TODAY

- HW 1 discussion
- USING AXIOMS — BAYES THEOREM
- RANDOM VARIABLES

PROPERTIES, CONNECTION TO SIMULATIONS

BERNOULLI(P)

p-coins prob "H" p

"T" 1-p

$\underline{X}$ = # coin tosses until 1st H

possible value $\{1, 2, \ldots\}$

$\mathbb{P}(X=5) = (1-p)^4 p$

$\underline{T}\ \underline{T}\ \underline{T}\ \underline{T}\ \underline{H}$

$(1-p)\ (1-p)\ (1-p)\ (1-p)\ p$

Y = # tails before 1st head

values $\{0, 1, 2, \ldots\}$

$P(Y=3) = \underset{T}{\underline{(1-p)}}\underset{T}{\underline{(1-p)}}\underset{T}{\underline{(1-p)}}\ \underset{H}{\underline{p}} = (1-p)^3 p$

$p = 1/2$

$P(Y=0) = \frac{1}{2}$

$P(Y \neq 0) = 1/2$

$Y_1, Y_2, \ldots, Y_{100}$

$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$

$\left(\frac{1}{2}\right)^{100}$

$p = \frac{1}{2} \leftrightarrow P(X=1) = \frac{1}{2}$

$P(Y=1) = \frac{1}{4}$

$P(D_i = k) = 1/6 \quad k = 1, 2, \ldots, 6$

D_1	D_2	D_3	D_4	D_5	D_6			
1	2	4	5	3	6	-	—	—

$k = 4$
$X = 12$

3 2 6 - — — —

$k = 1$
$X = 3$

Test

$P(+ \mid D)$ $\quad$ $P(D)$ $\quad$ KNOWNS

$P(+ \mid D^c)$ $\quad$ $P(D^c) = 1 - P(D)$

$P(D \mid +)$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

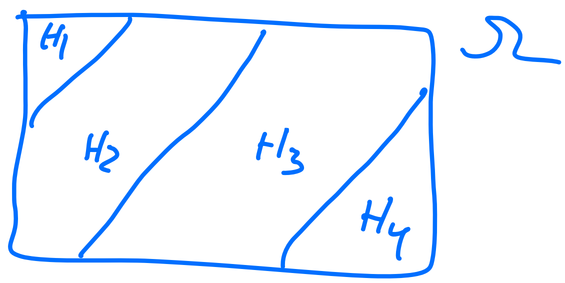
$$P(A \mid B) P(B) = P(AB)$$

$$\mathbb{P}(D \mid +) = \frac{\mathbb{P}(+D)}{\mathbb{P}(+)}$$

$$= \frac{\mathbb{P}(+\mid D)\,\mathbb{P}(D)}{\underbrace{\mathbb{P}(+\mid D)\,\mathbb{P}(D)}_{\mathbb{P}(+D)} + \underbrace{\mathbb{P}(+\mid D^c)\,\mathbb{P}(D^c)}_{\mathbb{P}(+D^c)}}$$

BAYES' FORMULA

Given $\begin{cases} H_1, H_2, \ldots, H_k & \text{disjoint, } \bigcup_{j=1}^{k} H_j = \Omega \\ \mathbb{P}(H_1) \ldots \mathbb{P}(H_k) & \\ \mathbb{P}(\text{DATA} \mid H_j) & j = 1, \ldots, k \end{cases}$



Want $\mathbb{P}(H_j \mid \text{DATA})$

$$\mathbb{P}(H_j \mid \text{DATA}) = \frac{\mathbb{P}(\text{DATA} \mid H_j)\,\mathbb{P}(H_j)}{\sum_{i=1}^{k} \mathbb{P}(\text{DATA} \mid H_i)\,\mathbb{P}(H_i)}$$

Experiment: toss p-coin 3 times

$\Omega = \{HHH, HHT, \ldots, TTT\}$

$p^3 \to x=3$, $p^2(1-p) \to X=2$, $(1-p)^3 \to X=0$

$X = \#\ H's$

$$\mathbb{P}(X=3) = p^3$$

$$\mathbb{P}(X=2) = 3p^2(1-p)$$

HHT $p^2(1-p)$
HTH $p^2(1-p)$
THH $p^2(1-p)$

Binomial random variable (n, p)

$n = \#$ coin tosses (independent)

$p = \mathbb{P}(\text{heads})$

$X = \#$ Heads

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Discrete or continuous

$\downarrow$

possible values can be enumerated

possible values Binomial (n,p) r.v.

$\{0, 1, 2, \ldots, n\}$

$$\mathbb{P}(X=k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, 2, \ldots, n$$

$$\mathbb{P}(X=0) = \binom{n}{0} p^0 (1-p)^n = (1-p)^n$$

$\leftarrow$ prob. mass function

$$\mathbb{P}(X \le 2) = \mathbb{P}(X=0) + \mathbb{P}(X=1) + \mathbb{P}(X=2)$$

$\mathbb{P}(G_L = 0) = \frac{1}{2}$

$\mathbb{P}(G_L = 2) = \frac{1}{2}$

$\mathbb{P}(G_L = x) = 0$ if $x \neq 0$ or 2

Can simulate $X_1, X_2, ----, X_N$ each a simulation X

$$\mathbb{P}(X=k) \simeq \frac{\#\text{ simulations w/ value } k}{N}$$

Law of large numbers! $N \to \infty$

$$\mathbb{P}(X=k) \leftarrow \frac{\#\text{ simulations w/ value } k}{N}$$