Confident in confidence intervals?

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t distribution reminder

Recall our AirBnB example:

```
airbnb <- read.csv("../Datasets/portland-airbnb-listings.csv")
airbnb$price <- as.numeric(gsub("$", "", airbnb$price, fixed=TRUE))
airbnb$instant_bookable <- (airbnb$instant_bookable == "t")
instant <- airbnb$price[airbnb$instant_bookable]
not_instant <- airbnb$price[!airbnb$instant_bookable]
(tt <- t.test(instant, not_instant))
```

```
##
##  Welch Two Sample t-test
##
## data:  instant and not_instant
## t = 3.6482, df = 5039.8, p-value = 0.0002667
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##   4.475555 14.872518
## sample estimates:
## mean of x mean of y
##  124.6409  114.9668
```

How's the t test work?

The *central limit theorem*.

In words:

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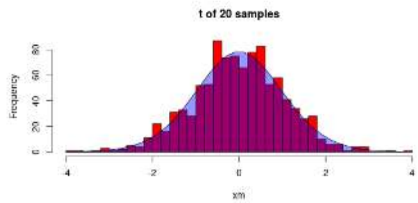

For instance, the probability that the sample mean is within 2 standard errors of the true mean is approximately

$$\int_{-2}^{2} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{(n-2)\pi}\Gamma\left(\frac{n-2}{2}\right)} \left(1+\frac{x^2}{n-2}\right)^{-\frac{n-1}{2}} dx.$$

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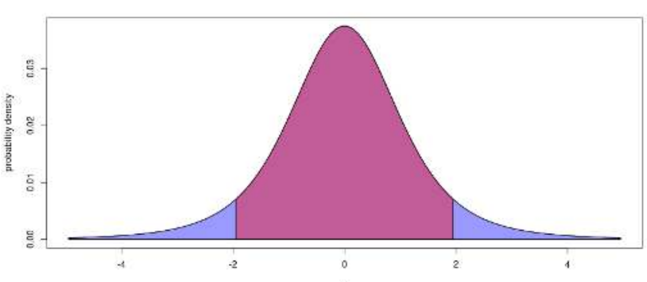

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3. Change mean of the simulated values to 1, and do the same.

Confidence intervals

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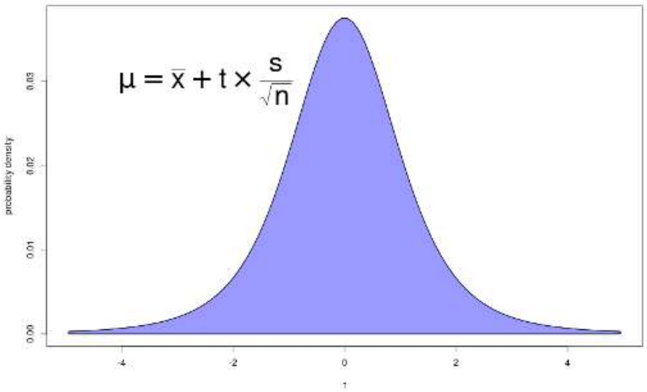
In other words, if we collect 100 independent samples from a population with true mean μ, and 95% construct confidence intervals for the mean from each, then about 95 of these should overlap μ.

How's that work?



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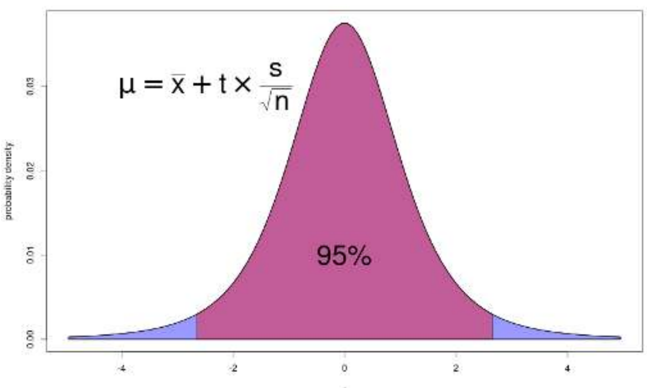


Check this.

> *if we collect 100 independent samples from a population with true mean μ, and construct 95% confidence intervals from each, then about 95 of these should overlap μ.*

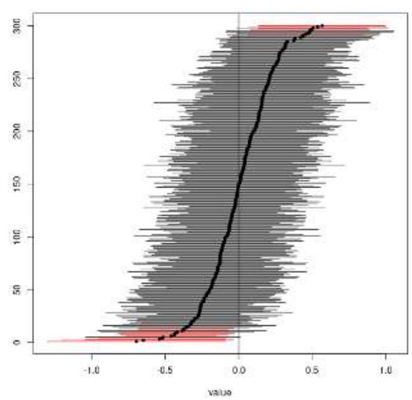
Let's take independent samples of size $n = 20$ from a Normal distribution with $\mu = 0$. Example:

```
n <- 20; mu <- 0
t.test(rnorm(n, mean=mu))$conf.int
```

```
## [1] -0.4019083  0.5862080
## attr(,"conf.level")
## [1] 0.95
```

```r
tci <- replicate(300, t.test(rnorm(n, mean=mu))$conf.int)
mean(tci[1,] > 0 | tci[2,] < 0)
```

```
## [1] 0.05
```

What’s that 95% mean?

Suppose we survey 100 random UO students and find that 10 had been to a party recently and so get a 95% confidence interval of 4%-16% for the percentage of UO students who have been to a party recently.

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> *There is a 95% chance that the true proportion of UO students who have been to a party recently is between 4% and 16%.*

Not so good: the true proportion is a *fixed* number, so it doesn't make sense to talk about a *probability* here.

Power analysis

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Example: Suppose two snail species' speeds differ by 3cm/h. What's the chance our experiment will identify the difference?

A prospective study

Suppose that we're going to do a survey of room prices of an AirBnB competitor. How do our power and accuracy depend on sample size? Supposing that prices roughly match AirBnB's: mean $\mu = \$120$ and SD $\sigma = \$98$, estimate:

1. The size of the difference between the mean price of a random sample of size n and the (true) mean price.
2. The probability that a sample of size n rooms has a sample mean within $10 of the (true) mean price.

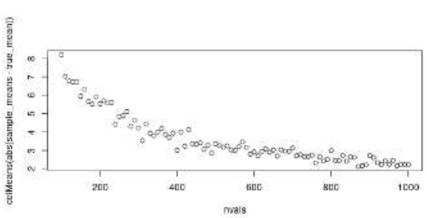
Group exercise

Answer those questions *empirically*: by taking random samples from the `price` column of the `airbnb` data, make two plots:

1. Expected difference between the mean price of a random sample of n Portland AirBnB rooms and the (true) mean price of *all* rooms, as a function of n.
2. Probability that a sample of size n of Portland AirBnB rooms has a sample mean within $10 of the (true) mean price of *all* rooms, as a function of n.

In class: part 1

```r
true_mean <- mean(airbnb$price, na.rm=TRUE)
n <- 20
sample_mean <- mean(sample(airbnb$price, n))
nvals <- 10 * 10:100
nreps <- 100
sample_means <- matrix(NA, nrow=nreps, ncol=length(nvals))
for (j in seq_along(nvals)) {
    n <- nvals[j]
    sample_means[,j] <- replicate(nreps, mean(sample(airbnb$price, n), na.rm=TRUE))
}
plot(nvals, colMeans(abs(sample_means - true_mean)))
```



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