fix a clear limit between the two classes, by applying the spherical harmonic analysis to the Earth's magnetic field time varia-

Fig. 1.16. Semi-logarithmic geomagnetic field power spectrum R (nT^2) at the satellite altitude for degree $n = 1$ to 60. Interpolating lines indicate best fit lines for the two emerging preferred slopes



tions, it is shown that variations on a time scale shorter than 5 years are generally considered to be of external origin. The variations on time scales longer than 5 years are commonly called *secular variation* (SV) and are of internal origin to the Earth. External origin time variations are clearly recognizable in magnetic observatory data, survey time variation recording stations, and data from rapid run magnetic time recording devices. Very long period external origin time variations, such as for example, those related to the solar cycle (about 11 yr), can generally be seen only in observatory data. Internal origin time variations, like SV, can be seen in observatory data but also in the archaeological or geological records when magnetic investigations are undertaken on dated archaeological samples or rocks. The amplitude of Earth's magnetic field SV for a given place of observation fluctuates between a few nT/year to several tens of nT/year for the magnetic intensity components and from a few minutes/year to several minutes/year for declination and inclination.

1.3.1 Secular Variation

Secular variation is clearly seen in geomagnetic observatory data, when several years for one or more field elements (generally by their annual or monthly means) are plotted against time. Starting from magnetic field observations, carried out for declina-

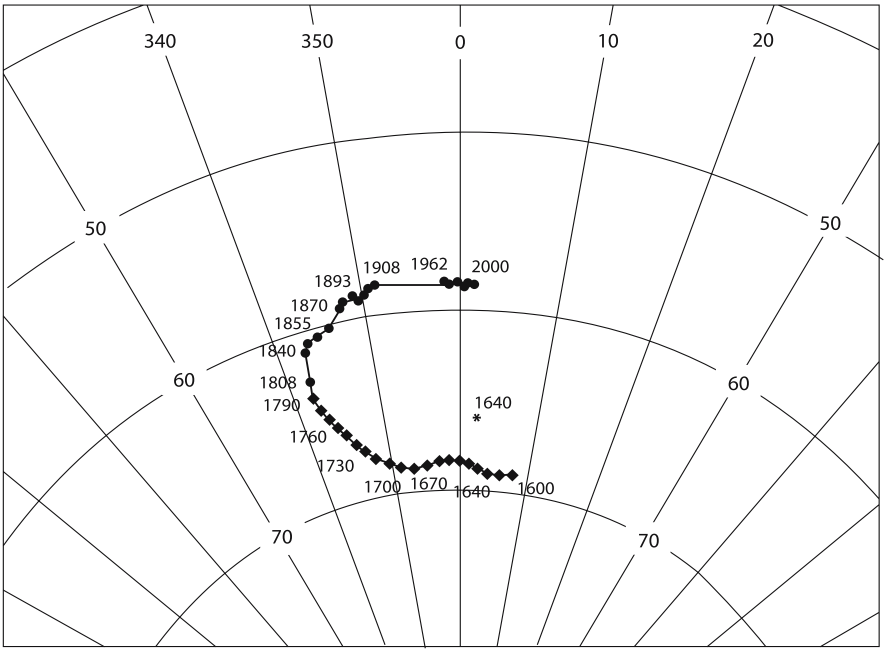


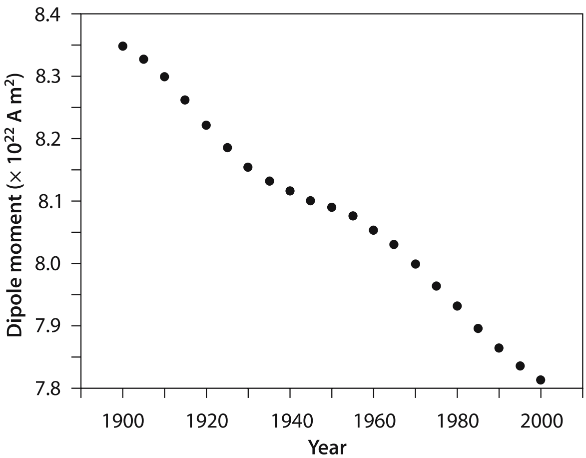
Fig. 1.17. Secular variation diagram for central Italy. In the diagram *D* and *I* time variation is reported on a stereographic projection for years 1600 to the present (from Lanza et al. 2005)

tion and inclination since the 16th century, the geomagnetic field has undergone a SV that, for instance in declination, has covered, in central Europe (Fig. 1.17), a maximum range of about 20°. The angle of declination, in the period under examination, has been predominantly negative, i.e. westerly. SV, thought to be closely linked to the dynamics of the Earth's core and to the phenomena that produce the field itself, occurs on two timescales which are related to two types of core process. One, evident on time scales of hundreds or thousands years, is related to the main dipole field variation, while the second one, clearly appreciable also on the shortest time scales, of the order of tens of years, is related to the non-dipole field variations. For this reason, even though SV shows a different behavior in a range of world observatories, it is typical of the main field, thus being representative of planetary phenomena.

1.3.1.1
Dipole Field Variation

The characteristics of the Earth's dipole field were determined for the first time with modern scientific accuracy by Gauss, by means of his spherical harmonic analysis in 1839, which subsequently allowed to infer the characteristics of the global variations of the field over the last 160 years. In particular the dipole magnetic moment has decreased from 9.6×10^{22} to 7.8×10^{22} A m^2 from the time of Gauss to 2000 (Fig. 1.18), while the angle between the dipole axis and the Earth's rotation axis has remained almost unchanged at about 11.5°. Moreover the dipole has shown a precessional motion around the rotation axis taking the north geomagnetic pole from its position of 63.5° W in 1830 to 71.6° W in 1990 with a precessional velocity of about 0.05° yr^{-1}. A third time variation of the dipole field consists in the slight displacement of the dipole along its axis towards the geographic north at a velocity of about 2 km yr^{-1}. Inspection of the data from 1600 onwards has shown that an ideal line of force of the purely dipole field has moved over the last 400 years in a westerly direction at a velocity of about 0.08° yr^{-1} and with a variation in latitude of about 0.01° yr^{-1}.

Fig. 1.18. Geomagnetic dipole moment time variation from years 1900 to 2000



The determination of the elements of the Earth's field before instrumental measurements are available, i.e. before roughly 1600, can be done with other techniques. By means of rock magnetism and artefacts' magnetic properties, attempts have been made to determine the value of the dipole moment in the past. For the last thousands years, in spite of all the uncertainties in measuring the paleointensity of the magnetic field, there is sound evidence that the present decrease of the magnetic moment began about 2 000 years ago when its value was around 11×10^{22} A m^2. It has been attempted to establish if the Earth's field has had in the past the same dipolar nature as it has today. It has been inferred that the field is due to an almost axial dipole at least for the last 500 000 years. But this axiality is only of a statistical nature as there are evident and repeated fluctuations of the geomagnetic poles around the geographic poles; the present situation must be considered as one of these fluctuations.

1.3.1.2
Variation of the Non-Dipole Field

Observing contour maps of declination for different epochs, geomagnetists have noted a clear drift of almost all declination contour lines toward the west. Halley was the first to show declination contours on his famous Atlantic Ocean map at the very beginning of the 18th century. This phenomenon is known as *westward drift* and can still today be clearly seen in the Atlantic and Europe, while it is not quite appreciable in the eastern Pacific, Australia and Antarctica. The westward drift, that in many studies on SV has been considered as its most evident characteristic, is due to the variation of the non-dipolar part of the field. Bullard was the first to estimate that the isoporic foci[3] of the non-dipole field undergo, in those regions where the phenomenon is evident, a westward drift motion of about 0.2° yr^{-1}. More in-depth studies have shown the possibility to discriminate two different behaviors of the non-dipolar magnetic field contribution to SV: a clear westward drift in some zones, as for example the so-called "African anomaly" (Fig. 1.19), and a geographically stationary effect with a strong intensity variation, for instance the "Mongolian anomaly". This circumstance has suggested to separate the contributions to the non-dipolar SV in two parts: a standing part and a drifting part. Therefore although the westward drift remains the most evident characteristic of SV of the non-dipole field, this seems to account for only one of its parts. For what concerns the geomagnetic field power, the various observations give evidence of a total decay in energy of its dipolar part, but at the same time the non-dipolar part of the field shows an energy increase so as to partly compensate for the dipolar decrease. However, recent estimates show that the total energy probably shows overall a slight decrease.

1.3.1.3
Rock Magnetism Results

As briefly mentioned above, the most ancient part of the history of the Earth's magnetic field comes from rock magnetization studies (paleomagnetism). Accurate analy-

[3] Isoporic lines are lines connecting points with the same SV values on a map.

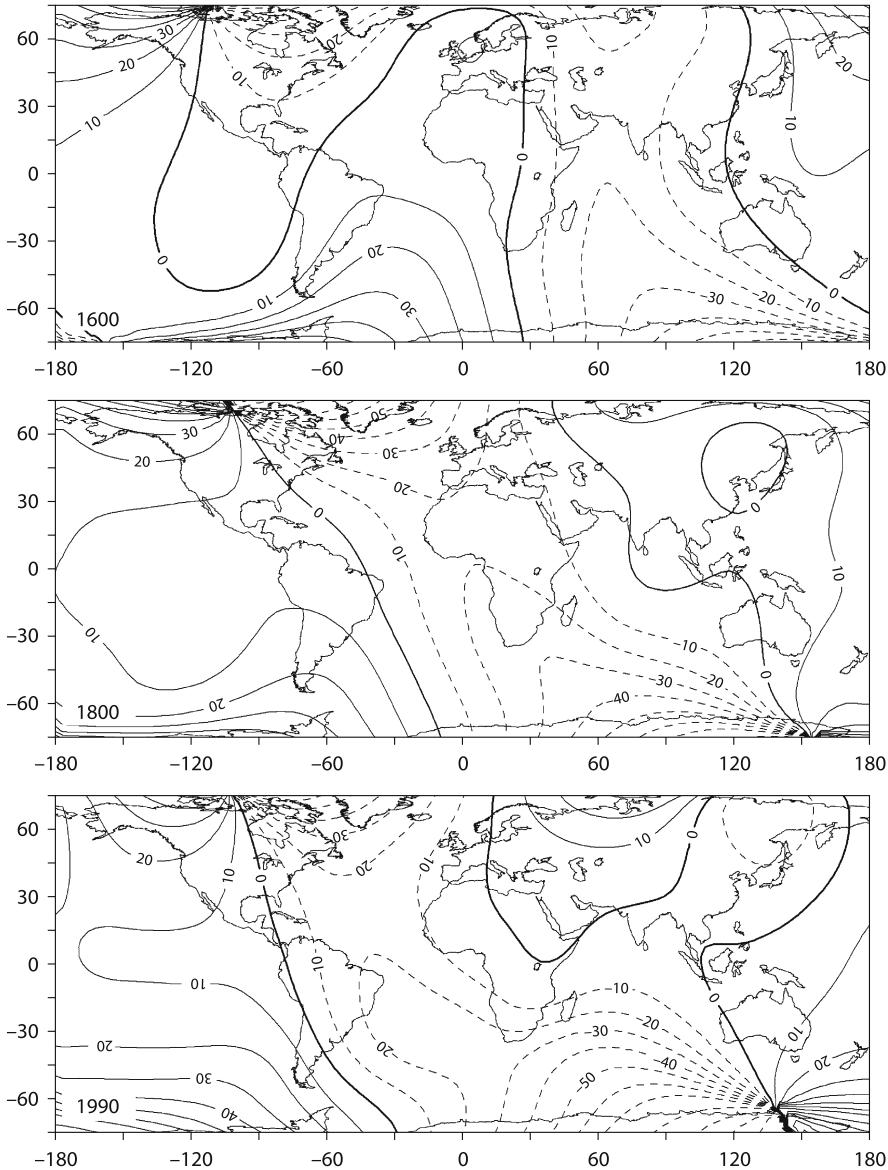


Fig. 1.19. Westward drift of Earth's magnetic field shown by three declination maps at years 1600, 1800 and 1990. Maps made using historical records database and model by Jackson et al. 2000

ses of volcanic rocks and lacustrine sediments has produced curves of paleosecular variation (PSV) extending back as far as hundred thousand years (Chap. 7). The accuracy is obviously lower than that of observatory data, but the whole of PSV data allows statistical analyses useful to outline the behavior of the main field.

However, the main contribution of paleomagnetism to the study of the Earth's field was the discovery of the most dramatic variation, the polarity reversal, that is the exchange of position between the north and south magnetic poles. This entails that the lines of force are directed towards south geographic pole (see Fig. 1.1 for a reference), declination changes by 180° and inclination reverses its sign, being negative in the northern and positive in the southern hemisphere. This feature was discovered thanks to the studies of young volcanic rocks that showed reverse polarity, i.e. the direction of their remanent magnetization was opposite to the direction of the present day field of the Earth. The study of Pleistocene rocks has confirmed that the last reversal occurred at about 0.78 Ma before the present day, and before that a series of periods characterized by normal or reverse polarity occurs for several million years. The reversals are distributed accidentally in time and periods of the two polarities are indicated in polarity time scales by white and black segments (Figs. 7.1, 7.5). There are however some patterns apparently systematic, for example a modest increase in the frequency of reversals in the last 100 Myr, also showing a quasi periodicity in the reversal rates for a million year time interval. Magnetic excursion is the term used to refer to those cases when the field direction has presented large and rapid variations of inclination without involving however a complete reversal. The transitions from one polarity to the other, takes place in a time interval of about 5 000–1 000 years whereas the details of the transitional field are still unknown.

Another important contribution of paleomagnetism regards the information on the longevity of the Earth's field. In the Australian and Greenland cratonic regions magnetized rocks as old as 4 Ga (1 Ga = 1 billion years) bear witness that a magnetic field already existed in the primeval times of the Earth's history.

1.3.1.4
Summary of Secular Variation Characteristics

In the previous sections it was shown that SV results from numerous complex causes. We can summarize SV main characteristics according to the relevant information derived from direct observations or analyses of magnetized materials. Other details of SV are discussed in Chap. 7. Summarizing what reported above on the basis of observational data, we can conclude that SV shows the following characteristics:

Results from Geomagnetic Observations (i.e. in the last 400 Years)

1. A mean annual decrease of the dipole moment of the order of 0.05% of its average value with a considerable acceleration in the last 30–40 years.
2. A westward precession of the dipole axis of 0.08° yr^{-1}.
3. A northward displacement of the dipole of the order of 2 km yr^{-1}.
4. A westward drift of the non-dipole field, or of a part of it, of 0.2–0.3° yr^{-1}, associated with a possible but not specified southward drift.
5. An intensity variation (increase or decrease) of the non-dipole field at a mean rate of about 10 nT yr^{-1}.

Results from Archaeo- and Paleomagnetic Studies

1. Archaeomagnetic information do not confirm in a clear way the existence of the westward drift and suggest therefore that it is not a permanent characteristic of SV.
2. The field fluctuates in direction and intensity, yet averaged on times greater than 100 kyr it is dipolar, geocentric and aligned with the Earth's rotation axis (GAD hypothesis).
3. The field reverses its polarity at a rate which looks random. The polarity remains constant for time periods of the order of 100 kyr to 1 Myr (Chap. 7).

1.3.2 Magnetic Tomography and Interpretation of Secular Variation

The interpretation of SV has closely followed studies and theories of the Earth's magnetic field generation. The existence and the time variation of the dipole field are attributed to a very deep source in the core, while the non-dipolar variations, in particular the westward drift, are attributed to more superficial sources, probably in relation to electrical currents flowing at the core-mantle boundary (a transition zone that we will denote from now on by CMB). This approach leads to consider that the physical conditions of the lower mantle could influence the dynamics of the core and therefore the geomagnetic field itself. By means of inverse numerical techniques and the spherical harmonic analyses, it is possible to obtain a picture of the magnetic field at the CMB. These maps have recently been produced by several authors and report the field at the CMB where, according to dynamo theory, the electric currents that generate the non-dipole field flow (Sect. 1.3.3). This technique has been named magnetic tomography. One of the most important results obtained by magnetic tomography is that the main contribution to the geometry of the dipole field is given by four patches of intense magnetic flux located symmetrically with respect to the equator, whose positions have not changed significantly from 1600 to nowadays. These four stationary spots have been interpreted as the extreme ends of two columns of fluid material that are tangential to the inner core and extend parallel to the Earth's rotation axis.

In order to get an interpretation of SV, and in particular of westward drift, we need to follow the historical as well as the most recent theories. To explain the presence of the significant westward drift of the field, Bullard in the fifties of the twentieth century, supposed that the core rotated at lower angular velocity with respect to the mantle and proposed as mechanism for this differential rotation the existence of an electromagnetic coupling. Hide in the same period proposed instead the existence of some undulating motions inside the core, able to generate the drift of the field measured on the Earth's surface. The results provided by the magnetic tomography, partly contradict both theories. According to the former the drift should occur everywhere on Earth, while it mainly concerns the region comprised between longitudes from 90° W and 90° E, roughly from Europe and Africa to North and South America. The latter theory, instead, does not explain the absence of SV observed on the Pacific Ocean.

Using magnetic tomographic maps and some approximations from magnetohydrodynamics, it is possible to obtain a configuration of the fluid flow at the CMB. In order to recreate a configuration of the flow, an approximate estimate of the value of the electrical conductivity of the core is necessary. Its mean value is commonly assumed to be $\sigma = 10^5–10^4$ S m^{-1} that is $10^3–10^4$ times higher than the conductivity estimated for the mantle. The maps of velocity and direction of the flow at the CMB, have shown two fluid circulations below the Atlantic, one toward north and the other toward the south of the equator; near the equator this flow goes in a westerly direction. Therefore the westward drift, observed between longitudes 90° W and 90° E could be the result of this circulation. With the intent of explaining the origin of the fluid flows, in his fundamental fifties works, Bullard was the first to propose a way to generate fluid circulations in the core. Bullard proposed the existence of flows coming from the deep core moving towards the CMB. In this vertical motion, the rising fluid, in the case of frozen field (see Sect. 1.4.2), tends to concentrate the magnetic lines of force against the CMB, so forming a magnetic flux spot alike those in the solar photosphere. Some bipolar magnetic structures form and move quickly, while others remain stationary. For this reason a number of authors have thought that their motion could be modified by some influence external to the core. According to Hide, a possible candidate is the topography of the CMB whose extensions in the core could be envisaged as upturned mountains. Bloxham and Jackson have afterwards excluded this hypothesis as well as that of an electromagnetic coupling. These authors propose that the flow in the core is coupled to the mantle in a thermal way instead. In practice the core fluid would flow upwards below the hot regions of the mantle and downward below the cold ones. Regarding the dipolar part of the Earth's field magnitude the tomographic analysis makes it clear that the present drastic decrease of the dipolar component can be due to the increase and propagation of structures with an opposite flow, below Africa and the Atlantic.

1.3.3 Geomagnetic Jerks

On average magnetic field elements, when plotted versus time at a geomagnetic observatory, show quasi-stable, slow changing time variations. However one peculiar feature of SV is represented by a clear tendency, for a given field element, to show at times rapid changes, observable as a variation in the slope, taking place in one or two years. This peculiar phenomenon that separates periods of reasonably steady SV patterns, i.e. constant slopes in the geomagnetic field time variation, is called a geomagnetic jerk (GJ). GJs are thus abrupt changes in the second time derivative (secular acceleration) of the geomagnetic field. In this sense a GJ separates periods of almost steady secular acceleration of the geomagnetic field.

Secular variation models and the role of the peculiar phenomenon of GJs, following their discovery at the beginning of the eighties of the twentieth century, have widely been investigated. According to the majority of scientists this phenomenon is of internal origin and this was shown clearly by the use of spherical harmonic analyses undertaken on the variation field. Of course the investigation of GJs' importance is mainly in terms of possible explanations of the mechanisms that produce and maintain the geomagnetic SV. We know in fact that SV is associated, via the induction equation in magnetohydrodynamics, to steady fluid flows at the top of the fluid Earth's core;

non-steady, time-varying flows could be associated with SV anomalies, including jerks. The very rapid time variation that is characteristic of GJs is an indication that can also have very important connections to the knowledge of the electrical conductivity of the Earth's mantle. In fact 2–4 years is the accepted upper limit for a magnetic time variation period that is able to to penetrate the full thickness of the mantle without being completely screened out.

Some authors have also associated jerks with decadal fluctuations in the length of day and the Chandler wobble, although this connection is still under examination.

In many of the world magnetic observatory data series, GJs can easily be identified. Jerks are often easier to be observed in D or Y component. For a quantitative study, direct recognizable patterns due to effects of external fields, are generally removed from data series by modeling external variations by means of so-called geomagnetic indices (K, Dst, aa; see Sect. 1.5.3). Last century widely observed geomagnetic jerks are reported in Fig. 1.20 for two observatories, Tucson and Chambon-la-Forêt. Evident slope changes are observed in SV at years 1901, 1925, 1932, 1949, 1969, 1978, 1991 and 1999. The 1969 jerk was discussed in detail in many papers and it was shown that it was observed worldwide although it was not always manifest in all magnetic elements. Other analyses have shown that the occurrence time for GJs, although global in scale, is not the same all over the globe. It is accepted now that for every GJ there is an average time of occurrence and that two to four years are required for the event to be observed all around the globe.

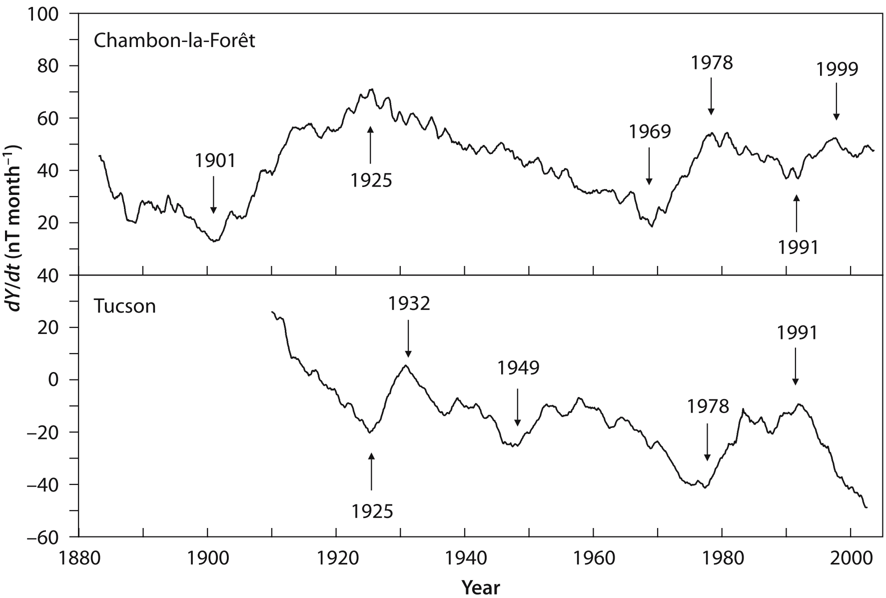


Fig. 1.20. Geomagnetic jerks for element Y the two magnetic observatories Tucson and Chambon-la-Forêt. Slope changes for declination SV are reported with *arrows*

1.3.4 External Origin Time Variations

Time variations of the Earth's magnetic field are constituted by a series of contributions, each of them having a different typology, intensity and typical timescale. As we have just seen, in the case of magnetic field variations, the term *slow* refers to SV; the term *rapid* is referred to external to the Earth origin time variations, mainly connected with the Sun and solar activity. The Sun is in fact the critical factor to interpret all rapid magnetic time variations.

1.3.4.1 *Solar Wind and the Magnetosphere*

The regular emission of solar electromagnetic radiation is accompanied by a continuous emission of an ionized gas, called the solar wind, which is essentially the expansion of the solar corona. Because of its high temperature, the solar wind leaves the immediate vicinity of the Sun, fills all the interplanetary space and also affects the Earth. The solar wind particle flux is composed mainly by electrons, protons, He nuclei, and also heavier elements and is constantly emitted by the Sun at an average speed of 1.5 million km hr^{-1}; the solar wind density at the Earth is ~7 ions cm^{-3}. The solar wind is not uniform, although always directed away from the Sun, it can change speed and carries a feeble magnetic field of solar origin, the interplanetary magnetic field (IMF). As a consequence of the solar wind expansion in the solar system, the Earth's magnetic field is confined by this low density plasma and is immersed in the IMF, both originating in the Sun. The cavity in which the Earth is bounded is called *magnetosphere* and shows a quite different shape with respect to the pure magnetic dipolar field (Fig. 1.21). The magnetosphere is limited on the sunward side by a paraboloidal surface called *magnetopause,* where a balance between the Earth's magnetospheric magnetic pressure and the solar wind particle pressure is reached. The distance of the magnetopause from the Earth ranges, according to varying solar activity, from 5 to 12 R_e (Earth radius), with a typical value around 10 R_e. At about 13 Earth radii a *bow shock* is formed by the incident solar wind that has a super magnetosonic velocity (i.e. a velocity larger than the propagation velocity of ordinary magnetosonic waves) relative to Earth. A long tail on the anti-sunward side extends the magnetosphere to a very long distances from Earth, typically for a hundred Earth radii or even more.

Inside the magnetosphere the motion of energetic ions and electrons is constrained by the local Earth's magnetic field. The basic mode is a rotation around magnetic field lines, with at the same time a drift along those lines, giving the particles a spiral trajectory. On typical field lines, attached to the Earth at both ends, such motion would soon lead the particles into the atmosphere, where they collide and would lose their energy. However, the sliding motion slows down as the particle moves into regions where the magnetic field is strong, and it may even stop and reverse; it is as if the particles were repelled from such regions. The magnetic field is much stronger near the Earth than far away, and on any field line it is greatest at the ends, where the line enters the atmosphere. Thus electrons and ions are trapped for a long time, bouncing back and forth from one hemisphere to the other. In this way the Earth is encircled by particle radiation belts.

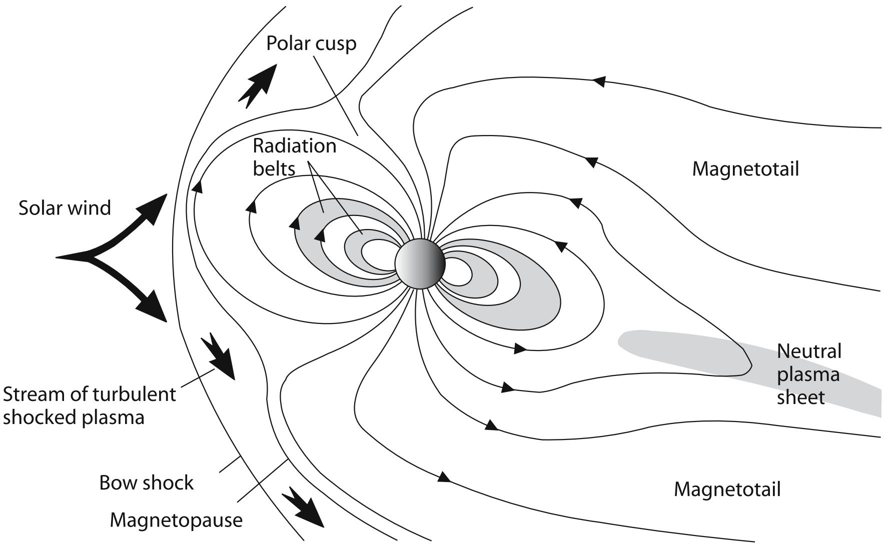


Fig. 1.21. The Earth's magnetosphere. Solar wind shapes the Earth's magnetic field differently from a purely dipolar structure. In the pictorial representation magnetic field lines are drawn to represent the real structure of the Earth's magnetic field around the Earth

From the first satellite investigations it was discovered that the Earth has two regions of trapped fast particles. The inner radiation belt discovered by Van Allen, is relatively compact, extending perhaps one R_e above the equator. It consists of very energetic protons, a by-product of collisions by cosmic ray ions with atoms of the atmosphere. The number of such ions is relatively small, and the inner belt therefore accumulates slowly, but because trapping near Earth is very stable, rather high intensities are reached, even though their build-up may take years. Further out is the large region of the outer radiation belt, containing ions and electrons of much lower energy; this population fluctuates widely, rising when magnetic storms inject fresh particles from the tail then gradually falling off again.

1.3.4.2
The Ionosphere

The magnetosphere has a lower boundary. Theoretically this boundary should be located where the medium passes from the condition of plasma, in which the control of the particle motion is determined mainly by the Earth's magnetic field, to that in which ion and electron densities make collision processes an important factor and the control by the magnetic field small. This layer, that has all the characteristics of a transition zone, can be approximately localized at a height between 200 and 600 km from the ground, to some extent variable with latitude, season, solar time, etc., and is called the ionosphere. The historical discovery of the ionosphere dates back to the early twentieth century, when Guglielmo Marconi succeeded in obtaining an ether connection between Europe and North America. Shortly afterwards, Kennelly and Heaviside attributed to a reflecting ionized layer in the upper atmosphere the discovery made by Marconi and called this part of the atmosphere the ionosphere.

The principal source of the ionization of the neutral atmospheric constituents, that gives rise to the formation of the ionosphere, is the solar radiation; according to the theory, proposed by Sidney Chapman, the electronic density varies with the solar zenithal angle and the sunspot cycle. In the ionosphere, ions and electrons are present in sufficient quantity to modify the propagation of electromagnetic waves and in particular of the so-called HF band radio waves. Ionospheric layers act, by subsequent refractions, as reflecting layers for the radio waves. During the daytime, starting from the lower altitudes, we find a region, between 50 and 80 km, called D layer, where all the free electrons assume a relevant role with regards to the so-called electromagnetic refractive index. This region is responsible for the absorption of the radiocommunication waves over long distance. At about 100 km altitude there is another relative electronic density maximum referred to as the normal E-region and also an intermittent zone of intense ionization, just called 'sporadic' E-region. Finally two maximum densities at about 300 km characterize the F-region in the two F1- and F2-zones. Above these altitudes the electronic density constantly decreases with heigth. At night, once the effect of the solar radiation, that is the main responsible of the ionization, is cut off the electronic density, at various altitudes, decreases because of a recombination process between ions and electrons; only one peak characterizing the F-region is in this case observed. Thanks to their improvement, development and the possibility to use them in various areas of the world, the survey instruments used to measure ionospheric characteristics, the so-called ionosondes, showed that the Chapman theory could be only applied to two ionospheric layers, E and F1, but not, for example, to the layer with the maximum electronic density, F2. Therefore other processes had to be sought to explain why the electronic density of the F2-layer was so high and capable of resisting also without solar ionization. The diffusive and transport processes taking place in the ionospheric plasma complete the Chapman's photoionization theory.

The presence of the ionosphere and of the ionization processes acting there give rise to other important geophysical phenomena, besides a radiopropagation by reflection. As a result of the absorption of the solar radiation, the gas in the upper atmosphere is subjected to an internal photochemical excitation process and also to a natural ion recombination. This last phenomenon produces a light emission in the infrared, visible and ultraviolet regions of the spectrum, called *airglow*. Even though it is always present, during the day the airglow cannot be seen easily, because of the strong sunlight; on the other hand it provides a dim light during the night. The airglow is a phenomenon that takes place at all latitudes without a particular geometric or temporal structure. On the contrary the aurora (Sect. 1.3.4.4) also caused by photochemical atmospheric processes, takes only place at high latitudes, where energetic particles of magnetospheric origin act as the ionizing agent.

1.3.4.3
Regular Time Variations

The existence of the ionosphere will help us to explain some of the time variations of the Earth's magnetic field. The sunlight daily variation is directly connected to the upper atmosphere electrical conductivity and to the motions of the atmospheric gas through the Earth's magnetic field lines of force. These motions and their complex interactions with the field, create an electrical current system in the ionosphere detectable on the Earth's surface as a slow modulation of the three components of the Earth's magnetic field, that can be observed clearly only when additional stronger disturbances produced by other phenomena in the magnetosphere, are not taking place. This variation is in fact called *Sq* from *Solar quiet,* where 'solar' indicates that the variation acts following the local solar time and 'quiet' indicates that it is typical of an unperturbed situation. As the ionospheric conductivity is proportional to the ionic mobility and to ion concentration, the most effective conductive layer is between 90 and 120 km of altitude in the E-region, where the currents responsible for the *Sq*, are assumed to be located in. The variation, known as *diurnal variation*, acts following the local time; each Earth's magnetic field element shows a time behavior, that can be interpreted as a superimposition of waves with periods of 24 and 12 hours, and their harmonics with an amplitude of the order of some tens of nT. The variation is restricted to the daylight hours; during the night it is negligible. As briefly mentioned on some days it can be clearly seen, but generally it is overlaid by irregular variations that partly deform it, allowing however at times to see its fundamental characteristics.

As it is simpler to study the diurnal variation on days without irregular perturbations, from the very beginning of the twentieth century, it has been agreed to determine, by appropriate methods, for each month, the five most quiet days, valid all over the world (international quiet days) and to calculate the mean diurnal variation during these five days for each observatory. A spherical harmonic study has shown that 2/3 of the daily variation is of external origin to the Earth and can be attributed to electrical currents circulating in the ionosphere and partly in the magnetosphere. In the ionosphere on the daytime side of the Earth these currents are constituted by a couple of fixed vortices one in each hemisphere whose position is fixed with respect to the Sun and thus follows its apparent rotation. The centers of these vortices are in the hemisphere illuminated by the Sun at latitudes of around 40° and very near to the meridian of the Sun. The ionosphere is the dynamo conductor and the Earth's field the magnet. In the nighttime hemisphere there are two vortices of opposite rotation to diurnal ones and of weaker intensity. The remaining 1/3 of the variation is of internal origin and is due to electrical currents produced by electromagnetic induction in the Earth's interior probably extending down to a depth of about 800 km, by the varying external magnetic field. The *Sq* amplitude changes during the seasons with a summer maximum and a winter minimum at high and mid latitudes, and with a maximum at the equinoxes in the inter-tropical zone for H and Z (Fig. 1.22). Moreover *Sq* amplitude depends on the phase of the sunspot cycle with the quietest levels occurring around minimum sunspot number years. The ratio between maximum and minimum amplitudes can reach 2 to 2.5. The actual value varies according to the solar cycle intensity as expressed by the sunspot number and the ratio between the amplitude at its maximum and that at its minimum is of the order of 1.5.

Above the equator there is an accumulation of the atmospheric tide in the west-east direction, the so-called equatorial electrojet (EEJ), that causes an enhancement of the diurnal variation in the geomagnetic field near the magnetic equator. This leads to a diurnal variation that can reach values of 200 nT. The width of the interested region is ≈4° in latitude, return currents north and south of the eastward current are a common feature of the EEJ. The intensity of the EEJ varies from day to day.

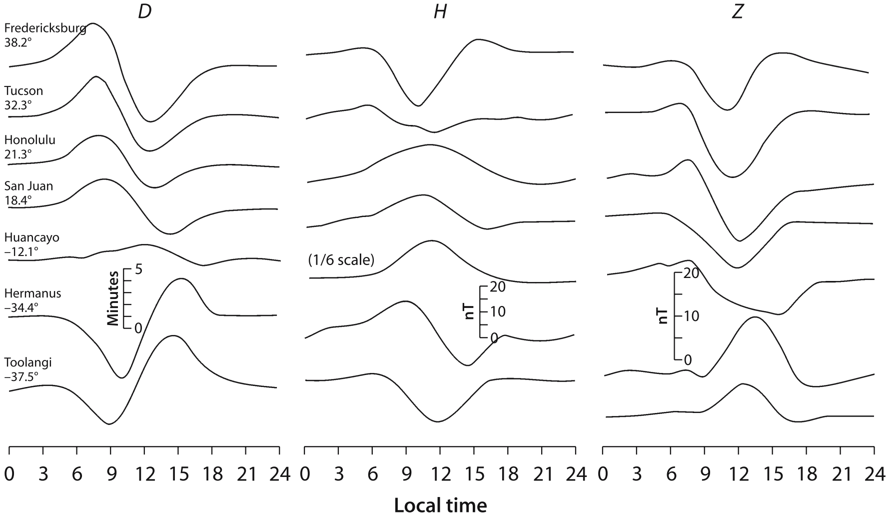


Fig. 1.22. Latitudinal variation of *Sq* for the three magnetic elements *D*, *H* and *Z* as observed on average at the indicated magnetic observatories (from Campbell 2003)

An in-depth study of the daily variation morphology has shown the existence of a weak component which is a function of the lunar time rather than solar. This variation, called *lunar variation L* has a period of 12 lunar hours (12.408 solar hours) and an amplitude of about 1–2 nT. The system of electrical currents generating these regular variations is triggered and sustained by a dynamo process. The atmospheric tides give rise to a wind system in the ionosphere and the motions of electrically conducting matter interacting with the Earth's magnetic field produce in turn electrical currents that are more intensive the higher is the ionization degree, caused by the photoionization in the ionosphere. The atmospheric tides due to the Sun are mainly of thermal origin, those due to the Moon are simply gravitational and for this reason the solar effect is larger than the lunar effect.

1.3.4.4
Irregular Time Variations

In the study of the Earth's magnetic field, in analogy with the weather conditions, the term magnetic storm is used to indicate a generally perturbed magnetic state, characterized by impulses and unpredictable rapid time variations. This irregular phenomenology is strongly influenced by the solar wind. When a magnetic storm starts with a clear impulse (also called a sudden storm commencement, SSC) a typical behavior of magnetic storm time variation evolution is observable globally in the magnetic elements (Fig. 1.23). For example in the initial phase the intensity of the horizontal component increases for a few hours with respect to the average level it had before the storm. Then it decreases quickly down to a lower level than average. This second phase is called the main storm phase and can last from several hours to one or two days. At

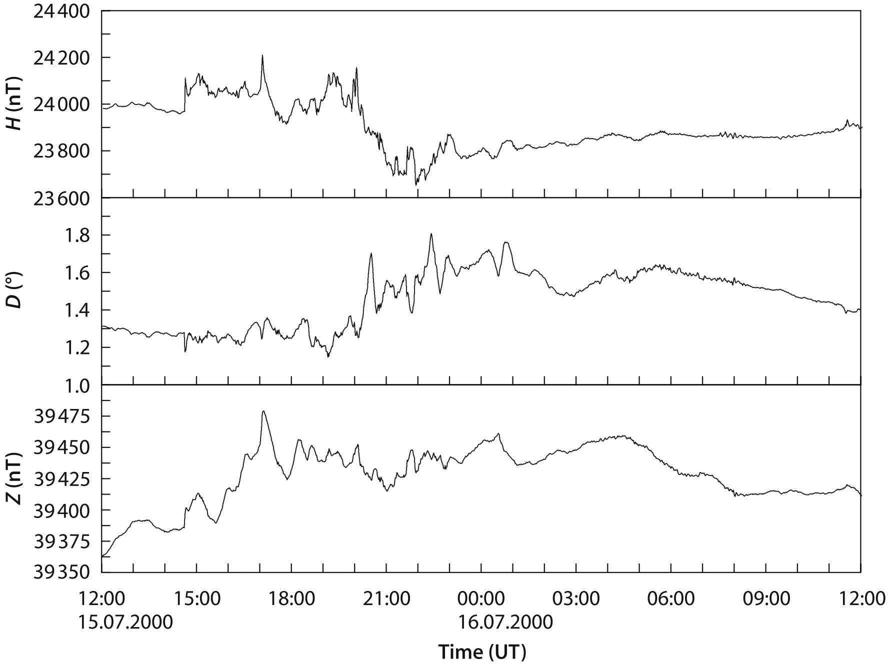


Fig. 1.23. Magnetic elements H, D and Z time variation in the magnetic storm (Bastille event) of July 15, 2000 at L'Aquila Italy geomagnetic latitude 36.3° N

mid latitudes this deviation reaches easily several tens of nT but can reach also hundreds of nT. Finally there is a gradual return to the average value, called recovery phase and lasting generally some hours to one day. The mean total duration of a SSC fluctuates between several hours and a few days. At mid latitudes during the initial and the main phases we can observe also an important activity due to the superimposition of a few more or less intense elementary disturbances, called *bays*. They generally take place during the nighttime hours and last between 1 and 2 hours. Bays are among the most important irregular variations recorded at the magnetic observatories. The bays often coincide with storms, but some times they are observed also without a global magnetic storm taking place. Accordingly they can be considered as elementary storms (also called substorms). From the observations it has been inferred that the bays are produced by electrical currents flowing in the ionosphere at latitudes between 65° and 70°. These currents are produced in the magnetosphere, flow into the ionosphere and return to the magnetosphere following the lines of force of the magnetic field. All irregular variations show up in intensity and forms depending on latitude.

After a careful analysis it was possible to identify different causes for the two main contributions to the irregular variations. The first that characterizes the planetary magnetic storms, consists of a systematic variation of the field that can be mainly attributed to the so-called *ring current*. This is one of the major current systems in the magnetosphere, it encircles the Earth in the equatorial plane and is generated by the longitudinal drift of energetic (10 to 200 keV) charged particles trapped on magnetic field lines between 2 and 7 Earth radii. The second contribution consists of a shorter, at times powerful, local effect, very evident at high latitudes, called *substorm*. The low latitude effect of substorms are the before mentioned bays.

Taken together the solar wind, the magnetosphere and the ionosphere form one dynamical system guided by the energy coming from the solar wind and transferred to the magnetosphere and ionosphere. This energy transfer is particularly evident in correspondence of the so-called CME (coronal mass ejection), solar plasma impulsive emissions that spread through interplanetary space, and is controlled by the interplanetary magnetic field, that is the magnetic field of solar origin transported by the solar wind. The most intense phenomena take place during the so-called *reconnection* periods, that consist in the cancellation of the Earth's magnetic field on the solar side of the magnetosphere, owing to the presence of an interplanetary magnetic field oriented in the opposite direction to the Earth's magnetic field on the side facing the Sun. Under these conditions the Earth loses its natural magnetic screen against the solar wind, thus becoming an open system for the solar wind. The magnetospheric plasma participates in a gigantic convection process, whose effect is the plasma transfer from the day side to the night side. The most considerable effects of this are the activation of large electrical currents in the polar ionosphere, that in turn cause high latitude magnetic substorms, and the large auroral display phenomena, also typical of the polar latitudes. Part of the energy transported in these explosive phenomena is also transferred to the Van Allen radiation belts thus causing the enrichment of the carriers of the so-called circum-terrestrial ring current, that is the cause of the main phase of magnetic storms.

The peaks of solar activity, characterized by an eleven-year cycle, coincide with conditions of greater magnetospheric activity and the intensification of the effects mentioned above. As many technological systems (satellites, high-voltage power lines, oil pipelines, etc.) are sensitive to conditions of high magnetospheric perturbation, most of applied research in this field is devoted to forecasting the magnetospheric activity, in order to prevent damage, in analogy with the weather forecasting.

1.4 Essentials on the Origin of the Earth's Magnetic Field

Following his spherical harmonic analysis of the Earth's magnetic field, in the 19th century, Gauss had analytically demonstrated that the origin of the field was internal to the Earth. Nevertheless not all the scientific community at that time was entirely convinced. For example, although very slow, the observed temporal variation of the elements of the field did not seem in agreement with other phenomena of internal origin to the Earth. Typical manifestations of internal origin showed on average an apparent temporal invariability and all known internal phenomena showed a high stability. Moreover the most rapid magnetic time variations that were discovered thanks to the introduction of photographic recording of variations of the elements of field, did not perfectly appear in tune with an internal origin. For example, the most rapid time variations showed evident correlations with the solar activity; the recorded magnetograms had a strong diurnal periodicity and showed other indications that seemed to suggest a possible external origin. Therefore the origin of the field remained an enigma and a possible source external to the Earth, even if in contrast with Gauss's results, could still not be entirely ruled out.

The most obvious among the hypotheses able to justify an internal origin, was that of a uniform magnetization of the whole Earth. However this hypothesis was set aside very soon. In fact temperatures measured in mines and other deep excavations, were showing that they were very high inside the Earth. Linearly extrapolating measured gradients showed that approximately at 25 km depth, the temperature is higher than the Curie temperature of almost all known ferromagnetic materials. If the magnetization hypothesis had to be followed, then only a thin outer layer of the whole Earth, with a maximum thickness of 25–30 km, should have been considered. In this case a simple calculation leads to an average magnetization of about 10^4 A m^{-1}, a value well above that of crustal rocks. Moreover a permanent magnetization could explain only a constant magnetic field, that contradicts the well-known variability of the Earth's magnetic field.

At the beginning of the 20th century a new impulse to the search for the origin of the Earth's magnetic field came from seismology. The hypothesis that our planet possesses a fluid core, mainly composed of a high-electrical conductivity material such as iron, revitalized the hypothesis for an internal origin of the field but of a different type: a magnetic field generated by an electrical current system. This theory was based in particular on the possible existence of deep Earth conductive fluid motions able to produce electric currents, which consequently generated a magnetic field. The present magnetic field could not be a remnant of an ancient process, a dynamo continuously operating is necessary. Briefly this dynamo can be assumed to operate through a distortion and amplification of an initial magnetic field due to a magnetohydrodynamic interaction with the motions of the plasma constituting the fluid Earth's core. The field so generated decays with time, owing to ohmic dissipation in the conductor. For its maintenance it is then necessary to hypothesize its continuous regeneration to the detriment of some other forms of energy.

The idea of a self-sustaining magnetic dynamo in the Earth was proposed by Larmor in 1919 but it was not immediately accepted; moreover in 1933 Cowling showed that a field with axial symmetry could not be sustained by means of fluid motions with the same symmetry. Towards the end of the 1950s it was possible to demonstrate the existence of a homogeneous self-sustaining dynamo, clearing the way to one of the most fascinating fields of theoretical geophysics. However it was also clear that the introduction of dynamo theories in planetary and stellar physics for the explanation of magnetic fields in the Earth and the Cosmos, represented an extremely complex subject.

1.4.1 Toroidal and Poloidal Fields

In order to get a better understanding of the mathematical scheme of dynamo theory, it is necessary to introduce several mathematical concepts, starting with: toroidal and poloidal magnetic fields. From Maxwell's equations, for a solenoidal magnetic field (hereafter referred to the magnetic induction $\boldsymbol{B}$), it is always possible to obtain a potential vector $\boldsymbol{A}$

$$\nabla \cdot \boldsymbol{B} = 0 \;\Rightarrow \boldsymbol{B} = \mathrm{curl}\boldsymbol{A} = \nabla \times \boldsymbol{A} \tag{1.48}$$

where

$$\boldsymbol{A} = T\boldsymbol{r} + (\nabla S) \times \boldsymbol{r} \tag{1.49}$$

being T and S two scalar functions introduced to express the potential vector $\boldsymbol{A}$ and $\boldsymbol{r}$ the radius vector, we have

$$\boldsymbol{B}_T = \mathrm{curl}(T\boldsymbol{r}) = \nabla \times (T\boldsymbol{r})$$

$$\nabla \times (T\boldsymbol{r}) = T(\nabla \times \boldsymbol{r}) + (\nabla T) \times \boldsymbol{r}$$

being by definition $\mathrm{curl}\boldsymbol{r} = \nabla \times \boldsymbol{r} = 0$,

$$\boldsymbol{B}_T = (\nabla T) \times \boldsymbol{r} \tag{1.50}$$

which is called the toroidal field; it is always perpendicular to $\boldsymbol{r}$ that is to say it lies on a spherical surface.

As regards the second function under study S,since for any scalar function $(\nabla S) \times \boldsymbol{r} = \nabla \times (S \cdot \boldsymbol{r})$; therefore we can write the second term in $\boldsymbol{B}$ as follows

$$\boldsymbol{B}_P = \nabla \times (\nabla \times (S \cdot \boldsymbol{r})) = \mathrm{curl}^2(S \cdot \boldsymbol{r}) \tag{1.51}$$

which is called the poloidal field and that can have a component along $\boldsymbol{r}$.

Therefore $\boldsymbol{B}$ can always be represented by the sum of a toroidal and a poloidal magnetic field:

$$\boldsymbol{B} = (\nabla T) \times \boldsymbol{r} + \mathrm{curl}^2(S \cdot \boldsymbol{r}) \tag{1.52}$$

The toroidal field has always components perpendicular to $\boldsymbol{r}$ and so it lies on spherical surfaces. It is also called "electrical mode" since it is similar to the distribution that electric currents normally take, for instance, on a sphere. The poloidal field, on the contrary, can have radial components and is also called "magnetic mode" because it is typical, for example, of dipolar magnetic fields. So what we can measure outside the core is only the poloidal part that represents for us the Earth's surface measurable magnetic field. In Fig. 1.24 low degree examples of poloidal and toroidal fields are shown.

1.4.2 Fundamental Equations of Magnetohydrodynamics

The first fundamental equation of magnetohydrodynamics is the equation of motion for an incompressible electric fluid conductor immersed in a magnetic field. This is then a typical Navier-Stokes equation modified for the existence of a magnetic field

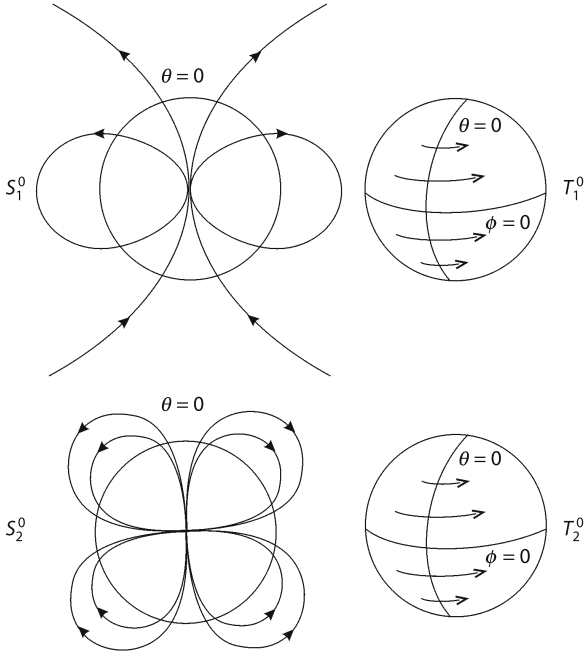


Fig. 1.24. Examples of low degree poloidal (S) and toroidal (T) field lines on a sphere with θ colatitude and φ longitude (redrawn from Merrill et al. 1996)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\rho}\nabla p - 2\Omega \times \boldsymbol{v} + \eta \nabla^2 \boldsymbol{v} + \Im + \frac{1}{\rho}\boldsymbol{J} \times \boldsymbol{B} \tag{1.53}$$

where p denotes the pressure; Ω is the Earth's rotation; $\Im$ denotes the volume forces; η is the coefficient of kinematic viscosity; ρ is density, $\boldsymbol{v}$ is fluid velocity and $\boldsymbol{J}$ the electrical current density.

To introduce a second fundamental equation we need now to go back to some electrodynamic principles. In general, Ohm's law can be written in a vectorial form as

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \sigma v \times \boldsymbol{B} \tag{1.54}$$

where σ is the fluid electrical conductivity. Consider the curl of both sides in this equation:

$$\frac{\nabla \times \boldsymbol{J}}{\sigma} = \nabla \times \boldsymbol{E} + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) \tag{1.55}$$

In case it is possible to neglect the contribution of the displacement current, as may reasonably be assumed in a high conductivity environment, from the Maxwell's equations we have

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu \boldsymbol{J}$$

These relationships can be applied to Eq. 1.55:

$$\frac{1}{\mu\sigma} \nabla \times \nabla \times \boldsymbol{B} = -\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) \tag{1.56}$$

$$\frac{1}{\mu\sigma} \mathrm{curl}^2 \boldsymbol{B} = -\frac{\partial \boldsymbol{B}}{\partial t} + \mathrm{curl}(\boldsymbol{v} \times \boldsymbol{B})$$

If we take in account that

$$\mathrm{curl}^2 \boldsymbol{B} = \mathrm{grad}\ \mathrm{div} \boldsymbol{B} - \nabla^2 \boldsymbol{B}$$

$$\nabla \times \nabla \times \boldsymbol{B} = \nabla (\nabla \cdot \boldsymbol{B}) - \nabla^2 \boldsymbol{B}$$

we obtain the second fundamental equation of magnetohydrodynamics:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \frac{1}{\mu\sigma} \nabla^2 \boldsymbol{B} + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) \tag{1.57}$$

According to this equation time variations of the Earth's magnetic field, its possible increase or decrease, are connected with the velocity of the motion of the fluid conductor present in the core. In order to illustrate this situation, we can say that the increase or decrease of the field is due to the conditions of instability represented by the two terms on the right side in Eq. 1.57. The first term is called *diffusive* and expresses the possibility that in a given place the magnetic field decays in a characteristic time; in fact the term $1/\mu\sigma$ is also called magnetic diffusion. The second term, on the contrary, can be called *advective* and expresses the close connection between the field and the motions of the fluid, which can in fact increase the magnetic field to the expense of the fluid's kinetic energy. This situation can be compared with that of meteorological advection which explains that an atmospheric temperature variation, at a given place, occurs for two distinct causes: a local variation, or the transport of warmer/colder air from other zones as a consequence of atmospheric motions. Briefly, the second magnetohydrodynamic equation, also called induction equation, tells us that, inside the Earth's core, a local modification of the magnetic field can be attributed to only two causes: diffusion and advection. Moreover we can simplify this by saying that, in the case of high-electrical conductivity, that we can reasonably assume to be present in the Earth's core, the diffusive term is far lower in importance than the advective term.

Under appropriate conditions of very high-electrical conductivity, the diffusive term in Eq. 1.57 could in the extreme case be neglected. Therefore the temporal variations of the field in this case are only connected with the fluid velocity configuration. In these conditions we obtain the so-called *frozen flux* magnetic field hypothesis. The concept of frozen magnetic field is attributed to the physicist Alfvèn who was the first to show that the magnetic field lines of force in a fluid in motion, in the case of perfect-electrical conductivity, can be considered as frozen in the fluid; therefore changes in magnetic field are completely due to advection of the field owing to fluid flow, where the magnetic field lines act as markers of fluid motion. Even if under different physical conditions, this situation has been found to apply also in the Sun and in the solar wind, where the motion of plasma transports the magnetic field of solar origin and leads it to fill the interplanetary space.

When a magnetic field is immersed in a fluid conductor, the magnetic field can be transported and deformed by the fluid motion, under appropriate conditions of dimensional factors, velocity and fluid electrical conductivity. It can also be said that, in a perfect conductor, the magnetic field flux out of a surface in motion with the fluid, is constant. This concept can also be visualized with the magnetic field lines of force. If the ideal surface in motion is deformed, the magnetic field lines can be used as tracers of the fluid motion.

The fundamental advantage of the introduction of the induction equation in dynamo theory is that it reduces the relevant quantities, such as all the dynamo problem variables, to only two vector quantities: the magnetic field and the fluid velocity or, to be more precise, the fluid velocity field. On the whole, four Maxwell equations and Ohm's law are replaced by only one equation. If we include in the analysis the equation of motion, we have two equations for two unknown quantities: the magnetic field and the fluid velocity field. However in order to eliminate the diffusive term in the induction equation, we have supposed that the fluid in the core has an infinite conductivity. This condition is called *magnetohydrodynamic condition*. Obviously, being this an unrealistic condition, it must be replaced by an extremely high but not infinite conductivity. An important consequence of the introduction of this restriction will be a slight attenuation of the field with time, that is a non-zero diffusive term, that must be balanced by a continuous regeneration of the field through the dynamo process.

1.4.3 Elementary Dynamo Models

A magnetic field can be generated if an interaction between the fluid motion and the magnetic field takes place, consequently an exchange of energy between the fluid kinetic energy and the magnetic field can take place. If the fluid motion geometry shows a velocity gradient, under the hypothesis of the magnetohydrodynamic condition, this leads to a concentration of the magnetic field lines and therefore to an increase in magnetic field intensity. We can observe that in this case the magnetic field energy has increased (larger density of field lines) where the velocity gradient is present to the expense of the fluid kinetic energy (Fig. 1.25). Several problems arise in dynamo

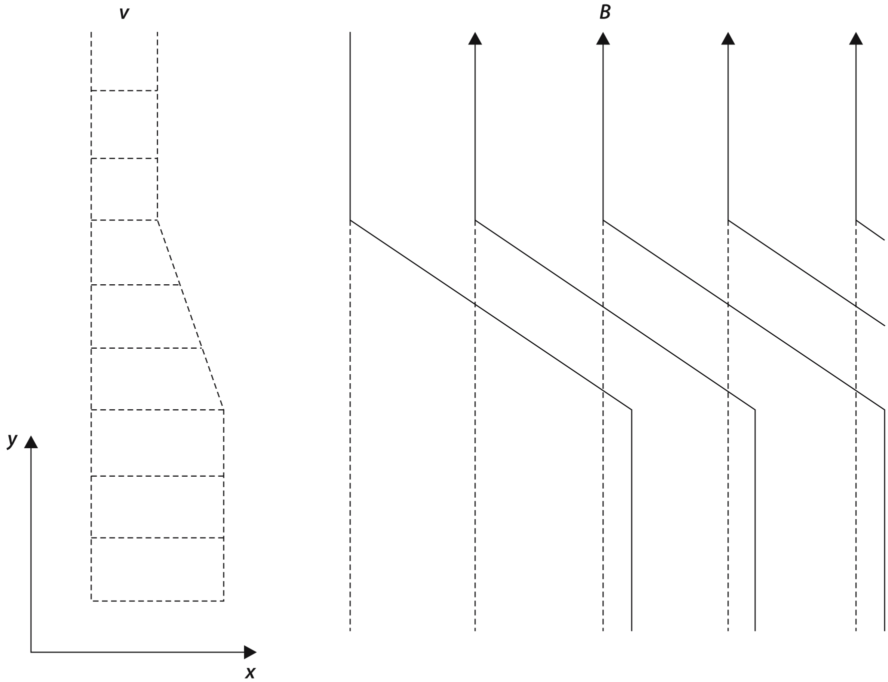


Fig. 1.25. Fluid velocity $\boldsymbol{v}$ gradients in a plane (x-, y-axes) and corresponding $\boldsymbol{B}$ magnetic field lines concentration leading to a field increase (redrawn from Merrill et al. 1996)

theories, we will here only mention a few of them. One is the origin of the energy necessary for the dynamo to work, another is to demonstrate if a certain configuration of motion in the fluid inside the core is able to generate the field as measured on the Earth's surface. This second "preliminary" problem is called that of the kinematic dynamo and has been already discussed since the 1950s; so far many theories had proved the possibility to produce magnetic fields with certain fluid motions. Some empirical numbers are used in the description of the complex dynamics of fluid motions. The Reynolds number is an example. The Reynolds number is defined in terms of some geometrical parameters of the fluid and, according to its value, the fluid motion results to be laminar or turbulent. In the case of a fluid immersed in a magnetic field the magnetic viscosity

$$\nu_{\mathrm{m}} = \frac{1}{\mu\sigma}$$

is used to define the so-called *magnetic Reynolds number*

$$R_{\mathrm{m}} = \frac{L \cdot V}{\nu_{\mathrm{m}}}$$

where L and V denote a geometric scale and a typical velocity of the fluid motion. In short, this number expresses the relationship between the geometric details of fluid motion and its velocity.

In the frame of the dynamo theory, motions that are thought to produce a dynamo effect are also thought to be turbulent. The turbulence takes place when, in a given point of the fluid, the velocity of one of its elements fluctuates randomly without showing a correlation with the velocity of other parts in the fluid, therefore in contrast to the typical laminar flow of slow and organized motions. Turbulent motions are characterized by particles, animated by rotational vortical motions, which create vortices. Fluid vortices have high stability and move in the fluid as if they were individual particles. The value of Reynolds number fixes a quantitative limit between the laminar and vortical regimes. Once the vortical regime has taken place, the presence of a close correlation between its velocity and the turbulence, mathematically expressed by $(\nabla \times \boldsymbol{V})$ inside the fluid, leads to another important dynamic effect, the helicity, i.e. $\boldsymbol{V} \cdot (\nabla \times \boldsymbol{V}) \neq 0$, which is a specific example of turbulence. The existence of helicity is the manifestation of a strong transfer of kinetic energy from fluid volume motions to fluid vortices. Another important factor in the development of a *turbulent dynamo* was the introduction of the electrodynamic *mean field* approximation. An α-dynamo is a dynamo based on the transformation of turbulent energy of the fluid in electrical energy. In an α-dynamo the electromotive force created by means of turbulent motion is parallel to the magnetic field. This means that the generation of electromotive force due to turbulence will be such that

$$\boldsymbol{E} = \alpha \overline{\boldsymbol{B}}_0 \tag{1.58}$$

where $\overline{\boldsymbol{B}}_0$ is the initial stationary magnetic field and α an appropriate constant. Only in the case of suitable geometries, this electromotive force can produce an electrical current that strengthens the initial field. The α-effect is fundamental for two of the most well known candidates for terrestrial possible dynamos called simply the α^2 and $\alpha\omega$ dynamos.

We will analyze the $\alpha\omega$-dynamo only in a schematic rather than a mathematical way. Consider an initial poloidal magnetic field $\boldsymbol{S}_1^0$ and a toroidal velocity field $\boldsymbol{T}_1^0$ to be present in the Earth's fluid core. In the frozen magnetic flux approximation, the fluid motion deforms the poloidal magnetic field, therefore the pre-existing magnetic field becomes more intense where, due to differential rotation, the fluid flow compresses the poloidal magnetic field lines to the detriment of the fluid's kinetic energy. The interaction produces a new toroidal magnetic field called $\boldsymbol{T}_2^0$, that has opposite signs in the two hemispheres; this can be schematically expressed as

$$\boldsymbol{S}_1^0(\text{mag}) + \boldsymbol{T}_1^0(\text{vel}) = \boldsymbol{T}_2^0(\text{mag}) \tag{1.59}$$

This effect is called the ω-effect.

Once the toroidal field has been produced a second fluid velocity field comes in the play. Owing to the reasonable presence of a fully developed convection in the core, a motion with a radial component is now assumed to exist. The convection cells, that can be visualized as columns consisting of fluid substance moving circularly with a volume radial velocity from the core bottom to the CMB and vice versa, will produce, as it happens in the Earth's atmosphere, some ascending rotational motions consisting of fluid substance (equivalent to tropical atmospheric cyclones). This torsion is the effect of Coriolis forces. The net result is a helical motion which interacts with the toroidal magnetic field lines previously generated, that we assume to be normal to the axis of the ascending columns. Original and toroidal field lines will be deformed and undergo a torsion due to helical motion in the column (Fig. 1.26). If electrodynamic forces are such as to limit this torsion to 90°, some circular magnetic coils will form perpendicular to the initial toroidal field line, i.e. in the meridian plane. Finally as a third step new coils will then be able to regenerate magnetic field lines of the initial poloidal field by a *coalescence process*.

Briefly, we have three processes also summarized in Fig. 1.27:

1. Generation of a toroidal magnetic field $\boldsymbol{T}_2^0$ by the ω-effect (differential rotation)
2. Ascending flows with helicity produce closed coils in meridian planes
3. Closed coils, by a coalescence process, regenerate the $\boldsymbol{S}_1^0$-magnetic field (α-effect)

The whole described process is intrinsically three-dimensional and can be mathematically explained by means of the differential equations and partial derivatives given in the above mentioned magnetohydrodynamic Eqs. 1.53 and 1.57.

1.4.4 Dynamo Energy

Even if many details of dynamo theory remain unsolved, or are still to be developed, at this time a sufficient schematic knowledge of the physics that determines the process of generation of the terrestrial magnetic field in the core, is available. Electric current systems, produced by a dynamo effect would decay within a time window of about 1 000 –10 000 years unless they were constantly regenerated by an energetic source. Different energy sources have been proposed for the maintenance of the electric current systems flowing in the core and in particular we will take three of them into consideration:

1. A gravitational descent of the heaviest elements from the fluid part of the core into the inner solid part; in this process the inner solid core size increases and a convection mechanism is mechanically generated in the fluid core. This solidification also releases energy due to latent heat of solidification.
2. A thermal convection produced by possible radioactive sources in the core. This effect is however considered less effective than the compositional convection mentioned above, in particular from a mechanical point of view.
3. A coupling between the mantle and the core due to Earth's axis precession (Poincaré had already dealt with this subject), even if many authors think it not to be entirely effective for dynamo from an energetic point of view.

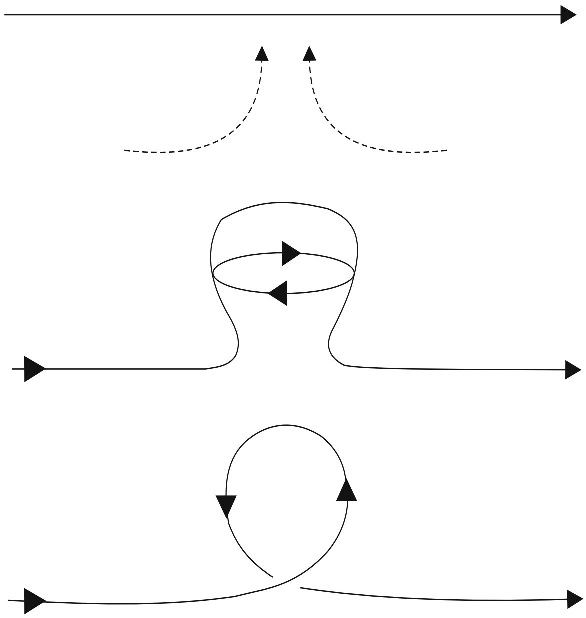


Fig. 1.26. From top to bottom, a magnetic field line, subjected to a fluid upwelling in rotation around a vertical axis in a helical motion (*dashed lines*), is distorted and undergoes a torsion (redrawn from Merrill et al. 1996)

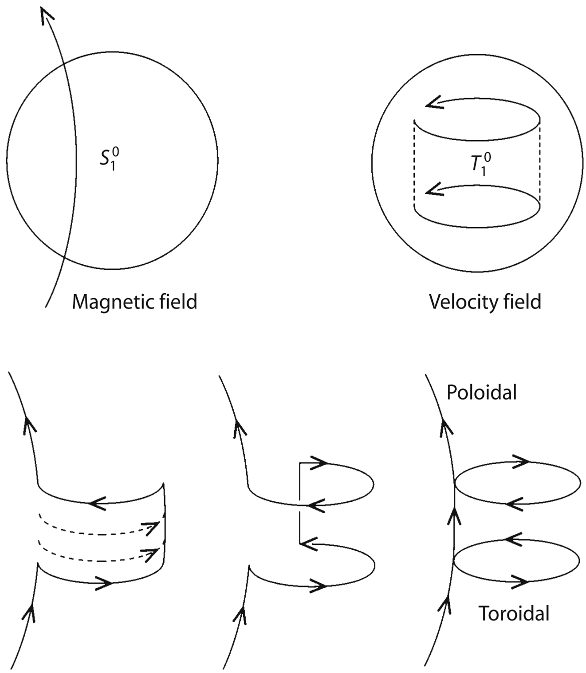


Fig. 1.27. Three processes that lead to the Earth's dynamo action (redrawn from Merrill et al. 1996)

In conclusion it can be said that the terrestrial dynamo is essentially based on the following principles:

a A solidification of the inner core due to compositional increase and corresponding release of lighter elements inside the fluid core; this "mechanical" convective motion power is estimated to about 2×10^{11} W.
b The combination of the convection produced inside the core and Earth's rotation produces complex motion patterns in the fluid core that are indispensable to self-sustaining dynamo action.
c The Reynolds magnetic number $R_m = (L\,V) / \nu_m$, that expresses the scalar factors from which dynamo depends on (dimensions, fluid velocity and magnetic viscosity) must be larger than 10. The motion must be turbulent with helicity.
d The generated magnetic field is essentially dipolar with a toroidal part that remains confined in the core. The dipolar part, i.e. the main component measured on the Earth's surface, is superimposed on higher-degree harmonics which make a significant contribution to the total field.

The particular geometry inside the Earth plays an important role in the complex dynamo process. The rapid rotation of the fluid inside the Earth forces its motion to be strongly driven toward axisymmetry. The presence of the inner core also forces to maintain convection along rolls that, in the purely hydrodynamic case (i.e. in absence of a magnetic field) would be approximately shaped as cylindrical columns parallel to the rotation axis and tangential to the inner core (Fig. 1.28). Conductive fluid motions are the currents that generate the geomagnetic field. It is likely, because of the nearly cylindrical symmetry of the fluid motions that the geomagnetic field is persistently dipolar.

From a physical point of view there is no reason why the Earth's magnetic field should prefer a particular polarity, and there is no fundamental reason why its polar-

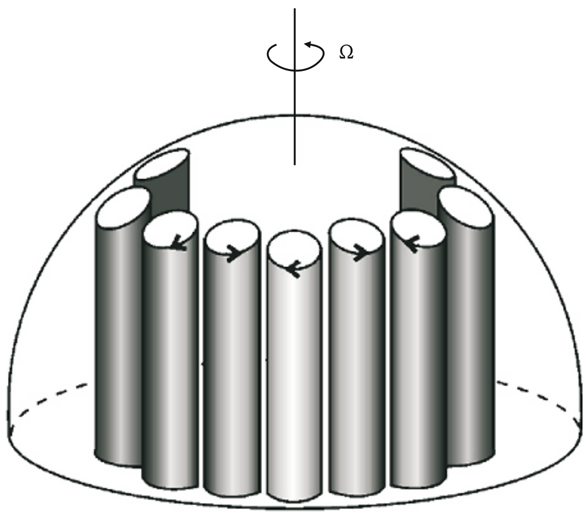


Fig. 1.28. Convection rolls in a rapidly rotating sphere with inner solid core (redrawn from Merrill et al. 1996)

ity should not change. Given the axisymmetry of core fluid dynamics it is possible to see that a dynamo can produce a field in either direction with respect to the rotation axis. A geomagnetic field reversal is in fact an implicit possibility because of the fundamental symmetry of the dynamo equations: if $\boldsymbol{B}$ is a solution, then so is $-\boldsymbol{B}$. Unprecedented details with impressive realizations of geomagnetic field lines and their behavior in terms of secular variation and geomagnetic reversals, are now the results of numerical integrations of fully three dimensional dynamos. The use of supercomputers has allowed a few very specialized groups of scientists to obtain detailed models and plots of the field lines for the Earth's magnetic field.

1.5 Magnetic Observatories, Reference Field Models and Indices

1.5.1 Geomagnetic Observatories

Geomagnetic observatories are the structures designed to undertake a continuous monitoring of the Earth's magnetic field. This is obtained by a standard recording of natural magnetic field time variations and by measuring the absolute level of the magnetic field in all its elements. In geomagnetic observatories this requirement is necessary in order to reveal all possible time variations of interest in geomagnetism, including secular variation, the longest time-scale observable variation. This long-term engagement of an observatory is of course difficult to maintain for several reasons, but when achieved it provides invaluable data for a variety of geophysical studies and in the case of geomagnetism it is the only way to investigate secular variation in detail.

Temporary variometer stations are less restricted structures where magnetic instruments are installed for limited time campaigns in different parts of the world, normally for the recording of rapid external origin time variations. One example is the data acquisition of rapid time variations that need to be known during the execution of magnetic crustal field surveys. Other applications of temporary magnetic stations are for example investigations of the space structure of geomagnetic storms or reconstruction of ionospheric or magnetospheric electrical currents. Conversely in the geomagnetic community, an observatory is considered a 'solid' long-term structure where not only a continuously high accuracy in the absolute level of magnetic field measurements is required, but also a commitment to long time working is necessary.

A continuous long-term monitoring of the Earth's magnetic field is carried out in many observatories all over the world. The number of locations where this is undertaken has, since the times of Gauss, grown to about 150 (Fig. 1.29). Many geomagnetic observatories have at this time collected more than one, or in a few remarkable cases, two centuries of magnetic data. Exceptionally remarkable cases are for example the very long time series of London and Paris, that for the Earth's magnetic field angular elements, go back to the mid-sixteenth century. Plots of magnetic elements versus time, using observatory data, are used for example to study secular variation, the slow unpredictable change of all magnetic elements. From a knowledge of secular variation, several studies have been conducted on the Earth's deep interior and particularly on the electrical conducting fluid motions in the Earth's core.

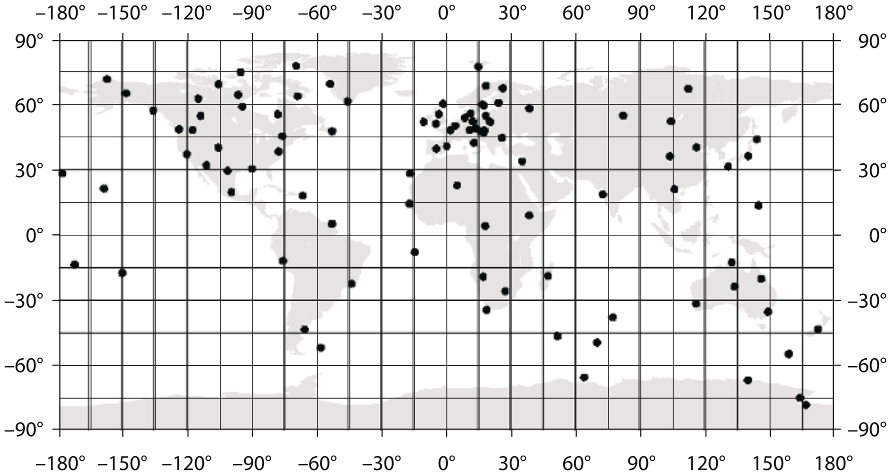


Fig. 1.29. Geomagnetic observatories in the World

From the beginning of magnetic field studies it was recognized that geomagnetic observatories needed to be located reasonably far away from strong natural disturbance such as volcanoes or magnetic mineral deposits, in order to be able to record the average level of the field representing a vast area. Of course, also the need of being away from artificial noise was soon recognized, something that was reasonably easy until only about a century ago. Nowadays the strong perturbation of the natural magnetic field introduced by electrification, especially tramways, underground trains and surface railways, has strongly limited the possibility of installing or even maintaining magnetic observatory activities in many developed countries. This is a problem faced in several geomagnetic institutes. At the beginning, in the few existing magnetic observatories the effort was only to measure the time variations of the angular elements such as *D* and *I*, from the mid-eighteenth century also intensive elements were recorded. During the years, magnetic measurement methods have of course changed considerably, especially in the last twenty to thirty years. Only until a few tens of years ago, magnetic operators were engaged with so-called absolute magnetic measurements that could take hours. These reference values for all magnetic field elements were used for the knowledge of the natural base line levels of the natural magnetic field. These procedures are to a great extent simplified today, but a certain skill is still required in the correct conduction of a geomagnetic observatory.

In general, a magnetic observatory is constituted by a few non-magnetic buildings where different tasks are performed. One building is hosting those instruments that run continuously, the variometers, for example fluxgates, in order to record magnetic elements time variations. A second building hosts the so-called absolute measurement devices. These are instruments with which magnetic operators undertake those measurements that are used to calibrate the time variations recorded by the variometers, for example proton precession magnetometers and *DI*-fluxgates for angular elements. *DI*-fluxgates are the new magnetic theodolites, consisting of a fluxgate magnetometer element and an optical theodolite. The fluxgate element is mounted axially on an optical theodolite telescope, when the fluxgate element is orthogonal to a magnetic field line, a zero field is measured by the fluxgate electronics. This instrument allows an accurate determination of declination and inclination angles by a null method. Generally in the absolute building a series of pillars host the absolute instruments and one pillar is taken as the fundamental one for the observatory.

Nowadays in order to facilitate data exchanges and the making of geomagnetic products available close to real time, an international coordination programme among all world magnetic observatories called INTERMAGNET is in operation. The programme has facilitated the establishment of a global network of cooperating digital magnetic observatories, adopting modern standard specifications for measuring and recording equipment. An INTERMAGNET Magnetic Observatory (IMO) is a modern magnetic observatory, having full absolute control, that provides one minute magnetic field values measured by a vector magnetometer, and an optional scalar magnetometer, all with a resolution of 0.1 nT. Vector measurements performed by a magnetometer must include the best available baseline reference measurement. Since the effort of running magnetic observatories is elevated, geomagnetic institutions also take magnetic measurements at discrete points, in so-called magnetic networks. These points are marked on maps and are reoccupied every few years (normally at intervals of 5 years), and measurements of all magnetic elements are there repeated. For this reason these points are generally called *repeat stations*. With this additional information geomagnetic observatory data are supplemented and the full information on secular variation behavior on large areas around observatories is obtained. In the case of Italy a network of about 100 repeat stations supplements the magnetic information from the two magnetic observatories, L'Aquila and Castello Tesino.

1.5.2
Geomagnetic Field Reference Models

Data from observatories as well as from repeat stations are only point data. In several occasions the need of magnetic field elements determination in a certain region, or even over the whole Earth, is required. For example the accurate estimate of the magnetic field elements, in particular the magnetic declination, for navigational purposes, is required. For what concerns the portion of the magnetic field generated in the Earth's core, the community of scientists that operates in geomagnetism, has decided to represent it by means of the so-called geomagnetic reference fields. Geomagnetic reference fields are mathematical models of the Earth's magnetic field that represent its space and time variations; time variations generally refer only to secular variation.

Magnetic observatory data, as well as land, marine, airborne and satellite survey field measurements are the fundamental contributions from which models can be made. In conjunction with magnetic observatory data the availability of measurements on a network of repeat stations provides the means to monitor the geomagnetic field and its long-term variations on a regional scale. Geomagnetic observatories and repeat stations are of course located in areas free of artificial disturbances, and not characterized by large surface anomalies and therefore satisfy the standard requirements for reasonably reflecting the main field.

A mathematical model is a representation, for example by means of power series in latitude and longitude, of the spatial time variations of the Earth's magnetic field over a certain area. Mathematical models representing the geomagnetic field at a certain epoch and its secular variation on a global Earth scale, are also possible, generally by means of spherical harmonic polynomials. If secular variation is represented in a certain area, with data from different observatories, for example in a given time interval, a map that shows contour lines of secular variation, can be obtained; these maps are generally called isoporic charts.

One of the most important applications of SV models is obtained when several magnetic surveys are collected in order to join all measurements to a central unifying epoch. Magnetic anomaly maps, for the study of the crustal contribution to the geomagnetic field, are a typical application. Magnetic anomaly maps are in fact constructed after the removal of the main part of the geomagnetic field. Time reduction of magnetic surveys relies on secular variation models.

Spherical harmonic analysis is suitable to model the longest-wavelength part of the geomagnetic field. The IGRF (International Geomagnetic Reference Field) is the accepted global model of the geomagnetic field. It is believed to contain all, or most of, the core field, i.e. the largest part of the field observed at the Earth's surface. The IGRF is given in the form of sets of spherical harmonic coefficients up to and including degree and order 10, except for the most recent main-field models, which extend to degree and order 13, and the predictive secular variation model which extends to degree and order 8. The current 10th generation, revised in 2005 and indicated by the short name IGRF-10, is composed of 12 definitive sets of main field models, ranging from 1945 to 2000.0[4], at 5-year intervals. The latest coefficients are the main field coefficients for 2000.0 and 2005.0 and the secular variation coefficients for 2005.0–2010.0. Since geomagnetic data do not cover all the Earth uniformly, the most suitable method of finding the spherical harmonic coefficients is that based on a least-squares procedure. Because of the irregular geographic distributions of lands and of different economical situations of countries, some regions (e.g. Europe and northern America) are better represented by the IGRF than others (e.g. the oceans).

1.5.3 Geomagnetic Indices

The term magnetic activity is referred to the amplitude variability of magnetic time variations associated with external origin fields. This activity, recorded on the ground by magnetic observatories, is difficult to be exactly quantified. Magnetic indices have therefore been introduced to provide a quantification of the Earth's magnetic field activity level. A few indices are now commonly in use to characterize magnetic activity

[4] 2000.0 is a standard use in geomagnetism when referring to maps. It means that all values used to draw the map (which individually have been measured at different times) have been corrected to the same time, i.e. hour 0:00 of January 1st, 2000 (2000.5 means: hour 0:00 of July 1st, 2000).

on a global scale and also on more specific scales. We will describe in what follows the most widely used.

1.5.3.1
K-, Kp- and ap-, Ap-Indices

The *K*-index summarizes geomagnetic activity generated by solar particle radiation injection into the magnetosphere as recorded at magnetic observatories. The index is expressed by assigning a code, an integer in the range 0 to 9, to each 3-hour Universal Time (UT) interval in a day. Therefore each day is characterized by 8 *K*-indices. The index for each 3-hour UT interval is determined from the ranges in *H* and *D* (scaled in nT), after the removal of the regular expected *Sq* diurnal variation, generated by solar electromagnetic radiation. In order to remove to an average *Sq* variation pattern, the five quietest days of the month are used to compute a reference standard variation.

The conversion from a range to a numeric value index, is made using a quasi-logarithmic scale, with the scale values dependent on the observatory's geomagnetic latitude. In fact the same planetary disturbance can show up with different amplitudes at different latitudes. As an example the conversion for L'Aquila, Italy, geomagnetic observatory (geomagnetic latitude 36.3° N) is given in Table 1.2.

In Fig. 1.30 the time variation of the horizontal component *H* (upper panel) and of declination *D* (lower panel) of the Earth's magnetic field as observed at L'Aquila observatory on January 23, 2004 is shown. Both magnetic elements (*H* and *D*) are measured in nanotesla (nT), while time is measured in minutes. The corresponding series of *K*-indices for the day is the following: $K = 4, 3, 3, 4, 3, 6, 6, 4$.

The planetary 3-hour-range index *Kp* is the mean standardized *K*-index from 13 geomagnetic observatories selected on purpose located between 44° and 60° northern or southern geomagnetic latitude. In this case the scale 0 to 9 is farther subdivided and expressed in such a way to include the thirds of a unit. As an example the symbol 5– represents 4 and 2/3; 5o is 5 and 5+ is 5 and 1/3. Being *K*- and *Kp*-indices based on a logarithmic scale a linearized scale index was also introduced. The 3-hourly *ap* (equivalent range) index is derived from the *Kp*-index as shown in Table 1.3.

The daily index *Ap* is obtained by averaging the eight values of *ap* for each day.

1.5.3.2
AE-Index

The *AE*-index is an auroral electrojet index obtained from a number (usually greater than 10) of stations distributed in local time in the latitude region typical of the northern hemisphere auroral zone. For each station the north-south magnetic perturbation *H* is recorded as a function of universal time. A superposition of these data from all the stations enables a lower bound or maximum negative excursion of the *H* component to be determined; this excursion is called the *AL*-index. Similarly, an upper bound or maximum positive excursion in *H* is determined; this is called the *AU*-index. The difference between these two indices, *AU*-*AL*, is called the *AE*-index. Notice that nega-

Table 1.2. Conversion from a range to a numerical index value for L'Aquila, Italy (geomagnetic latitude 36.3° N)

Difference in nT	*K*-index
0 – 4	0
4 – 8	1
8 – 16	2
16 – 30	3
30 – 50	4
50 – 85	5
85 – 140	6
140 – 230	7
230 – 350	8
> 350	9

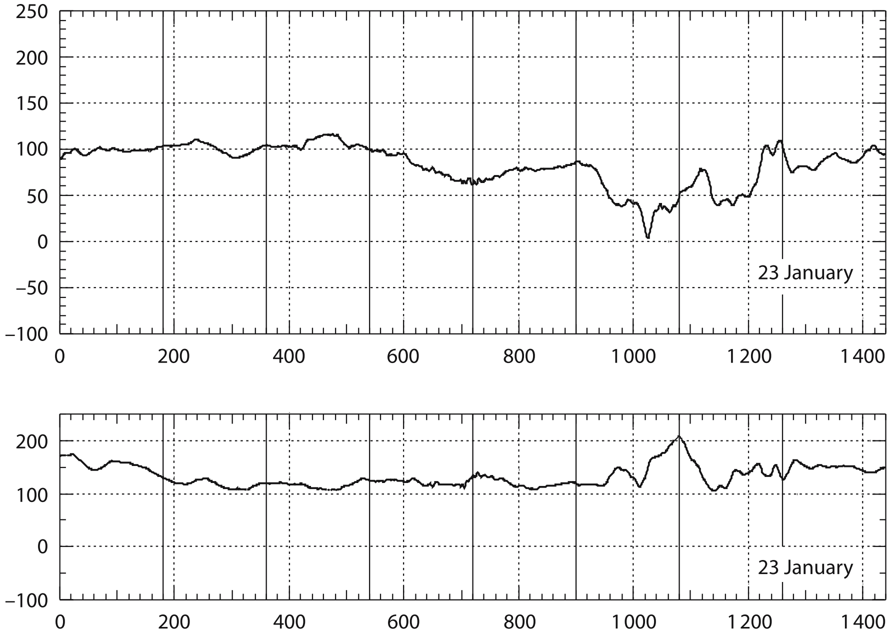


Fig. 1.30. Example of a magnetogram for day January 23, 2004, for L'Aquila Observatory, geomagnetic latitude 36.3° N, and *K*-indices corresponding values. $K = 4, 3, 3, 4, 3, 6, 6, 4$. *H* and *D* are measured in nT and time is measured in minutes

tive *H* perturbations occur when stations are under a westward-flowing electrical current flowing in the auroral electrojet.

Thus the indices *AU* and *AL* give some measure of the individual strengths of the eastward and westward electrojets, while *AE* provides a measure of the overall

Table 1.3. 3-hourly *ap* (equivalent range) index derived from the *Kp*-index

Kp	*ap*
0o	0
0+	2
1–	3
1o	4
1+	5
2–	6
2o	7
2+	9
3–	12
3o	15
3+	18
4–	22
4o	27
4+	32
5–	39
5o	48
5+	56
6–	67
6o	80
6+	94
7–	111
7o	132
7+	154
8–	179
8o	207
8+	236
9–	300
9o	400

horizontal current strength. The ordinary time resolution for *AE*-indices is one hour but higher time resolution indices (5 minutes or so) are at times computed for special purposes.

1.5.3.3
Dst-Index

Planetary perturbations characterized by a marked decrease in the *H* (northward) component at mid latitude observatories are called magnetic storms. In order to quantify the *H* depression effect, an hourly *Dst* (Disturbance storm time) index is obtained from magnetometer stations near the equator, but not so close that the E-region equatorial electrojet dominates the magnetic perturbations observed on the ground. At such latitudes the horizontal intensity and the vertical intensity of the magnetic perturbation are dominated by the effects of the magnetospheric ring current. The *Dst*-index is a direct measure of the hourly average of this perturbation. Large negative perturbations in *H* are indicative of an increase in the intensity of the ring current and typically appear on time scales of about an hour. The intensity decrease may take much longer, on the order of several hours or even one or two days. Since during a magnetic storm several isolated or one prolonged substorm signature in the *AE*-index are sometimes observed, specific high time resolution (5 min or so) versions of the *Dst*-index are computed to study the relationship between storms and substorms.

Magnetic indices are in conclusion the quantification of magnetic activity; different indices have been introduced for different purposes. Several applications in geomagnetism make a large use of magnetic indices, however the meaning of geomagnetic indices has changed during the years. For example in the case of *Dst*, it is well known now that this index does not exactly describe the ring current activity only; in fact the decrease in horizontal component at low latitude magnetic observatories is also caused by the existence of other current systems in the magnetosphere and is not only due to the ring current. Nowadays a major objective is the prediction of the state of magnetic activity on Earth and then the prediction of magnetic indices. In space weather the effects begin on the Sun and ultimately affect the Earth, as geomagnetic variations and all related effects. From observations of the Sun and data from interplanetary space-probes the chain of effects is followed and short-term predictions are already available from several space weather services.

Suggested Readings and Sources of Figures

Books

Backus G, Parker R, Constable C (1996) Foundations of geomagnetism. Cambridge University Press, Cambridge, 369 pp

Campbell WH (2001) Earth magnetism. A guided tour through magnetic fields. Harcourt Academic Press, San Diego, CA, USA, 151 pp

Campbell WH (2003) Introduction to geomagnetic fields. Cambridge University Press, Cambridge, 337 pp

Jankowsky J, Sucksdorff C (1996) IAGA guide for magnetic mesurements and observatory practice. Warsaw, 235 pp

Kivelson MG, Russell CT (1996) Introduction to space physics. Cambridge University Press, Cambridge, 568 pp

Lowrie W (1997) Fundamentals of geophysics. Cambridge University Press, Cambridge, 354 pp

Merrill RT, McElhinny MW, McFadden PL (1996) The magnetic field of the Earth: Paleomagnetism, the core and the deep mantle. Academic Press, San Diego, California, 531 pp

Newitt LR, Burton CE, Bitterly J (1996) IAGA guide for magnetic repeat station surveys. Boulder, 112 pp

Tauxe L (1998) Paleomagnetic principles and practice. Kluwer Academic Publishers Group, 312 pp

Articles

Buffett BA (2000) Earth's core and the geodynamo. Science 288:2007–2012

Cain JC, Wang Z, Schmitz DR, Meyer J (1989) The geomagnetic spectrum for 1980 and core crustal separation. Geophys J 97:443–447

De Michelis P, Cafarella L, Meloni A (2000) A global analysis of the 1991 geomagnetic jerk. Geophys J Int 143:545–556

Gonzales WD, Tsuritani B, Clua De Gonzales A (1999) Interplanetary origin of geomagnetic storms. Space Sci Rev 88:529–562

http://solid_earth.ou.edu/ notes/potential/legendre.gif (Copyright 2004, J. Ahern)

Jackson A, Jonkers ART, Walker MR (2000) Four centuries of geomagnetic secular variation from historical records. Philos T Roy Soc A 358:957–990

Kono M, Roberts HR (2002) Recent geodynamo simulations and observations of the geomagnetic field. Rev Geophys 40:4

Lanza R, Meloni A, Tema E (2005) Historical measurements of the Earth's magnetic field compared with remanence directions from lava flows in Italy over the last four centuries. Phys Earth Planet Int 148:97–107

Malin SRC, Barraclough DR (1982) 150th anniversary of Gauss's first absolute magnetic measurement. Nature 297:285

McMillan S and Maus S (2005) International Geomagnetic Reference Field the 10th generation. Earth Planets Space 57(12):1135–1140

Valet JP (2003) Time variations in geomagnetic intensity. Rev Geophys 4:1–44