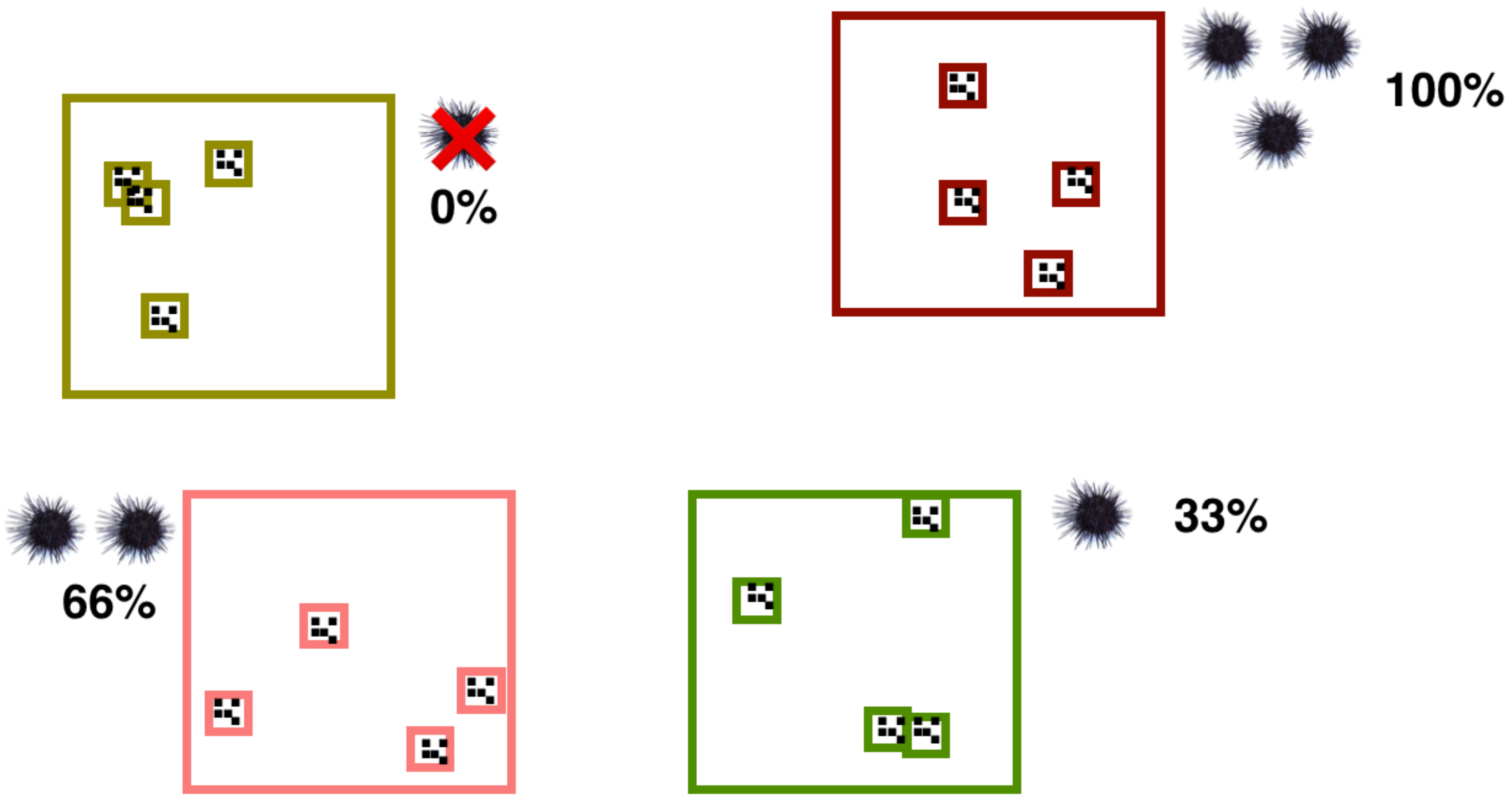
Foundational Statistics

Multi-factor ANOVA Models



Factor A (A_i)	Density	Density mean $\bar{y}_i$ est μ_i	Factor B ($B_{j(i)}$)	Patch	Patch mean $\bar{y}_{j(i)}$ est $\mu_{j(i)}$
A_1	0%	39.2	$B_{1(1)}$	1	34.2
			$B_{2(1)}$	2	62.0
			$B_{3(1)}$	3	2.2
			$B_{4(1)}$	4	58.4
A_2	33%	19.0	$B_{1(2)}$	5	2.6
			$B_{2(2)}$	6	0.0
			$B_{3(2)}$	7	37.6
			$B_{4(2)}$	8	35.8
A_3	66%	21.6	$B_{1(3)}$	9	28.4
			$B_{2(3}$	10	36.8
			$B_{3(3)}$	11	1.0
			$B_{4(3}$	12	20.0
A_4	100%	1.3	$B_{1(4)}$	13	1.6
			$B_{2(4)}$	14	0.0
			$B_{3(4)}$	15	1.0
			$B_{4(4)}$	16	2.6

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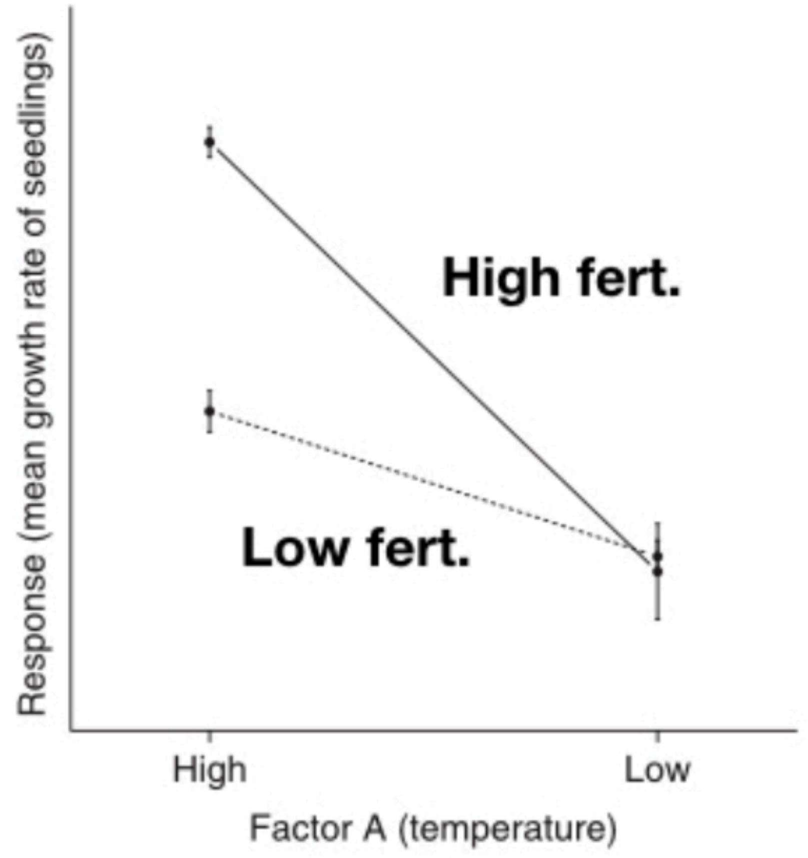
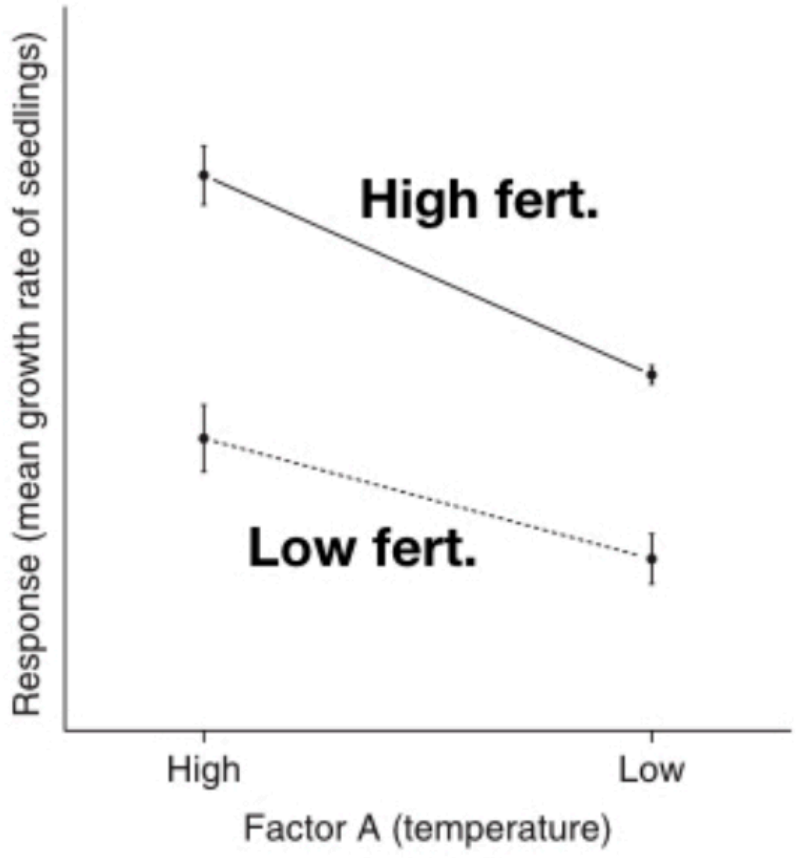




A note from last time on nonparametric ANOVA alternatives

Randomization Tests: Repeatedly shuffle observation labels, calculate an *F*-ratio each time, and use these to approximate a null sampling distribution.

Kruskal-Wallis Test: Rank-based test that is similar to the Mann-Whitney U test, but for >2 groups.

(For post-hoc tests, can do pairwise Mann-Whitney U tests, and correct p-values for multiple comparisons)

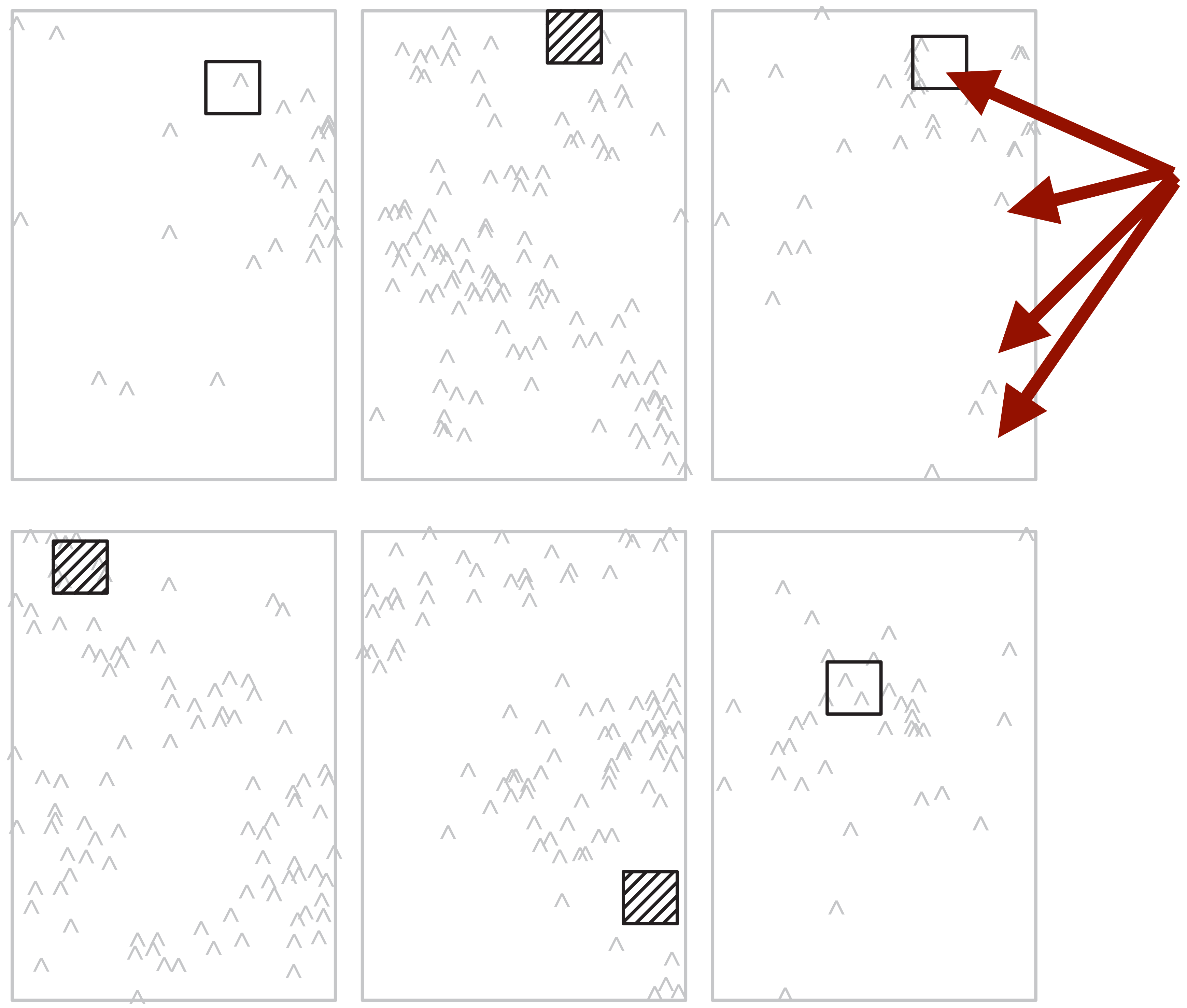
Multi-factor ANOVAs can take different forms

- ***Nested*** - the levels of one factor are contained exclusively within a level of another factor
- Factors are “hierarchical”

- ***Factorial*** - two or more factors and their interactions
- Useful for investigating multiple factors of interest at the same time.
- Factor level combinations are required
- Used to investigate **interactions**: the effect of one variable on the response depends on the state of a second variable.

Nested ANOVA Models

No Nesting



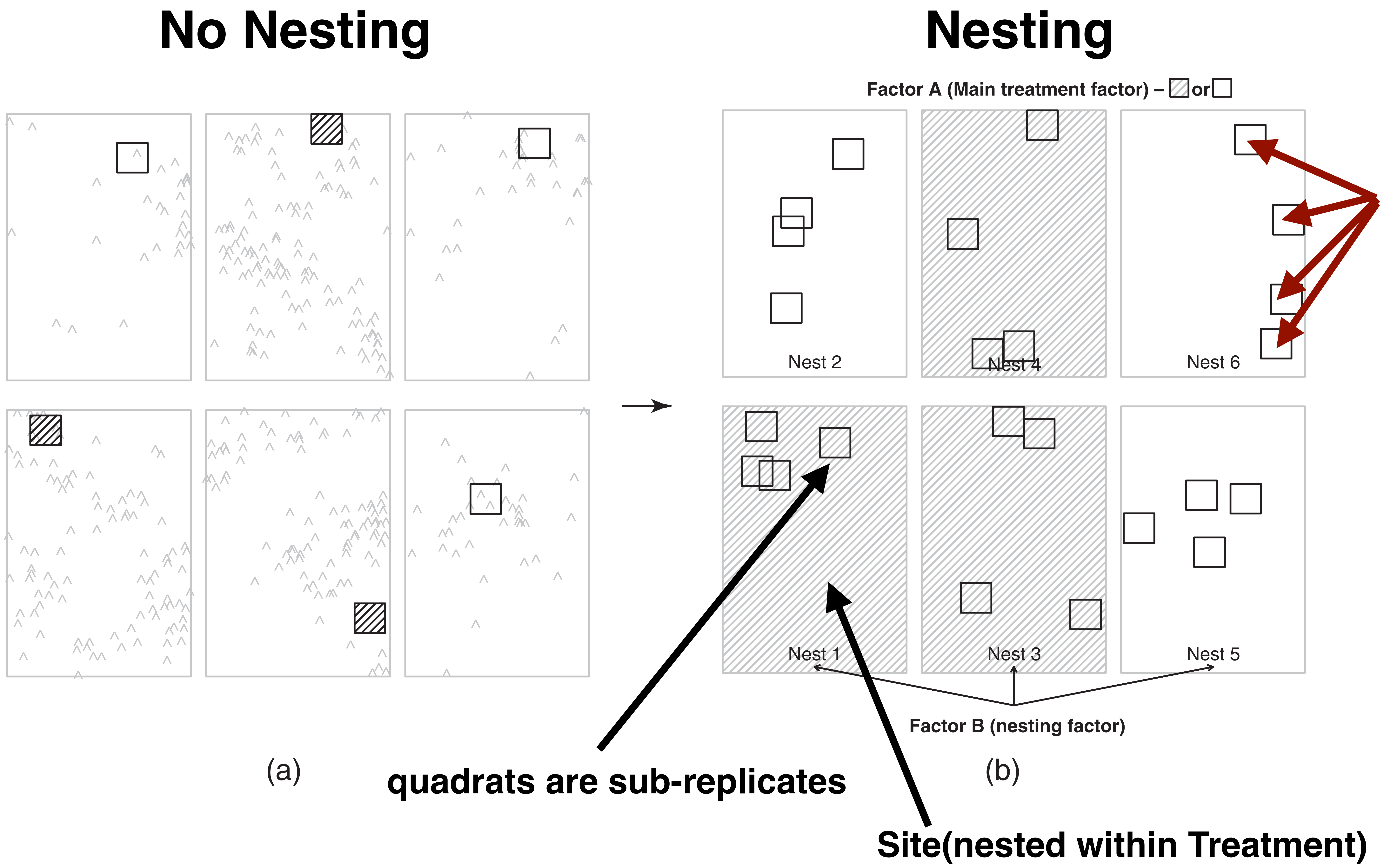
(a)

Sites are highly heterogeneous.

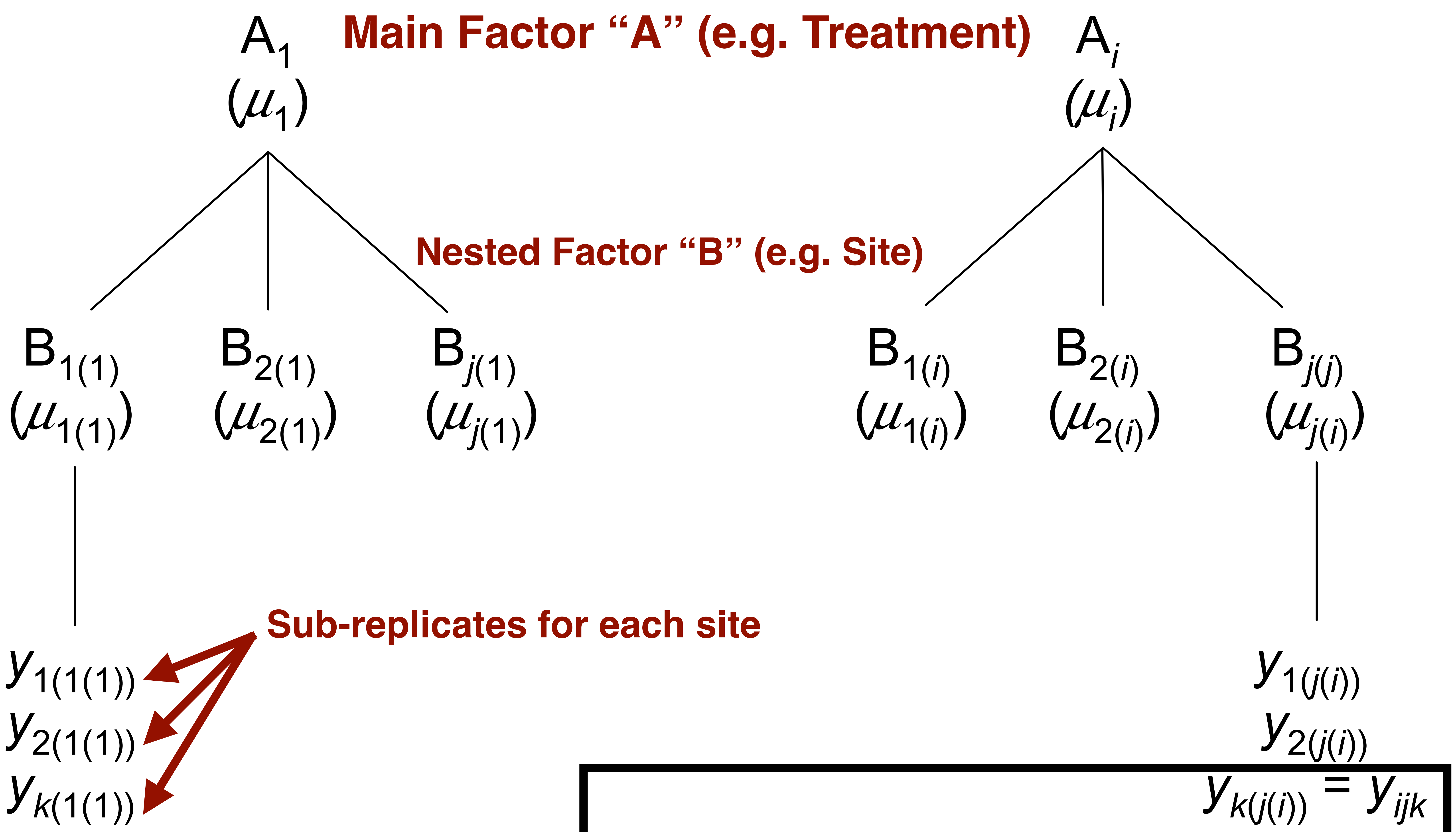
How will this affect our ability to detect an effect of burning?

Nested ANOVA Models

Nesting allows us to "subsample" quadrats per site, and therefore average over site heterogeneity.



Nested ANOVA Models



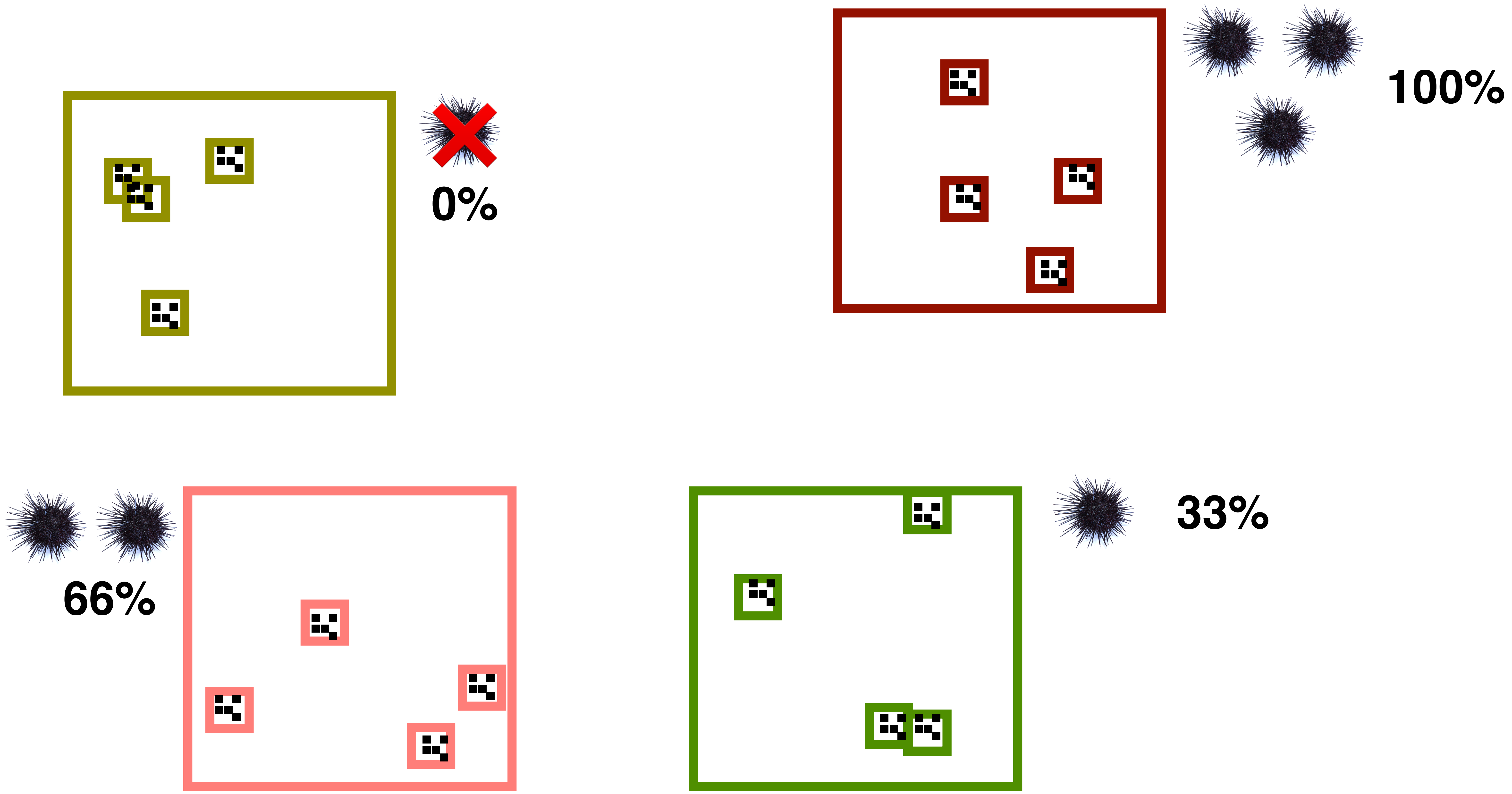
$y_{k(j(i))} = y_{ijk}$

observation *k* within Site *j* within Treatment *i*

Nested ANOVA Models

Data from Andrew and Underwood (1993)

Goal was to test for an effect of sea urchin density on filamentous algae abundance.



Nested ANOVA Models

One level of nesting:

$$y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk}$$

Effects of Factor A levels
(e.g. sea urchin density)

Effects of Factor B levels nested
within A levels (e.g. patches)

Fixed effects (for the main factor)

$$\mathrm{H}_0(A): \mu_1 = \mu_2 = \cdots = \mu_i = \mu$$

Means for all Factor A levels are equal

Random effects (for the nested factor)

$$\mathrm{H}_0(B): \sigma_\beta^2 = 0$$

There is no added variance due to all possible levels of B within all possible levels of A

Factorial ANOVA Models

Experiment from Relyae et al. (2003)

Studied tadpole survival rate as a response to 1. carbaryl pesticide treatment and 2. presence of a newt predator.

Factor level combinations:
1.6 mg/L carbaryl with predator
0.0 mg/L carbaryl without predator
1.6 mg/L carbaryl without predator
0.0 mg/L carbaryl with predator

4 replicates for each combination

All 4 possible factor level combos are represented: “2x2 full factorial”

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

Main Effects

Interaction term

Factorial ANOVA Models -
What does an “interaction” look like?

Factorial ANOVA Hypotheses

Factor A

$$\mathrm{H}_0(A)\colon \mu_1 = \mu_2 = \cdots = \mu_i = \mu$$

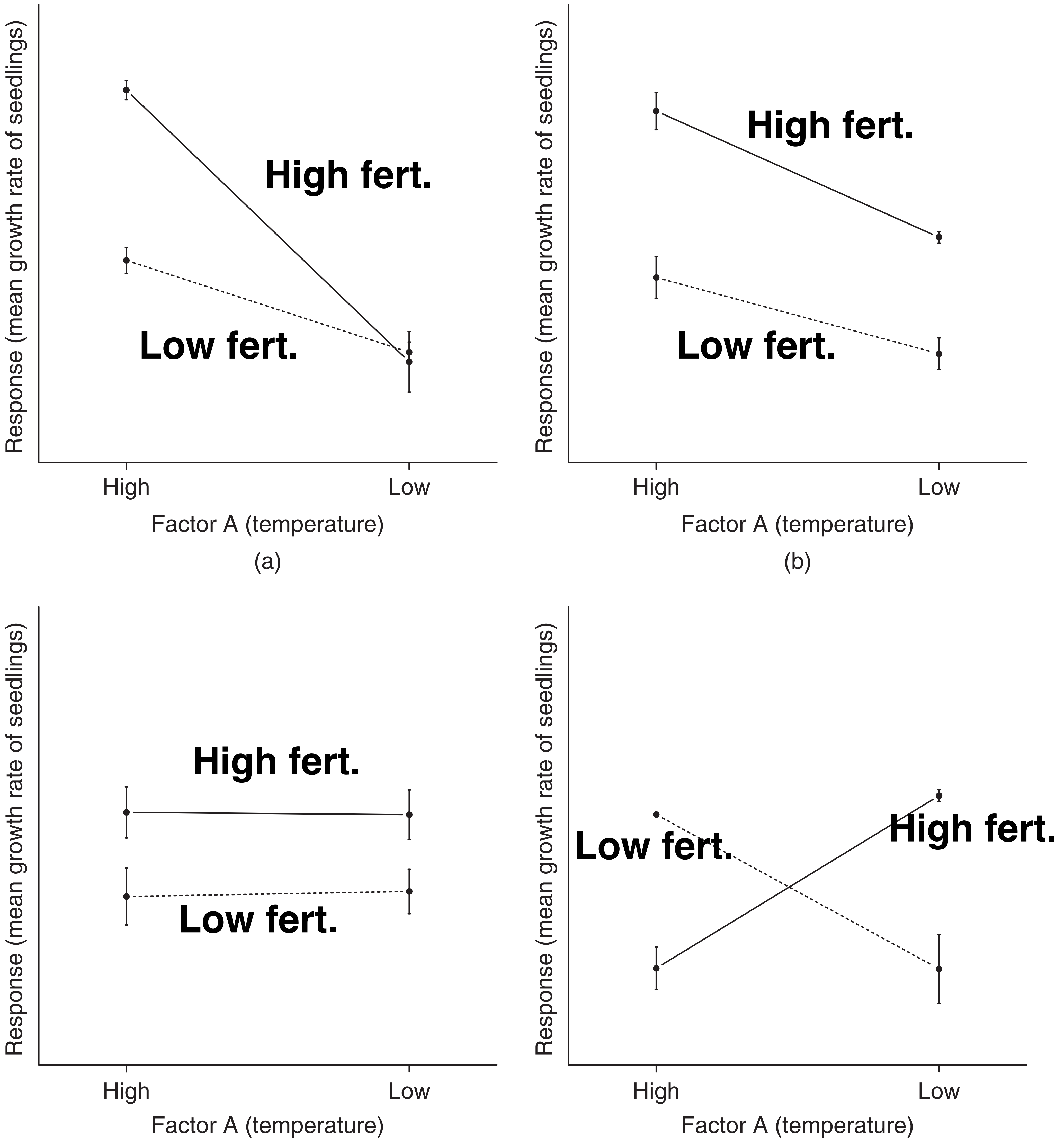
Factor B

$$\mathrm{H}_0(B)\colon \mu_1 = \mu_2 = \cdots = \mu_i = \mu$$

A:B Interaction

$$\mathrm{H}_0(AB)\colon \mu_{ij} = \mu_i + \mu_j - \mu$$

Interpreting significant main and interaction effects: **Interaction Plots**



Multi-factor ANOVA **Assumptions**

- <u>Variation within groups follows a normal distribution</u> **(needs to be satisfied for all factor-level combinations for factorial design)**

- <u>**Equal variance** among groups, and no strong mean-variance relationship</u> (satisfied for all **factor-level combinations for factorial design)**

- <u>**Measurements/observations within groups are random samples, and independent**</u> **(unless there is nesting)**