WEEK 8

WED NOV 15

HW7 DUE THURS (TOMORROW) 11:59 PM

NEXT HW DUE TUES (HW 8)

FINAL (TAKE HOME) DUE FRI FINALS WEEK

GLM's, NONLINEAR MODELS

Review: Linear models (MODEL, DATA ESTIMATION, PERFORMANCE) GOALS?

DATA MODEL

$$
\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}
\qquad
\begin{bmatrix} X_{11} & \cdots & X_{1k} \\ X_{21} & & \\ \vdots & & \\ X_{n1} & \cdots & X_{nk} \end{bmatrix}
\qquad \underline{X}
$$

$n \times 1$ $\qquad$ $n \times k$

$$E[Y_i] = b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{ik}$$

$$E[Y] = Xb \qquad b = \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix}$$

Y, X observed

b is unknown parameters

Goal: Find estimates of b

Linear model

$$\underset{n\times 1}{Y} = Xb + \underset{n \times 1}{\varepsilon} \qquad \varepsilon \sim N(0, \sigma^2 \underset{n\times n}{I})$$

$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ independent $N(0, \sigma^2)$

$$Y \sim N(Xb, \sigma^2 I)$$

Goals: Estimates of b $\rightarrow$ MLE / LEAST-SQUARES (SAME FOR NORMAL)

ONCE ESTIMATED b, what good is it?

Make predictions:

If you want to predict y for $x_1^*, x_2^*, \ldots, x_k^*$

y_i $\hat{y}_i$

$\hookrightarrow \hat{b}_1 x_{i1} + \dots + \hat{b}_k x_{ik}$

$\frac{1}{n} \sum (y_i - \hat{y}_i)^2 = MSE$ = est of σ^2

is estimate of how well model fits data

$E y_i$

Generalized Linear Model

$\eta_i = (Xb)_i$ linear predictor

Ingredients to GLM

(1) DIST'N OF RESPONSE Y_i

Y_i Bernoulli(p_i)

$E Y_i = p_i$

$$Y_i = \begin{cases} 1 & \text{w.p. } p_i \\ 0 & 1-p_i \end{cases} \qquad E Y_i = p_i$$

(2) $g(E Y_i)$ = linear predictor $\eta_i = (X\beta)_i$

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & \vdots \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \qquad X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a + b x_1 \\ \vdots \\ a + b x_n \end{bmatrix}$$

$\eta_i = +5 - x_i$ (here $a = 5$ $b = -1$)

$$p_i = \frac{1}{1 + e^{-\eta_i}}$$

Likelihood function

i^{th} data point $\quad \theta_i = a + b a_i + c s_i$

$p(y_i \mid p_i)$

$$p_i = \frac{1}{1+e^{-\theta_i}}$$

$$y_i = \begin{cases} 1 \\ 0 \end{cases}$$

$p(1 \mid p_i) = p_i$

$p(0 \mid p_i) = 1 - p_i$

$$p(y_i \mid p_i) = p_i^{y_i}(1-p_i)^{1-y_i}$$

$$-\log p(y_i \mid p_i) = -y_i \log p_i - (1-y_i)\log(1-p_i)$$

$$-\log L(a,b,c) = \sum_i -\log p(y_i \mid p_i)$$

minimize to find a, b, c

$$y_i = 2 + \cos(t_i) + \varepsilon_i \qquad \varepsilon_i \sim N(0, \sigma^2)$$

$$y_i = a + b\underbrace{\cos(t_i)}_{x_i} + \varepsilon_i \qquad a, b \text{ unknown}$$

CLAIM: can use what we know to est a, b

$$y_i = a + b x_i + \varepsilon_i \qquad \text{linear model}$$

est a, b least-squares

$$y = X\beta + \varepsilon$$

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

LEAST-SQUARES SOL'N FOR MATRIX LINEAR MODEL