

WED NOV 22

LAST TIME CROSS-VALIDATION

METHOD FOR ESTIMATING PREDICTIVE ERROR

ONCE A MODEL HAS BEEN FIT

$(y_i, x_{i1}, \dots, x_{ik}) \quad i=1, \dots, n$  once fit a model  
 $\hat{y}_i$  is the fitted value when plugging these inputs  
 $x_{i1}, \dots, x_{ik}$  into fitted model

Linear model

$$y_i = b_0 + b_1 x_{i1} + \dots + b_k x_{ik} + \varepsilon_i$$
$$\hat{y}_i = \hat{b}_0 + \hat{b}_1 x_{i1} + \dots + \hat{b}_k x_{ik}$$

Estimate predictive error  
over  $n$  data  
points

$$\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

MEAN SQUARED ERROR  
OVER  $n$  data points

TEST DATA

$n$  points

TRAINING DATA

$$x_i \sim \begin{cases} 1 \\ 0 \end{cases} \quad \text{w.p.} \quad p(\eta_i)$$

$$p(x) = \frac{1}{1 + e^{-x}}$$

$$\eta_i = b_0 + b_1 x_{i1} + \dots + b_k x_{ik}$$

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$y_i = \text{coffee cups}$

$$y_i \sim \text{Poisson}(\lambda_i)$$

$$\lambda_i = e^{\eta_i}$$

$$\log \lambda_i = \eta_i$$

$$\lambda_i = E(y_i)$$

$$\eta_i = b_0 + b_1 x_{i1}$$

$x_i = \text{amount of sleep}$

Estimator  $\hat{\beta}$  of parameter  $\beta$

$$\underset{\text{MSE}}{E[(\hat{\beta} - \beta)^2]} = E[(\hat{\beta} - E\hat{\beta} + E\hat{\beta} - \beta)^2]$$

$$= E[(\hat{\beta} - E\hat{\beta})^2] + (E\hat{\beta} - \beta)^2$$
$$= \text{Var}(\hat{\beta}) + \text{BIAS}^2$$



In practice: how to choose  $\alpha$ ?

Run the model w/ different values

Compute  $\text{MSEP}_{\text{ESTIMATED}}$  using cross-validation

Choose  $\alpha$  to minimize MSEP

CROSS VALIDATION

INPUT DATA, MODEL SPECIFICATION, METHOD  
FOR FIT

OUTPUT MSEP

$CV(\text{DATA}, \text{MODEL}, \text{FIT})$

