## 大整数模余运算举例说明

- $3185^{2753} \mod 3233 = (-48)^{2753} \mod 3233$
- =- ( $3^{2753}$  mod3233\*  $2^{4}*^{2753}$  mod3233 ) mod 3233
- 由于这里的模数3233太大,指数2753也太大,没有什么特殊情况可以利用。所以计算比较困难。
- 一种减负的做法是,利用模指数运算(参见下一页)。
- 2753 = (1010110000001)<sub>2</sub>, 借助计算器或者计算机, 利用 这个分别将每项的模余求出来, 再求乘积的模余。

## Modular Exponentiation 模指数运算 (自己看看)

- In cryptography it is important to be able to find  $b^n \mod m$  efficiently, where b, n, and m are large integers.
- It is impractical to first compute  $b^n$  and then find its remainder when divided by m because  $b^n$  will be a huge number. Instead, we can use an algorithm as follows.
- Assume  $n = (a_{k-1} \dots a_1 a_0)_2$ , we can get

$$b^n = b^{a_{k-1} \cdot 2^{k-1} + \dots + a_1 \cdot 2 + a_0} = b^{a_{k-1} \cdot 2^{k-1}} \cdot \dots \cdot b^{a_1 \cdot 2} \cdot b^{a_0}$$

- This shows that to compute  $b^n$ , we need only compute the values of b,  $b^2$ ,  $(b^2)^2 = b^4$ ,  $(b^4)^2 = b^8$ , . . . ,  $b^2$ . Once we have these values, we multiply the terms  $b^2$  in this list, where  $a_i = 1$ .
- The algorithm successively finds  $b \mod m$ ,  $b^2 \mod m$ ,  $b^4 \mod m$ , . . . ,  $b^{2k-1} \mod m$  and multiplies together those terms  $b^{2j} \mod m$  where  $a_j = 1$ ,