OCT 18 $\qquad\qquad\qquad\qquad$ $\Gamma(n) = (n-1)!$

Review: Maximum likelihood

example: Gamma distribution

$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)} \quad x \geq 0$$

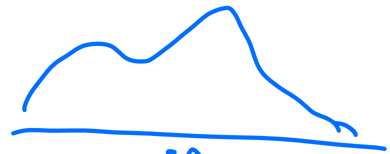
sum of exponential random vr.

Note: If $X_1, \ldots, X_r \sim \exp(\lambda)$

$\sum_{i=1}^r X_i \sim \text{Gamma}(r, \lambda)$

$X_1, X_2, \ldots, X_n$ IID from this distribution

How do I estimate r and λ



NEW

TODAY: p-values

$$\Gamma(r) = \int_0^\infty \lambda^r x^{r-1} e^{-\lambda x} dx$$

Background

Given data $X_1, \ldots, X_n$

- Pose model for data (e.g. appropriate distribution)
- Identify unknown parameters
- Use data to estimate parameters

Example: Want to estimate probability of engine failure in first 100k miles

Data: 250 engines, 31 failures

Model: ? Binomial(n, p) $n = 250$ $\qquad$ $X = \#$ failures

$X \sim \text{Binomial}(250, p)$

Estimate: ? p unknown, estimate by $31/250$

$\boxed{\text{Bonus: How certain are we?}}$ Simulation / Theory

$$X_i = \begin{cases} 1 & \text{if failure} \\ 0 & \text{o/w} \end{cases}$$

$X_1, \ldots, X_{250}$

$S_n = \sum_{i=1}^{250} X_i$

Bernoulli(p) $\qquad$ Bin$(1, p)$

$\mathbb{E} X_i = 1 \cdot p + 0 (1-p)$ $\qquad$ (Expected value) $\qquad$ sample mean

$= p$

mean of r.v. X_i

Can always estimate expectation $\mathbb{E} X_i$ by $\boxed{\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i}$

In this case $\bar{x} = \frac{\# \text{ failures}}{250}$

Simulate 1000 times this model

$S \sim \text{Binomial}(250, p)$

$\frac{S_1}{250}$ $\quad$ $S_{1000}/250$

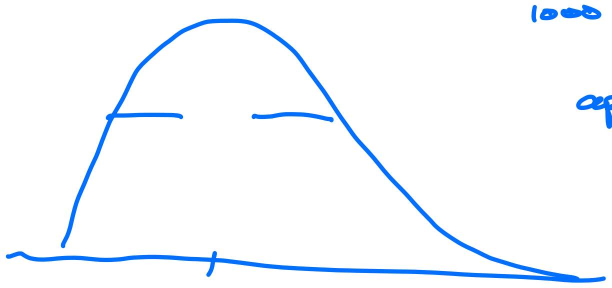
$\uparrow$ $\quad$ $\uparrow$

$S_1, \ldots, S_{1000}$

$\bar{S} =$

↳ DON'T KNOW, BUT....
WE USE ESTIMATE $\hat{p} = 31/250$

1000 simulated values of $\hat{p}$
approx to distn of $\hat{p}$

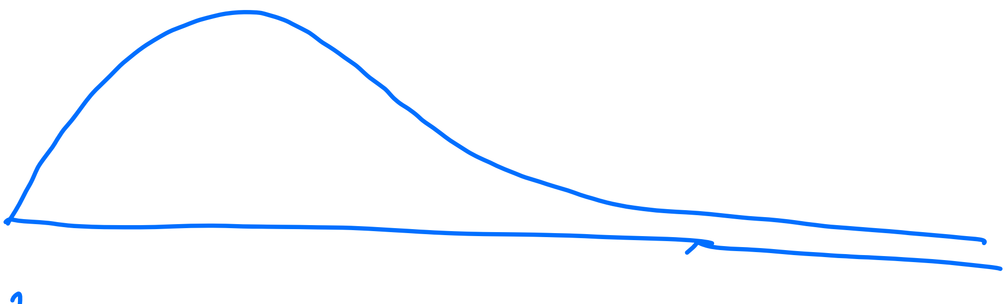


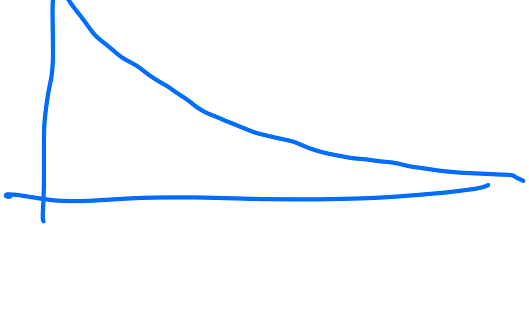
Computing sample standard deviation

$x_1, \ldots, x_n$ $\quad \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \hat{\sigma}$

Estimate of σ = STANDARD DEVIATION OF X_i

$\sqrt{Var(X_i)} = \sqrt{E(X_i - EX_i)^2}$





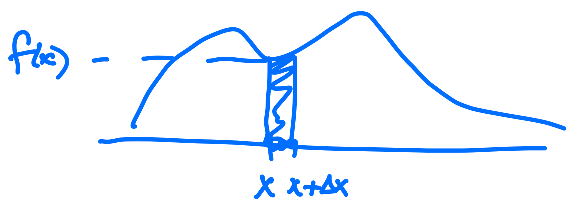
Likelihood function   observed values $x_1, \ldots, x_n$

1st write   "$\mathbb{P}(\text{DATA} \mid r, \lambda)$" = "$\mathbb{P}(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n \mid r, \lambda)$"

$= f(x_1) \cdots f(x_n)$

$n = 4$ $X_1 = 2.1$, $X_2 = 3.5$, $X_3 = 3.1$, $X_4 = 1.8$

If X has density $f(x)$ $f(x)dx \simeq \mathbb{P}(x \le X \le x + dx)$



$$f(x_1) \cdots f(x_n) = \prod_{i=1}^{n} \frac{1}{\Gamma(r)} \lambda^r x_i^{r-1} e^{-\lambda x_i}$$

$\mathbb{P}(\text{DATA} \mid r, \lambda)$

Regarding as a function of parameters (i.e. $x_1, \ldots, x_n$ fixed)
r, λ

$$L(r, \lambda) = \frac{1}{\Gamma(r)^n} \lambda^n \left(\prod x_i\right)^{r-1} e^{-\lambda \sum_i x_i}$$

$$\log \prod_{i=1}^{n} f(x_i) = \sum_{i=1}^{n} \log f(x_i)$$

Comment: maximizing $\log L$ same as maximizing L