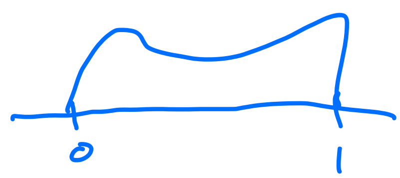
Oct 23

Homework 4

Problem 2



Data: 0.04, 0.55, ...

proportions (numbers in [0,1])

MODEL $X \sim \text{Beta}(\alpha, \beta)$

density $f_X(x) = \dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$

Problem 1

$\underset{k_1}{4}, \underset{k_2}{5}, \underset{k_3}{4}, 2, \ldots\ldots, 5$ DATA

USE MLE to fit Poisson distribution

MODEL

λ parameter $X_i \sim \text{Poisson}(\lambda)$ $\lambda = E(X_i)$

Estimate λ from the DATA

We already know λ is estimated by $\bar{X}$

Make a plot of Poisson likelihood as a function of λ

What is likelihood function?

$$p(k) = P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$p(4) = \frac{e^{-\lambda}\lambda^4}{4!}$$

$$\prod_{i=1}^{27} p(k_i) = \frac{e^{-\lambda}\lambda^4}{4!}\,\frac{e^{-\lambda}\lambda^5}{5!}\ \ldots\ldots$$

$$\log\left(\prod_{i=1}^{27} p(k_i)\right) = \sum_i \log p(k_i)$$
$$= \sum_{i=1}^{27}\left[-\lambda + k_i \log\lambda - \log k_i!\right]$$

$L(\lambda)$

Plot $L(\lambda)$ as a function of λ

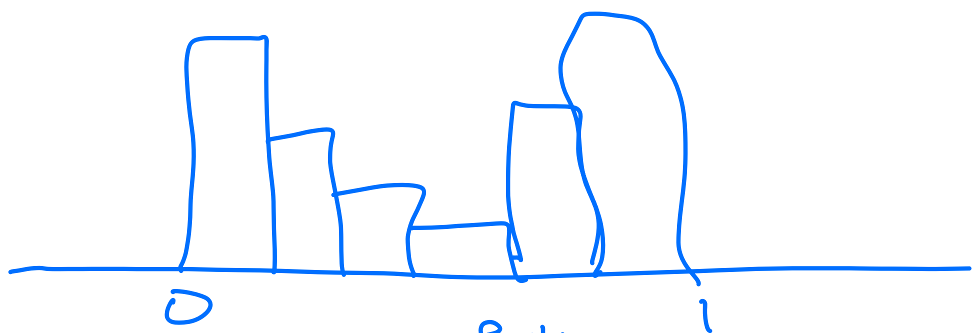
$$P(k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$L(\lambda) = \prod_{i=1}^{27} p(k_i)$$

Estimate λ from data

$$P(X=k) = \frac{e^{-\hat\lambda}\hat\lambda^k}{k!}$$

Proportion of data having value k



$$f(0.4) = (0.4)^{\alpha-1}(1-0.4)^{\beta-1}/B(\alpha,\beta)$$

$$B(\alpha,\beta) = \int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du$$

$$f(u) = \frac{u^{\alpha-1}(1-u)^{\beta-1}}{B(\alpha,\beta)}$$

Define a function
input is α, β
output

$$L(\alpha,\beta) = \prod_{i=1}^{100} f(u_i)$$

Beta pdf (α,β)

Estimate α and β from data

$\hat{\alpha}$ $\hat{\beta}$

$\mathbb{P}(X \le 0.20)$ $\leftarrow$ cdf of $\text{Beta}(\alpha,\beta)$ at 0.20

$\uparrow$ $\text{Beta}(\alpha,\beta)$

$L(\alpha,\beta)$ is a function of α,β

[The data is fixed at values given to you]

optimize: Find α,β so that $L(\alpha,\beta) = \max\limits_{a,b} L(a,b)$

General procedure

DATA $x_1, \dots, x_n$

MODEL $\rightarrow$ $\text{Poisson}(\lambda)$ $\leftarrow$ λ

$\rightarrow$ $\text{Beta}(\alpha,\beta)$ $\leftarrow$ α,β

IDENTIFY PARAMETERS

WRITE DOWN LIKELIHOOD FUNCTION!

$$L(\lambda) = \prod_{i=1}^{27} \frac{e^{-\lambda}\lambda^{k_i}}{k_i!}$$

$$L(\alpha,\beta) = \prod_{i=1}^{100} \frac{u_i^{\alpha-1}(1-u_i)^{\beta-1}}{B(\alpha,\beta)}$$