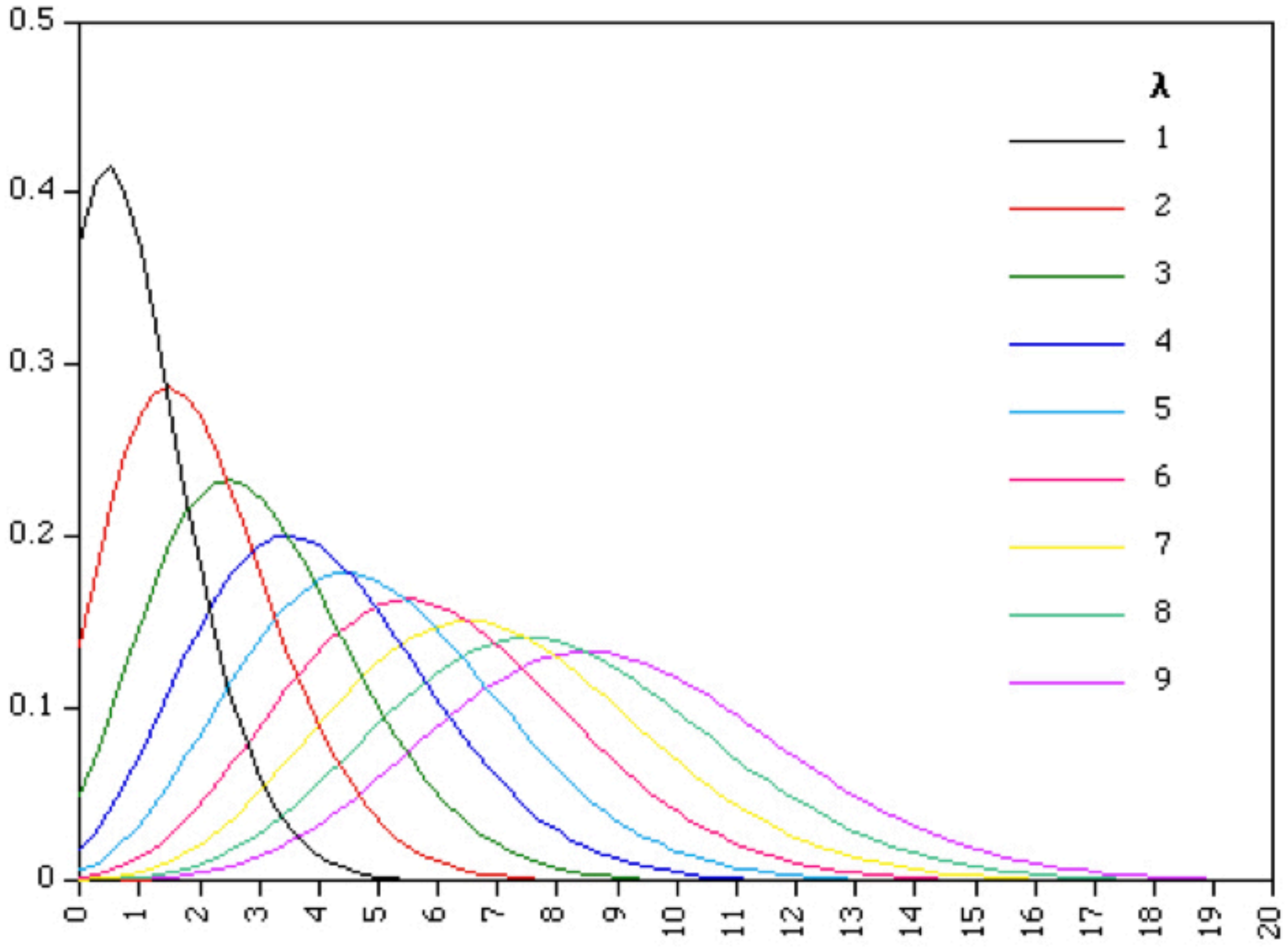
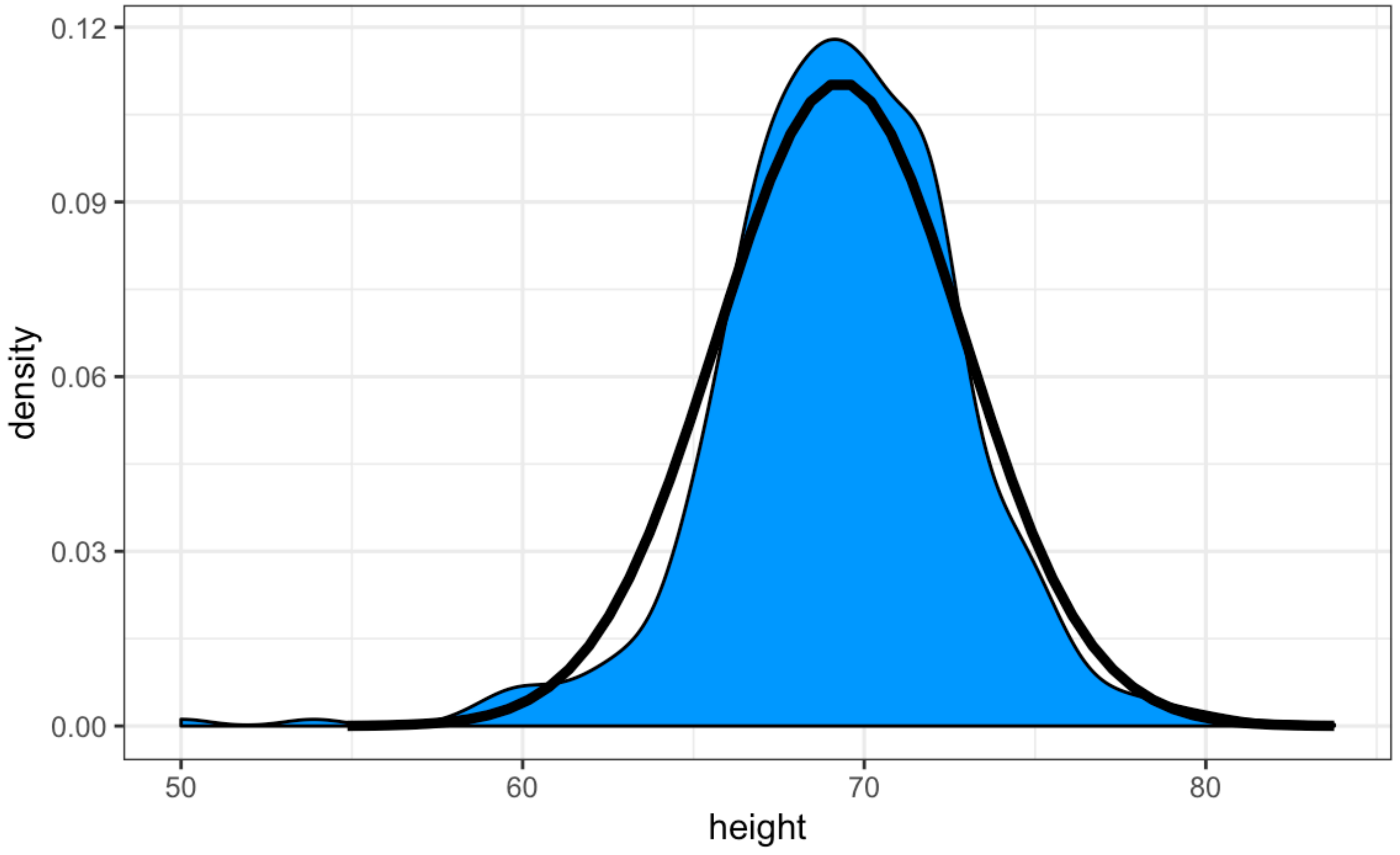
Foundational Statistics

Introduction to Random Variables and Probability Distributions





Probability (and probability distributions) help us:

1. Model the phenomena that give rise to the properties of our data

2. Understand and quantify uncertainty in our estimates and hypothesis tests

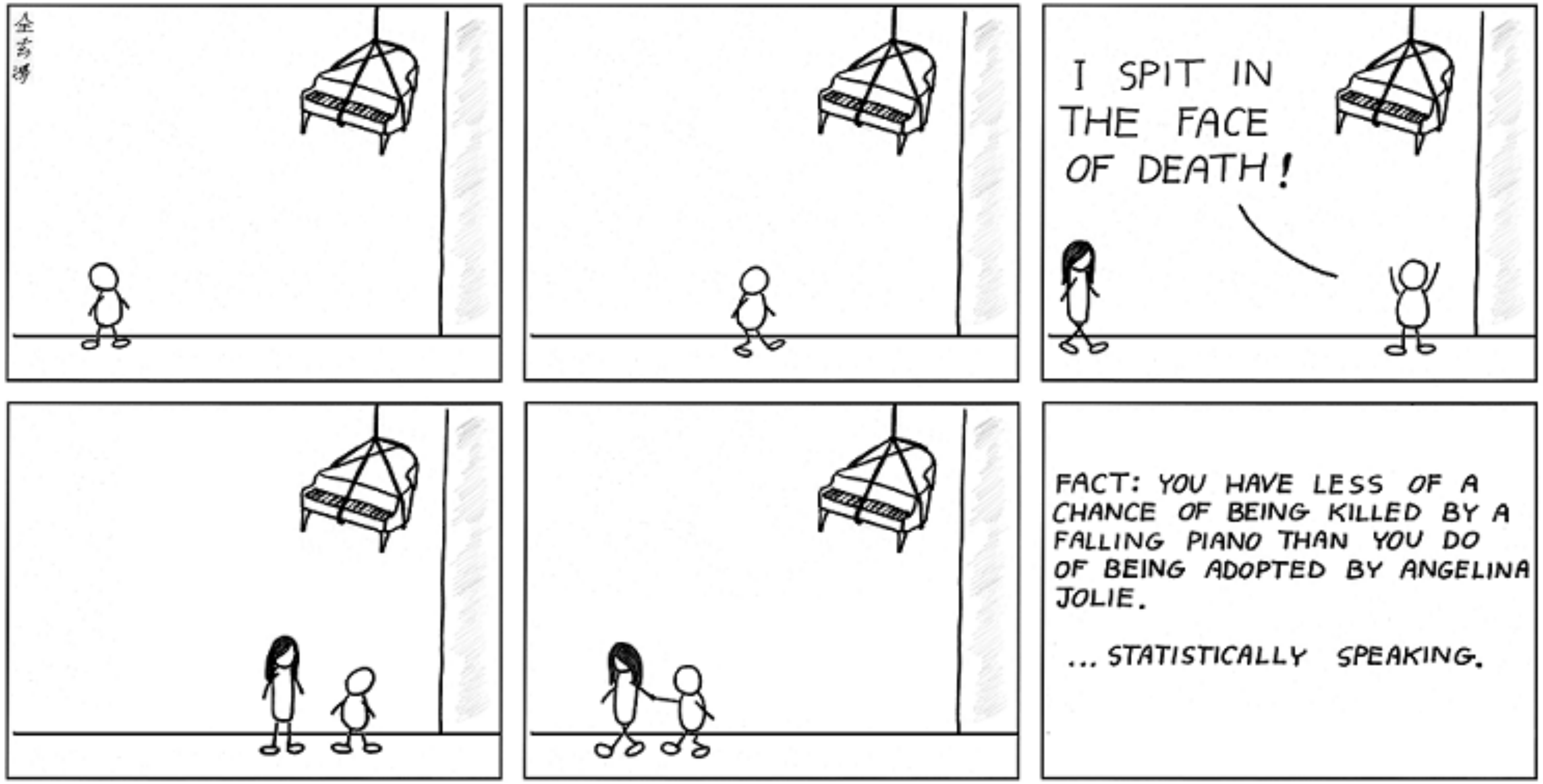
Probability conceptualizations

Frequency-based or "frequentist"

Observed frequencies approximate mathematical expectations

Bayesian

Statements about the certainty of an event or events, as informed by contextual knowledge



Random variables have probability distributions

Probability is the expression of belief in some future outcome

Pr(x=H) = 1/2

A **random variable** can take on different values with different probabilities (e.g. Heads or Tails, at p=0.5 and 1-p=0.5)

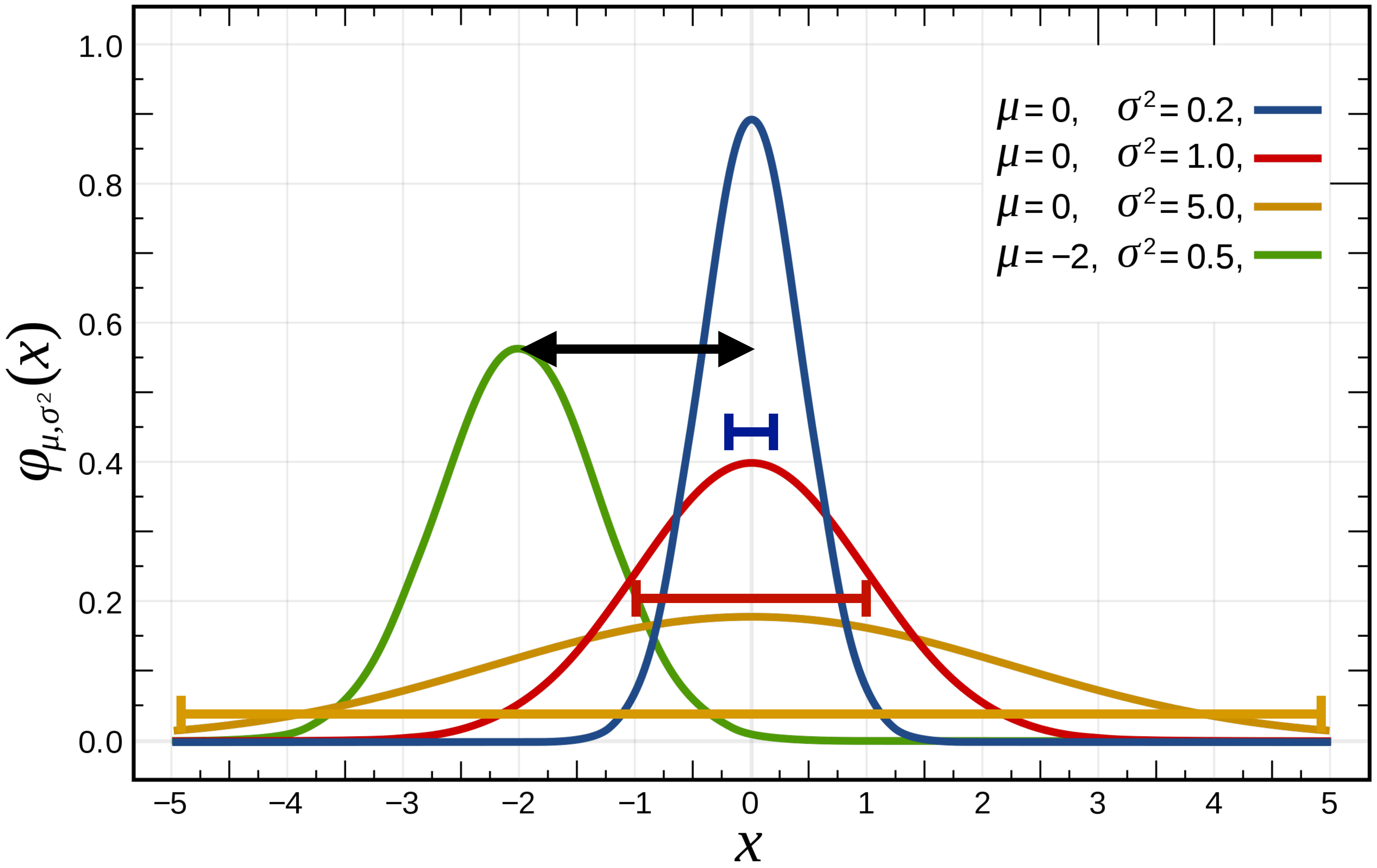
The **sample space** of a random variable is the universe of possible values

The sample space can be more richly described by a **probability distribution** (discrete) or **probability density function** (PDF - continuous)

Probability distributions are functions with features called "moments"

1st moment: Expected value or "mean"

2nd moment: Variance



4 different normal distributions

The Bernoulli probability distribution as a model for a single binary outcome

A Bernoulli distribution is the expected outcome of a single event with probability **p**

Example - flipping of a fair coin

$$
\Pr(X = x) = \begin{cases} p & \text{if } x = \text{Head} \\ 1-p & \text{if } x = \text{Tails} \end{cases}
$$

(A “fair” coin could be modeled by this dist. with, $p = 0.5$)
(If coin not fair, $p \neq 0.5$)

The Bernoulli probability distribution as a model for a single binary outcome

How many Bernoulli experiments would we have to do before for our observations consistently approximate the theoretical probability distribution?

If $p = 0.5$, would 1 coin flip show a good approximation?

How about 8 coin flips?

1000 coin flips?