Planar Graphs 平面图

Def: ***Planar graphs*** are graphs that can be drawn in the plane without edges having to cross. 能不交叉地画在一个平面上的图。否则就叫非平面图。

Understanding planar graph is important:

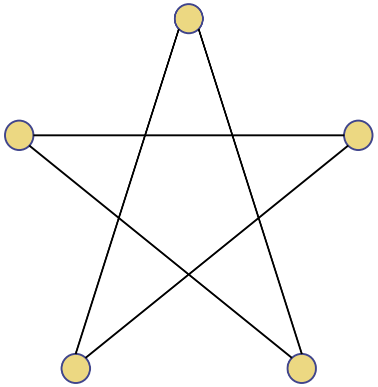
- Any graph representation of maps/ topographical information(地形图) is planar.
  - graph algorithms often specialized to planar graphs (e.g. traveling salesperson)

- Circuits usually represented by planar graphs

Planar Graphs -Common Misunderstanding

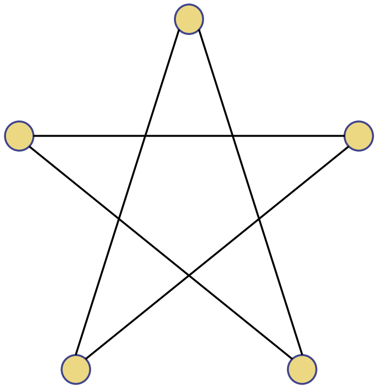
Just because a graph is drawn with edges crossing doesn’t mean its not planar.

Q: Why can’t we conclude that the following is non-planar?



Planar Graphs -Common Misunderstanding

A: Because it is isomorphic to a graph which *is* planar:



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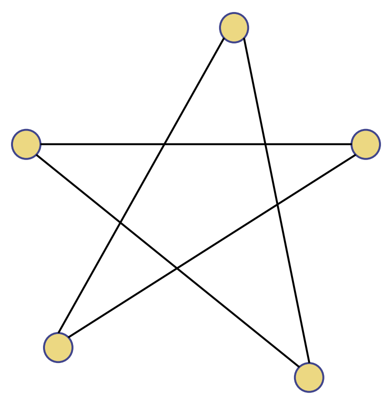

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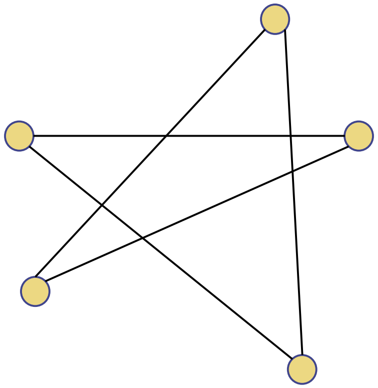

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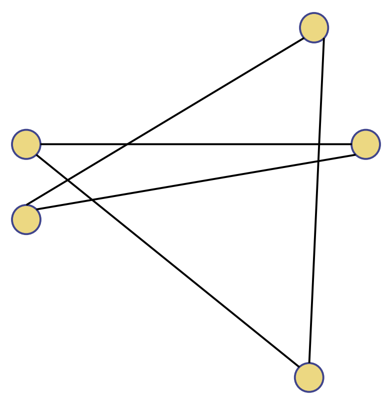

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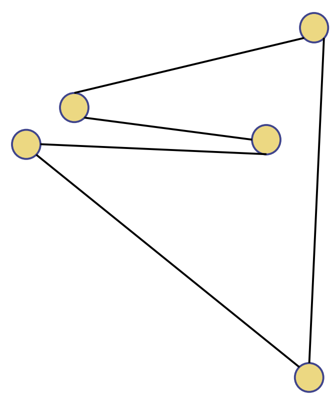

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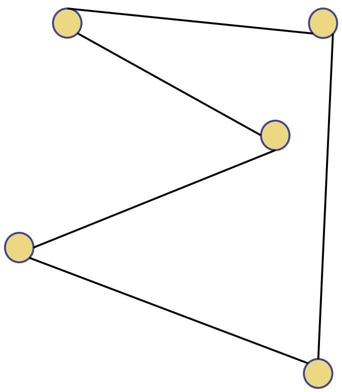

Planar Graphs -Common Misunderstanding

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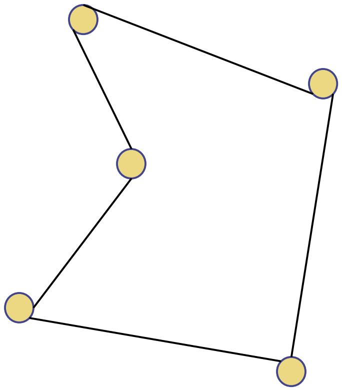

Planar Graphs
-Common Misunderstanding

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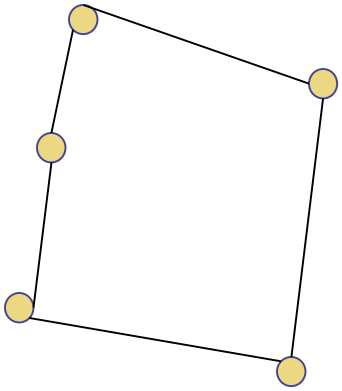

Planar Graphs -Common Misunderstanding

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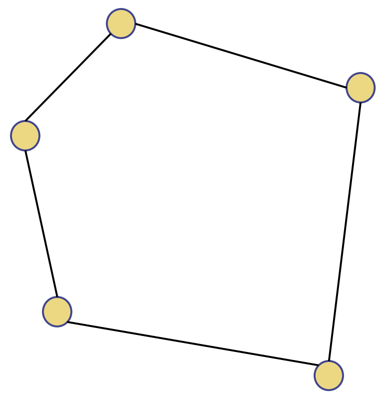
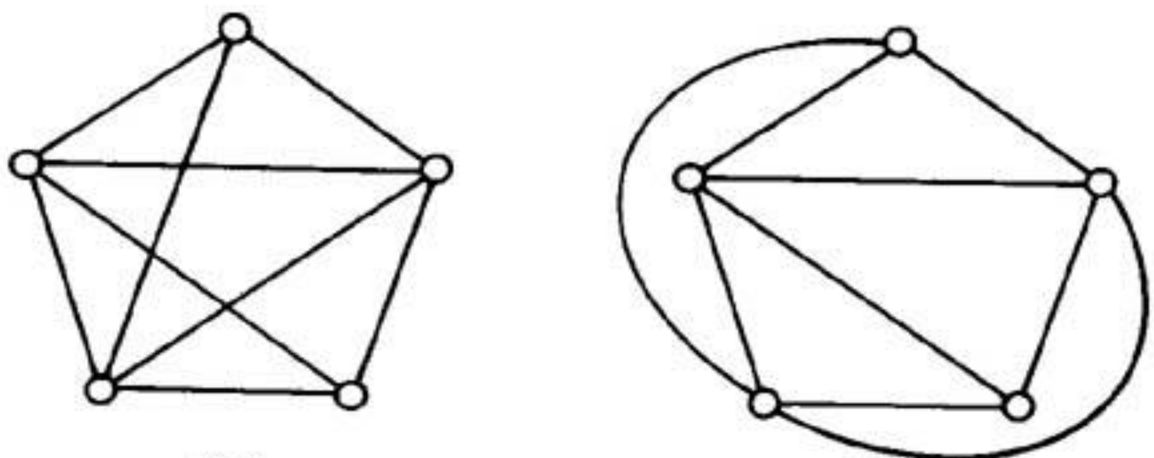


Do Edges Intersect?
下面图是平面图吗

- Planar graphs can sometimes be drawn as non-planar graphs. It is still a planar graph, because they are isomorphic.
- 以非平面图的方式画出来的图，仍然有可能是平面图。

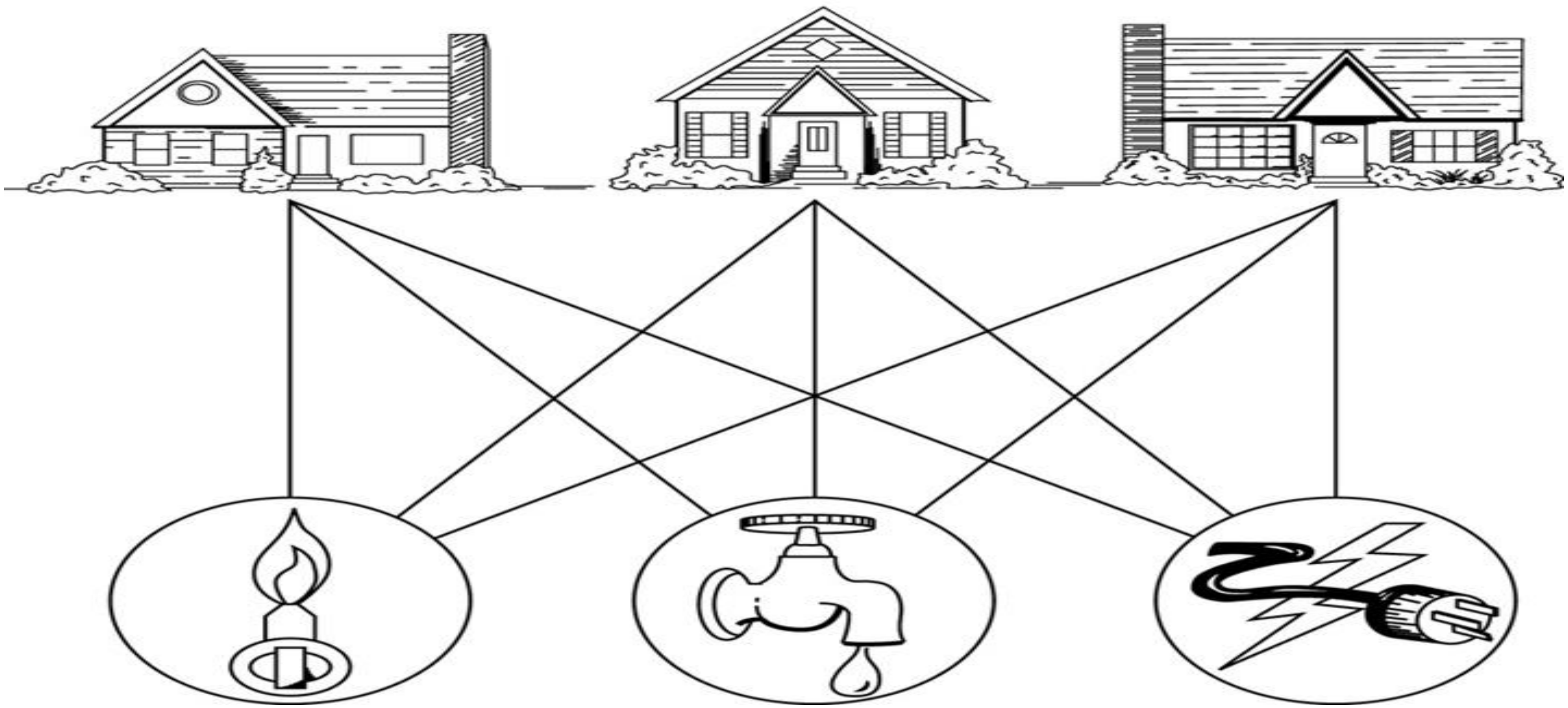


(a) (b)

Figure 9.3

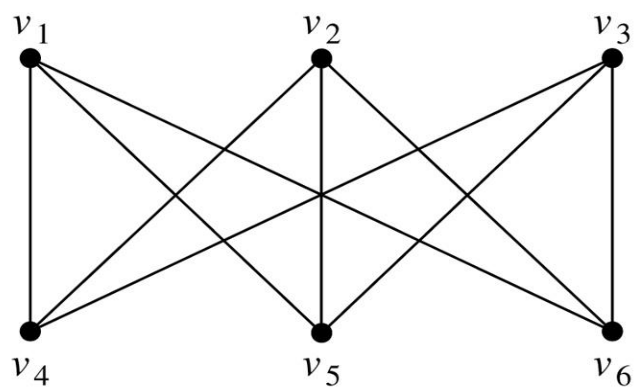
Three Houses / Three Utilities

- Q. Suppose we have three houses and three utilities. Is it possible to connect each utility to each of three houses without any lines crossing?
- Planar or Non-Planar ?
- This is also known as K(3,3) bipartite graph

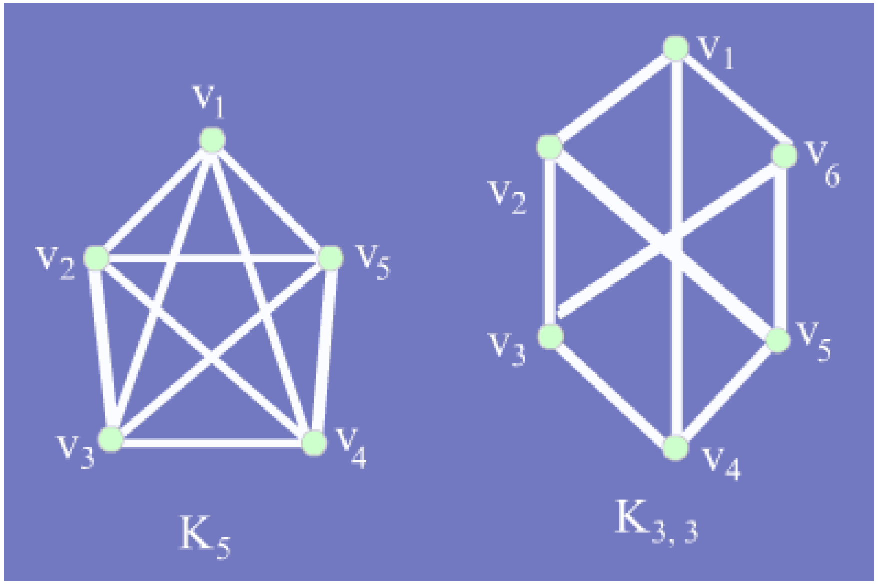


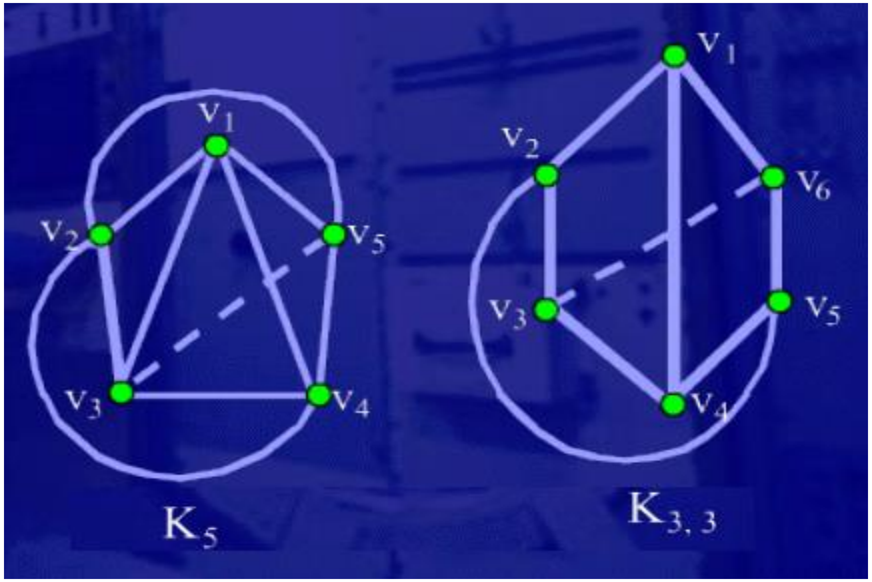
Two Examples of Non-planar

两个典型的非平面图的例子



The graph $K_{3,3}$ above is actually same with the $K_{3,3}$ below, it is just different way to draw on the plane.





K_5, $K_{3,3}$ are non-planar

平面图判断（难题）

思考问题：如果一个图有某个子图是非平面图，那么该图还可能是平面图吗？ 对于平面图又如何？

从上面六边形里有3条对角线$K_{3,3}$是非平面图的判断方法中，总结出一种判断非平面图的方法：

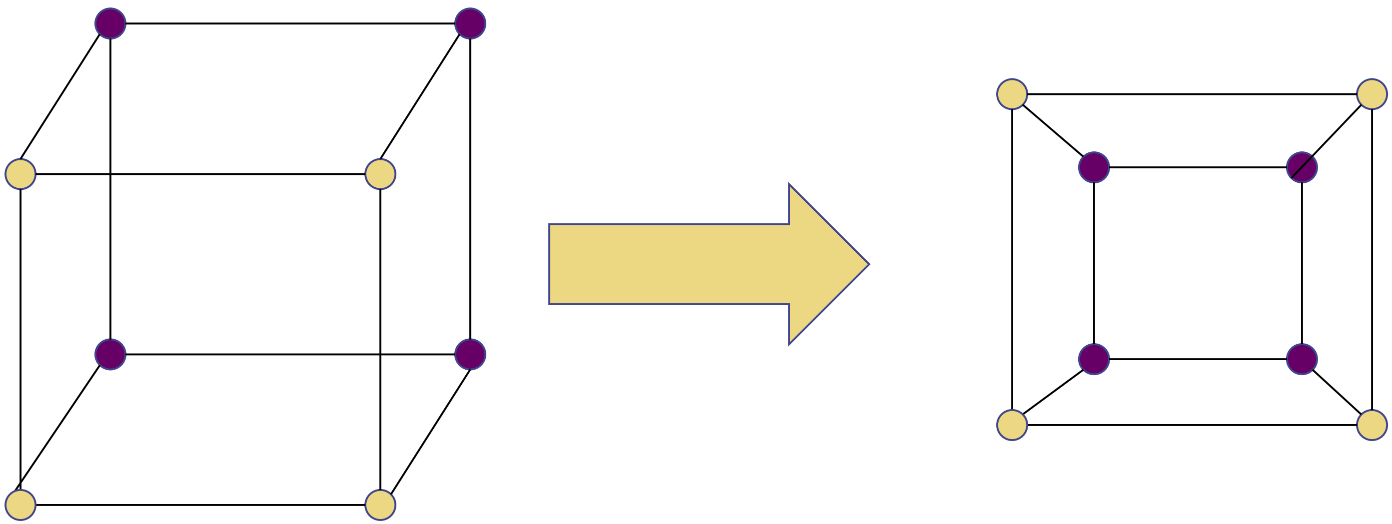
如果一个图里面有一个子图（圈）C_n, n>=6. 而且对于这个子图，存在至少3条及以上的类似于$K_{3,3}$交错的对角线，那么这个图一定是非平面图。
大家思考，为什么？

Proving Planarity 平面性

To prove that a graph is planar amounts to redrawing the edges in a way that no edges will cross. It may need to move vertices around and the edges may have to be drawn in a very indirect fashion. 要证明一个图是平面图，往往需要重画图的边；往往需要重新布局结点。

Proving Planarity 3-Cube

E.G. show that the 3-cube is planar:



Disproving Planarity

有一些方法用来判断一个图是或者不是平面图，可以根据具体情况选择任意一种。 总体上,平面性的判断是一个难题

Disproving Planarity

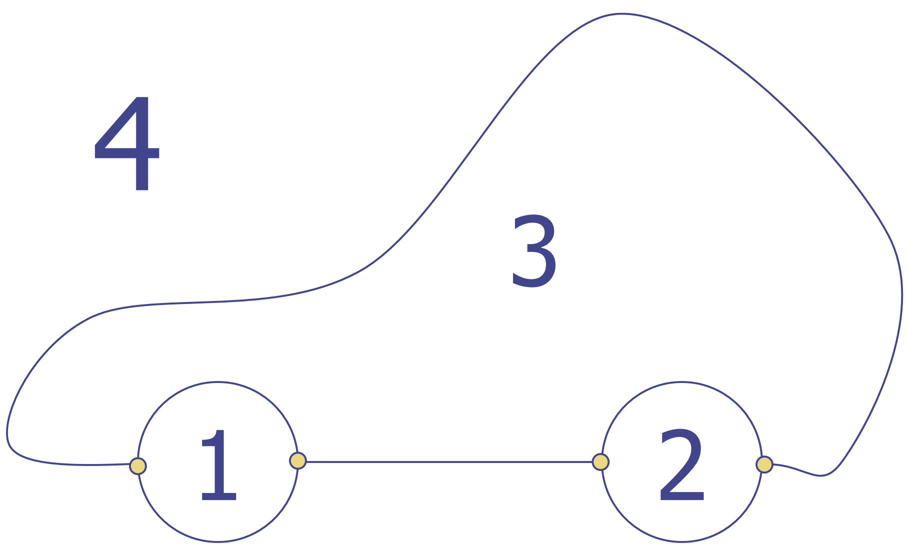
The idea tries to find some invariant quantities (不变量) possessed by graphs which are constrained to certain values, for planar graphs. Then to show that a graph is non-planar, compute the quantities and show that they do not satisfy the constraints on planar graphs.

一种想法是试图去寻找平面图的某些或某种内在的不变量（不变性），然后通过计算确定某些图不满足，从而否定一个图是平面图。

Regions 区域(面）

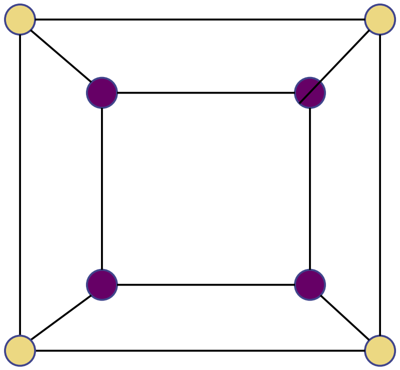
平面图的区域数：当一个图能不交叉地画于一个平面时，由它的所有边将平面分割成互不相交重叠不同的部分（块、区域），由一些边围成的封闭的或者是一个无限的，跟其它部分不重叠的区域（或者称为面），这些区域的个数，对于一个给定的图，只要是不交叉地画出来，无论画法如何，这个数是确定的不变的。

EG: the car graph has 4 regions:



Regions 区域

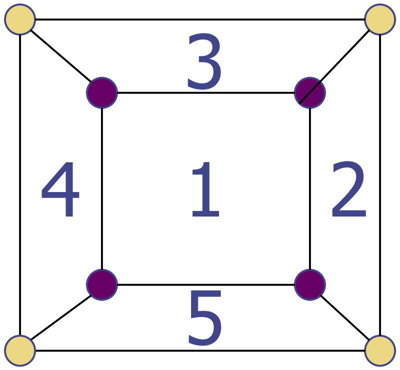
Q: How many regions does the 3-cube have?



Regions

A: 6 regions

6



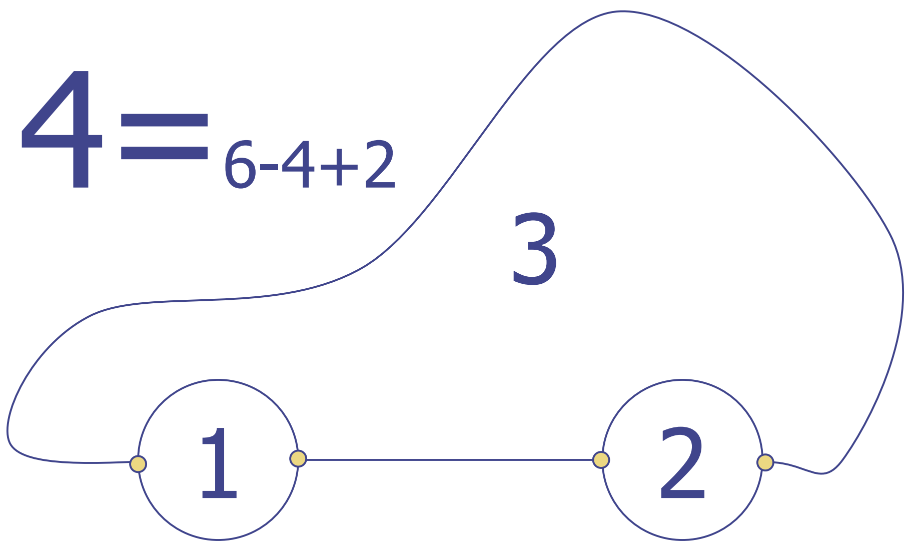
欧拉公式

Theorem: 一个**连通**的平面图所围成的区域数是一个与画法无关的不变量，这个区域数与结点数、边数的关系满足下面的公式（欧拉公式）

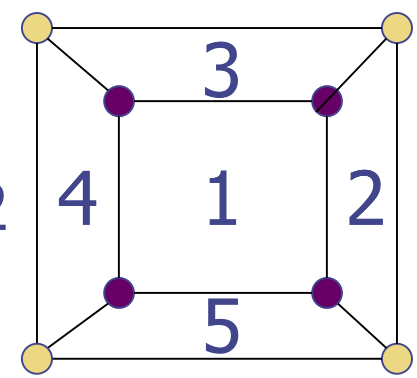
$$r = |E| - |V| + 2 \quad (\textbf{Euler Formula})$$

EG: Verify formula for car and 3-cube:

$4 =_{6-4+2}$



$6 =_{12-8+2}$



Euler Characteristic

The formula is proved by showing that the quantity $\chi = r - |E| + |V|$ must equal 2 for planar graphs. χ is called the ***Euler characteristic*** *欧拉特征值*.

注:这个特征值2就是一个所有连通平面图的不变特性(不变特征)

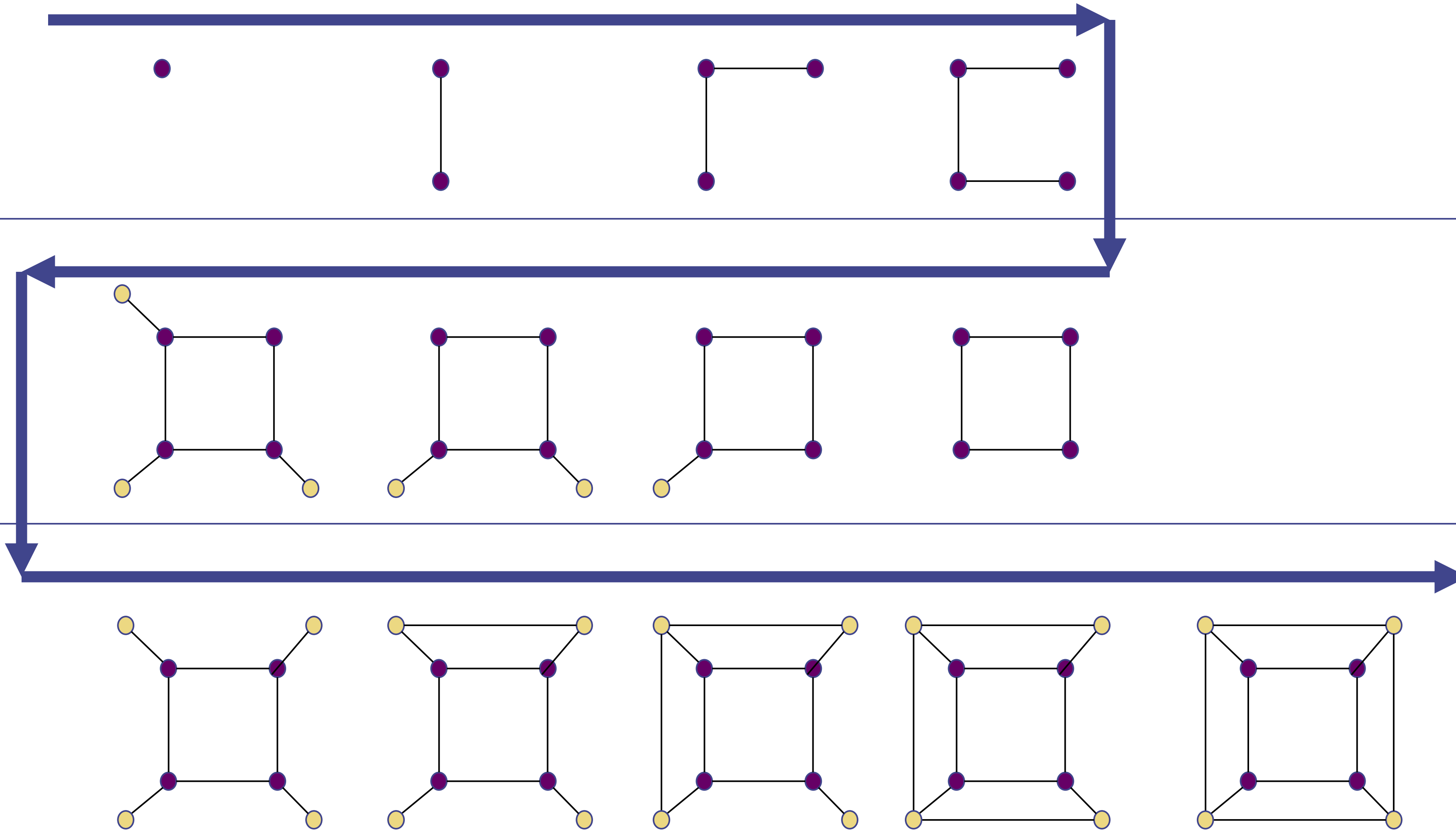
The idea is that any connected planar graph can be built up

From a vertex through a sequence of vertex and edge

additions. 一个连通的平面图可以从一个结点开始，再通过逐个加入点和不交叉地加入边的思路，画出来，然后总结分析其中边数、点数、面数的变化

For example, build 3- Cube as follows:

Euler Characteristic



Euler Characteristic

Thus to prove that χ is always 2 for planar graphs, one calculate χ for the trivial vertex graph:

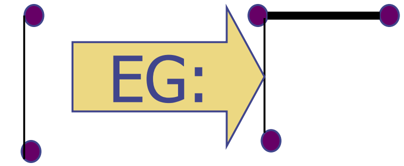
$$\chi = 1-0+1 = 2$$

and then checks that each possible move does not change χ .

Euler Characteristic 的证明思路

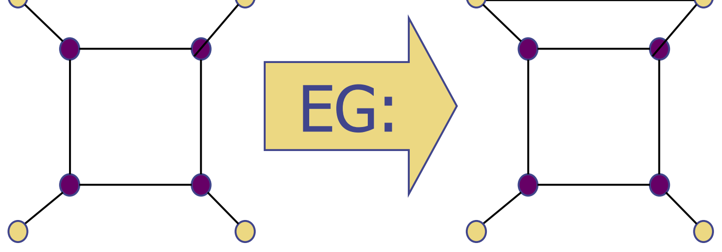
Check that moves don't change χ :

1) Adding **a degree 1 vertex**:



r is unchanged. $|E|$ increases by 1. $|V|$ increases by 1. $\chi = \chi + (0-1+1)$

2) Adding **an edge between pre-existing vertices**:



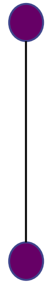
r increases by 1. $|E|$ increases by 1. $|V|$ unchanged. χ += (1-1+0)

（想象一下:如果这里增加单边弧，或者增加多重边会如何？ ）

Animated Invariance of Euler Characteristic

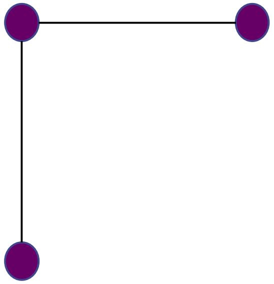
$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
1	0	1	2

Animated Invariance of Euler Characteristic



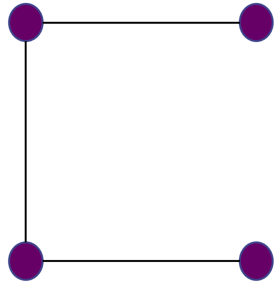
$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
2	1	1	2

Animated Invariance of Euler Characteristic



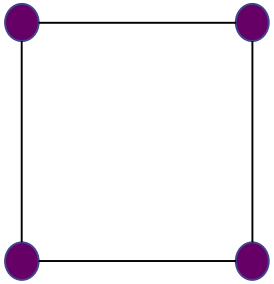
$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
3	2	1	2

Animated Invariance of Euler Characteristic



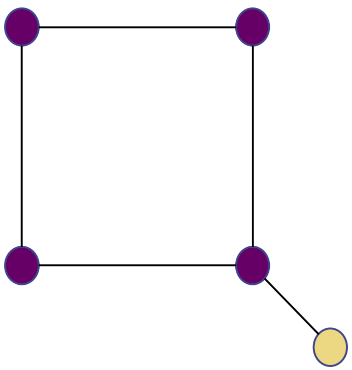
$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
4	3	1	2

Animated Invariance of Euler Characteristic



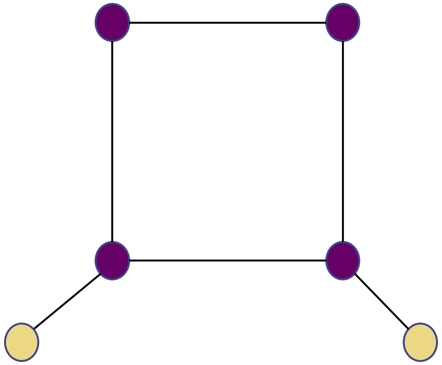
$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
4	4	2	2

Animated Invariance of Euler Characteristic



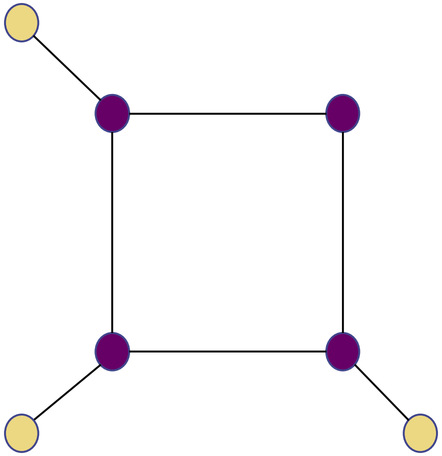
$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
5	5	2	2

Animated Invariance of Euler Characteristic



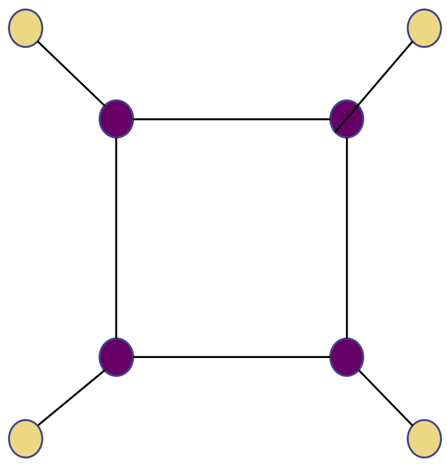
$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
6	6	2	2

Animated Invariance of Euler Characteristic



$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
7	7	2	2

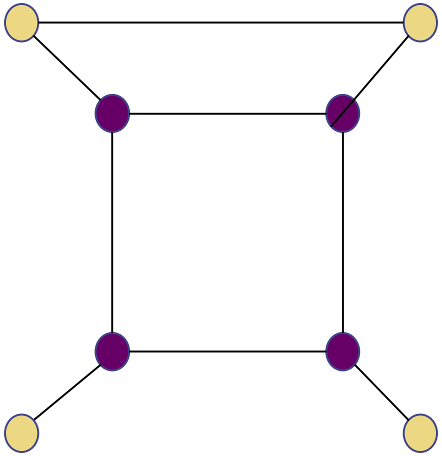
Animated Invariance of Euler Characteristic



$\|V\|$	$\|E\|$	r	$\chi = r - \|E\| + \|V\|$
8	8	2	2

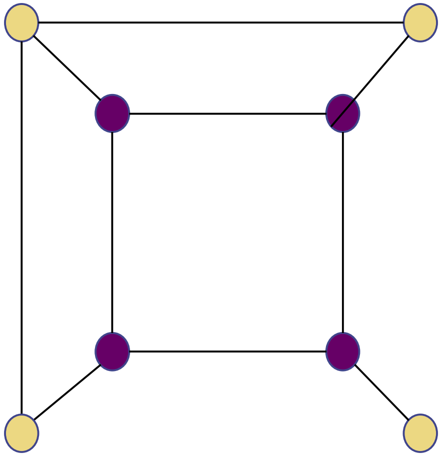
依次加入度为一的点，以及在现有点间加入边。 观察点数、边数、面数的变化

Animated Invariance of Euler Characteristic



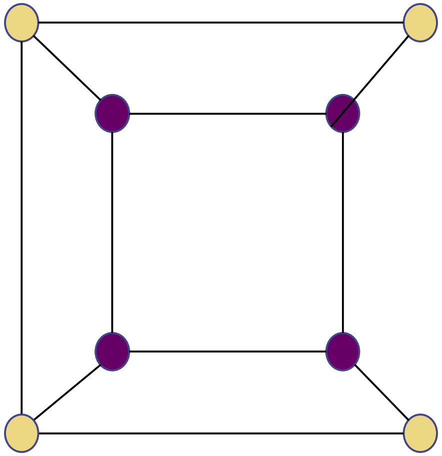
$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
8	9	3	2

Animated Invariance of Euler Characteristic



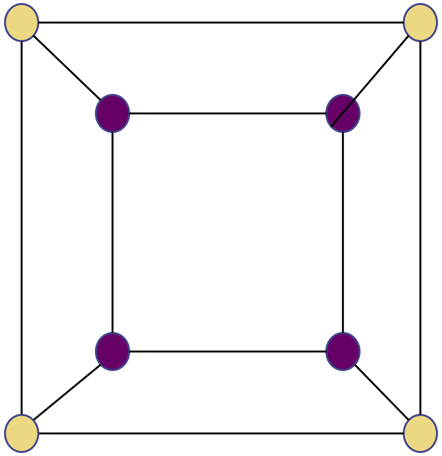
$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
8	10	4	2

Animated Invariance of Euler Characteristic



$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
8	11	5	2

Animated Invariance of Euler Characteristic



$\lvert V \rvert$	$\lvert E \rvert$	r	$\chi = r - \lvert E \rvert + \lvert V \rvert$
8	12	6	2

平面图的必要条件

- **推论 1**: 如果图G是一个连通的简单平面图，那么当结点数 $v\geq3$时，有不等式 $e\leq3v-6$.
- *证明思路:*
- Concept “the degree of a region”: the number of edges on the boundary of this region. N: total degree of all regions.
- (0): 特殊情况，没有围成有限区域时，只有一个无限面。此时 $e = v-1$， $e\leq3v-6$ 是成立的。
- (1):For simple graph (no loop, no multi-edge), the degree of each region is at least 3. $N \geq 3r$ (r is the number of regions)
- (2) because each edge occurs on the boundary of a region exactly twice. $N \leq 2e$
- (3) Using Euler formula. $r=e-v+2$

K_5 is non-planar 非平面图

- n=5
- e= n * (n – 1) / 2 = 10
- Using necessary conditions of planar graphs:
- e <= 3n – 6
- 10 <= 3(5) – 6
- 10 <= 9 ???
- By contradiction, K_5 must be non-planar

平面图的必要条件

- **推论 2**: 一个**简单连通的**平面图，至少有一个结点的度不大于5 **($deg(v_i) \le 5$)**.
- ***证明思路:*** v denotes the number of vertices.
  - When v=1,2 or ≥3
- If *G* has one or two vertices only, the result is true. If *G* has at least three vertices, by Corollary 1
- we know that $e \le 3v - 6$, so $2e \le 6v - 12$. If the degree of every vertex were at least six, then…

- because $2e =$ 总度数 (by the handshaking theorem), we would have $2e \ge 6v$. But this contradicts the inequality $2e \le 6v - 12$. It follows that there must be a vertex with degree no greater than 5.

平面图的必要条件

- 推论3：若连通的简单平面图有e条边，v个结点，v>2, 并且没有长度为3的回路，则e<=2v-4

- 请同学们自己分析证明

$K_{3,3}$ is Non-Planar 非平面图

- Proof by contradiction of theorems 反证法思路

- Since graph is bipartite, no edge connects two edges within same subset of vertices.
- The total degrees of all regions N >= 4r must be true, since graph contains no simple triangle regions of 3 edges, where r is the number of distinct regions. 每个区域至少由4条边围成
- N <= 2e must be true, since no edge can be used more than twice in forming a region

没有哪条边会对N贡献2次以上，顶多贡献2.

(con't) Proof of $K_{3,3}$

- For K(3,3) v=6, e= 9, r= ??
- 4r <= N <= 2e
- 4r <= (2e = 2 * 9 = 18)
- r <= 4.5

- Using 欧拉定理, v – e + r = 2
- 6 – 9 + r = 2
- r = 5

- Proof by contradiction:
- r cannot be both equal to 5 and less than 4.5
- Therefore, K(3,3) is a non-planar graph

$K_{3,3}$ 是非平面图

- 更简单的证明：
- 前面学习的必要条件中的最后一条：连通的简单平面图，如果没有长度为3的回路，在v>2的情况下，一定有e<2v-4.

在此：e=9, v=6.

$K_{3,3}$ 是二分图，没有奇数长的回路，所以满足上面必要条件的前提。

Complete Graphs

- Denoted by K_n
- All vertices are connected to all vertices
- $e = n * (n - 1) / 2$

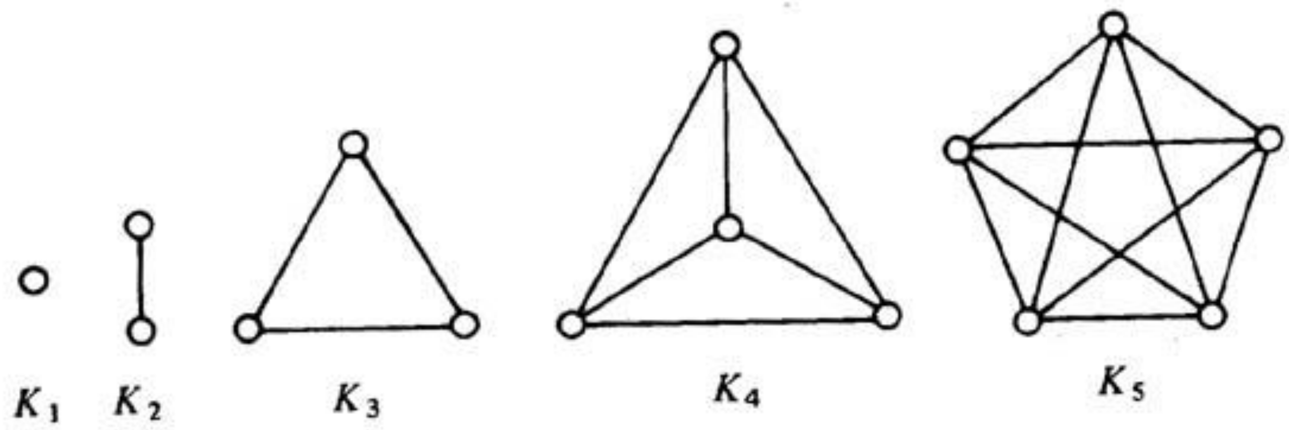


Figure 2.2

Question about K_n

- For the complete graph K_n with $n>5$, is it planar? Why?

- For $n>5$, the K_5 is a subgraph of K_n, so…

How about $K_{n,n}$?

思考问题

- 欧拉定理有一个前提条件---图是连通图。那么如果一个图不是连通图，如何？

假定有t个连通分支, 引导学生自己得出一个结论或者公式。

如果性质1中的连通条件去掉，能有什么类似结论吗？

- 如果一个图的有某个子图是非平面图，那么有什么结论？
- 如果一个图有一个子图为平面图，又如何？
- 如果一个图以边交叉的方式画出来，如何计数“区域数”？非平面图的区域数的问题怎么考虑？
- 注：对于非平面图，本身不能以不交叉的方式画出来，所有不能以不满足欧拉公式为理由，判断不是平面图

Subdivisions of graph G

- **Elementary Subdivision** – a graph obtained from a graph G, by inserting vertices of degree two into any edge
- (H is a valid subdivision of G, while F is not)
- 插入或者删除度为2的结点

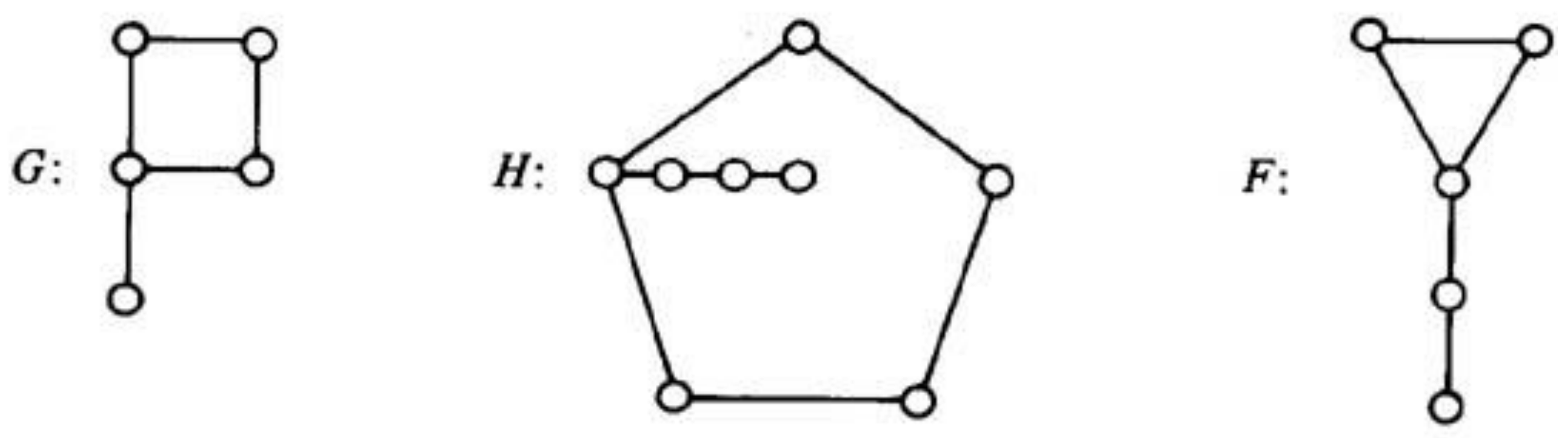


Figure 9.5

Kuratowski Reduction Theorem

- Homeomorphic: G1=(V_1,E_1), G_2=(V_2,E_2) are

called homeomorphic**同胚** if they can be obtained from the same graph by a sequence of elementary subdivisions.

- 中文翻译:增减度为2的结点变换意义下同构 （教材:**同胚**）：

- 通过一系列的删除或者添加度为2的结点使得图发生变化，变到另一个图, 但平面性不变。

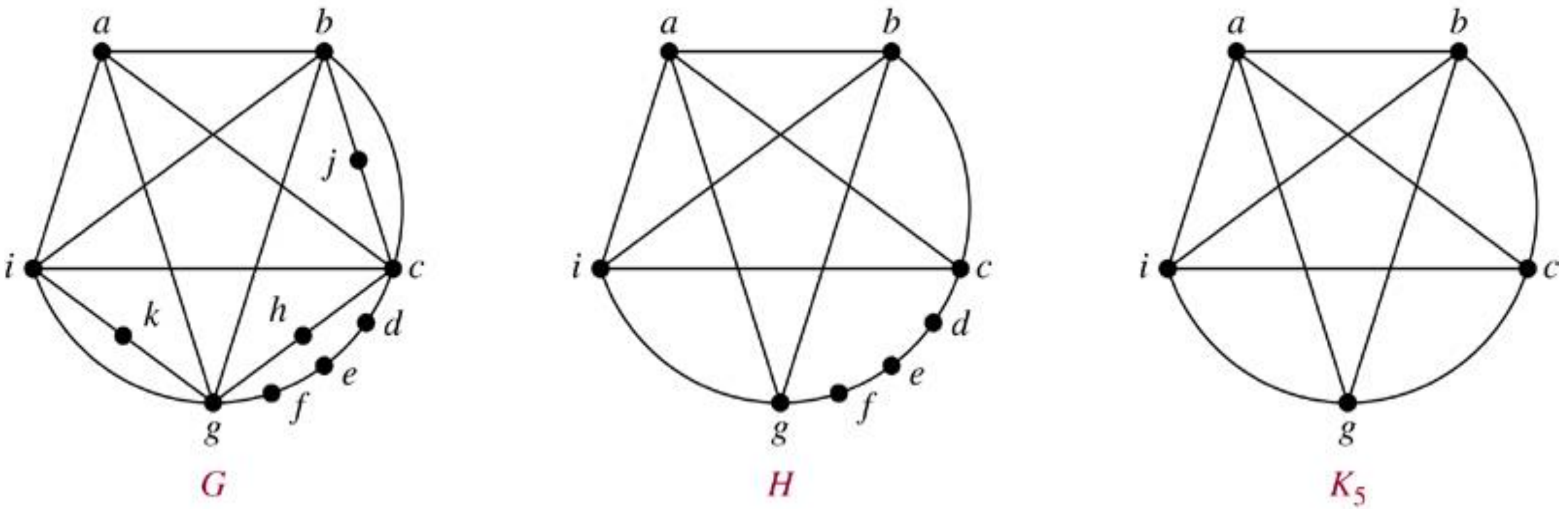
总结利用"**同胚**"概念进行平面性判断的思维方法....

Kuratowski Reduction Theorem

- **Kuratowski定理:** 一个图为非平面图的充分必要条件是它包含有与$K_{3,3}$ or K_5同胚（在增减度为2的结点变换意义下同构的子图）。
- *The proof of it is very complicated, will not shown here.*
- *Kuratowski's theorem– in principle always works, though in practice can be quite unwieldy.)*
- *思考:一个图为平面图的充分必要条件又是什么？*

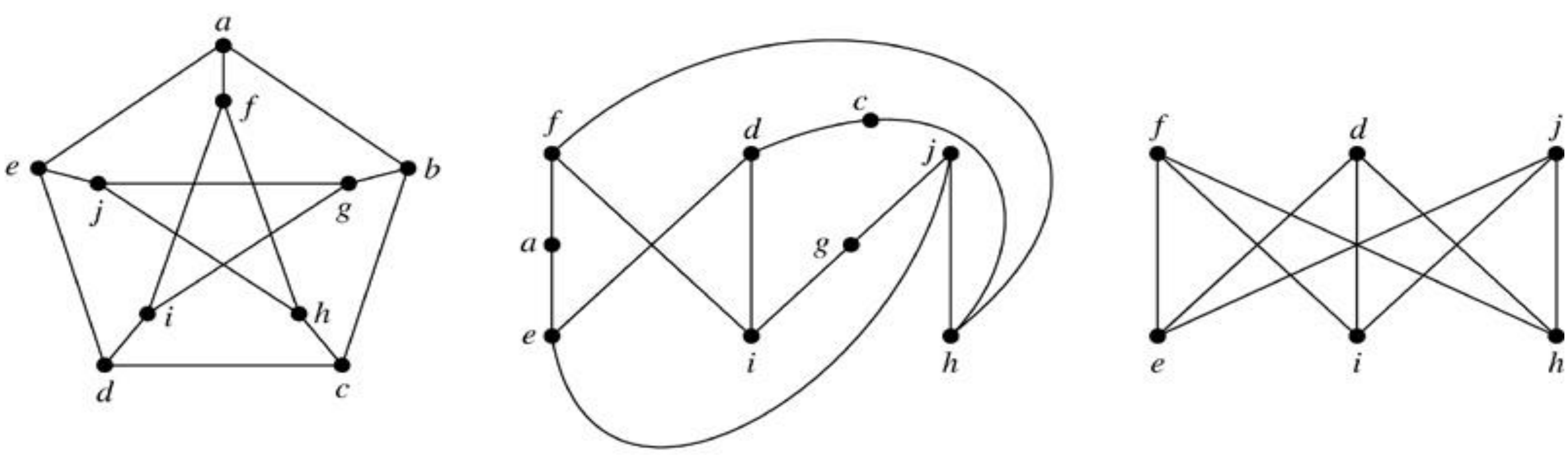
Examples using **Kuratowski THM**

- Determine whether the graph G is planar



Examples using **Kuratowski THM**

- Determine whether the Petersen graph (a) is planar
- Solution: to obtain H by removing vertex b and the three edges have b as a endpoint



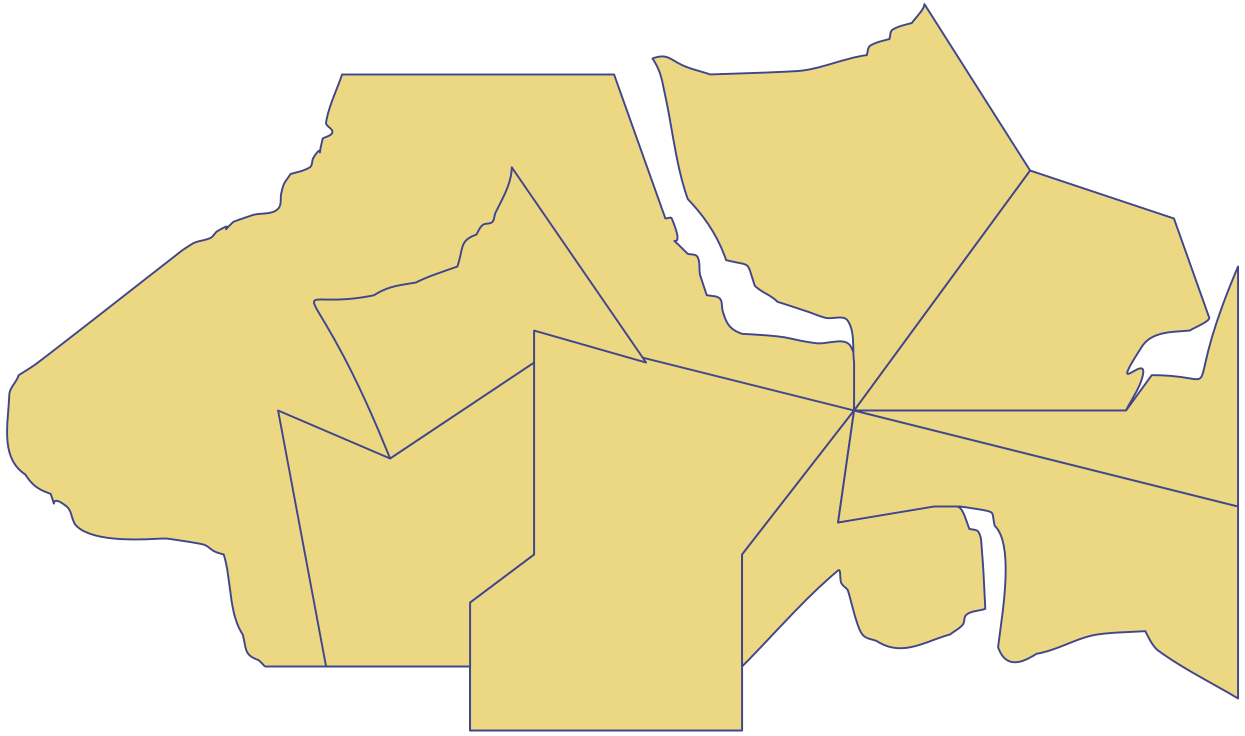
(a) (b) H (c) $K_{3,3}$

Planar Graph Exercises

- 6.7节 T5, T7， T13， T17

图着色

- Consider a fictional continent分析下面这个虚构的陆地图：要把不同地区区分（分割开来）可以用分割线.

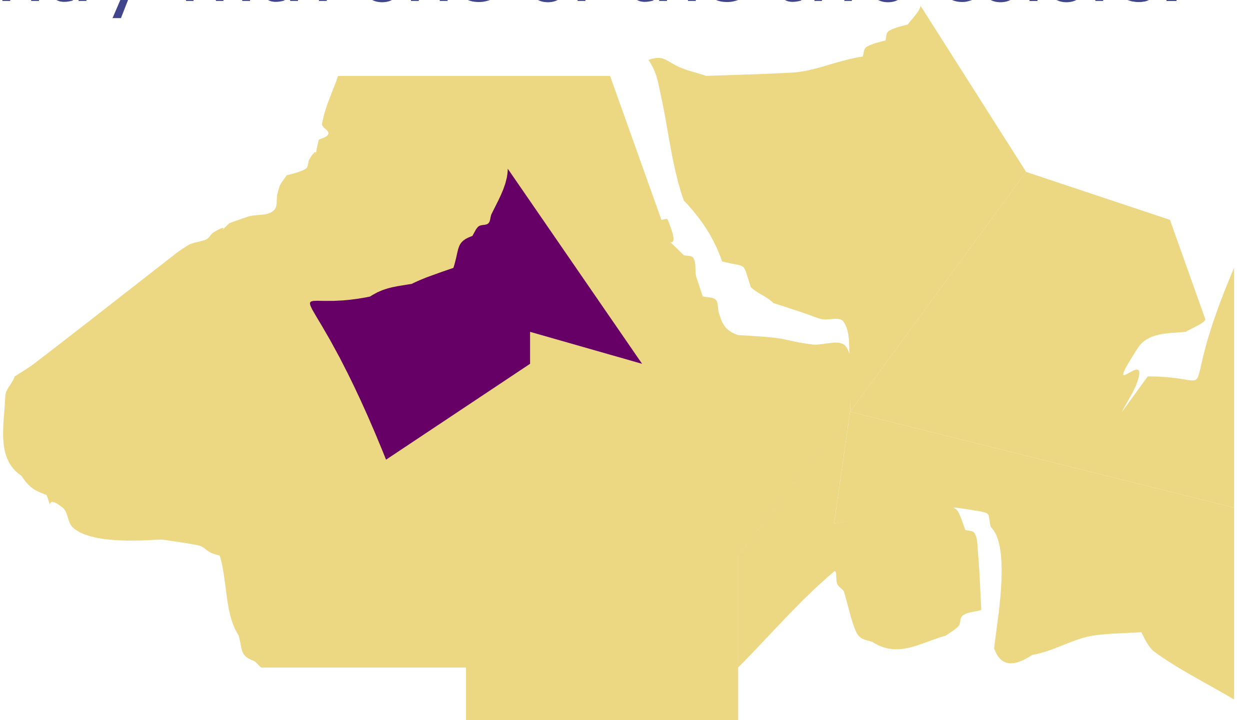


Map Coloring

- Suppose removed all borders but still wanted to see all the countries. 如果用颜色来区分的话， 1 color insufficient.

Map Coloring

So add another color. Try to fill in every country with one of the two colors.

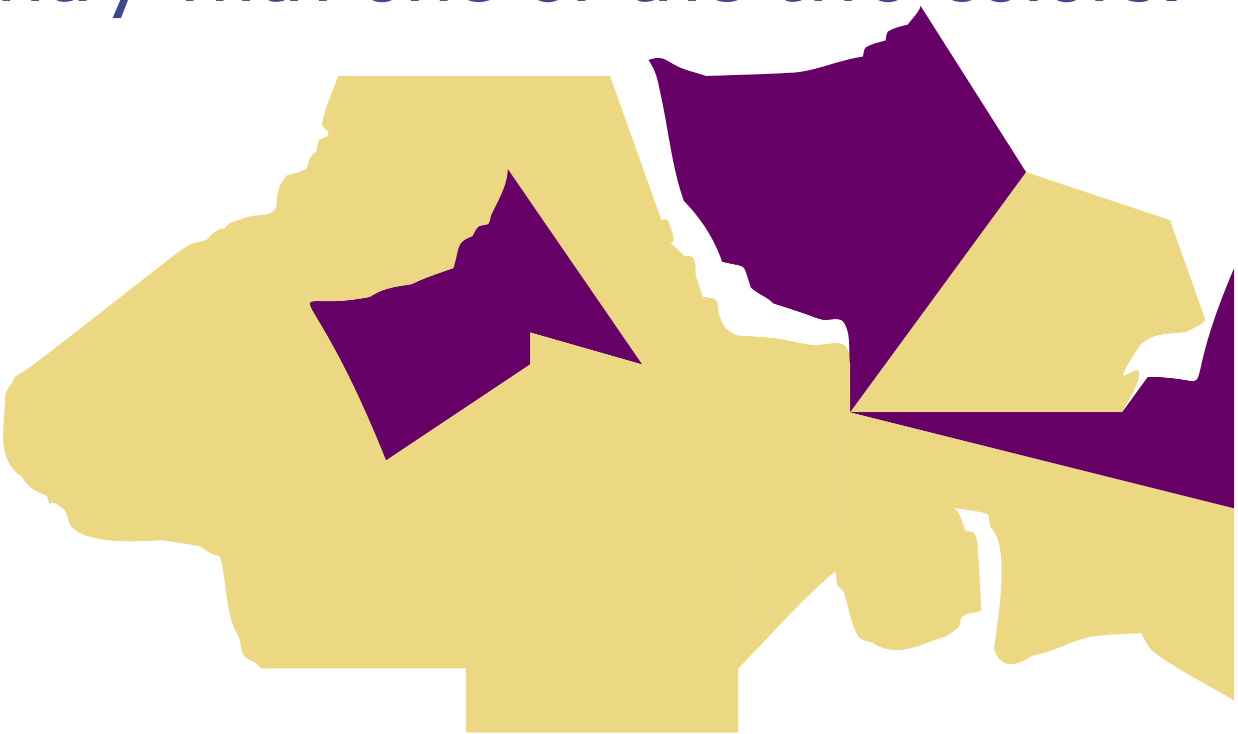


Map Coloring

So add another color. Try to fill in every country with one of the two colors.

Map Coloring

So add another color. Try to fill in every country with one of the two colors.



Map Coloring

So add another color. Try to fill in every country with one of the two colors.

Map Coloring

PROBLEM: Two adjacent countries forced to have same color. Border unseen.

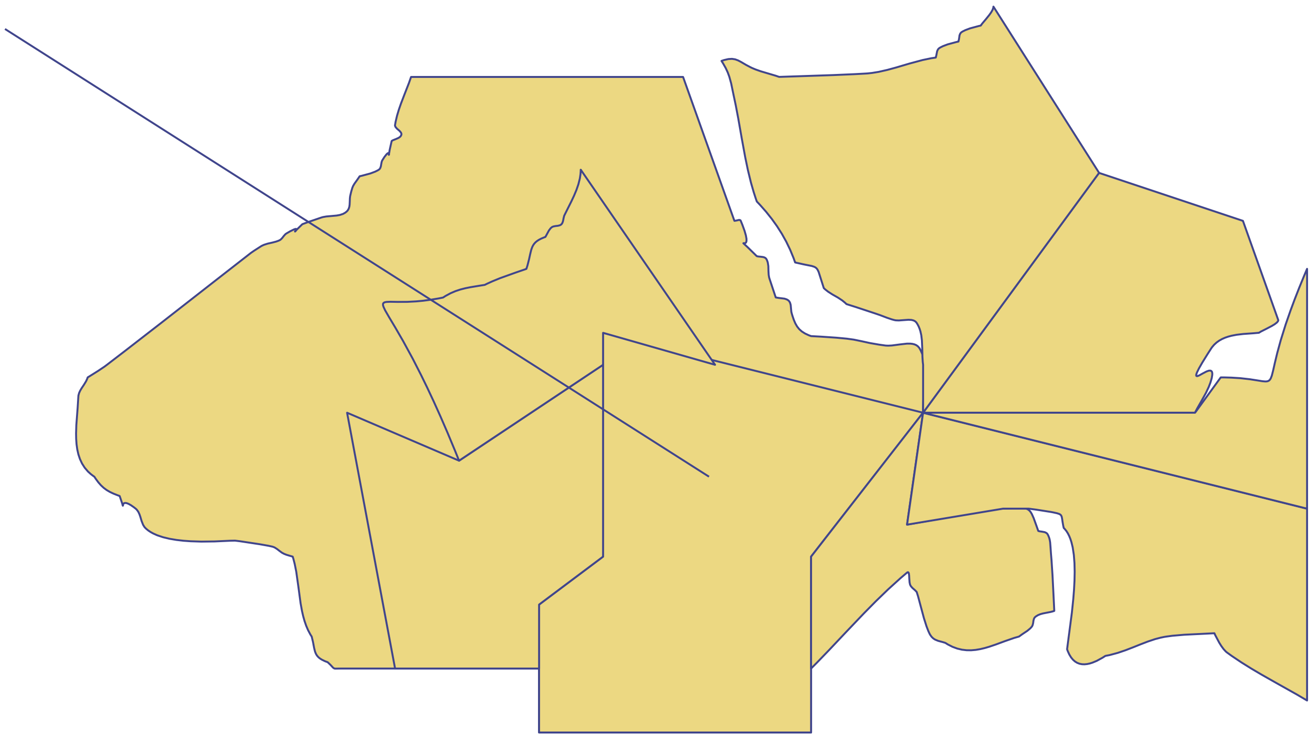
Map Coloring

So add another color:

Map Coloring

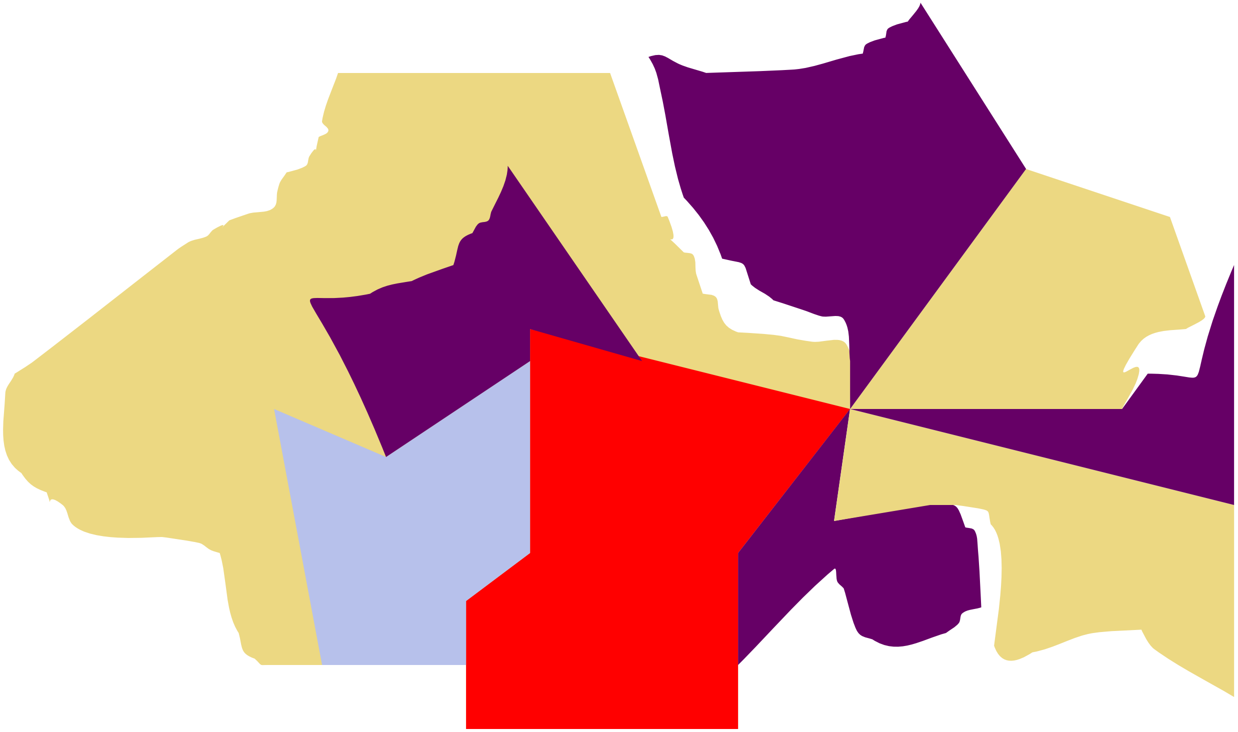
Insufficient. Need 4 colors because of this country.

4种不同颜色才能区分开来



Map Coloring

With 4 colors, could do it.



4-Color Theorem—四色定理

Theorem：任何平面图的区域都可以用4种颜色足够将所有区域分割开来，使得有共享边界的区域之间颜色不一样。

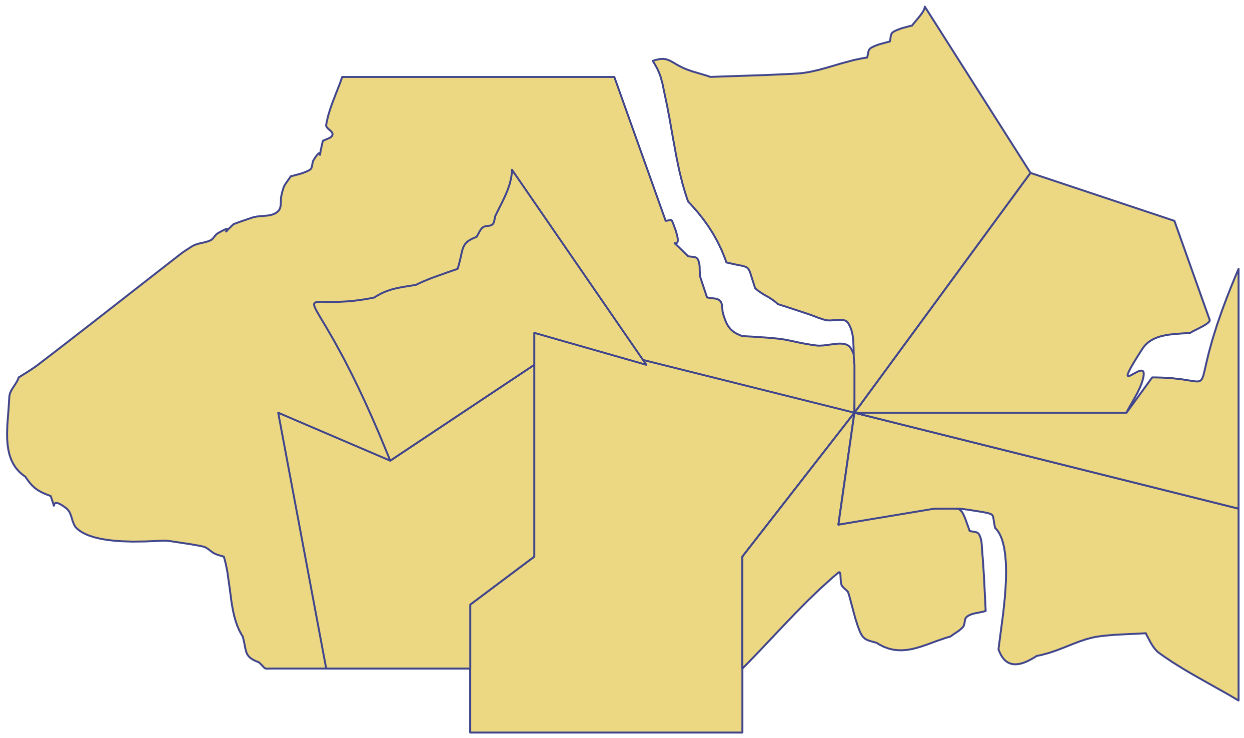
（也就是说最多是4色图）

Proof by Haaken and Appel used exhaustive computer search. (四色猜想)

It took more than 100 years to get the correct prove.

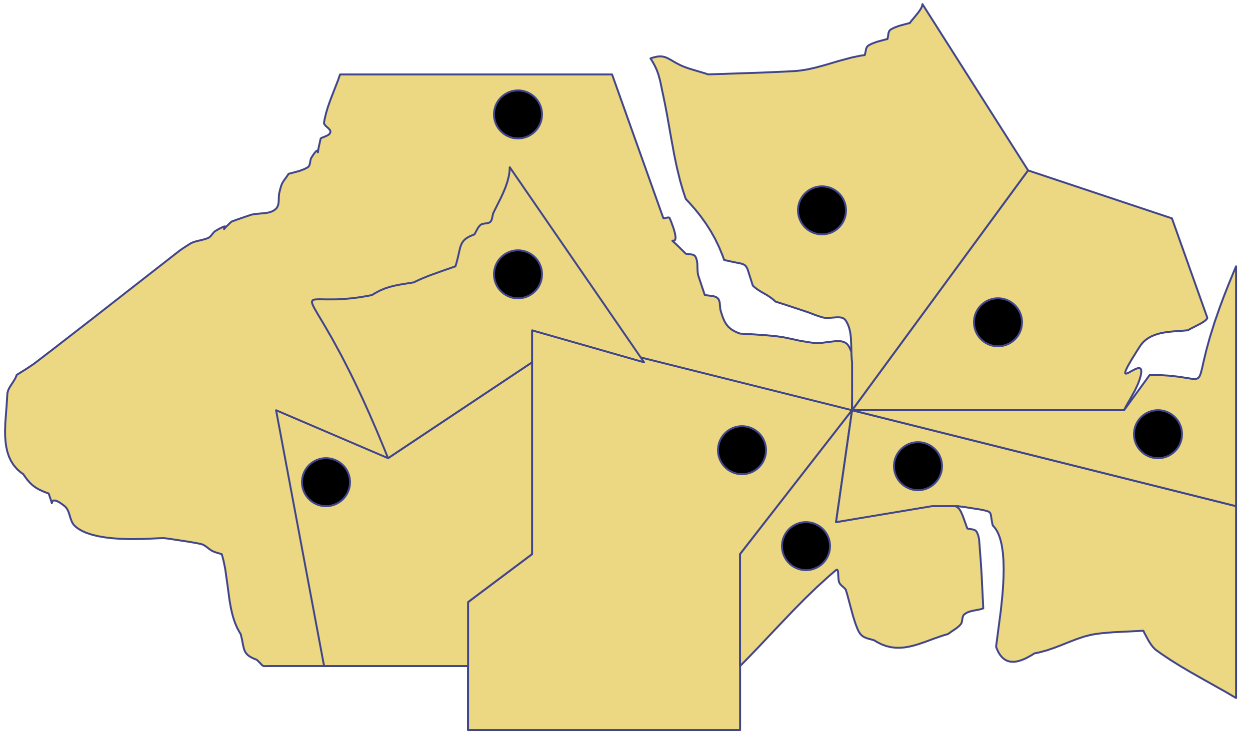
从地图着色到图着色的问题建模

对地图着色的问题建模转化成图着色的问题



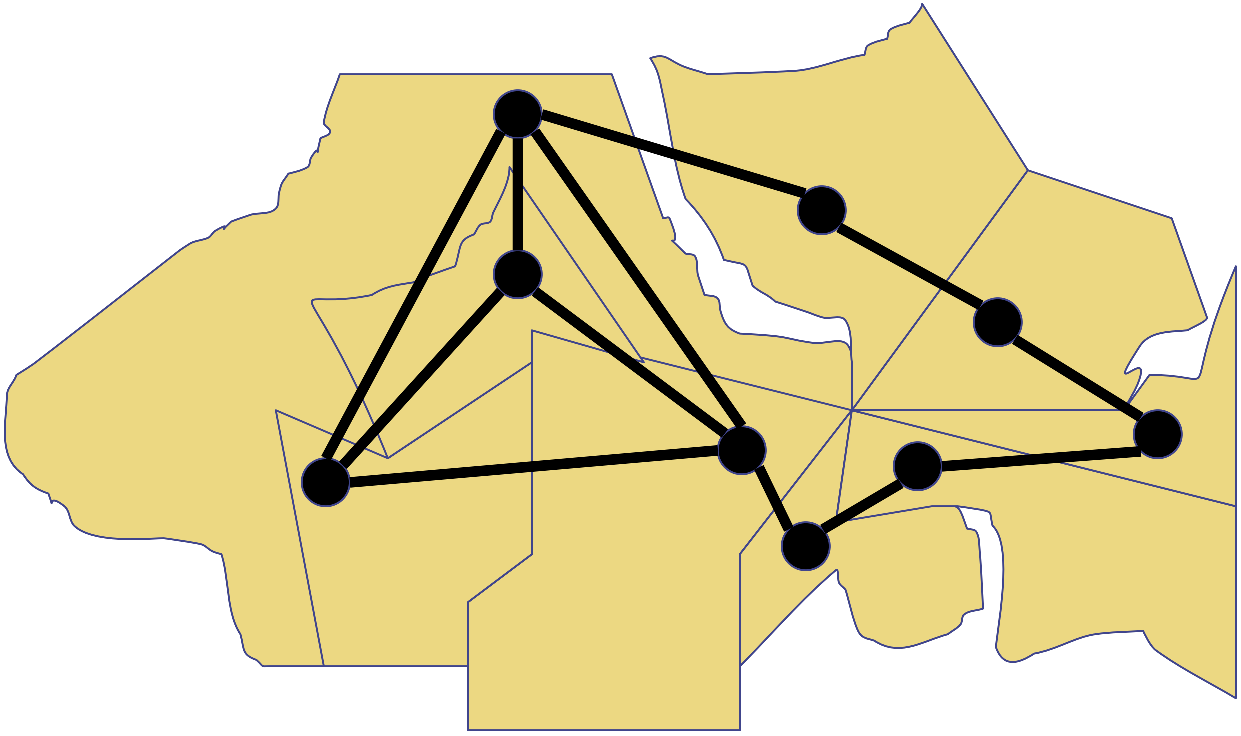
From Map Coloring to Graph Coloring

For each region introduce a vertex:



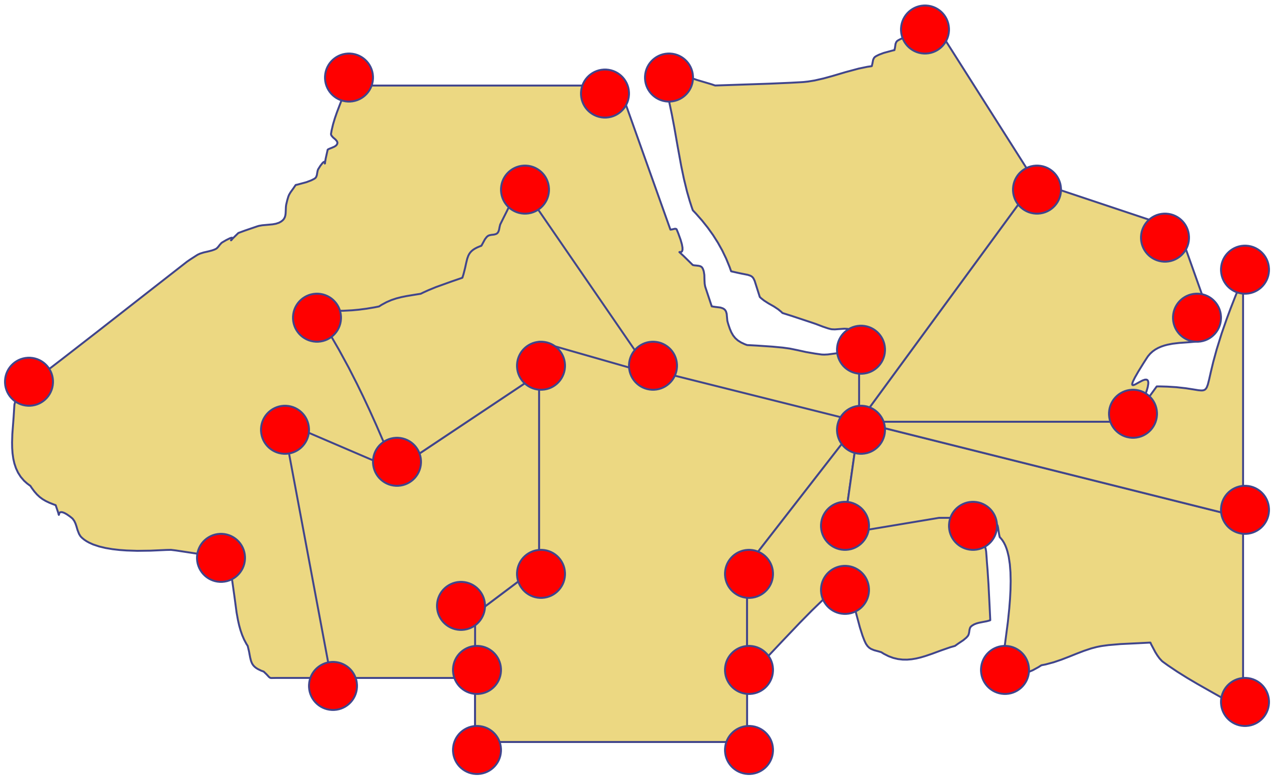
From Map Coloring to Graph Coloring

For each pair of regions with a positive-length common border introduce an edge:



From Maps to Graphs to Dual Graphs (对偶图)

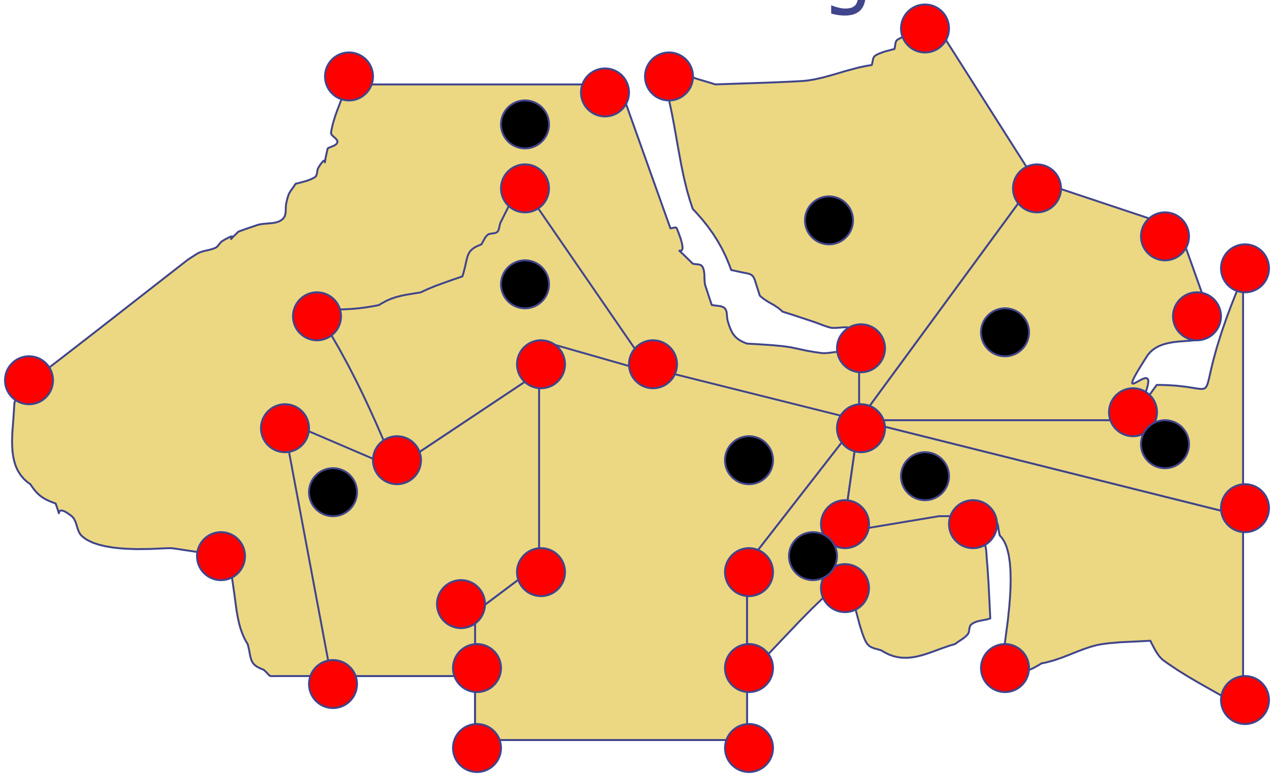
- think of original map as a graph, and we are looking at *dual graph*:



From Maps to Graphs to Dual Graphs (对偶图)

Dual Graphs :

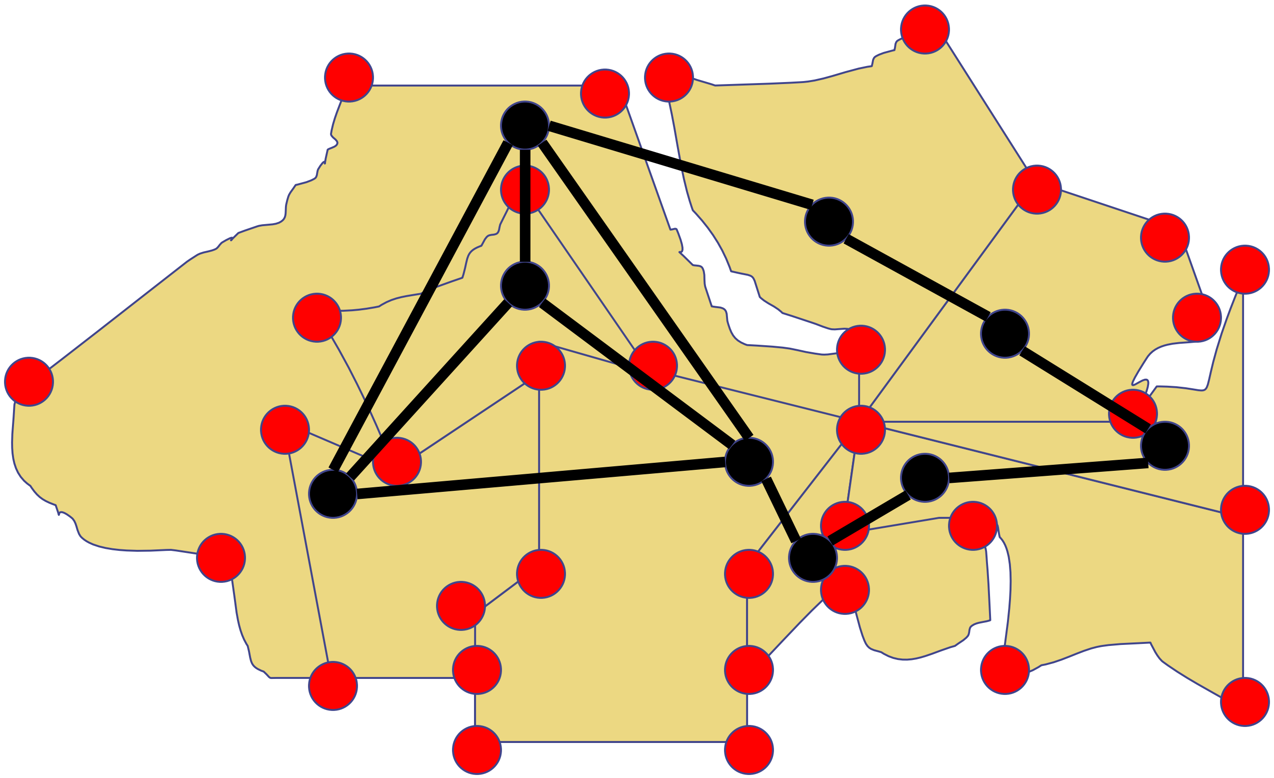
1) Put vertex inside each region:



From Maps to Graphs to Dual Graphs (对偶图)

Dual Graphs :

2) Connect vertices across common edges:



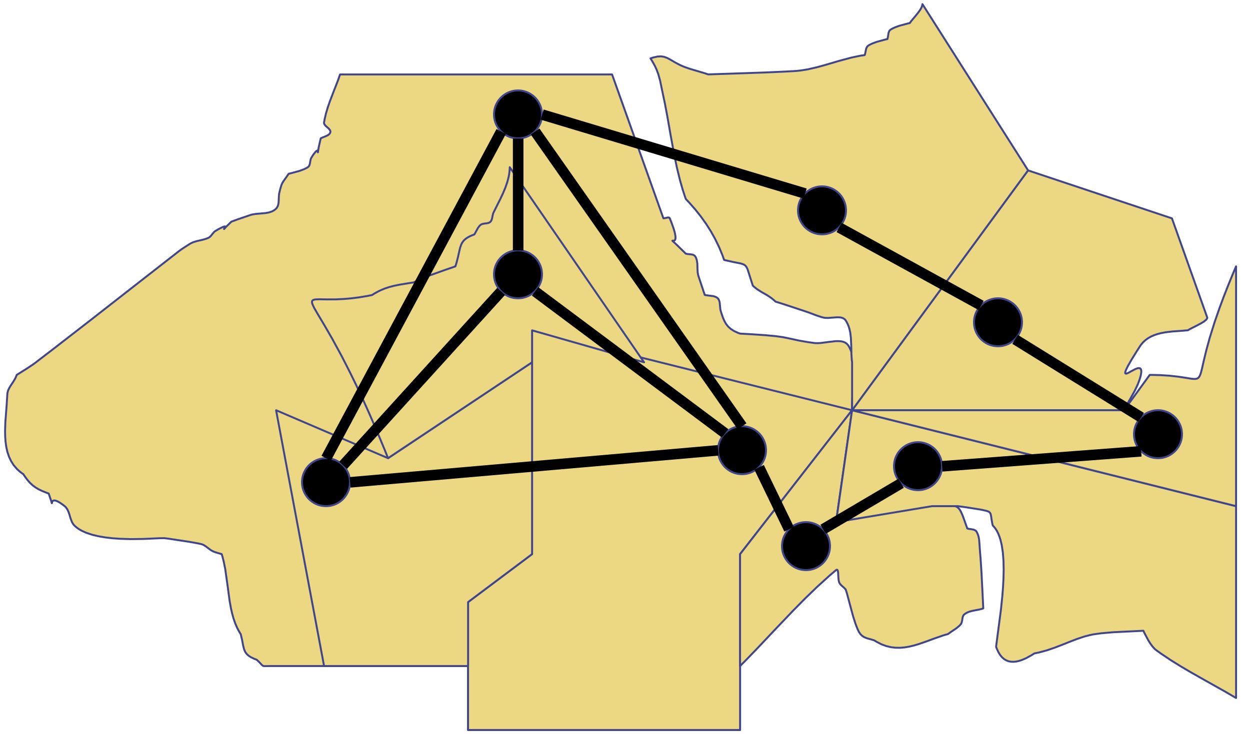
Def. of Dual Graph对偶图定义

DEF:一个平面图 $G = (V, E, R)$ [Vertices, Edges, Regions]的对偶图G^定义为如下的图:

- Vertices of G^: $V(G\hat{\ }) = R$
- Edges of G^: $E(G\hat{\ })$ = set of edges of the form $\{F_1,F_2\}$ where 区域F_1 and *区域*F_2 share a common edge.

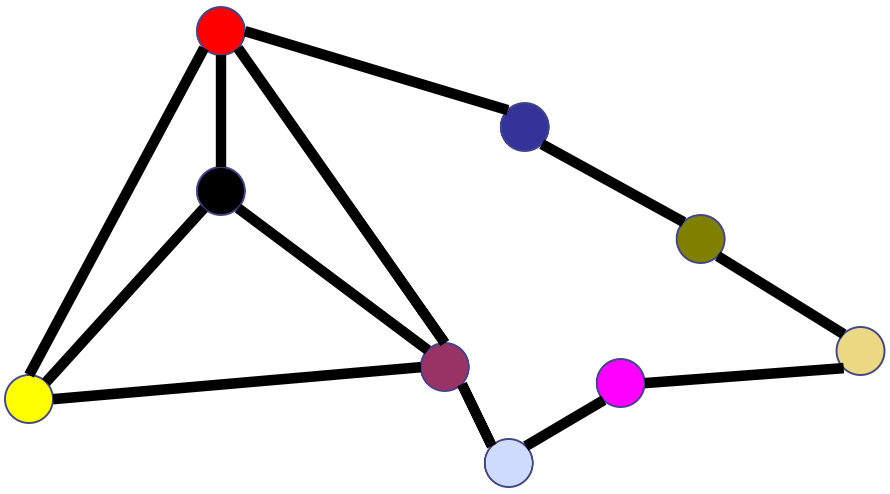
From Maps to Graphs to Dual Graphs

So take dual graph:



地图着色到图着色

Coloring regions is equivalent to coloring vertices of dual graph.



Definition of Colorable

DEF: Let *n* be a positive number. 一个简单图称为n-色图或者说可n-色图，如果能用不超过n种颜色标记所有结点，使得任意邻接的结点都有不同的颜色。 *(n种颜色不一定要用完）*

The ***chromatic number*** 颜色数 is smallest number *n* for which it is *n* -colorable.

EG: A graph is bipartite if and only if it is 2-colorable.

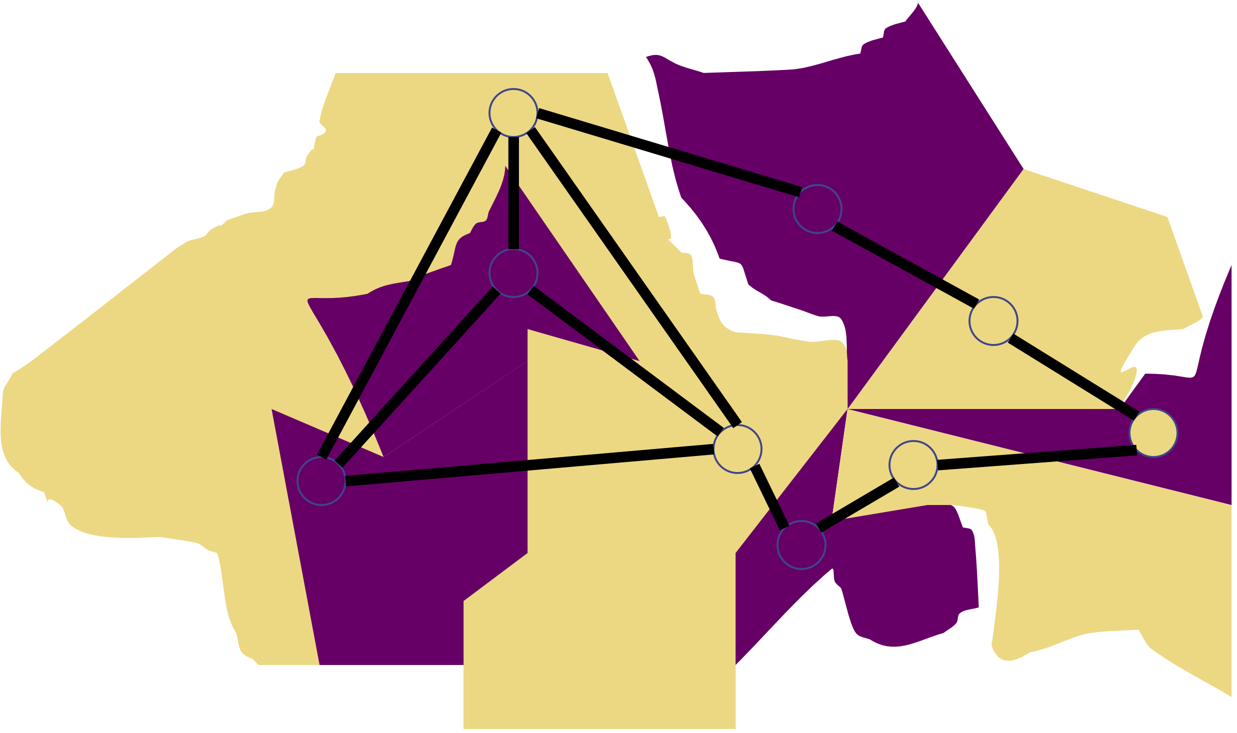
一个图为偶图当且仅当图是2-色图。

Think about why?

注：图的颜色数的寻找是一个非常难的NP-COMPLETE问题

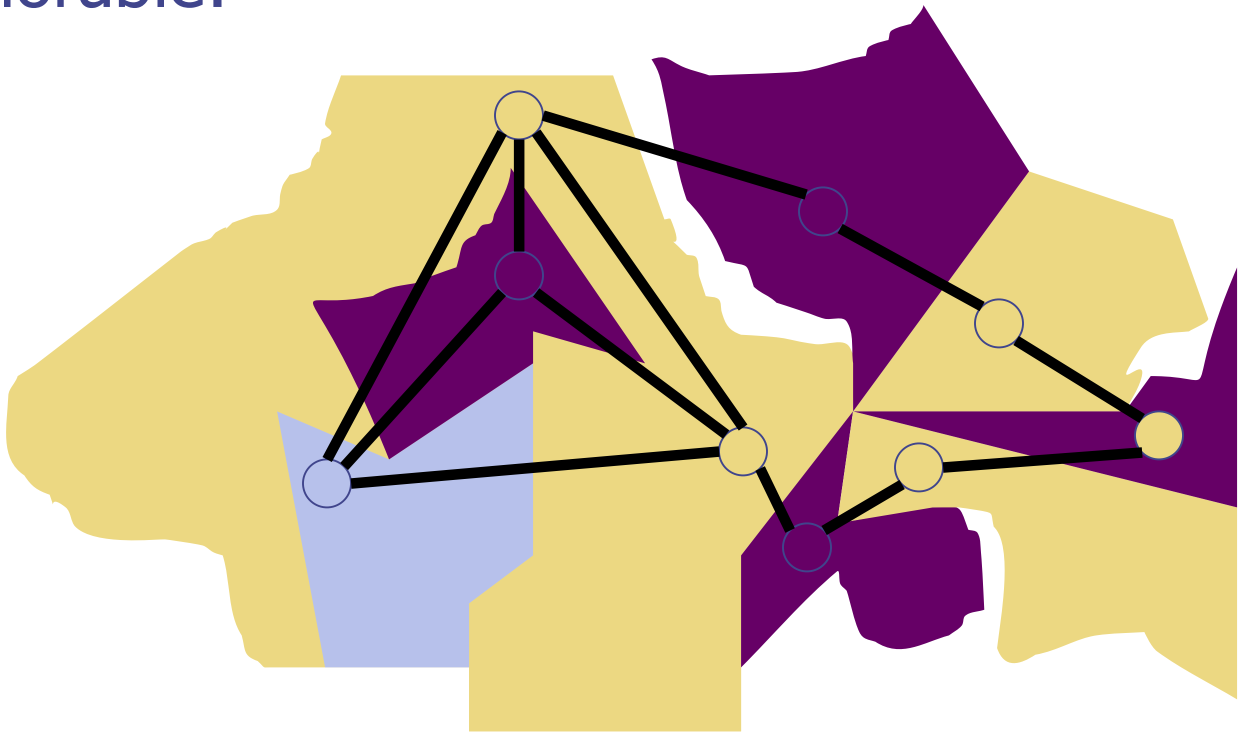
From Map Coloring to Graph Coloring

This map is not 2-colorable, so dual graph not 2-colorable:



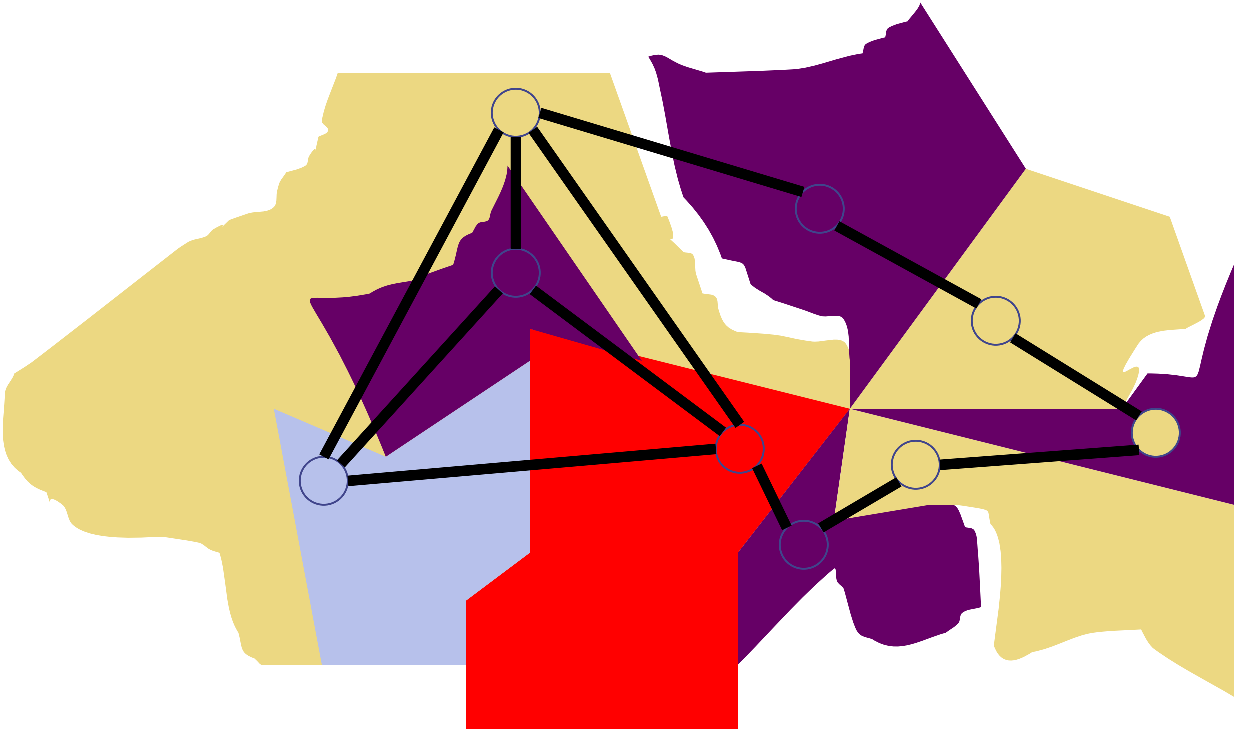
From Map Coloring to Graph Coloring

The following map is not 3-colorable, so graph not 3-colorable:



From Map Coloring to Graph Coloring

Graph is 4-colorable, so map is as well:



4-Color Theorem

四色定理：　任何平面图的颜色数不大于 4。

Note: it had been more than 100 years before a correct proof was given.

注：目前有的四色定理的证明是依赖计算机的，脱离计算机的人工证明还只有**五色定理。**

五色定理：也就是任何平面图的颜色数≤5

思考：　K_n的颜色数是多少？

颜色数是n．因为任何两个点都是邻接的，所以颜色数不可能小于n．否则就会有两个点颜色相同，然而是邻接的。

反过来，如果一个n个结点的简单图的颜色数是n，那么必然是完全图。

颜色数

- 性质：图G的颜色数不小于任何一个子图的颜色数。
- 求颜色数是一个难题，至今没有一个已知的好算法。
- 定理：假设G是一个简单图，其所有结点的最大度数是D_{max}. 那么G的颜色数≤ $D_{max}+1$

- Brooks定理：对一个图进行顶点着色，使颜色数χ(G)=$1+D_{max}$的图只有两类：或者是奇回路，或者是完全图。

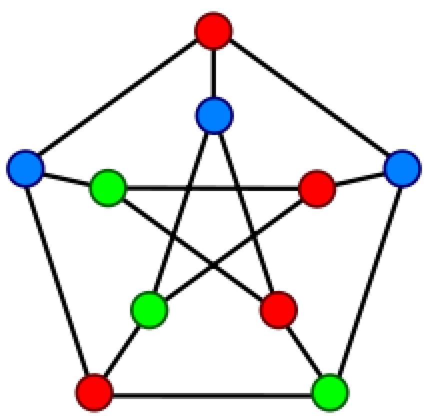
计算一个图的色数

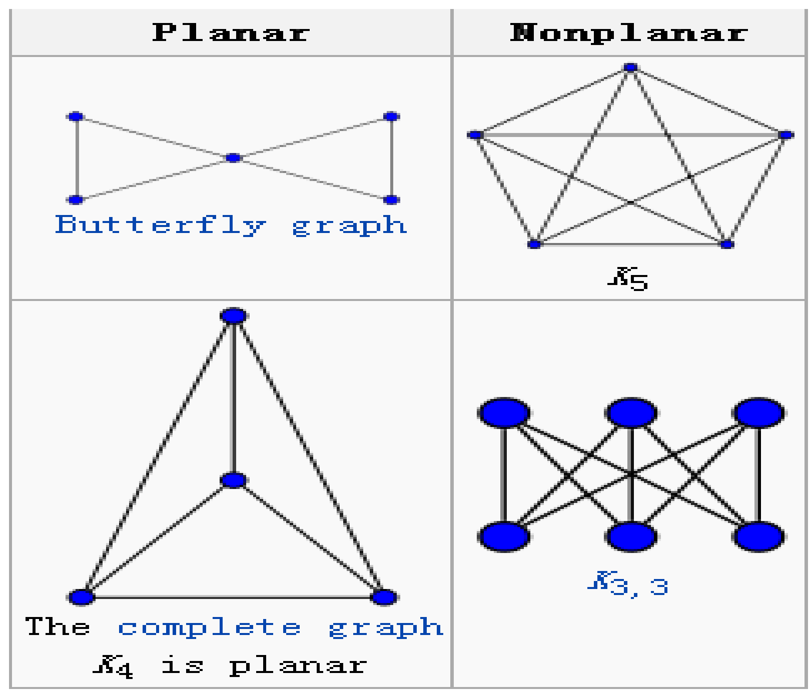
计算颜色数 χ(G)是一个NP-Complete问题。

但是对于一些特定的图，有一些现成的结论可以使用。

例如对一个平面图进行顶点着色，χ(G)≤4。

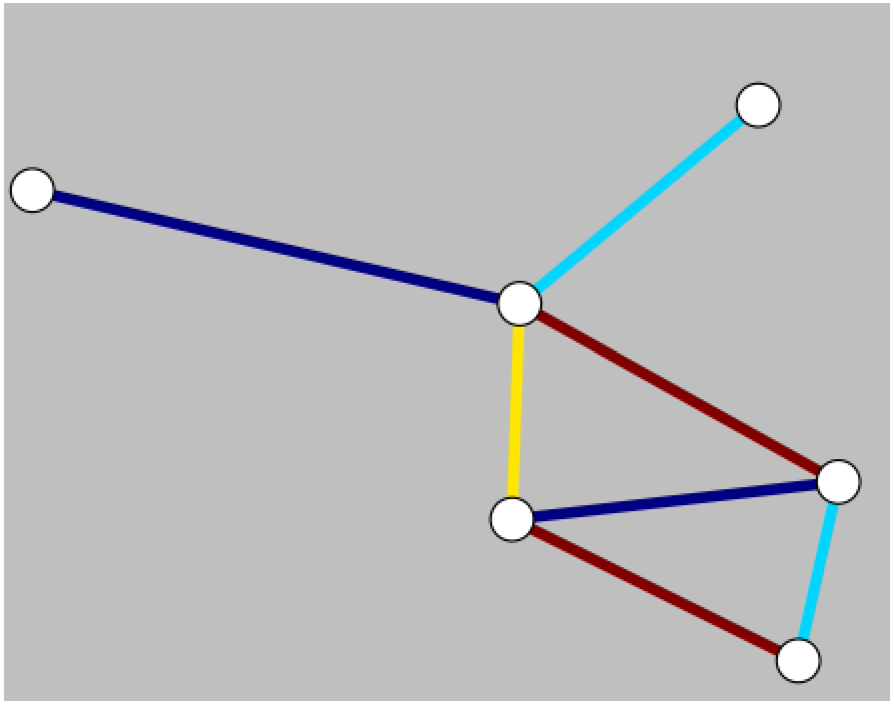
考察下列图的颜色数：





边着色(EdgeColoring)

- 任意相邻的边的颜色不能相同，也就是有公共结点的边颜色不能相同。边着色要求图不能有单边环，但是可以是多重图



- Vizing定理：设G是一个简单图，**对它进行边着色**，它的顶点最大度数是D_{max}，则颜色数 $\chi(G)=D_{max}$或者 $\chi(G)=D_{max}+1$。

星着色(Star Coloring)

- Star Coloring是一种特殊的Vertex Coloring，但是它的要求更严格。
- A star coloring of a graph G is a (proper) vertex coloring in which every path on four vertices uses at least three distinct colors. Equivalently, in a star coloring, the induced sub graphs formed by the vertices of any two colors has connected components that are star graphs. The star chromatic number of G is the least number of colors needed to star color G.
- 上述定义不太好理解：任意4个结点的路径，必须至少有3种不同的颜色。还可以定义成：任意3个结点的路径，3个结点颜色都不相同。
- 通俗的说，就是
- (1)一个结点与它的相邻结点有不同颜色
- (2)一个结点的相邻结点之间也不能同色

星着色(Star Coloring)应用举例

- **应用：IP溯源**
- 对网络进行星着色，如果我们知道一个IP包经过的路由器的颜色序列以及知道包走过的最后一个路由器，那么我们就能把整个路径找出来。

图着色应用—规划问题

EG: Suppose we want to schedule some final exams for CS courses with following course numbers(课程代号):

1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156

Suppose also that there are **no common students** in the following pairs of courses because of prerequisites:

1007-3137

1007-3157, 3137-3157

1007-3203

1007-3261, 3137-3261, 3203-3261

1007-4115, 3137-4115, 3203-4115, 3261-4115

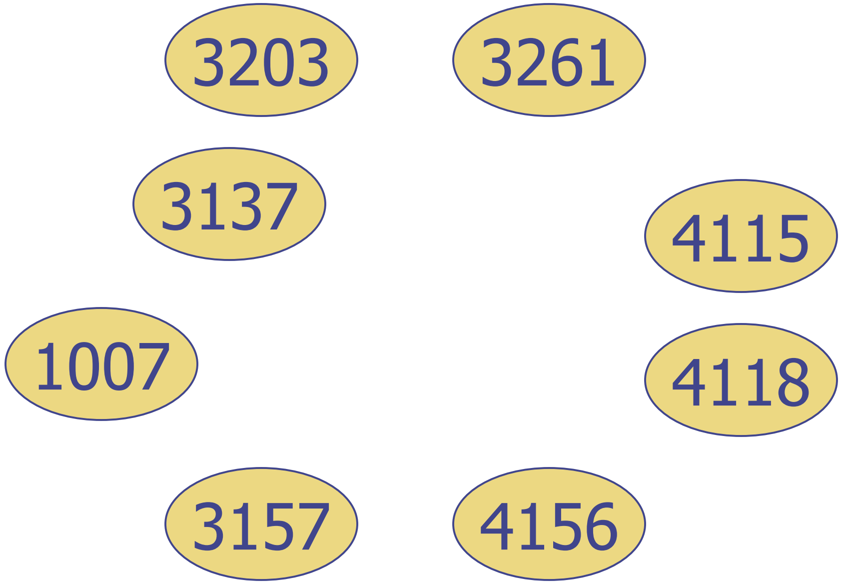
1007-4118, 3137-4118

1007-4156, 3137-4156, 3157-4156

How many exam slots are necessary to schedule exams?

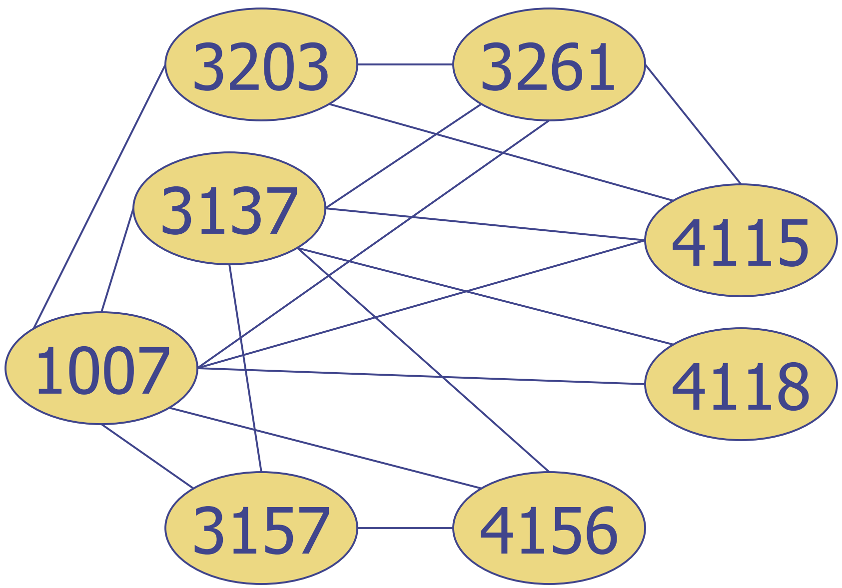
Graph Coloring and Schedules

Turn this into a graph coloring problem. Vertices are courses, and edges are courses which *cannot* be scheduled simultaneously because of possible students in common:



Graph Coloring and Schedules

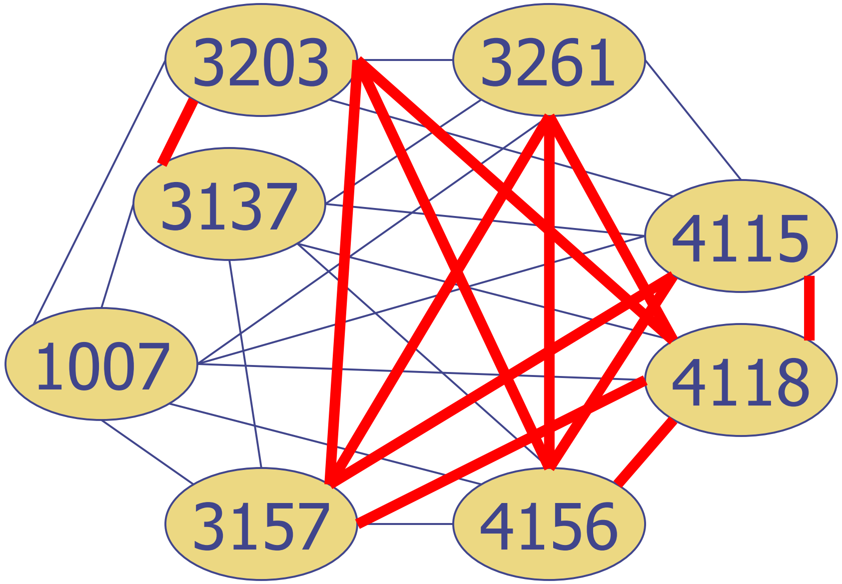
One way to do this is to put edges down where students mutually excluded (有冲突的）...



这个图中的边代表没有共同的学生，是可以安排在同一个时间段考试的

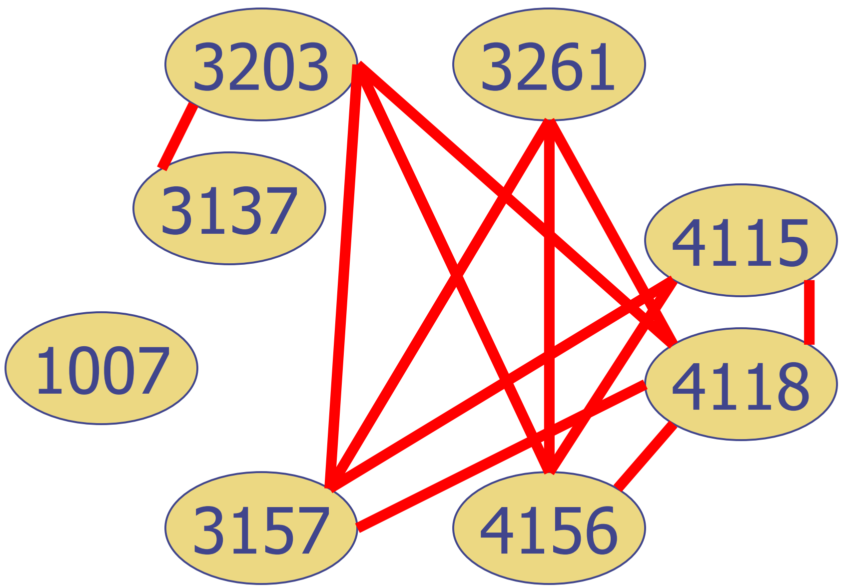
Graph Coloring and Schedules

…and then compute the complementary graph（补图）



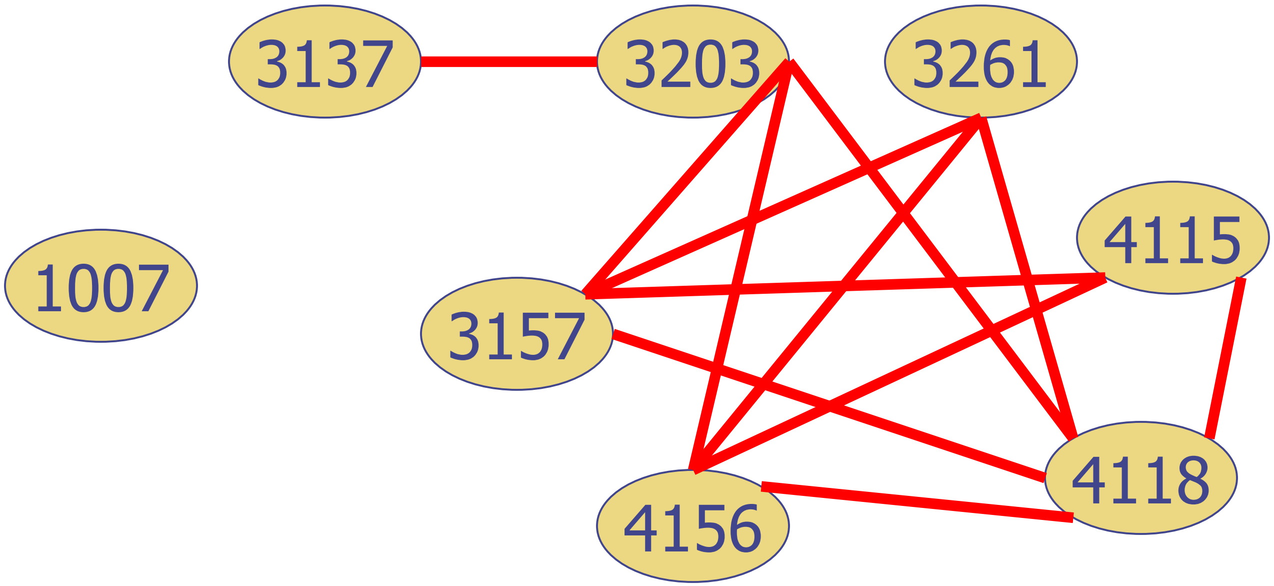
Graph Coloring and Schedules

…and then compute the complementary graph（补图）:



Graph Coloring and Schedules

Redraw:



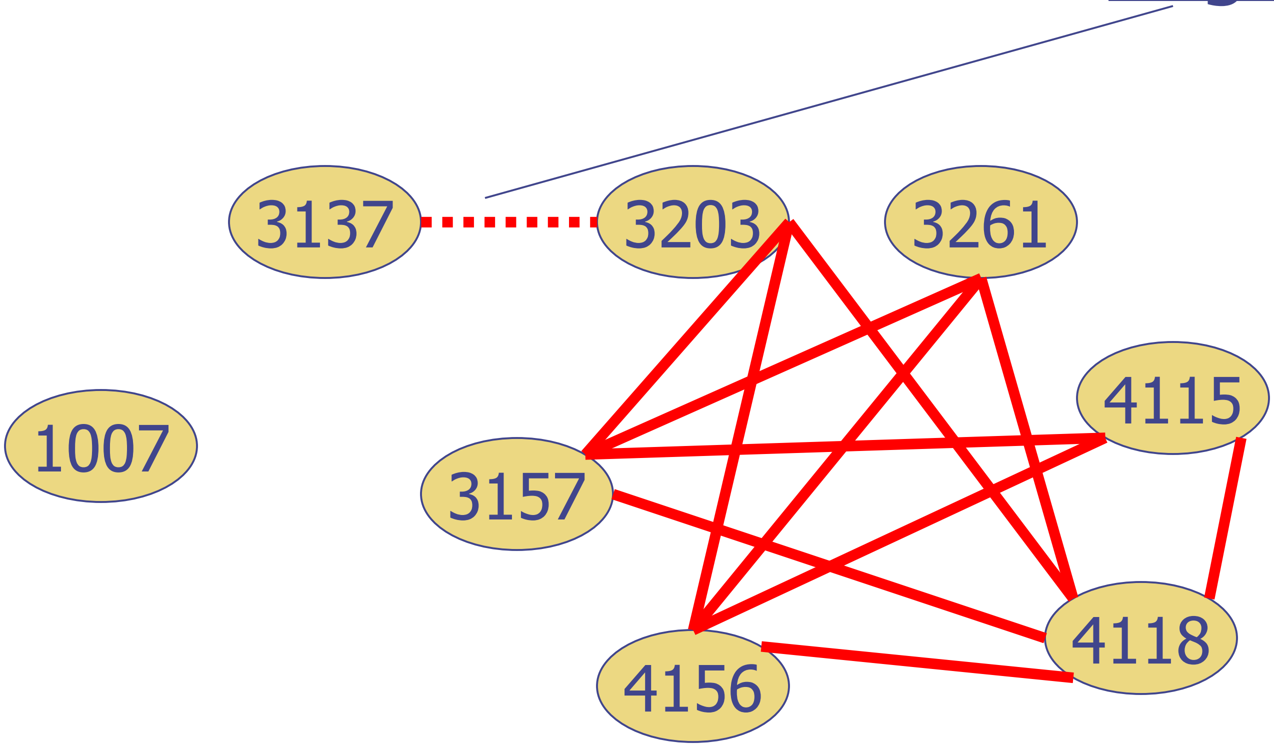
这个补图中的邻接的点对应的课程是不能同时间考试的，是有冲突的。

思考

- 能否用图着色的方法，寻找颜色数的方法来解决这个问题？

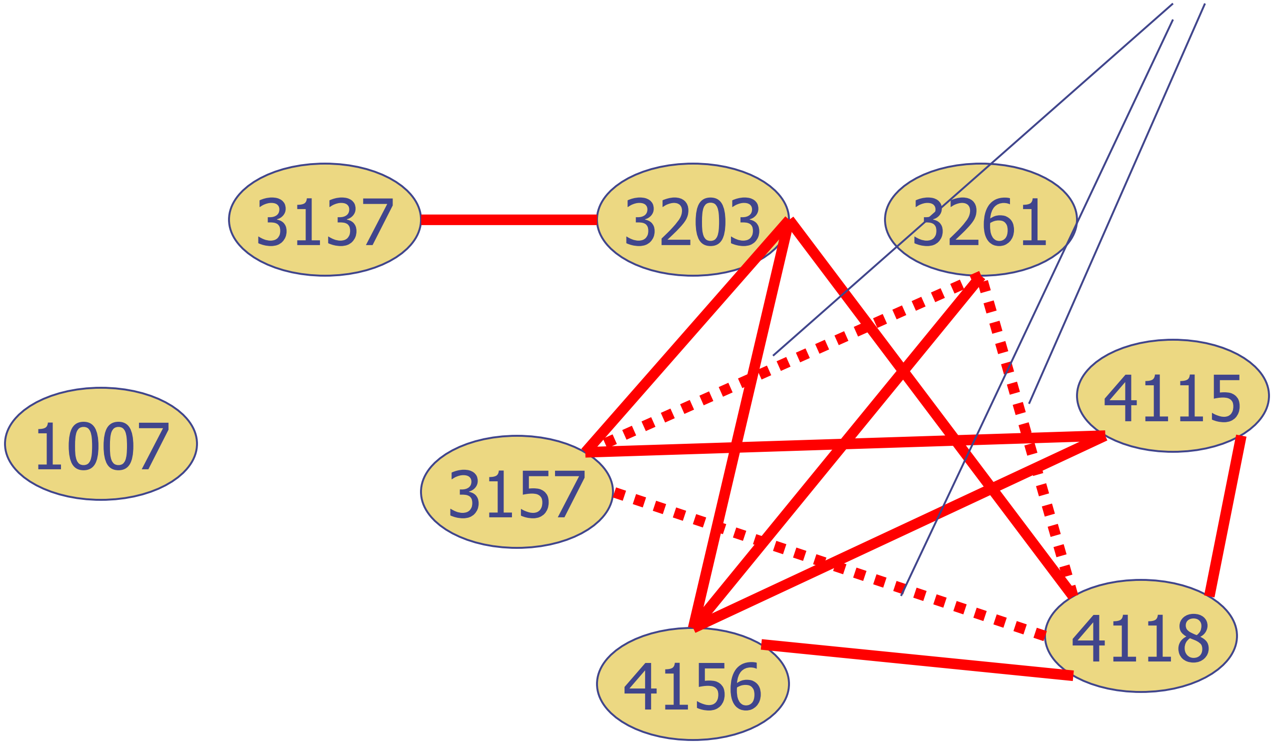
Graph Coloring and Schedules

Not 1-colorable because of edge



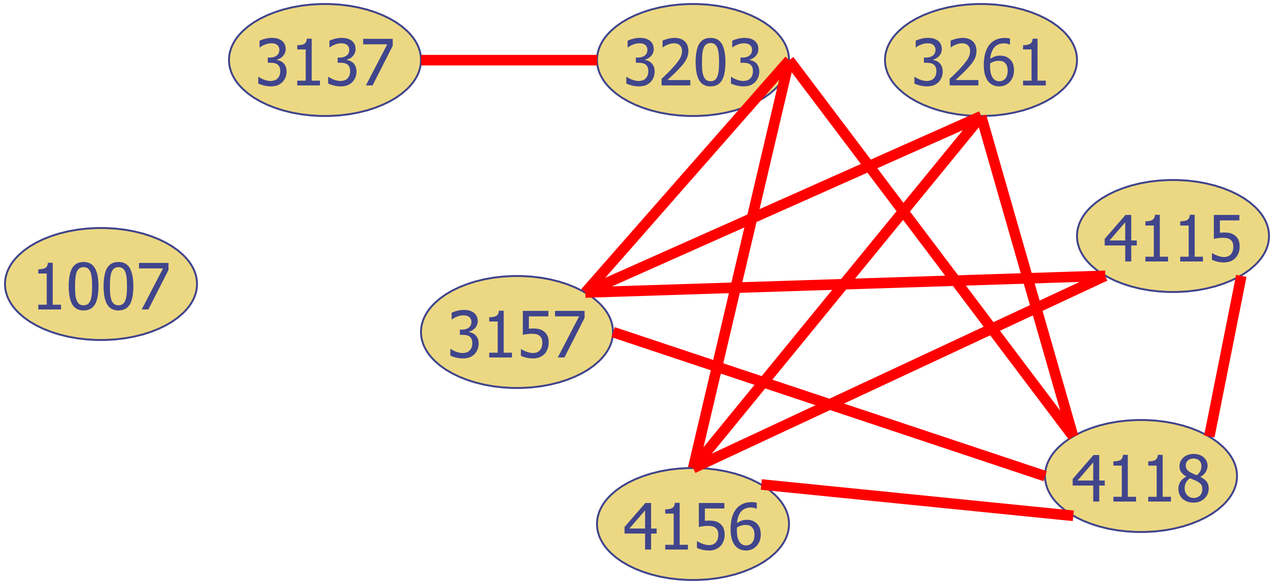
Graph Coloring and Schedules

Not 2-colorable because of triangle



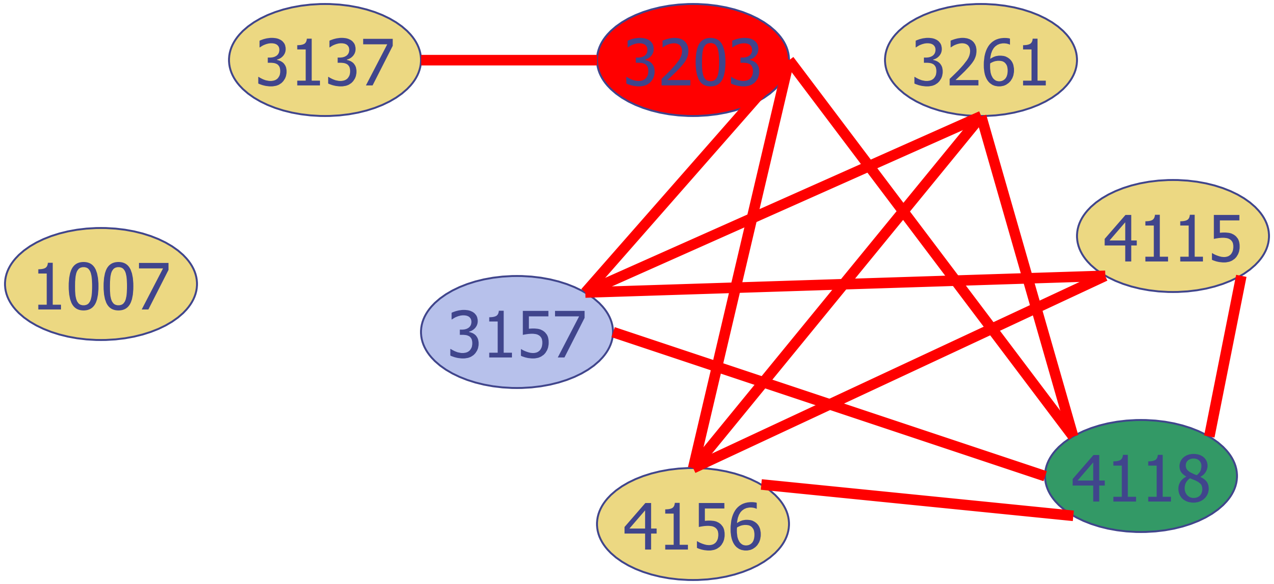
Graph Coloring and Schedules

Is 3-colorable. Try to color by Red, Green, Blue.



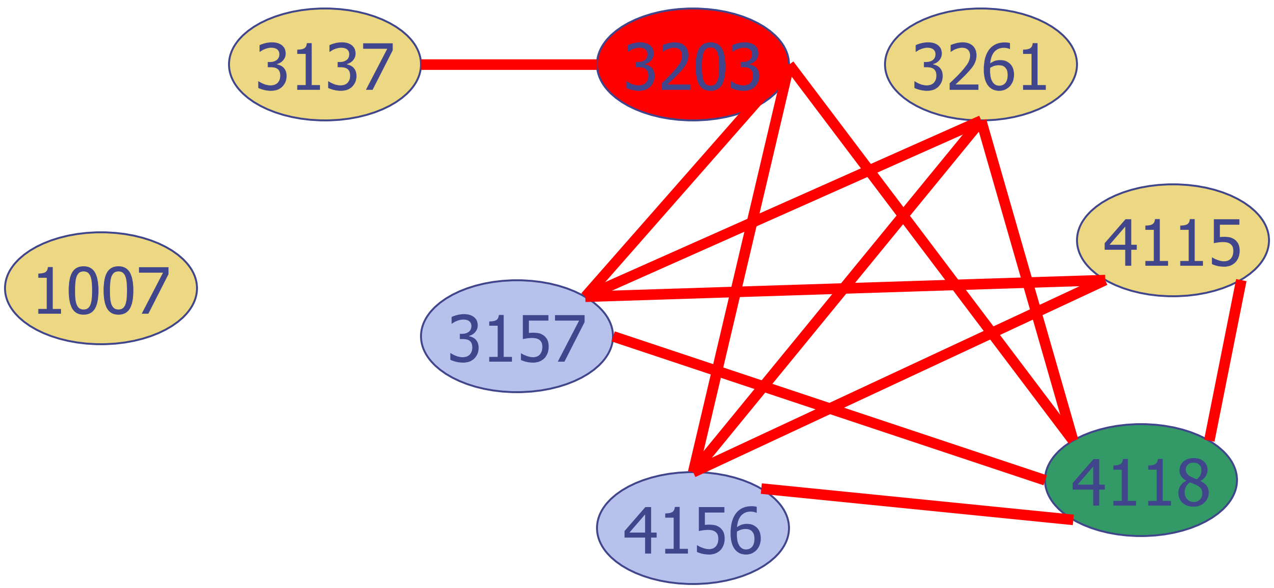
Graph Coloring and Schedules

假设：3203-Red, 3157-Blue, 4118-Green:



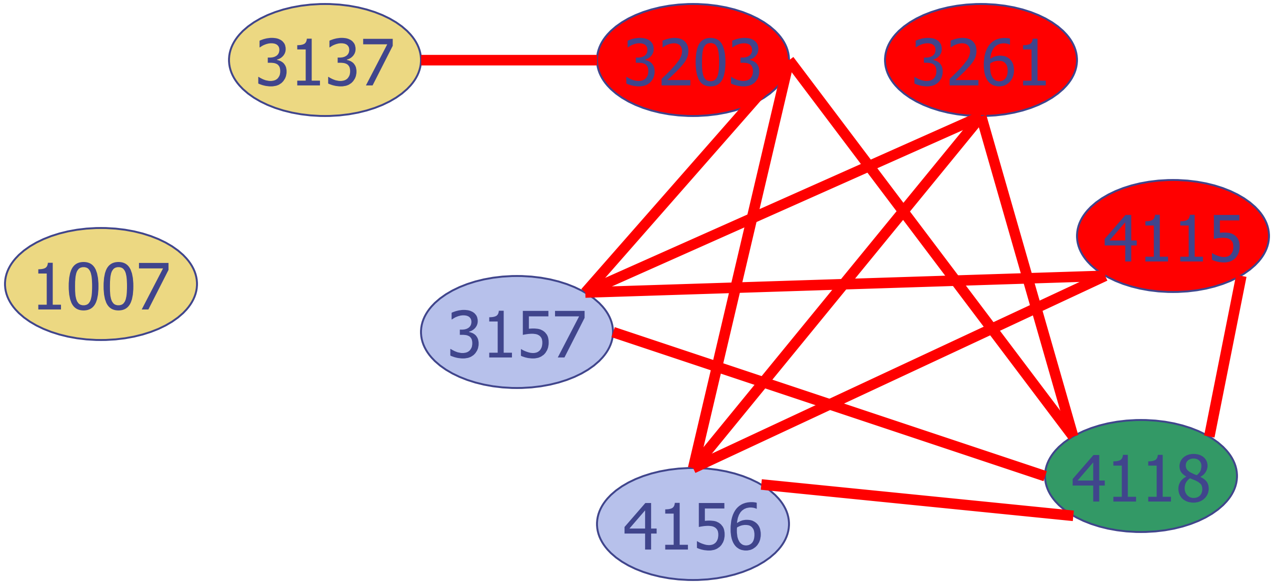
Graph Coloring and Schedules

So 4156 must be Blue:



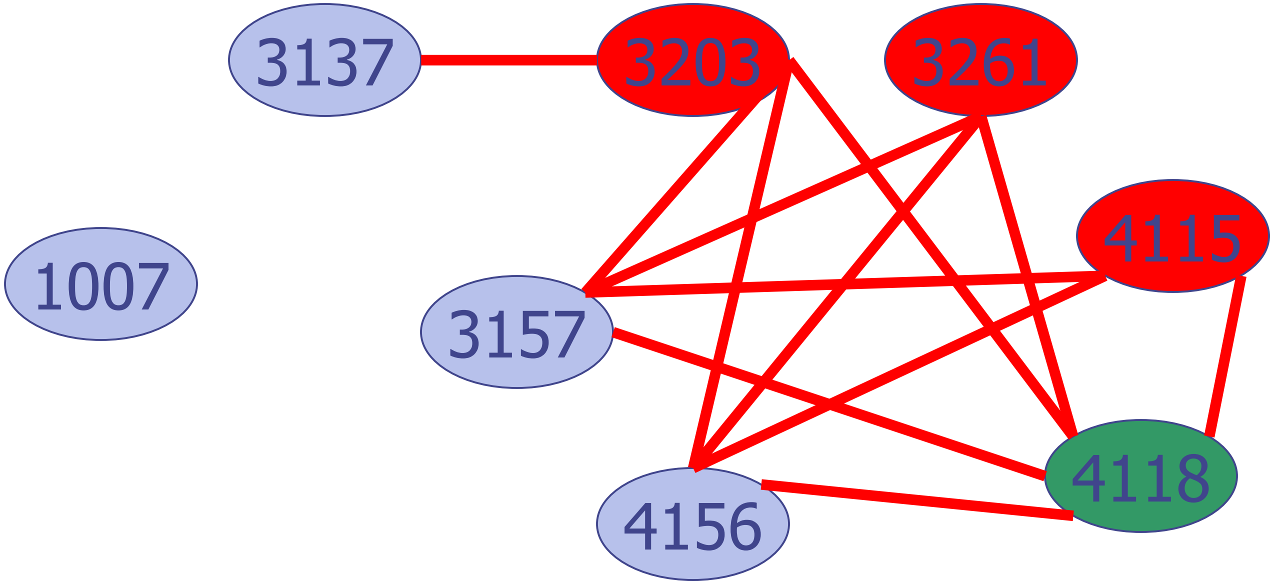
Graph Coloring and Schedules

So 3261 and 4115 must be Red.



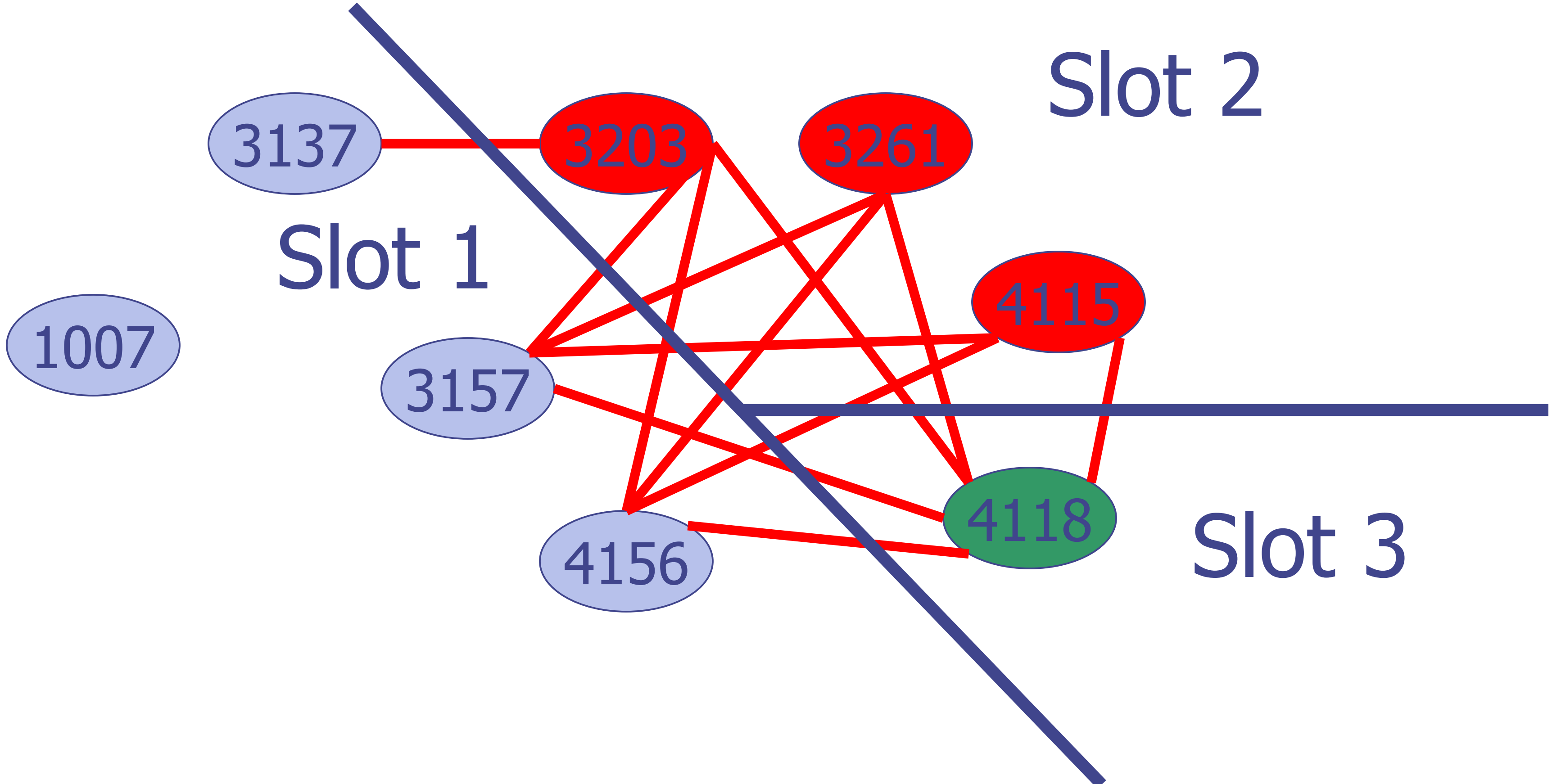
Graph Coloring and Schedules

3137 and 1007 easy to color.



Graph Coloring and Schedules

So need 3 exam slots:



图着色应用举例

- **例题2**：　无线广播电台频率管制问题。

某些距离太近的点不能有相同的频率，要避免频率干扰，就需要合理规划频率。类似的如电视频道分配问题等。

例题3：假定一个工厂需要生产n种化学品，某些化学品是不能在同一车间生产，否则会酿成事故。那么根据这些化学品的不相容情况，需要安排多少车间才能生产？

例题4：变址寄存器的分配问题：寄存器分配不仅仅是图着色的问题。当寄存器数目不足以分配某些变量时，就必须将这些变量溢出到内存中，该过程成为*spill*。最小化溢出代价的问题，也是一个NP-complete问题。如果简化该问题——假设所有溢出代价相等，那么最小化溢出代价的问题，等价于k着色问题，仍然是NP-complete问题。

类似的有货物装箱安全问题，有n个动物放到笼子里的冲突问题，或者有n个学生分班规划问题，还有如任务调度、集成电路布线问题等等。

图着色应用

以上这些问题都可以通过图建模，然后将实际问题转化为vertex coloring问题来解决。

建模：假设n个物品为一个图的顶点，如果物品n_i和n_j不能在一起，那么就在n_i和n_j之间连一条边，构成一个图模型，再编程求解该图的点着色的色数。

总结

- 图着色非常有用，但也非常难。
- 在未来的工作和学习研究中，如果需要用到图着色问题建模解决实际问题时，建议同学们跟踪、查找和学习届时的图着色最新、最优的求解颜色数的算法。

Coloring Exercises

- 6.8节 T3， T19

习题选讲

- 习题选讲：
- 6.5节 T55
- 补充练习 T8 （思考简单图结论又如何？）