OFFICE HRS

MW $12^{30} - 1^{30}$

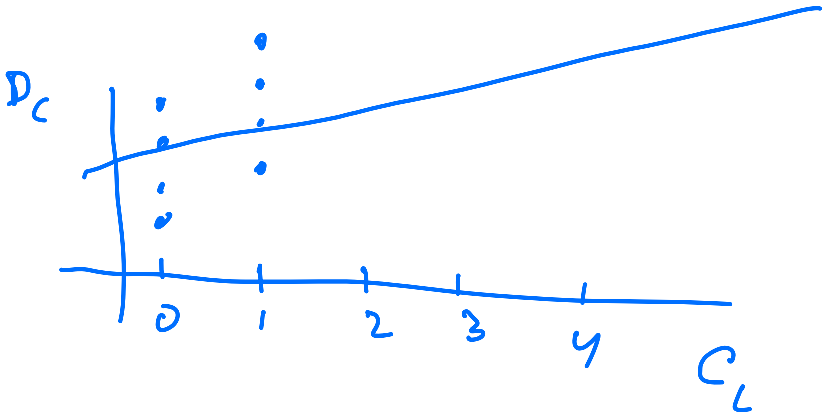
Monday Nov 13

HW: Due Thurs 11:59 PM

Properties of cov operator X, Y, Z any random variables

$$\mathrm{Cov}(X, Y + Z) = \mathrm{Cov}(X,Y) + \mathrm{Cov}(X,Z)$$

$$\mathrm{Cov}(aX, Y) = a\,\mathrm{Cov}(X,Y)$$

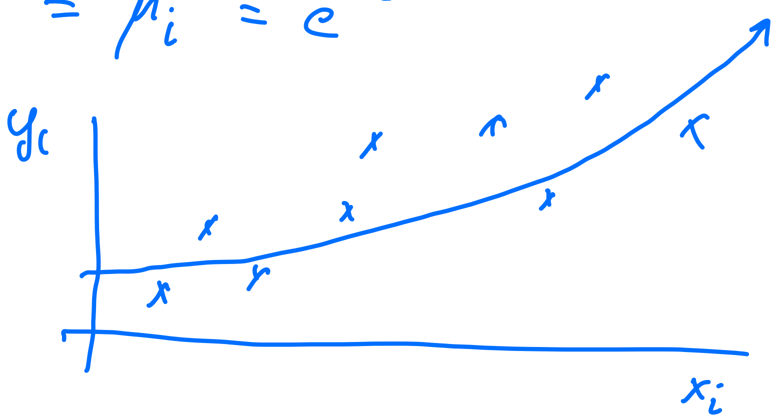


Problem 3

$Y_i \,,\; X_i \qquad\qquad Y_i \sim \mathrm{Poisson}(\mu_i)$

$\mu_i \updownarrow x_i$

$$\mathbb{E} Y_i = \mu_i = e^{a x_i + b}$$



$$\mathrm{Poisson}(k|\mu) = \frac{e^{-\mu}\mu^k}{k!}$$

$$\log p(y_i) = -\mu + y_i \log \mu_i - \log y_i.$$

$$\log p(y_i) = -e^{(a x_i + b)} + y_i(a x_i + b) - \log y_i$$

$$\log \mu_i = a x_i + b$$

$$\sum_{i=1}^{n} \log p(y_i) = L(a,b) \qquad \text{optimize to find } \hat{a}, \hat{b}$$

$y_i = a + b x_i + \varepsilon_i$

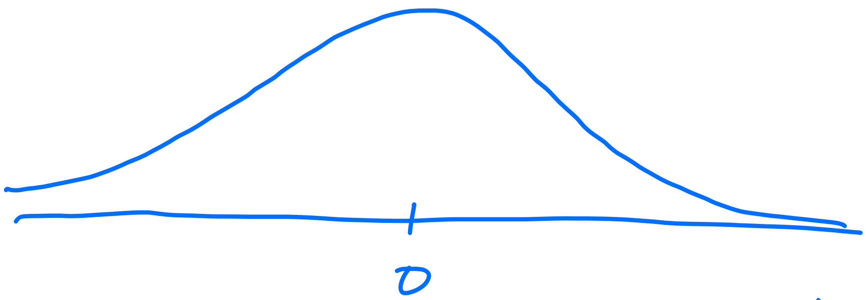
$\hookrightarrow$ $\cancel{N(0,\sigma^2)}$ Cauchy(scale $= \sigma$)

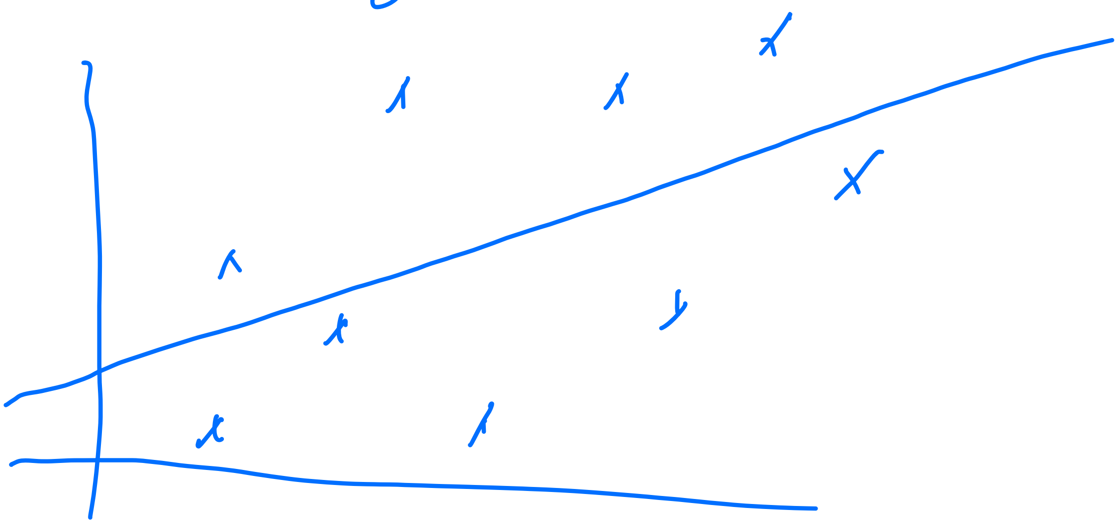
$y_i \sim N(a + b x_i, \sigma)$

MLE of a, b $\Longleftrightarrow$ intercept + slope of LS line

$\varepsilon_i \sim$ other distribution

$$f(u) = \frac{1}{\pi\sigma\left(1 + (u/\sigma)^2\right)}$$





<u>Fact</u>

$f(z)$ is density of Z

$W = \mu + Z$

density of W is $f(w - \mu)$

$$\mu_i = \mathbb{E} Y_i$$

Here $\mu_i = a + b x_i$

estimate a, b from data (β)

$\hat{a}, \hat{b}$ $\qquad \hat{\beta}$

$$y_i = a + b x_i + \varepsilon_i$$

predicted $y_i = \hat{a} + \hat{b} x_i = \hat{y}_i$

Normal

MLE $\Leftrightarrow$ minimize $\quad \sum_i (y_i - \hat{y}_i)^2$

$$\hat{y}_i = a + b x_i$$

Cauchy

MLE minimize $\quad \sum_i \log(1 + (y_i - \hat{y}_i)^2)$

$$Y \sim \text{Poisson}(\exp(Xb))$$

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}$$

$$Y_i \sim \text{Poisson}(\mu_i)$$

$$\mu_i = \exp((Xb)_i)$$

$$X = \begin{bmatrix} X_{11} & \cdots & X_{1K} \\ & & \\ X_{n1} & & X_{nK} \end{bmatrix}$$

$$\mathbb{E} Y_i = e^{a + b x_i} \qquad h(\mu) = \log(\mu)$$

$$\log \mathbb{E} Y_i = a + b x_i$$

$$p(x) = \frac{1}{1 + e^{-(a+bx)}}$$

a, b parameters fit from data

$Y_i \sim \text{Binomial}(1, p(x_i))$

x_i = height of flower

Data: 1 and 0's for y's

x_i = linear measurements