

1. From the symmetry of the flexure structure (the two flexures have the same material, geometry, and boundary conditions), we can simplify the problem by equivalently considering each flexure acted upon by an applied load $P/2$. Because the two flexures are rigidly connected, each flexure will have the same deflection.

From a global force balance in the y-direction and a global moment balance, we find:

$$R_A = P/2 \quad \text{and} \quad M_A + PL/2 = M_o$$

Making an arbitrary slice of the flexure at x and doing a force and moment balance, we find:

$$\begin{aligned} V(x) &= -P/2 \\ M(x) &= M_A + xR_A = M_o - PL/2 + Px/2 \end{aligned} \quad (1)$$

Using the moment-curvature relation and (1):

$$\begin{aligned} EI \frac{d^2v(x)}{dx^2} &= M(x) \\ EI \frac{d^2v(x)}{dx^2} &= M_o - \frac{P}{2}(L - x) \\ \Rightarrow \frac{dv(x)}{dx} &= \frac{1}{EI} \left[M_o x - \frac{P}{2} \left(Lx - \frac{x^2}{2} \right) + D_1 \right] \end{aligned} \quad (2)$$

$$\Rightarrow v(x) = \frac{1}{EI} \left[\frac{M_o x^2}{2} - \frac{P}{2} \left(\frac{Lx^2}{2} - \frac{x^3}{6} \right) + D_1 x + D_2 \right] \quad (3)$$

where M_o , D_1 and D_2 are constants of integration to be determined from the boundary conditions, $v(0) = 0$, $v'(0) = 0$ and $v'(L) = 0$. From the first boundary condition and (3), $D_1 = 0$. From the second boundary condition and (2), $D_2 = 0$. From the third boundary condition, $M_o = PL/4$. Substituting the constants into (3), we obtain the deflection:

$$v(x) = \frac{1}{EI} \left[\frac{PLx^2}{8} - \frac{P}{2} \left(\frac{Lx^2}{2} + \frac{x^3}{6} \right) \right] \quad (4)$$

$$v(x) = \frac{P}{EI} \left[\frac{x^3}{12} - \frac{Lx^2}{8} \right] \quad (5)$$

The deflection, δ , at $x = L$ is:

$$\delta = v(L) = -\frac{PL^3}{24EI}$$

The magnitude of the stiffness $k = P/\delta$ of the flexure system is then:

$$k = \left| \frac{P}{\delta} \right| = \frac{24EI}{L^3}$$

2. Plastic deformation in the flexure occurs when the maximum stress in the flexure reaches the tensile yield strength, σ_y . The stress in the flexure is calculated from the axial stress equation:

$$\sigma_{xx} = -\frac{M(x)y}{I} \quad (6)$$

For a rectangular cross-section of height h and width b , $I = bh^3/12$. To obtain the maximum stress, we find the maximum moment from the moment distribution found in part (a):

$$M(x) = \frac{P}{2} \left(x - \frac{L}{2} \right)$$

Due to the symmetry of the deformation in the flexure, the maximum stress occurs at four points along the flexure, but the maximum stress is compressive at $x = 0, y = -h/2$ and $x = L, y = h/2$, and tensile at $x = 0, y = h/2$ and $x = L, y = -h/2$. The maximum moment can be calculated from any of the four points, so at $x = L, y = -h/2$, the maximum moment is:

$$M(L) = \frac{PL}{4}$$

Substituting the expressions for $M(L)$ and y into (6), the maximum stress is:

$$\sigma_{max} = \sigma_y = \frac{PLh}{8I}$$

Rearranging:

$$P = \frac{8I\sigma_y}{Lh} \quad (7)$$

From part (a),

$$\delta_{max} = -\frac{PL^3}{24EI}$$

Substituting (7),

$$\delta_{max} = -\frac{\sigma_y L^2}{3Eh}$$

3.

$$\begin{aligned} \delta_{max} &= -\frac{(350 \times 10^6 \text{ N/m}^2)(0.05 \text{ m})^2}{(3)(100 \times 10^9 \text{ N/m}^2)(0.0015 \text{ m})} \\ \delta_{max} &= \boxed{-1.94 \times 10^{-3} \text{ m}} \end{aligned}$$