Convolution Neural Networks

Week 3

COMP6252 (Deep Learning Technologies)

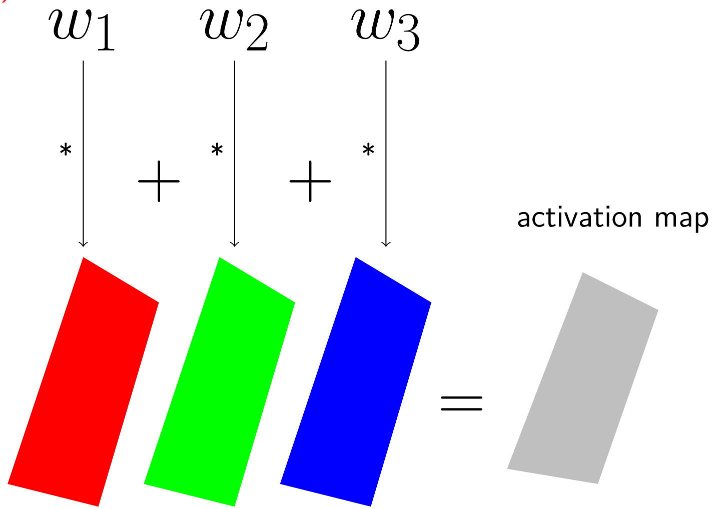
ECS, University of Southampton

28 April 2022

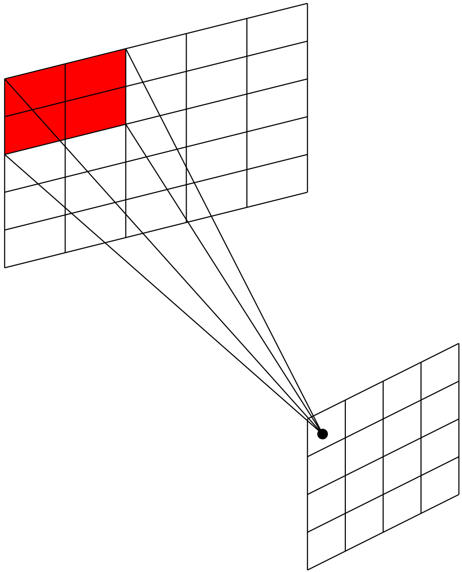
Introduction

- A convolution network usually has at least one convolution layer
- A convolution operation (slightly different from the usual definitio) is done by multiplying weights element-wise with a portion of the input
- The same operation with the same weights is repeated over all the input
- The set of weights are usually referred to as the **kernel**
- The result of the convolution is referred to as the **feature map** or **activation map**

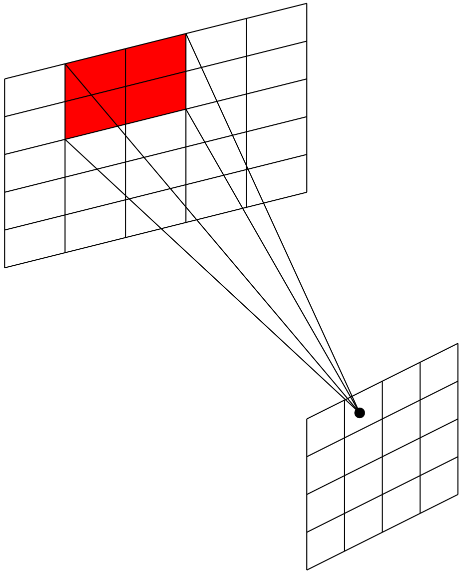
Convolution operation on an input image with 3 channels (RGB)



Conv operation on one channel



Conv operation on one channel



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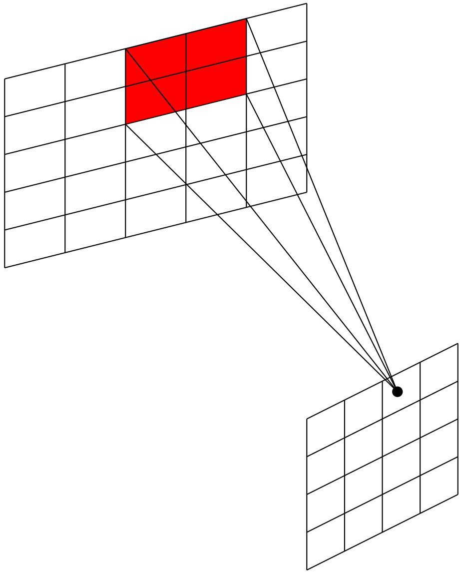

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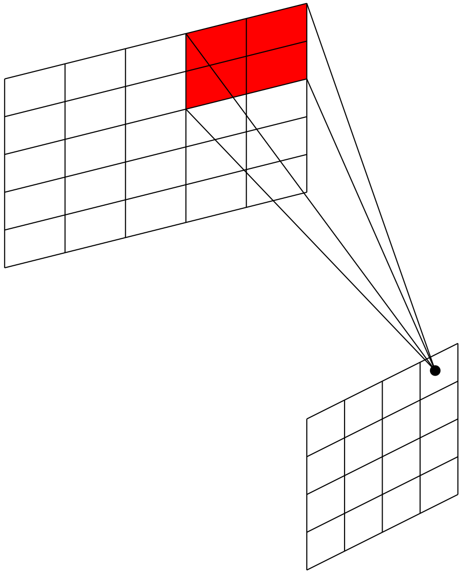

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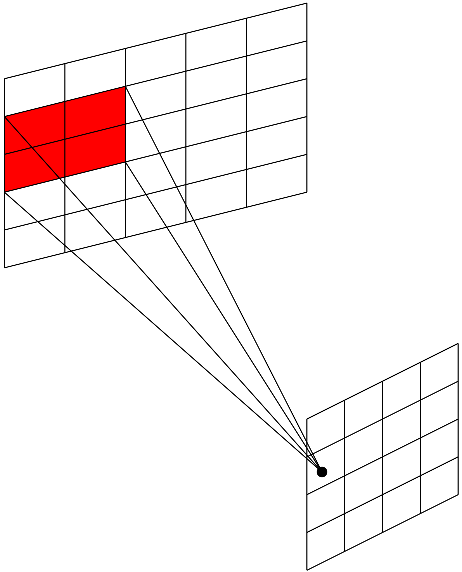

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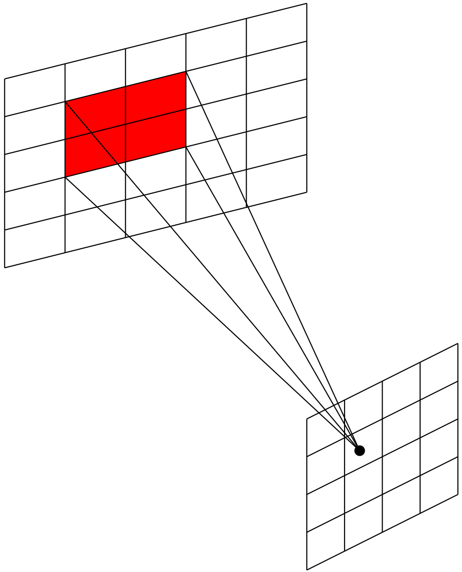

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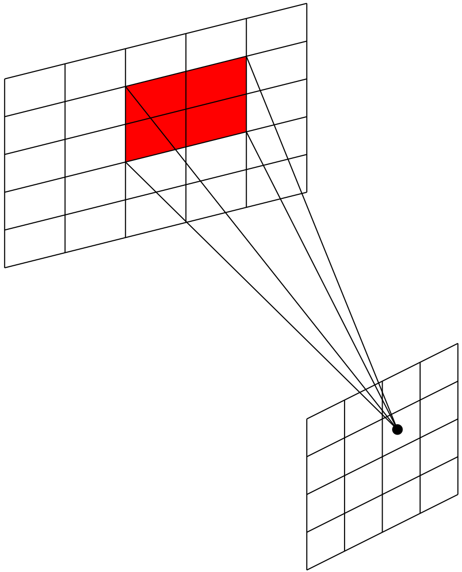

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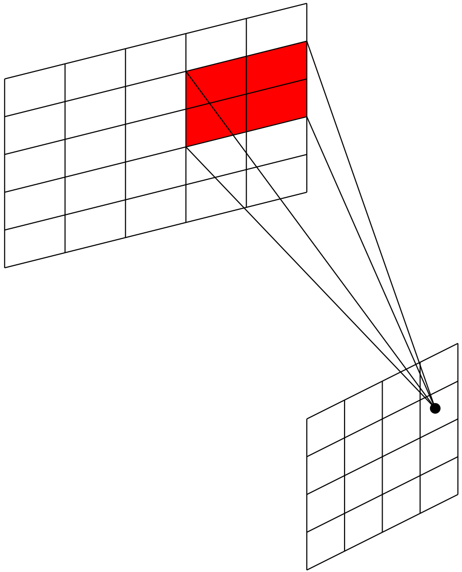

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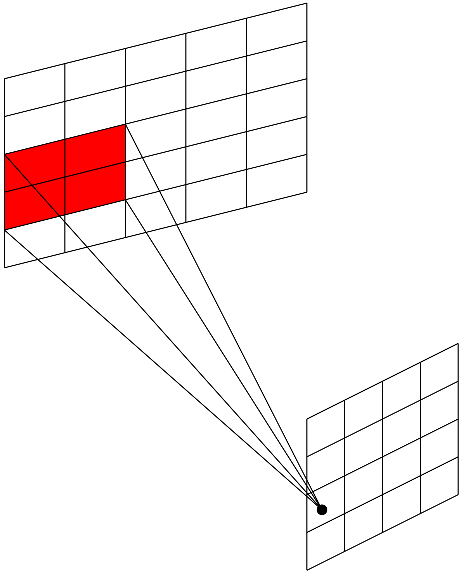

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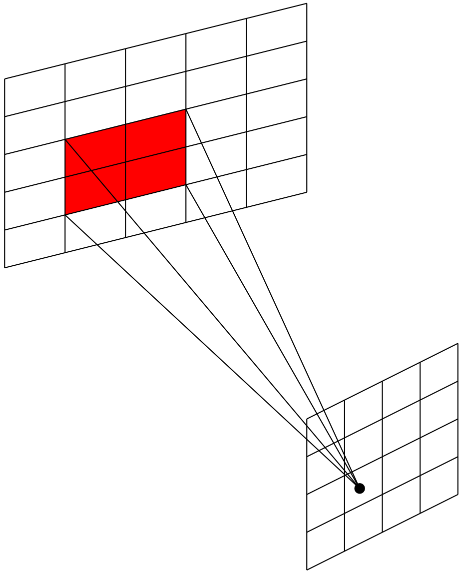

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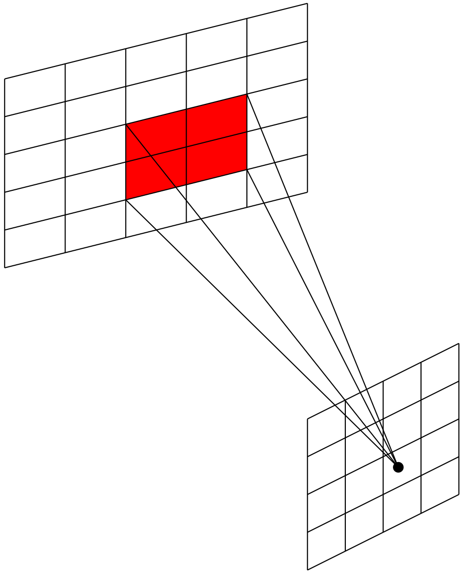

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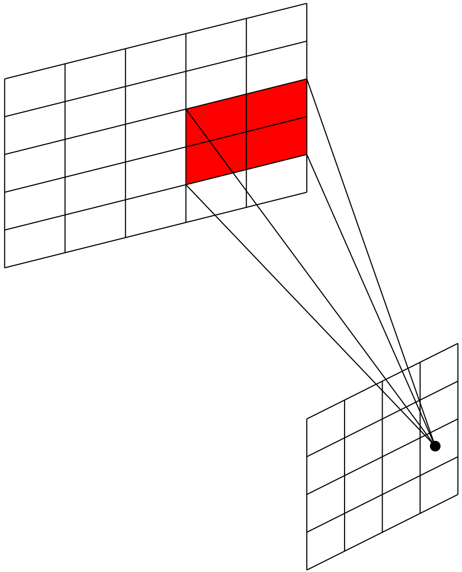

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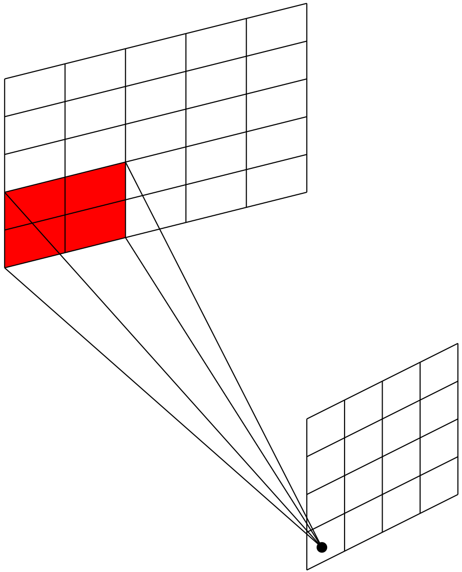

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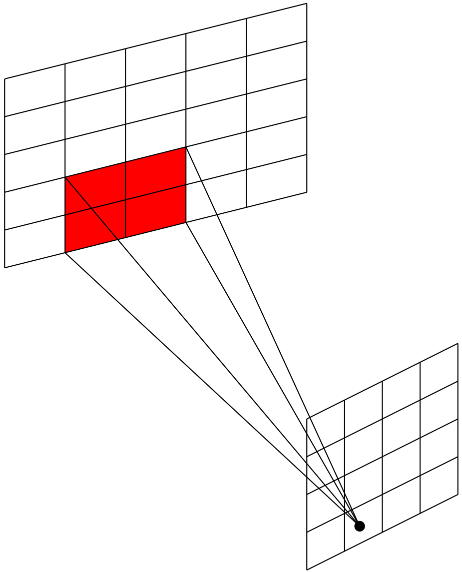

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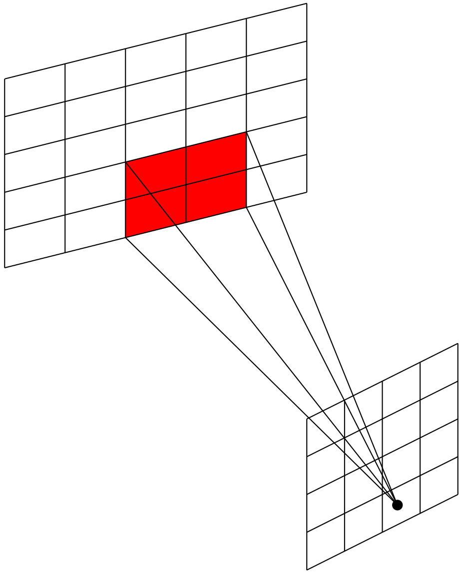
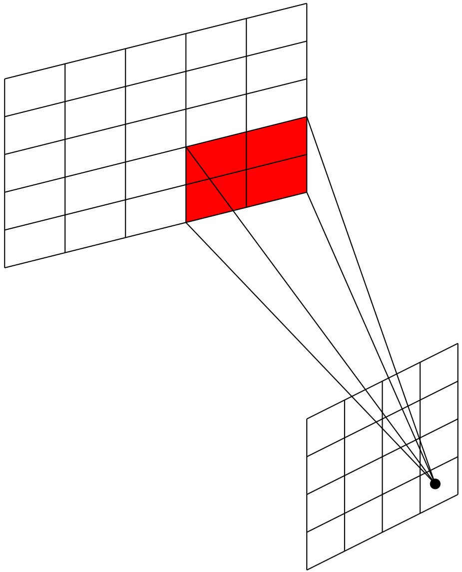


Conv operation on one channel



Example: Conv. operation on one channel

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

$\odot$

Kernel

1	-2
-3	4

activation map

$=$

6		

Example: Conv. operation on one channel

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1	2	4	3
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9	10	12	11
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$\odot$

Kernel

1	-2
-3	4

activation map

$=$

6	8	

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6	8	2
6	8	

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9	10	12	11
13	14	16	2

$\odot$

Kernel

1	-2
-3	4

activation map

$=$

6	8	2
6	8	2
6	8	-50

Add stride 2 example here

Stride and receptive field

- In the previous example we used a **stride** equal to 1
- The kernel **receptive field** was equal to 2x2
- The size of the output (activation map) was 3x3
- Are those numbers the same in all applications? No

Stride and receptive field

- In general one can use a stride of any size S. Stride of size 1 is the most common.
- Let $F \times F$ be the kernel **receptive field**
- Let $H_i \times W_i \times C_i$ be the size of the input
- The size of the resulting activation map is $H_o \times W_o$ where

$$H_o = \left\lfloor \frac{H_i - F}{S} \right\rfloor + 1$$
$$W_o = \left\lfloor \frac{W_i - F}{S} \right\rfloor + 1$$

Example

- Consider an input image of size (3,28,28)
- We are using the PyTorch convention with the channel dimension first.
- What would be the size of the activation map if a kernel of size 3x3 and stride 2 were used?
- Height and width are the same and equal to $\left\lfloor \frac{28-3}{2} \right\rfloor + 1 = 13$
- So the activation map has size 13x13
- Note that the output of **one** kernel is a 2-d object: the activation map.
- The number of output channels is determined by the **number** of kernels

Parameters

- Consider an image of size (3,28,28) used as input to 32 filters of size 3x3
- Since the input has 3 channels each filter has 3x3x3+1(bias)=28 Parameters
- With a stride of 1 the output has size (32,26,26), i.e., the layer has 32x26x26=21632 nodes
- Total number of parameters=32x28=896
- Contrast the above with a feedforward with the first layer having 32x26x26=21632 nodes
- It would have (3x28x28)*21632=50878464 parameters !(without bias)

Padding

- We saw previously that convolution reduces the size of the input
- When multiple layers are used the size is reduced at each layer
- Sometimes we don't want the size to change
- more importantly the "edges" of the input do not contribute as much as the "middle"
- One solution is to pad the input with zeros.

Zero padding

Input

1	2	1
4	2	3
2	1	1

0 padded

0	0	0	0	0
0	1	2	1	0
0	4	2	3	0
0	2	1	1	0
0	0	0	0	0

- The output size in the presence of padding

$$H_o = \left\lfloor \frac{H_i + 2P - F}{S} \right\rfloor + 1$$
$$W_o = \left\lfloor \frac{W_i + 2P - F}{S} \right\rfloor + 1$$

- Same formula as before if we consider the **effective** height $=H_i + 2P$ and width$=W_i + 2P$

Convolution operation mathematically

- Let f, i, j be the filter index,output height index, and output width respectively
- For a stride of 1 (most common), and bias per filter b_f
- The convolution operation is defined as

$$O_{f,i,j} = b_f + \sum_c \sum_{m,n} X_{c,i+m,j+n} * K_{f,c,m,n}$$

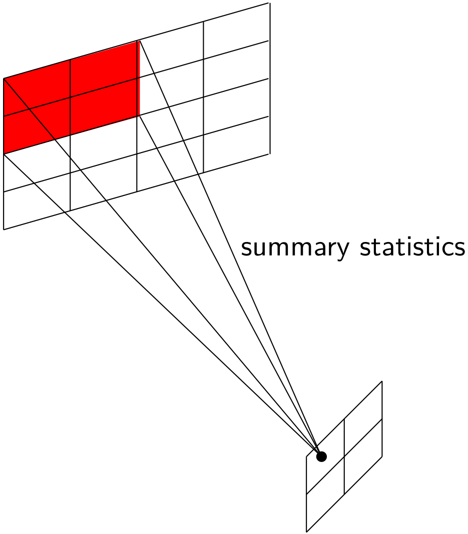
- If one includes the sample index s, needed in the case of batches

$$O_{s,f,i,j} = b_f + \sum_c \sum_{m,n} X_{s,c,i+m,j+n} * K_{f,c,m,n}$$

Pooling

- Typically, convolutional networks performs three steps
  1. Convolution operation
  2. followed by a Non-linear activation such as ReLU
  3. followed by **pooling**
- Pooling computes a summary statistics for a small area of the result
- Taking the max value is the most common pooling operation.

Pooling size 2x2, stride=2



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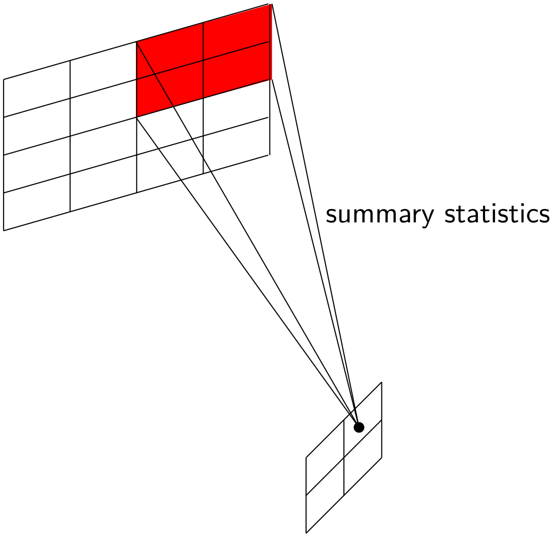

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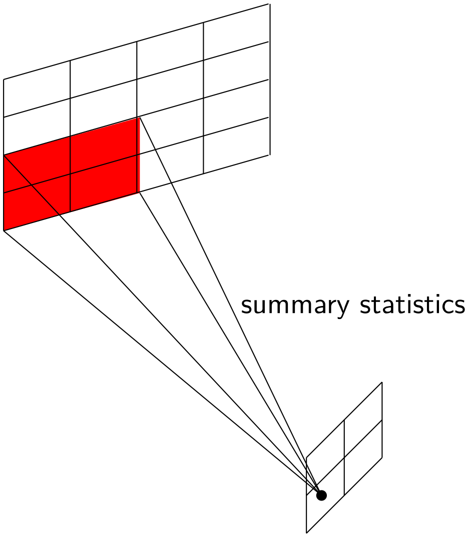
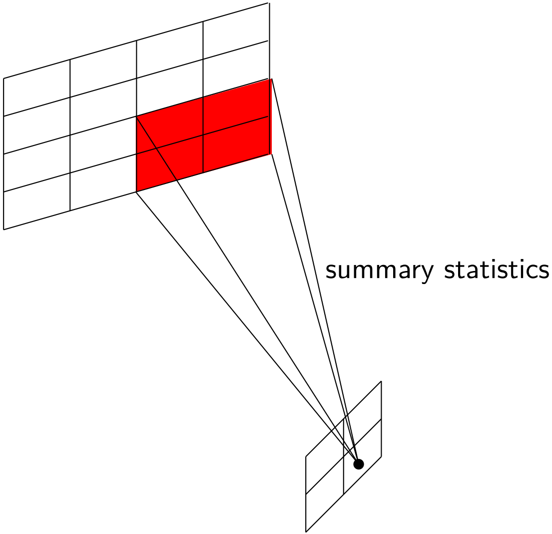


Pooling size 2x2, stride=2



Max pooling

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

Max Pool=

Result

6	

Max pooling

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

Max Pool=

Result

6	8

Max pooling

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

Max Pool=

Result

6	8
14	

Max pooling

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

Max Pool=

Result

6	8
14	16