Patterns Overview & Map Pattern

Parallel Computing

CIS 410/510

Department of Computer and Information Science

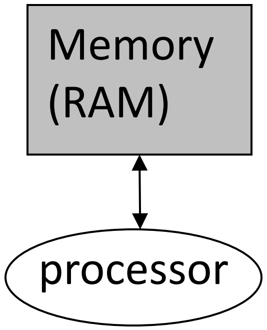
UNIVERSITY OF OREGON

Overview

- Dependencies
- Patterns
  - Serial/Parallel Control Patterns
  - Serial/Parallel Data Management Patterns
- Map Pattern
  - Map
  - Optimizations
  - Related Patterns
  - Example

Parallel Models 101

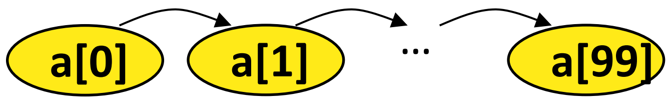
- Sequential models
  - von Neumann (RAM) model
- Parallel model
  - A parallel computer is simple a collection of *processors interconnected* in some manner to *coordinate* activities and *exchange data*
  - Models that can be used as general frameworks for describing and analyzing parallel algorithms
    - *Simplicity*: description, analysis, architecture independence
    - *Implementability*: able to be realized, reflect performance
- Three common parallel models
  - Directed acyclic graphs, shared-memory, network



Directed Acyclic Graphs (DAG)

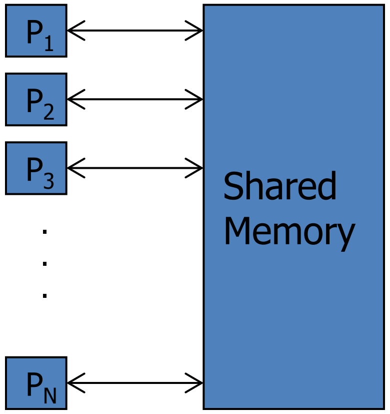
- Captures data flow parallelism
- Nodes represent operations to be performed
  - Inputs are nodes with no incoming arcs
  - Output are nodes with no outgoing arcs
  - Think of nodes as tasks
- Arcs are paths for flow of data results
- DAG represents the operations of the algorithm and implies precedent constraints on their order

```
for (i=1; i<100; i++)
      a[i] = a[i-1] + 100;
```



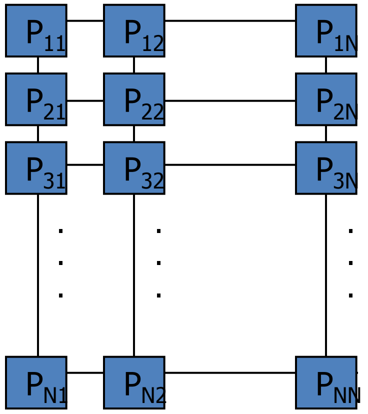
Shared Memory Model

- Parallel extension of RAM model (PRAM)
  - Memory size is infinite
  - Number of processors in unbounded
  - Processors communicate via the memory
  - Every processor accesses any memory location in 1 cycle
  - Synchronous
    - All processors execute same algorithm synchronously
      - READ phase
      - COMPUTE phase
      - WRITE phase
    - Some subset of the processors can stay idle
  - Asynchronous



Network Model

- G = (N,E)
  - N are processing nodes
  - E are bidirectional communication links
- Each processor has its own memory
- No shared memory is available
- Network operation may be synchronous or asynchronous
- Requires communication primitives
  - Send (X, i)
  - Receive (Y, j)
- Captures message passing model for algorithm design



Parallelism

- Ability to execute different parts of a computation concurrently on different machines
- Why do you want parallelism?
  - Shorter running time or handling more work
- What is being parallelized?
  - Task: instruction, statement, procedure, …
  - Data: data flow, size, replication
  - Parallelism granularity
    - Coarse-grain versus fine-grainded
- Thinking about parallelism
- Evaluation

Why is parallel programming important?

- Parallel programming has matured
  - Standard programming models
  - Common machine architectures
  - Programmer can focus on computation and use suitable programming model for implementation
- Increasing portability between models and architectures
- Reasonable hope of portability across platforms
- Problem
  - Performance optimization is still platform-dependent
  - Performance portability is a problem
  - Parallel programming methods are still evolving

Parallel Algorithm

- Recipe to solve a problem "in parallel" on multiple processing elements
- Standard steps for constructing a parallel algorithm
  - Identify work that can be performed concurrently
  - Partition the concurrent work on separate processors
  - Properly manage input, output, and intermediate data
  - Coordinate data accesses and work to satisfy dependencies
- Which are hard to do?

Parallelism Views

- Where can we find parallelism?
- Program (task) view
  - Statement level
    - Between program statements
    - Which statements can be executed at the same time?
  - Block level / Loop level / Routine level / Process level
    - Larger-grained program statements
- Data view
  - How is data operated on?
  - Where does data reside?
- Resource view

Parallelism, Correctness, and Dependence

- Parallel execution, from any point of view, will be constrained by the sequence of operations needed to be performed for a correct result
- Parallel execution must address control, data, and system dependences
- A *dependency* arises when one operation depends on an earlier operation to complete and produce a result before this later operation can be performed
- We extend this notion of dependency to resources since some operations may depend on certain resources
  - For example, due to where data is located

Executing Two Statements in Parallel

- Want to execute two statements in parallel
- On one processor:

    Statement 1;

    Statement 2;

- On two processors:

Processor 1:	Processor 2:
Statement 1;	Statement 2;

- Fundamental (*concurrent*) execution assumption
    - Processors execute independent of each other
    - No assumptions made about speed of processor execution

Sequential Consistency in Parallel Execution

- Case 1:

  Processor 1: Processor 2: time ↓

  statement 1;

  statement 2;

- Case 2:

  Processor 1: Processor 2: time ↓

  statement 2;

  statement 1;

- Sequential consistency
  - Statements execution does not interfere with each other
  - Computation results are the same (independent of order)

Independent versus Dependent

- In other words the execution of

  statement1;

  statement2;

  must be equivalent to

  statement2;

  statement1;

- Their order of execution must not matter!
- If true, the statements are *independent* of each other
- Two statements are *dependent* when the order of their execution affects the computation outcome

Examples

- Example 1
  S1: a=1;
  S2: b=1;
- Statements are independent

- Example 2
  S1: a=1;
  S2: b=a;
- Dependent (*true (flow) dependence*)
  - Second is dependent on first
  - Can you remove dependency?

- Example 3
  S1: a=f(x);
  S2: a=b;
- Dependent (*output dependence*)
  - Second is dependent on first
  - Can you remove dependency? How?

- Example 4
  S1: a=b;
  S2: b=1;
- Dependent (*anti-dependence*)
  - First is dependent on second
  - Can you remove dependency? How?

True Dependence and Anti-Dependence

- Given statements S1 and S2,

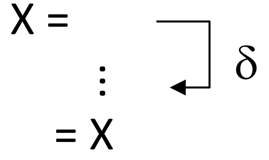
  S1;

  S2;

- S2 has a *true (flow) dependence* on S1

  if and only if

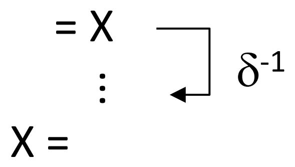
  S2 reads a value written by S1



- S2 has a *anti-dependence* on S1

  if and only if

  S2 writes a value read by S1



Output Dependence

- Given statements S1 and S2,

  S1;

  S2;

- S2 has an *output dependence* on S1

  if and only if

  S2 writes a variable written by S1

$$\begin{tikzcd} X = \arrow[dd, bend left, "\delta^0"] \\ \vdots \\ X = \end{tikzcd}$$

- Anti- and output dependences are “name” dependencies
  - Are they “true” dependences?
- How can you get rid of output dependences?
  - Are there cases where you can not?

Statement Dependency Graphs

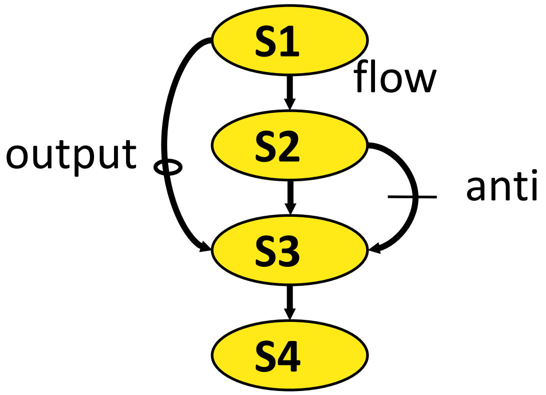
- Can use graphs to show dependence relationships
- Example

  S1: a=1;

  S2: b=a;

  S3: a=b+1;

  S4: c=a;



- $S_2 \ \delta \ S_3$: S_3 is flow-dependent on S_2
- $S_1 \ \delta^0 \ S_3$: S_3 is output-dependent on S_1
- $S_2 \ \delta^{-1} \ S_3$: S_3 is anti-dependent on S_2

When can two statements execute in parallel?

- Statements S1 and S2 can execute in parallel if and only if there are *no dependences* between S1 and S2
  - True dependences
  - Anti-dependences
  - Output dependences
- Some dependences can be remove by modifying the program
  - Rearranging statements
  - Eliminating statements

How do you compute dependence?

- Data dependence relations can be found by comparing the IN and OUT sets of each node
- The IN and OUT sets of a statement S are defined as:
  - IN(S) : set of memory locations (variables) that may be used in S
  - OUT(S) : set of memory locations (variables) that may be modified by S
- Note that these sets include all memory locations that may be fetched or modified
- As such, the sets can be conservatively large

IN / OUT Sets and Computing Dependence

- Assuming that there is a path from S1 to S2 , the following shows how to intersect the IN and OUT sets to test for data dependence

$$out(S_1) \cap in(S_2) \neq \emptyset \quad S_1\ \delta\ S_2 \quad \text{flow dependence}$$

$$in(S_1) \cap out(S_2) \neq \emptyset \quad S_1\ \delta^{-1}\ S_2 \quad \text{anti-dependence}$$

$$out(S_1) \cap out(S_2) \neq \emptyset \quad S_1\ \delta^{0}\ S_2 \quad \text{output dependence}$$

Loop-Level Parallelism

- Significant parallelism can be identified <u>within</u> loops

```
for (i=0; i<100; i++)
    S1: a[i] = i;
```

```
for (i=0; i<100; i++) {
    S1: a[i] = i;
    S2: b[i] = 2*i;
}
```

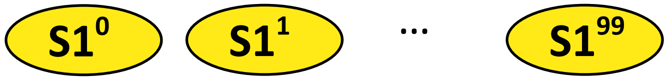
- Dependencies? What about *i*, the loop index?
- *DOALL* loop (a.k.a. *foreach* loop)
  - All iterations are independent of each other
  - All statements be executed in parallel at the same time
    - Is this really true?

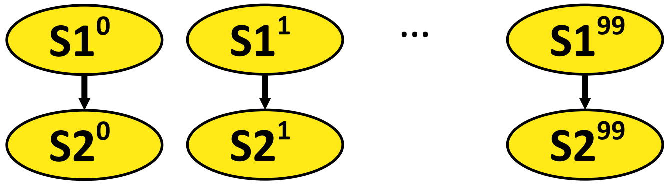
Iteration Space

- Unroll loop into separate statements / iterations
- Show dependences between iterations

```
for (i=0; i<100; i++)
    S1: a[i] = i;
```

```
for (i=0; i<100; i++) {
    S1: a[i] = i;
    S2: b[i] = 2*i;
}
```

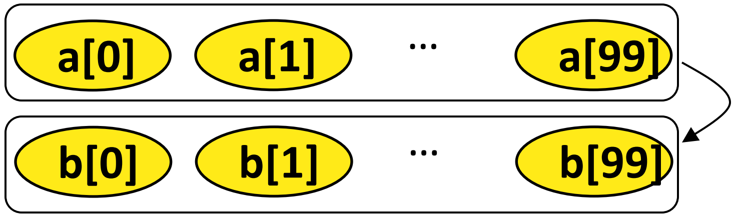




Multi-Loop Parallelism

- Significant parallelism can be identified <u>between</u> loops

```
for (i=0; i<100; i++) a[i] = i;
for (i=0; i<100; i++) b[i] = i;
```

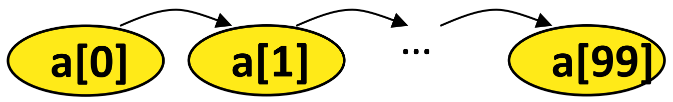


- Dependencies?
- How much parallelism is available?
- Given 4 processors, how much parallelism is possible?
- What parallelism is achievable with 50 processors?

Loops with Dependencies

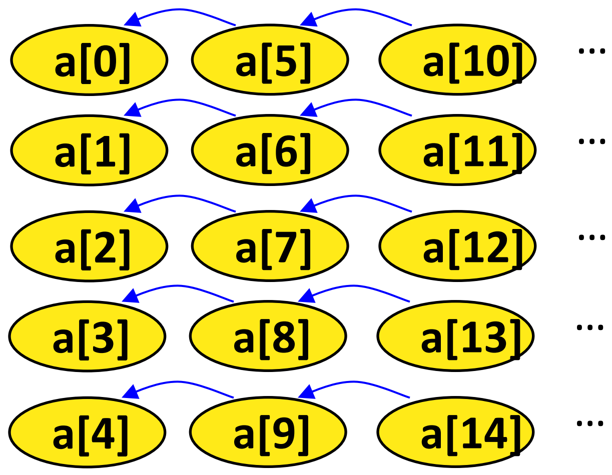
Case 1:

```
for (i=1; i<100; i++)
    a[i] = a[i-1] + 100;
```



Case 2:

```
for (i=5; i<100; i++)
    a[i-5] = a[i] + 100;
```



- Dependencies?
  - What type?
- Is the Case 1 loop parallelizable?
- Is the Case 2 loop parallelizable?

Another Loop Example

```
for (i=1; i<100; i++)
    a[i] = f(a[i-1]);
```

- Dependencies?
  - What type?
- Loop iterations are not parallelizable
  - Why not?

Loop Dependencies

- A *loop-carried* dependence is a dependence that is present only if the statements are part of the execution of a loop (i.e., between two statements instances in two different iterations of a loop)
- Otherwise, it is *loop-independent*, including between two statements instances in the same loop iteration
- Loop-carried dependences can prevent loop iteration parallelization
- The dependence is *lexically forward* if the source comes before the target or *lexically backward* otherwise
  - Unroll the loop to see

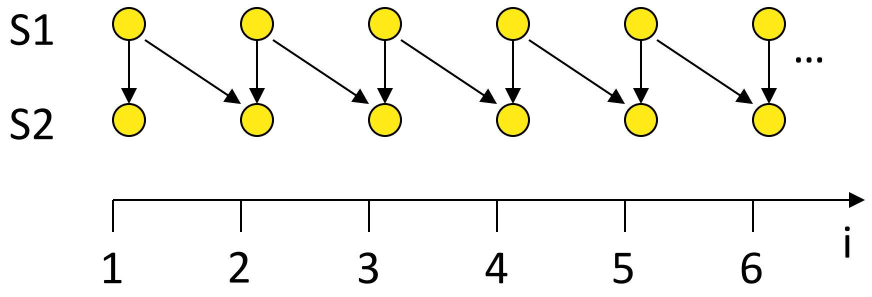
Loop Dependence Example

```
for (i=0; i<100; i++)
    a[i+10] = f(a[i]);
```

- Dependencies?
  - Between a[10], a[20], …
  - Between a[11], a[21], …
- Some parallel execution is possible
  - How much?

Dependences Between Iterations

```
for (i=1; i<100; i++) {
    S1: a[i] = …;
    S2: … = a[i-1];
}
```



- Dependencies?
  - Between a[i] and a[i-1]
- Is parallelism possible?
  - Statements can be executed in "pipeline" manner

Another Loop Dependence Example

```
for (i=0; i<100; i++)
    for (j=1; j<100; j++)
        a[i][j] = f(a[i][j-1]);
```

- Dependencies?
  - Loop-independent dependence on i
  - Loop-carried dependence on j
- Which loop can be parallelized?
  - Outer loop parallelizable
  - Inner loop cannot be parallelized

Still Another Loop Dependence Example

```
for (j=1; j<100; j++)
    for (i=0; i<100; i++)
  a[i][j] = f(a[i][j-1]);
```

- Dependencies?
  - Loop-independent dependence on i
  - Loop-carried dependence on j
- Which loop can be parallelized?
  - Inner loop parallelizable
  - Outer loop cannot be parallelized
  - Less desirable (why?)

Key Ideas for Dependency Analysis

- To execute in parallel:
  - Statement order must not matter
  - Statements must not have dependences
- Some dependences can be removed
- Some dependences may not be obvious

Dependencies and Synchronization

- How is parallelism achieved when have dependencies?
  - Think about concurrency
  - Some parts of the execution are independent
  - Some parts of the execution are dependent
- Must control ordering of events on different processors (cores)
  - Dependencies pose constraints on parallel event ordering
  - Partial ordering of execution action
- Use synchronization mechanisms
  - Need for concurrent execution too
  - Maintains partial order

Parallel Patterns

- **Parallel Patterns**: A recurring combination of task distribution and data access that solves a specific problem in parallel algorithm design.
- Patterns provide us with a "vocabulary" for algorithm design
- It can be useful to compare parallel patterns with serial patterns
- Patterns are universal – they can be used in *any* parallel programming system

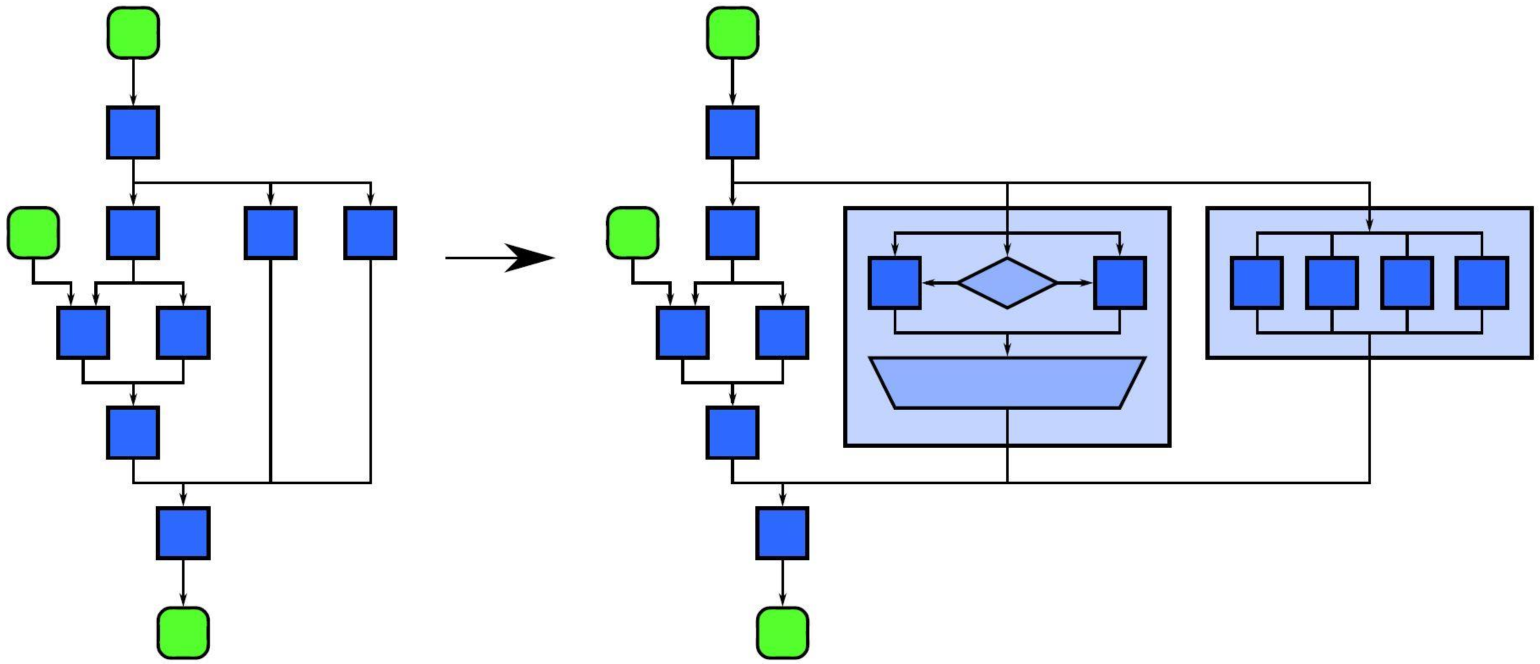
Parallel Patterns

- Nesting Pattern
- Serial / Parallel Control Patterns
- Serial / Parallel Data Management Patterns
- Other Patterns
- Programming Model Support for Patterns

Nesting Pattern

- **Nesting** is the ability to hierarchically compose patterns
- This pattern appears in both serial and parallel algorithms
- In our “pattern diagrams” (see next slide), each “task block” is a location of general code in an algorithm
- Each “task block” can in turn be another pattern – this is the **nesting pattern**

Nesting Pattern



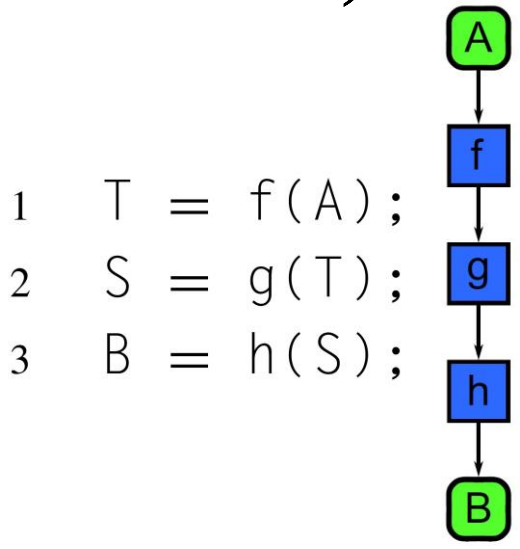
Nesting Pattern: A compositional pattern. Nesting allows other patterns to be composed in a hierarchy so that any task block in the above diagram can be replaced with a pattern with the same input/output and dependencies.

Serial Control Patterns

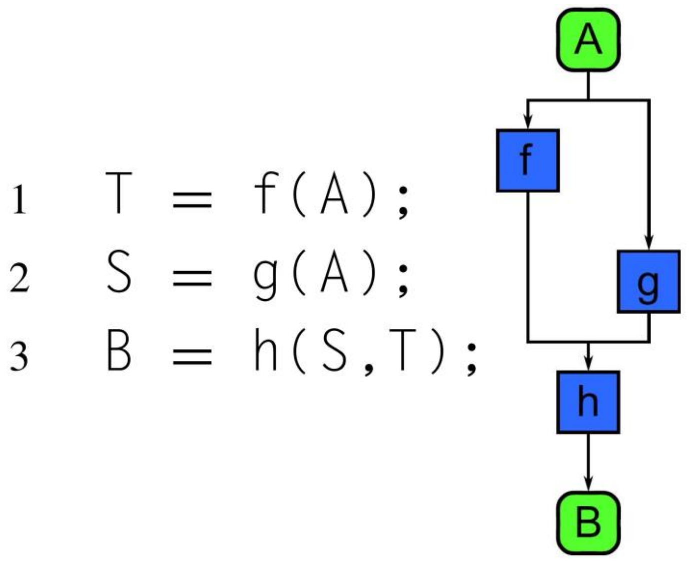
- Structured serial programming is based on these patterns: **sequence**, **selection**, **iteration**, and **recursion**
- The **nesting** pattern can also be used to hierarchically compose these four patterns
- Though you should be familiar with these, it's extra important to understand these patterns when parallelizing serial algorithms based on these patterns

Serial Control Patterns: Sequence

- **Sequence**: Ordered list of tasks that are executed in a specific order
- Assumption – program text ordering will be followed (obvious, but this will be important when parallelized)



```
T = f(A);
S = g(T);
B = h(S);
```

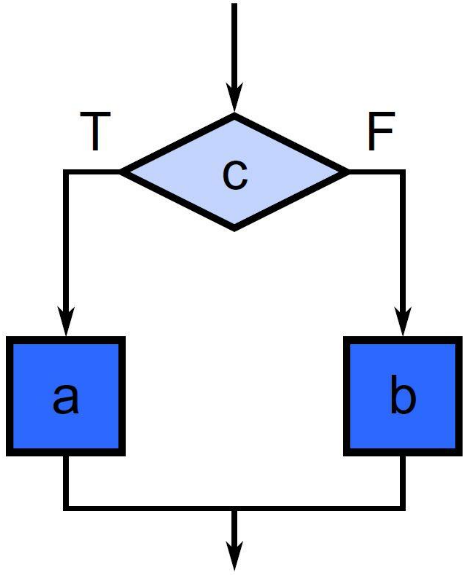


```
T = f(A);
S = g(A);
B = h(S,T);
```

Serial Control Patterns: Selection

- **Selection**: condition *c* is first evaluated. Either task *a* or *b* is executed depending on the true or false result of *c*.
- Assumptions – *a* and *b* are never executed before *c*, and only *a* or *b* is executed - never both

```
if (c) {
  a;
} else {
  b;
}
```

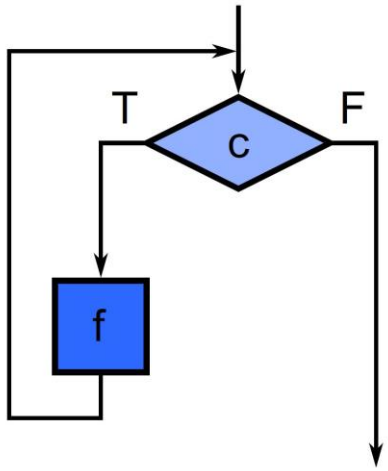


Serial Control Patterns: Iteration

- **Iteration**: a condition *c* is evaluated. If true, *a* is evaluated, and then *c* is evaluated again. This repeats until *c* is false.
- Complication when parallelizing: potential for dependencies to exist between previous iterations

```
for (i = 0; i < n;
  a;
}
```

```
while (c) {
  a;
}
```



Serial Control Patterns: Recursion

- **Recursion**: dynamic form of nesting allowing functions to call themselves
- Tail recursion is a special recursion that can be converted into iteration – important for functional languages

Parallel Control Patterns

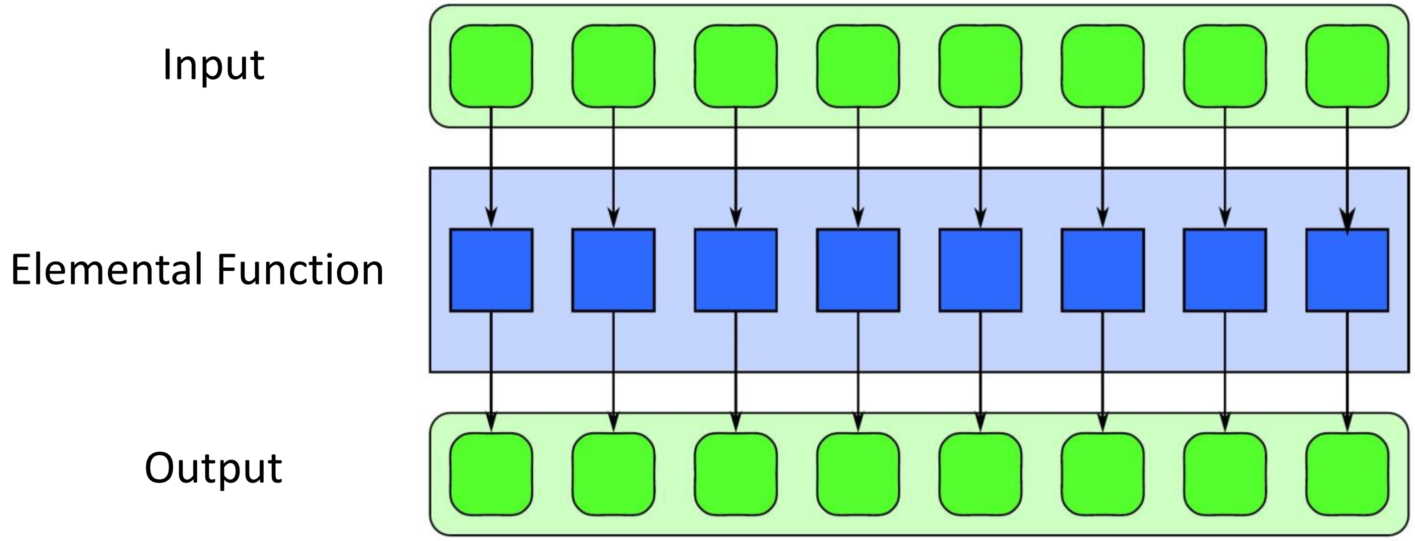
- Parallel control patterns extend serial control patterns
- Each parallel control pattern is related to at least one serial control pattern, but relaxes assumptions of serial control patterns
- Parallel control patterns: **fork-join**, **map**, **stencil**, **reduction**, **scan**, **recurrence**

Parallel Control Patterns: Fork-Join

- **Fork-join**: allows control flow to fork into multiple parallel flows, then rejoin later
- Cilk Plus implements this with **spawn** and **sync**
  - The call tree is a parallel call tree and functions are spawned instead of called
  - Functions that spawn another function call will continue to execute
  - Caller syncs with the spawned function to join the two
- A "join" is different than a "barrier
  - Sync – only one thread continues
  - Barrier – all threads continue

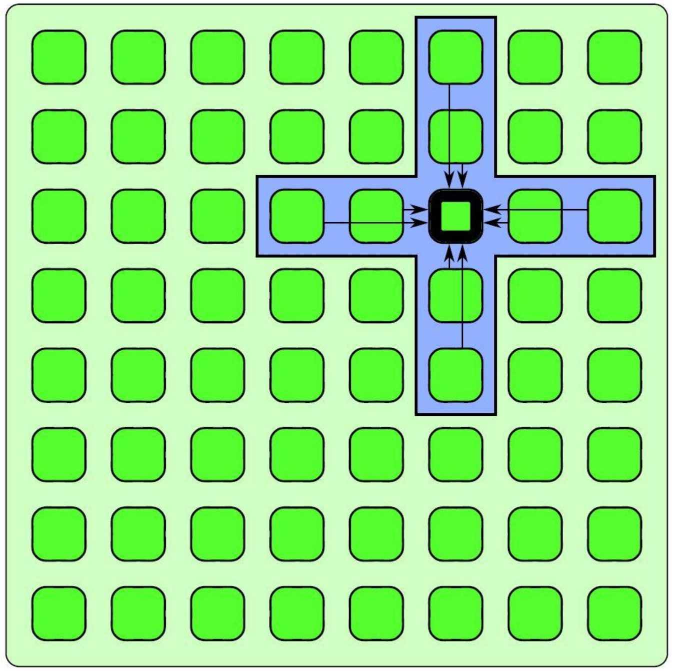
Parallel Control Patterns: Map

- **Map**: performs a function over every element of a collection
- Map replicates a serial iteration pattern where each iteration is independent of the others, the number of iterations is known in advance, and computation only depends on the iteration count and data from the input collection
- The replicated function is referred to as an "elemental function"



Parallel Control Patterns: Stencil

- **Stencil**: Elemental function accesses a set of "neighbors", stencil is a generalization of map
- Often combined with iteration – used with iterative solvers or to evolve a system through time
- Boundary conditions must be handled carefully in the stencil pattern. More on this in the stencil lecture…

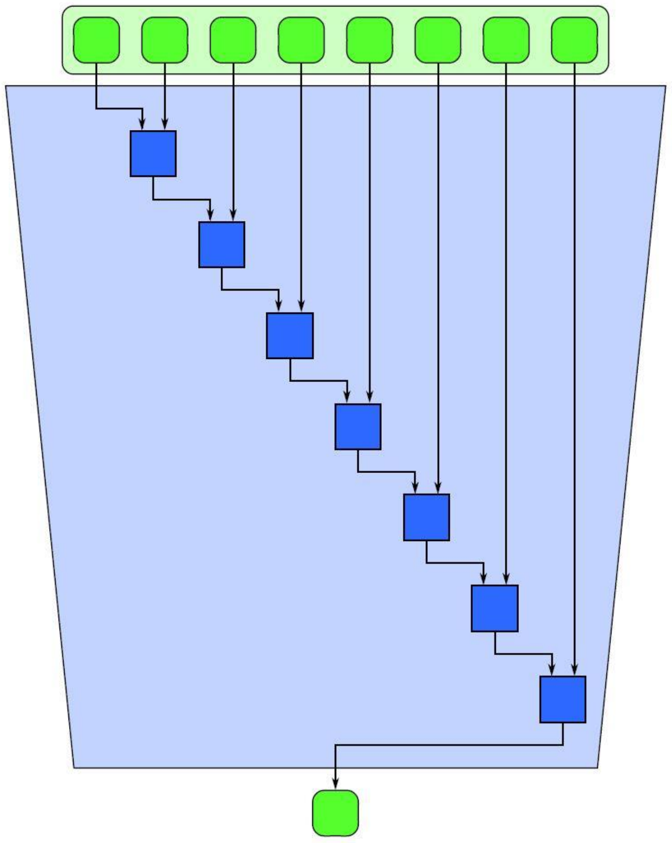


Parallel Control Patterns: Reduction

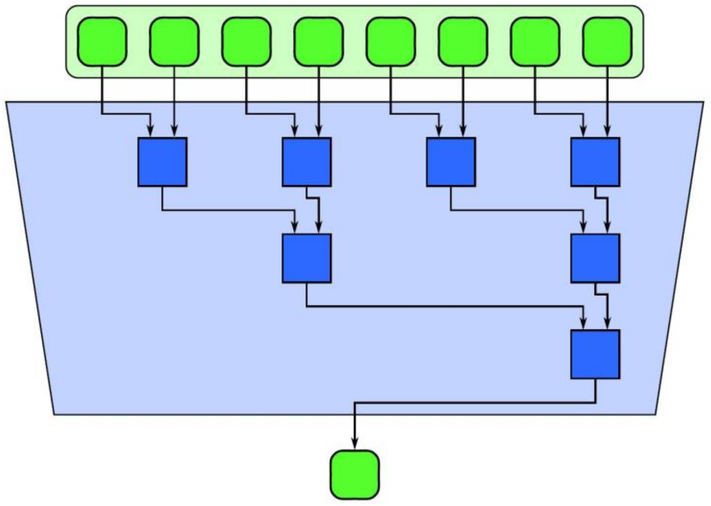
- **Reduction**: Combines every element in a collection using an associative “combiner function”
- Because of the associativity of the combiner function, different orderings of the reduction are possible
- Examples of combiner functions: addition, multiplication, maximum, minimum, and Boolean AND, OR, and XOR

Parallel Control Patterns: Reduction

Serial Reduction



Parallel Reduction

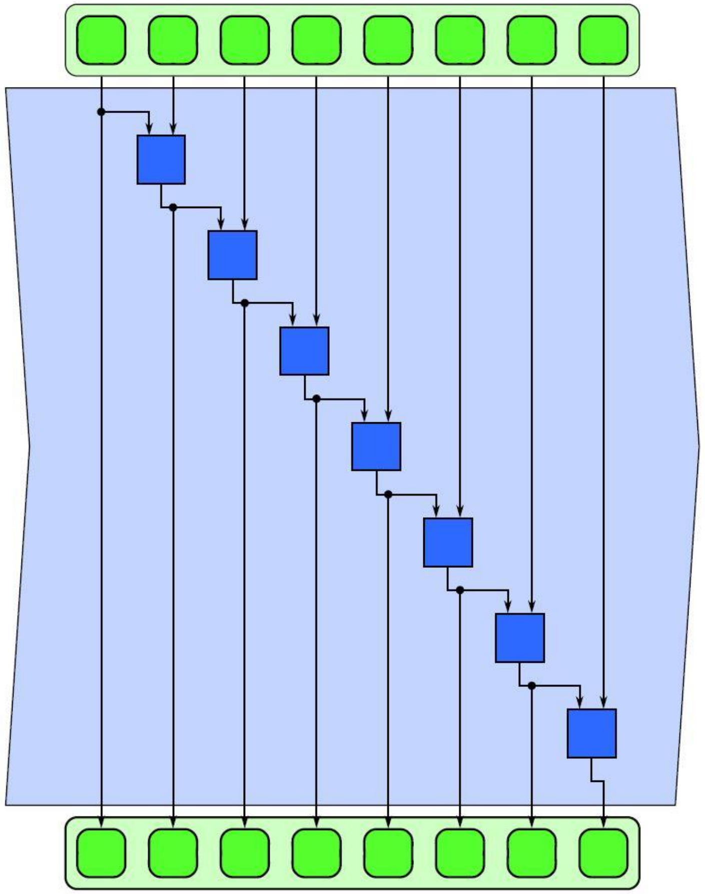


Parallel Control Patterns: Scan

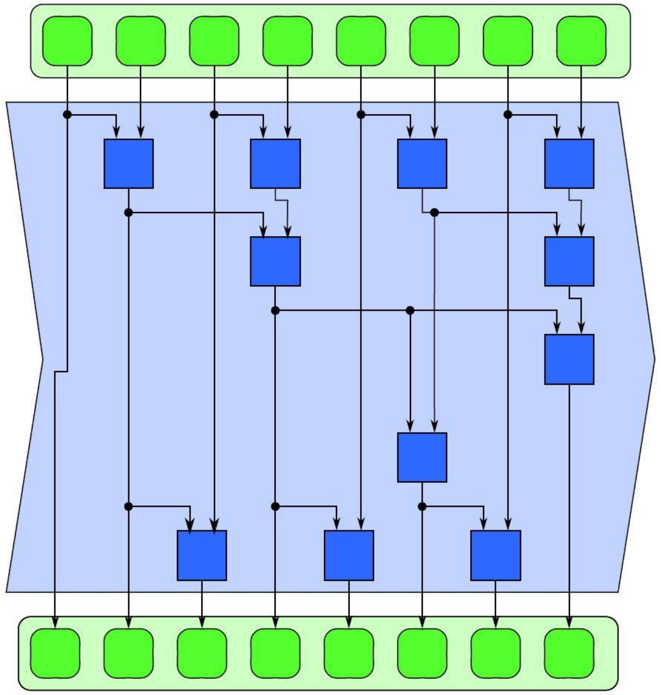
- **Scan**: computes all partial reduction of a collection
- For every output in a collection, a reduction of the input up to that point is computed
- If the function being used is associative, the scan can be parallelized
- Parallelizing a scan is not obvious at first, because of dependencies to previous iterations in the serial loop
- A parallel scan will require more operations than a serial version

Parallel Control Patterns: Scan

Serial Scan



Parallel Scan



Parallel Control Patterns: Recurrence

- **Recurrence**: More complex version of map, where the loop iterations can depend on one another
- Similar to map, but elements can use outputs of adjacent elements as inputs
- For a recurrence to be computable, there *must* be a serial ordering of the recurrence elements so that elements can be computed using previously computed outputs

Serial Data Management Patterns

- Serial programs can manage data in many ways
- Data management deals with how data is allocated, shared, read, written, and copied
- Serial Data Management Patterns: **random read and write**, **stack allocation**, **heap allocation**, **objects**

Serial Data Management Patterns: random read and write

- Memory locations indexed with addresses
- Pointers are typically used to refer to memory addresses
- Aliasing (uncertainty of two pointers referring to the same object) can cause problems when serial code is parallelized

Serial Data Management Patterns: Stack Allocation

- Stack allocation is useful for dynamically allocating data in LIFO manner
- Efficient – arbitrary amount of data can be allocated in constant time
- Stack allocation also preserves locality
- When parallelized, typically each thread will get its own stack so thread locality is preserved

Serial Data Management Patterns: Heap Allocation

- Heap allocation is useful when data cannot be allocated in a LIFO fashion
- But, heap allocation is slower and more complex than stack allocation
- A parallelized heap allocator should be used when dynamically allocating memory in parallel
    - This type of allocator will keep separate pools for each parallel worker

Serial Data Management Patterns: Objects

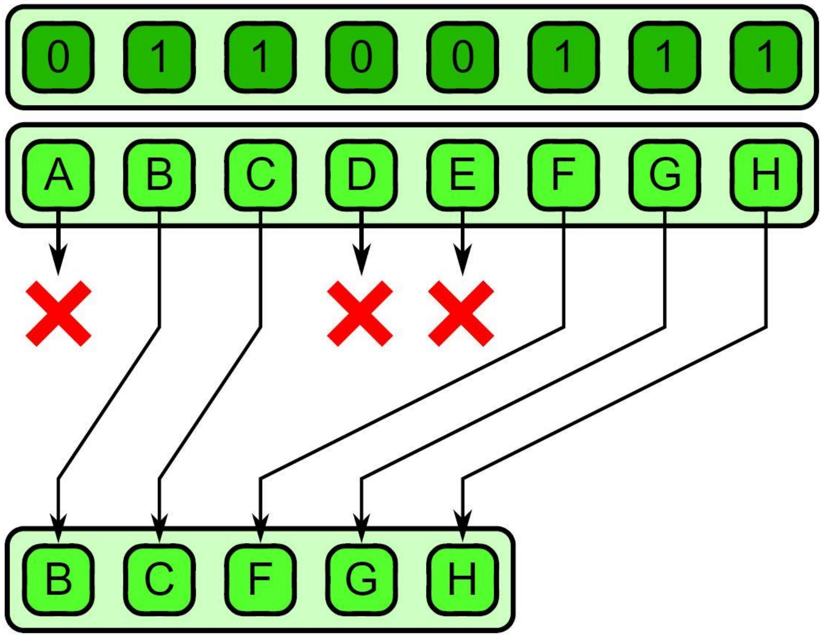
- Objects are language constructs to associate data with code to manipulate and manage that data
- Objects can have member functions, and they also are considered members of a class of objects
- Parallel programming models will generalize objects in various ways

Parallel Data Management Patterns

- To avoid things like race conditions, it is critically important to know when data is, and isn't, potentially shared by multiple parallel workers
- Some parallel data management patterns help us with data locality
- Parallel data management patterns: **pack**, **pipeline**, **geometric decomposition**, **gather**, and **scatter**

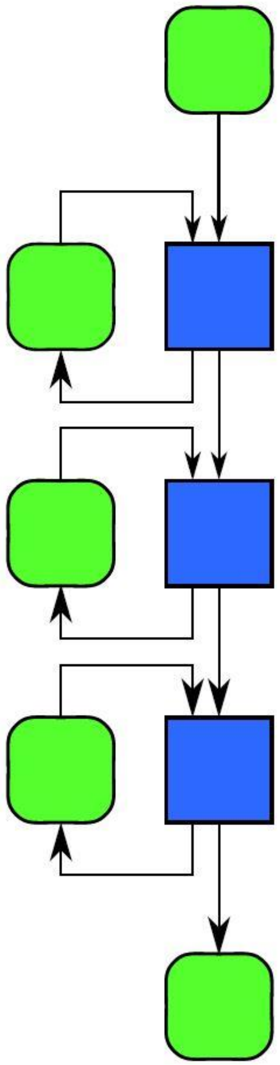
Parallel Data Management Patterns: Pack

- **Pack** is used eliminate unused space in a collection
- Elements marked *false* are discarded, the remaining elements are placed in a contiguous sequence in the same order
- Useful when used with map
- **Unpack** is the inverse and is used to place elements back in their original locations



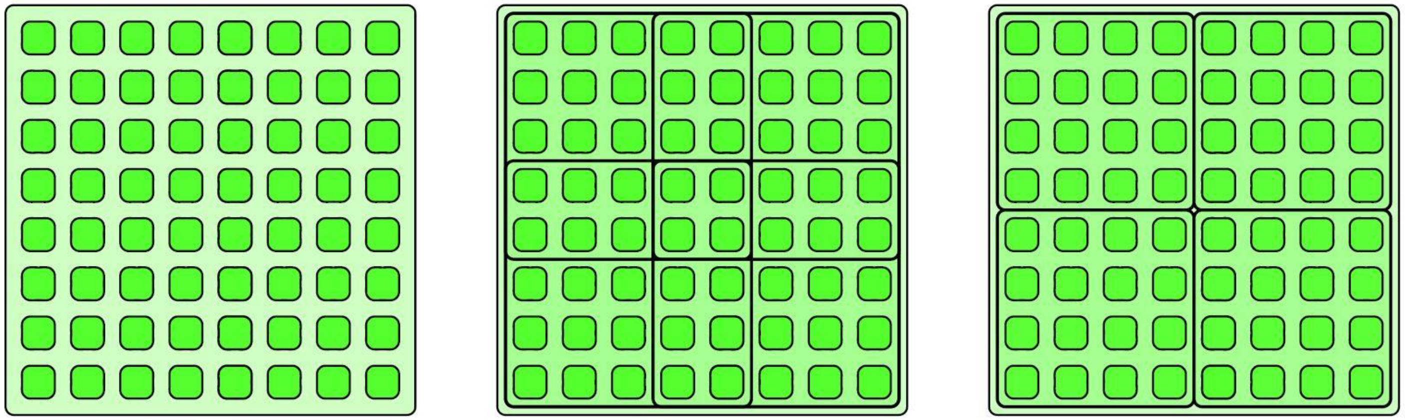
Parallel Data Management Patterns: Pipeline

- **Pipeline** connects tasks in a producer-consumer manner
- A linear pipeline is the basic pattern idea, but a pipeline in a DAG is also possible
- Pipelines are most useful when used with other patterns as they can multiply available parallelism



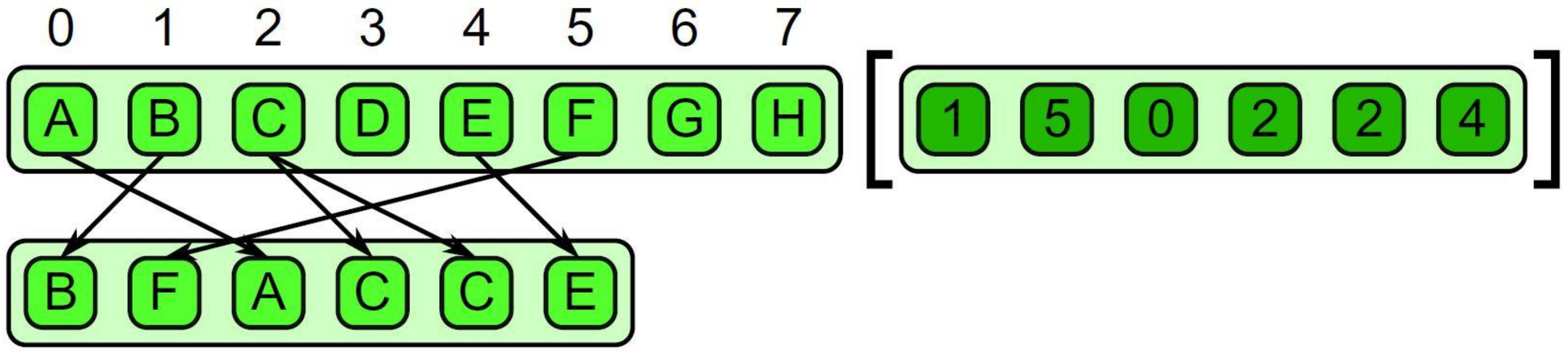
Parallel Data Management Patterns: Geometric Decomposition

- **Geometric Decomposition** – arranges data into subcollections
- Overlapping and non-overlapping decompositions are possible
- This pattern doesn't necessarily move data, it just gives us another view of it



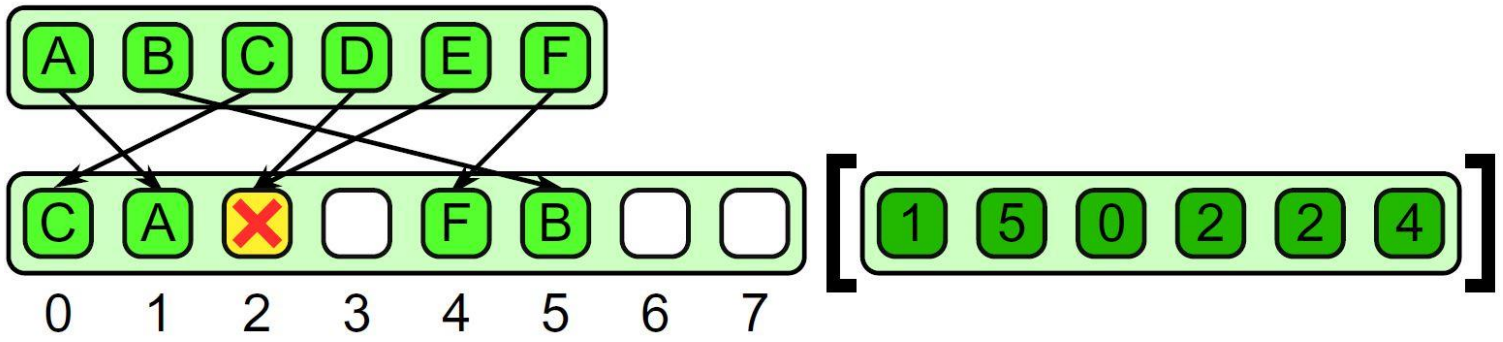
Parallel Data Management Patterns: Gather

- **Gather** reads a collection of data given a collection of indices
- Think of a combination of map and random serial reads
- The output collection shares the same type as the input collection, but it share the same shape as the indices collection



Parallel Data Management Patterns: Scatter

- **Scatter** is the inverse of gather
- A set of input and indices is required, but each element of the input is written to the output at the given index instead of read from the input at the given index
- Race conditions can occur when we have two writes to the same location!



Other Parallel Patterns

- **Superscalar Sequences**: write a sequence of tasks, ordered only by dependencies
- **Futures**: similar to fork-join, but tasks do not need to be nested hierarchically
- **Speculative Selection**: general version of serial selection where the condition and both outcomes can all run in parallel
- **Workpile**: general map pattern where each instance of elemental function can generate more instances, adding to the “pile” of work

Other Parallel Patterns

- **Search**: finds some data in a collection that meets some criteria
- **Segmentation**: operations on subdivided, non-overlapping, non-uniformly sized partitions of 1D collections
- **Expand**: a combination of pack and map
- **Category Reduction**: Given a collection of elements each with a label, find all elements with same label and reduce them

Programming Model Support for Patterns

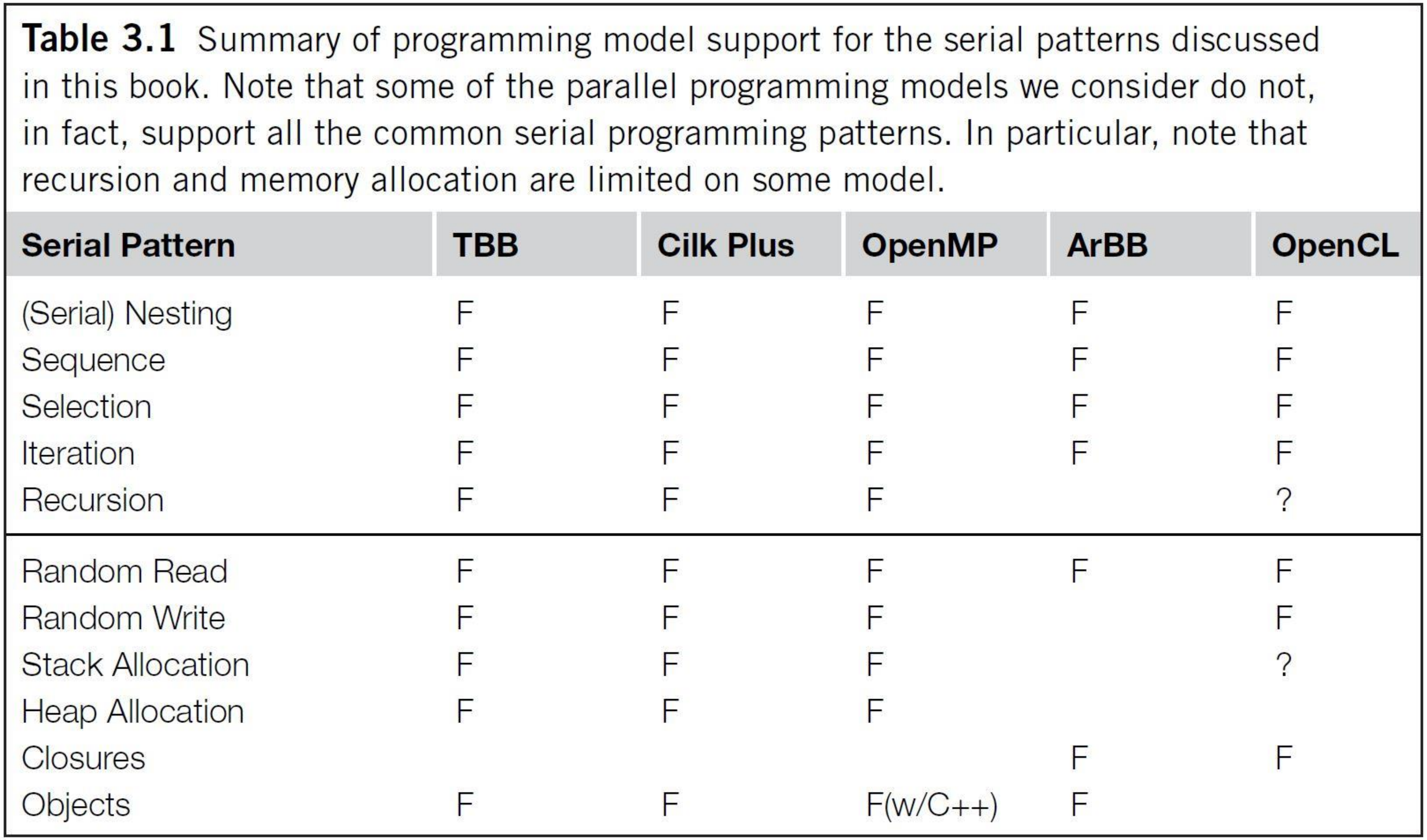


Table 3.1 Summary of programming model support for the serial patterns discussed in this book. Note that some of the parallel programming models we consider do not, in fact, support all the common serial programming patterns. In particular, note that recursion and memory allocation are limited on some model.

Serial Pattern	TBB	Cilk Plus	OpenMP	ArBB	OpenCL
(Serial) Nesting	F	F	F	F	F
Sequence	F	F	F	F	F
Selection	F	F	F	F	F
Iteration	F	F	F	F	F
Recursion	F	F	F		?
Random Read	F	F	F	F	F
Random Write	F	F	F		F
Stack Allocation	F	F	F		?
Heap Allocation	F	F	F		
Closures				F	F
Objects	F	F	F(w/C++)	F	

Programming Model Support for Patterns

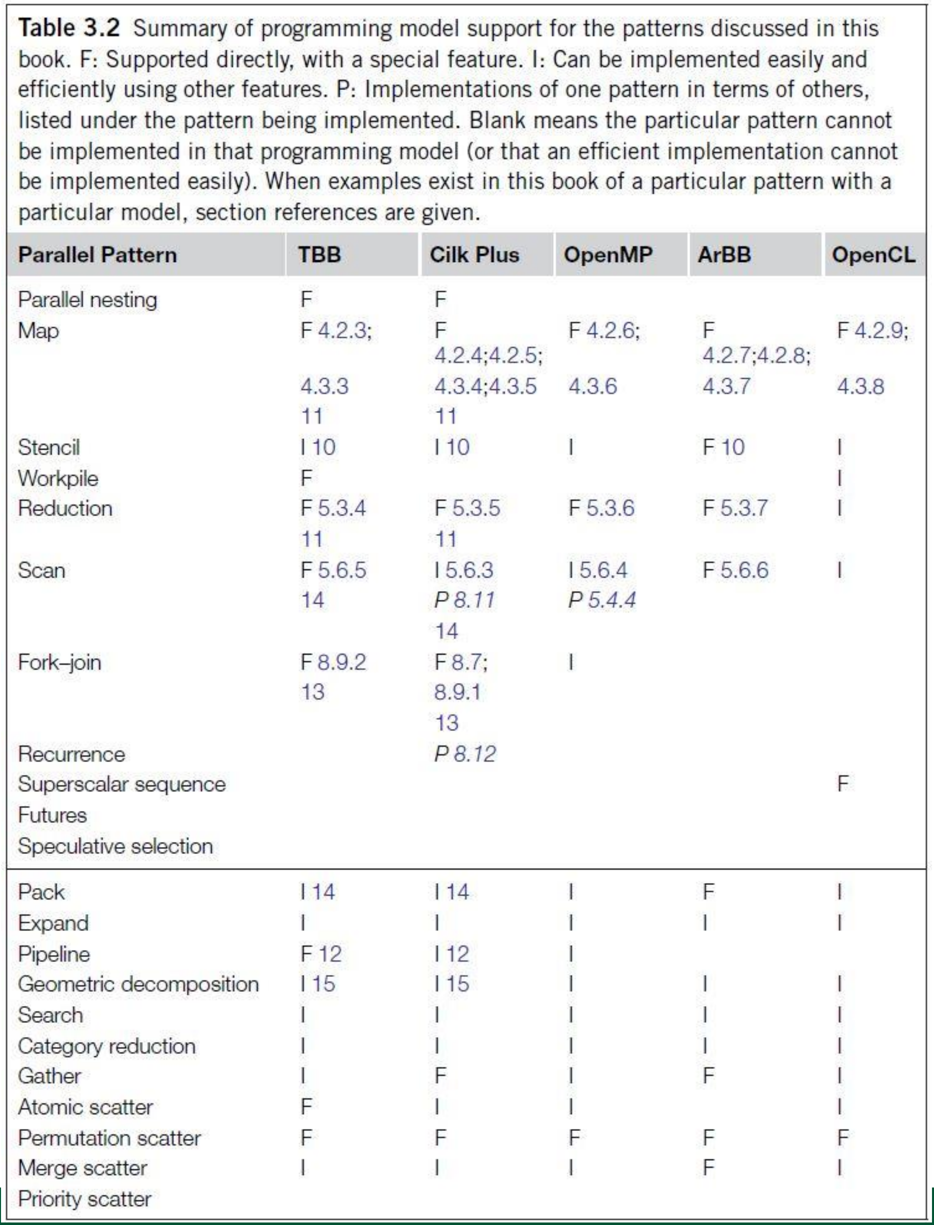


Table 3.2 Summary of programming model support for the patterns discussed in this book. F: Supported directly, with a special feature. I: Can be implemented easily and efficiently using other features. P: Implementations of one pattern in terms of others, listed under the pattern being implemented. Blank means the particular pattern cannot be implemented in that programming model (or that an efficient implementation cannot be implemented easily). When examples exist in this book of a particular pattern with a particular model, section references are given.

Parallel Pattern	TBB	Cilk Plus	OpenMP	ArBB	OpenCL
Parallel nesting	F	F			
Map	F 4.2.3; 4.3.3 11	F 4.2.4;4.2.5; 4.3.4;4.3.5 11	F 4.2.6; 4.3.6	F 4.2.7;4.2.8; 4.3.7	F 4.2.9; 4.3.8
Stencil	I 10	I 10	I	F 10	I
Workpile	F				I
Reduction	F 5.3.4 11	F 5.3.5 11	F 5.3.6	F 5.3.7	I
Scan	F 5.6.5 14	I 5.6.3 *P 8.11* 14	I 5.6.4 *P 5.4.4*	F 5.6.6	I
Fork–join	F 8.9.2 13	F 8.7; 8.9.1 13	I		
Recurrence		*P 8.12*			
Superscalar sequence					F
Futures					
Speculative selection					
Pack	I 14	I 14	I	F	I
Expand	I	I	I	I	I
Pipeline	F 12	I 12	I		
Geometric decomposition	I 15	I 15	I	I	I
Search	I	I	I	I	I
Category reduction	I	I	I	I	I
Gather	I	F	I	F	I
Atomic scatter	F	I	I		I
Permutation scatter	F	F	F	F	F
Merge scatter	I	I	I	F	I
Priority scatter					

Programming Model Support for Patterns

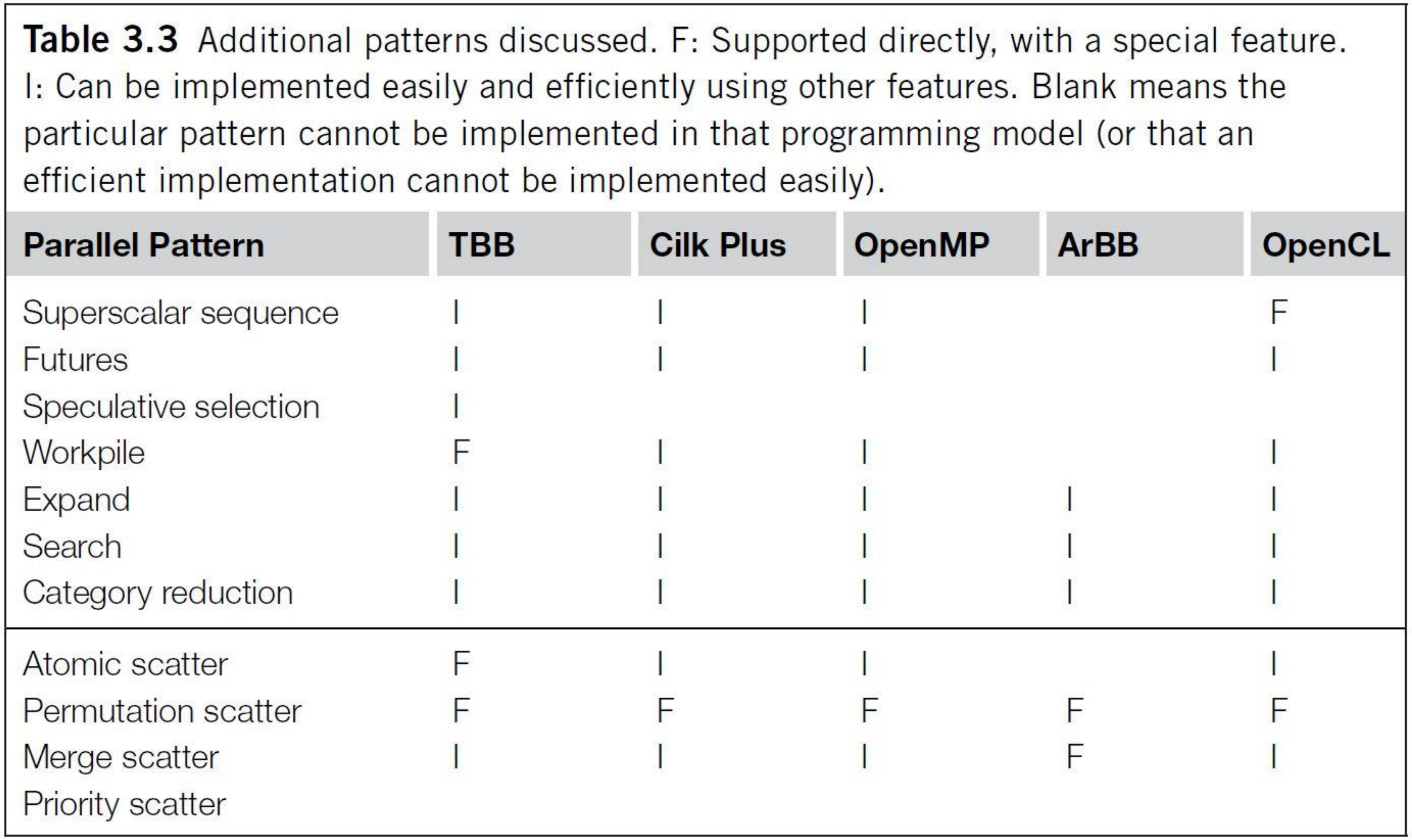


Table 3.3 Additional patterns discussed. F: Supported directly, with a special feature. I: Can be implemented easily and efficiently using other features. Blank means the particular pattern cannot be implemented in that programming model (or that an efficient implementation cannot be implemented easily).

Parallel Pattern	TBB	Cilk Plus	OpenMP	ArBB	OpenCL
Superscalar sequence	I	I	I		F
Futures	I	I	I		I
Speculative selection	I				
Workpile	F	I	I		I
Expand	I	I	I	I	I
Search	I	I	I	I	I
Category reduction	I	I	I	I	I
Atomic scatter	F	I	I		I
Permutation scatter	F	F	F	F	F
Merge scatter	I	I	I	F	I
Priority scatter					

Map Pattern

Map Pattern - Overview

- Map
- Optimizations
  - Sequences of Maps
  - Code Fusion
  - Cache Fusion
- Related Patterns
- Example Implementation: Scaled Vector Addition (SAXPY)
  - Problem Description
  - Various Implementations

Map

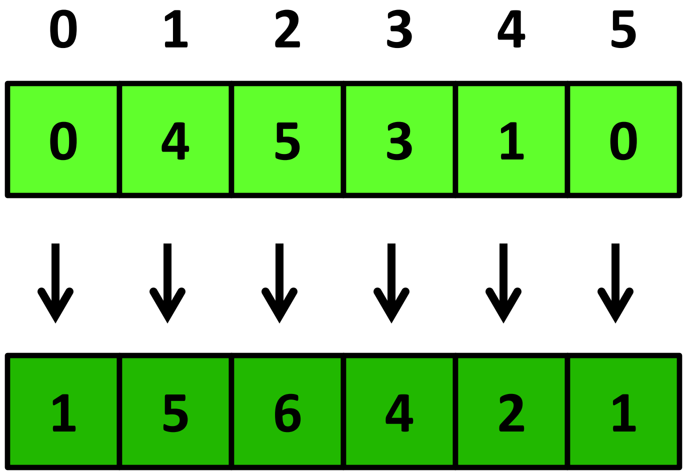
- "Do the same thing many times"

```
foreach i in foo:
    do something
```

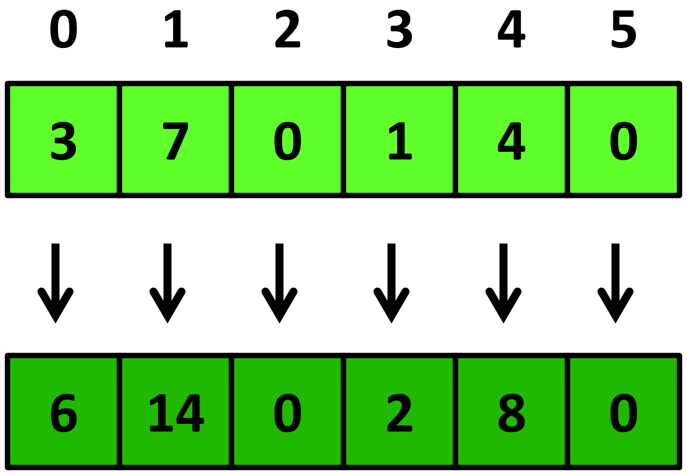
- Applies a function to each element in a list and returns a list of results

Example Maps

Add 1 to every item in an array



Double every item in an array



Key Point: An operation is a map if it can be applied to each element without knowledge of neighbors.

Key Idea

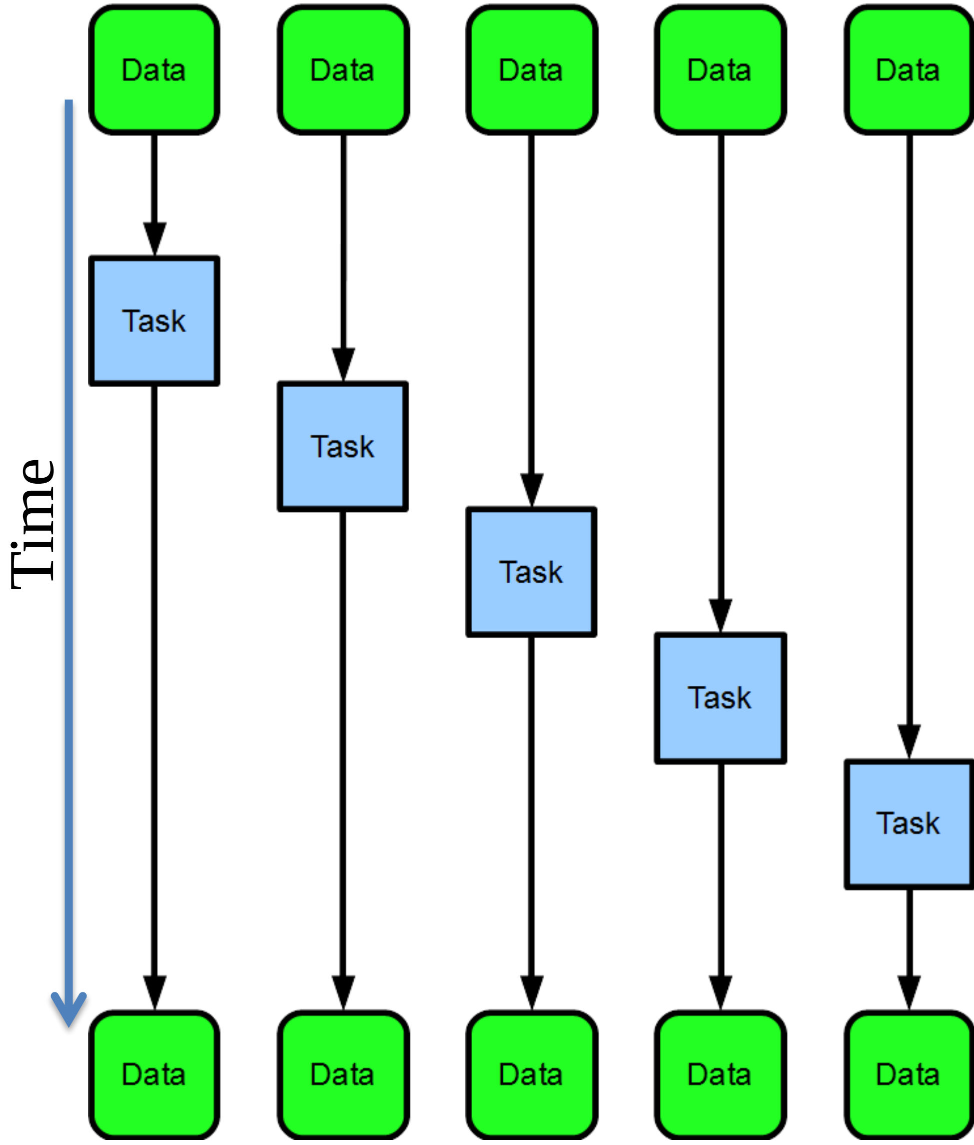
- Map is a "foreach loop" where each iteration is independent

Embarrassingly Parallel

Independence is key to map. We can run map completely in parallel. Significant speedups! More precisely: $T(¥)$ is O(1) plus implementation overhead that is O(log n)…so $T(¥)$ Î $O(\log n)$.

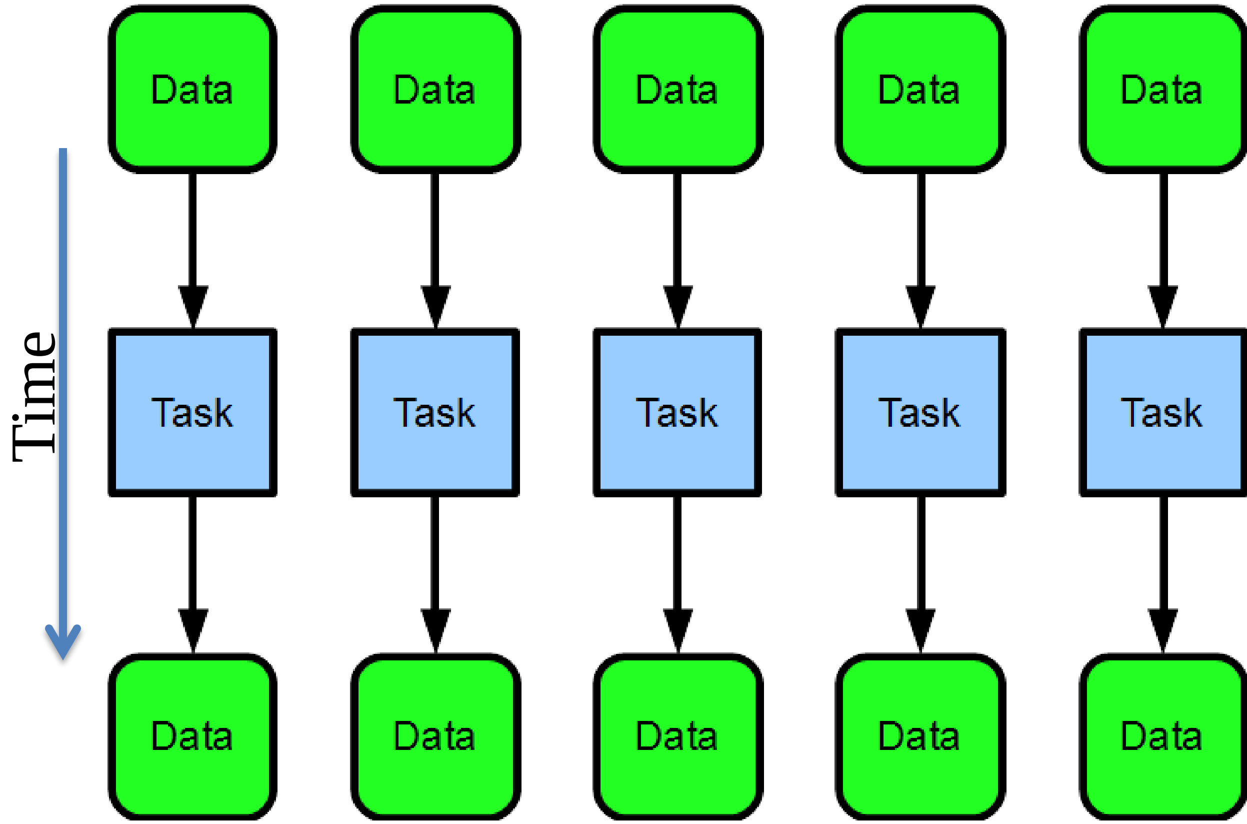
Sequential Map

```
for(int n=0;
    n< array.length;
    ++n){
         process(array[n]);
}
```



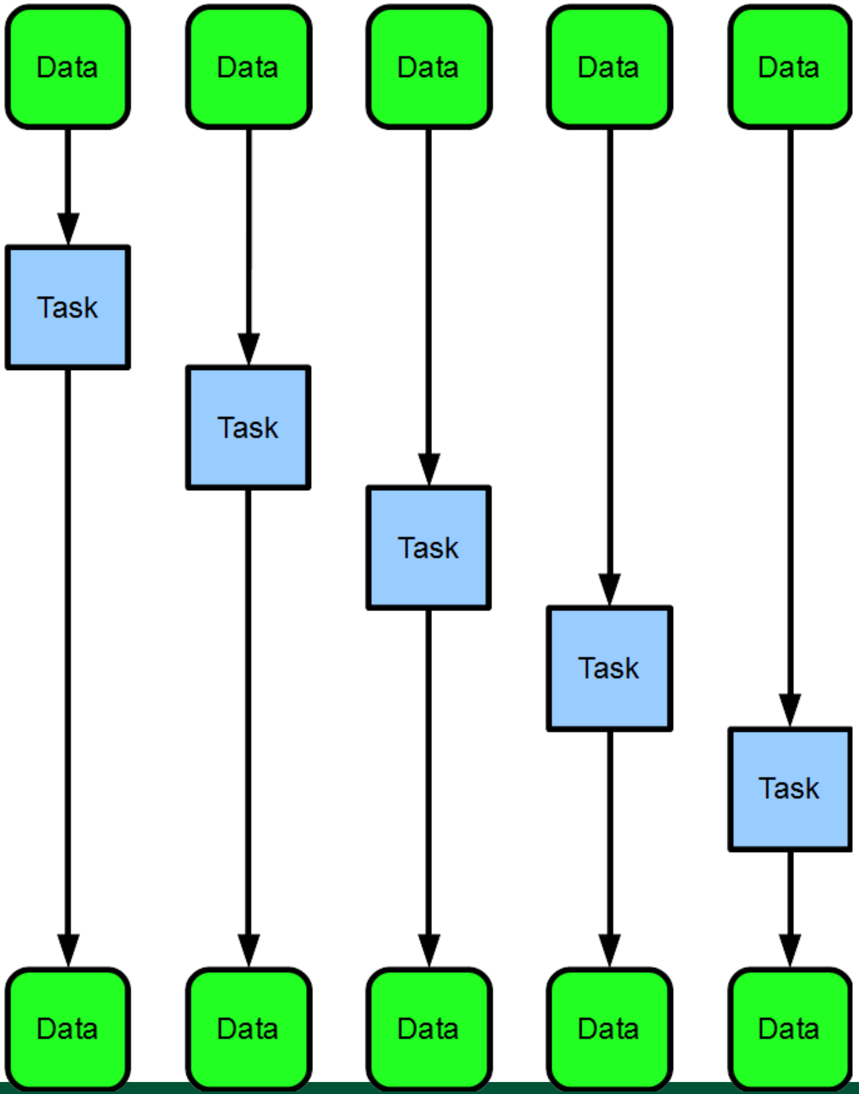
Parallel Map

```
parallel_for_each(
   x in array){
      process(x);
}
```

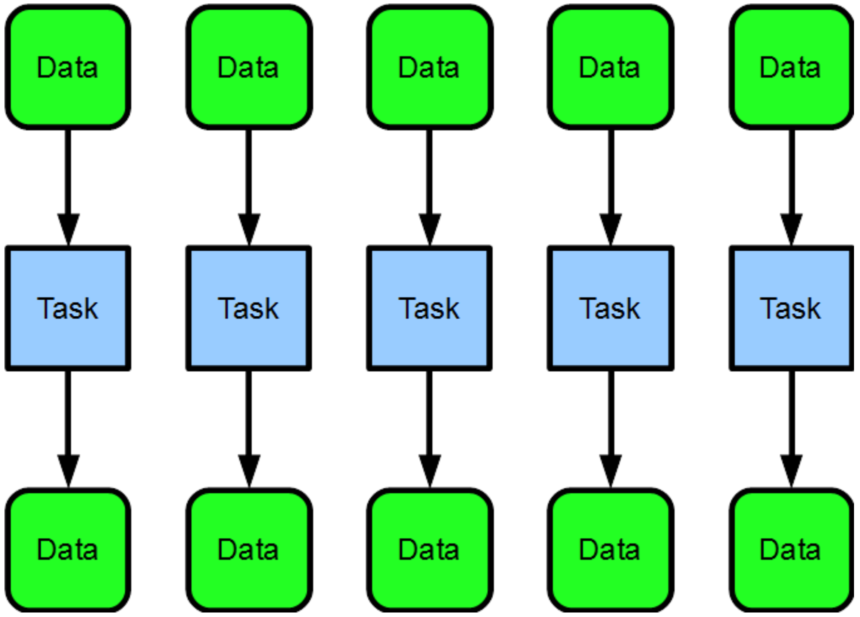


Comparing Maps

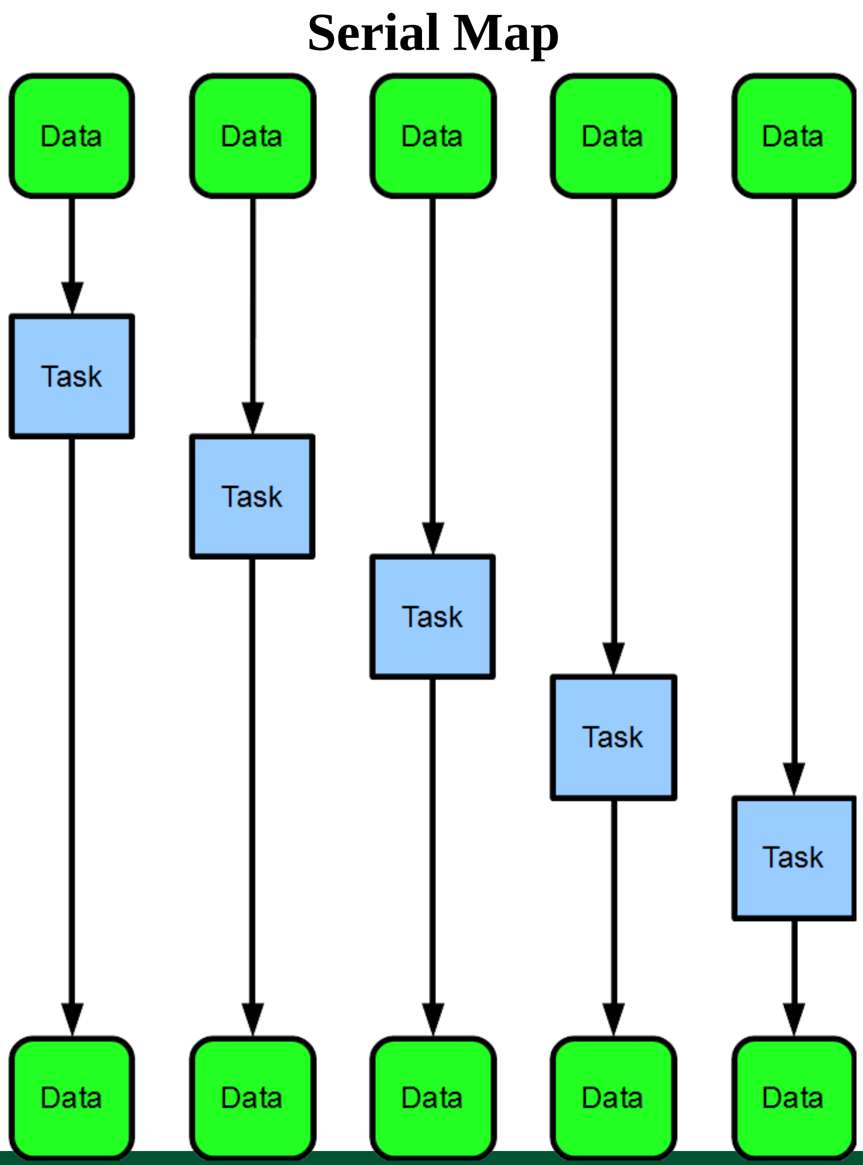
Serial Map

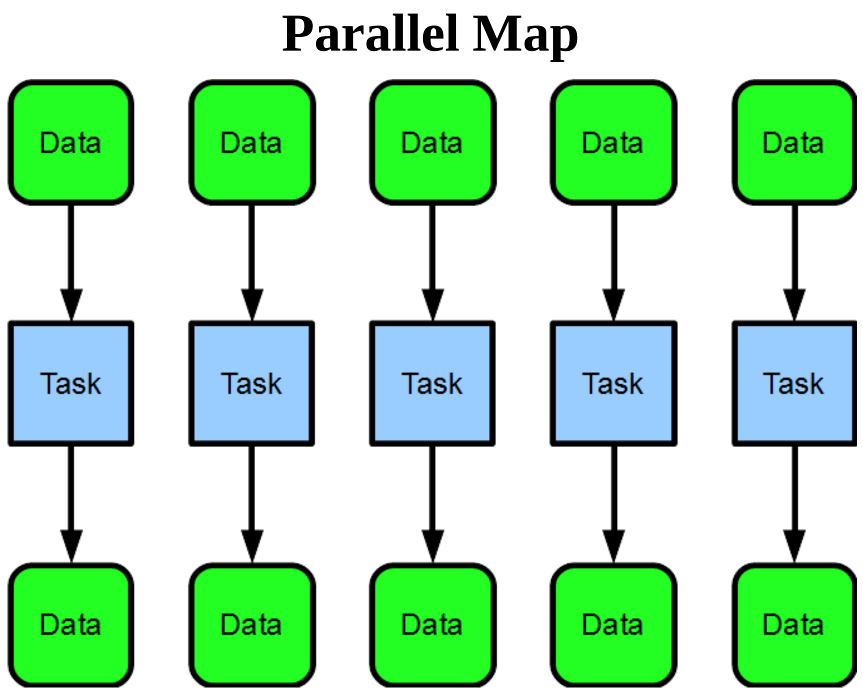


Parallel Map



Comparing Maps





Speedup

The space is speedup. With the parallel map, our program finished execution early, while the serial map is still running.

Independence

- The key to (embarrassing) parallelism is independence

Warning: No shared state!

Map function should be "pure" and should not modify shared states

- Modifying shared state breaks perfect independence
- Results of accidentally violating independence:
  - non-determinism
  - data-races
  - undefined behavior
  - segfaults

Implementation and API

- OpenMP and CilkPlus contain a parallel ***`for`*** language construct
- Map is a mode of use of parallel ***`for`***
- TBB uses **higher order functions** with lambda expressions/“funtors”
- Some languages (CilkPlus, Matlab, Fortran) provide **array notation** which makes some maps more concise

Array Notation

```
A[:] = A[:]*5;
```

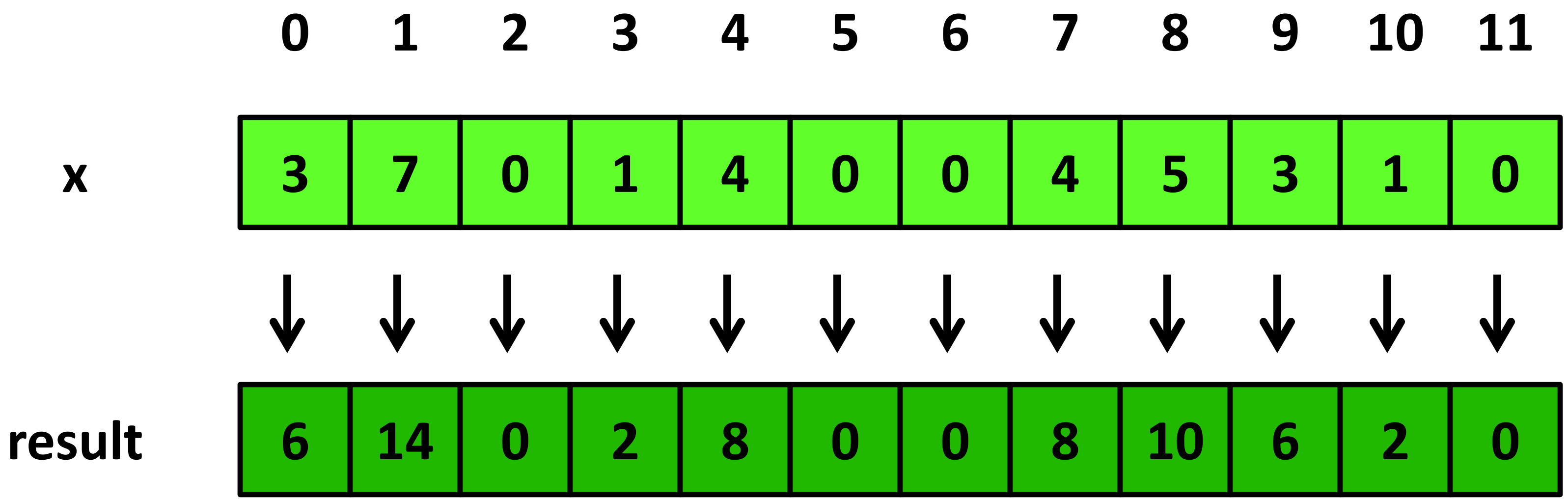
is CilkPlus array notation for “multiply every element in *A* by 5”

Unary Maps

Unary Maps

So far we have only dealt with mapping over a single collection…

Map with 1 Input, 1 Output



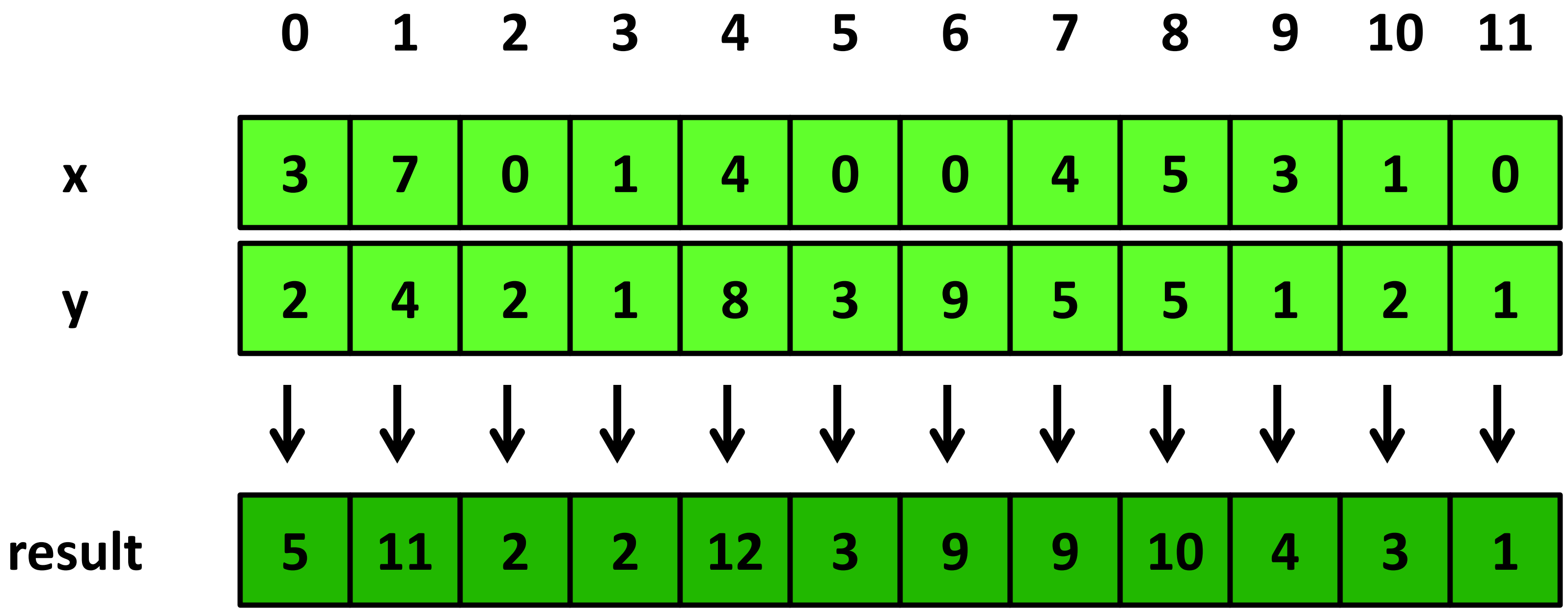
```
int  oneToOne ( int x[11] ) {
        return x*2;
}
```

N-ary Maps

N-ary Maps

But, sometimes it makes sense to map over multiple collections at once…

Map with 2 Inputs, 1 Output


0 1 2 3 4 5 6 7 8 9 10 11
x 3 7 0 1 4 0 0 4 5 3 1 0
y 2 4 2 1 8 3 9 5 5 1 2 1
result 5 11 2 2 12 3 9 9 10 4 3 1

```
int  twoToOne ( int x[11], int y[11] ) {
        return x+y;
}
```

Map Pattern - Overview

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  - Code Fusion
  - Cache Fusion
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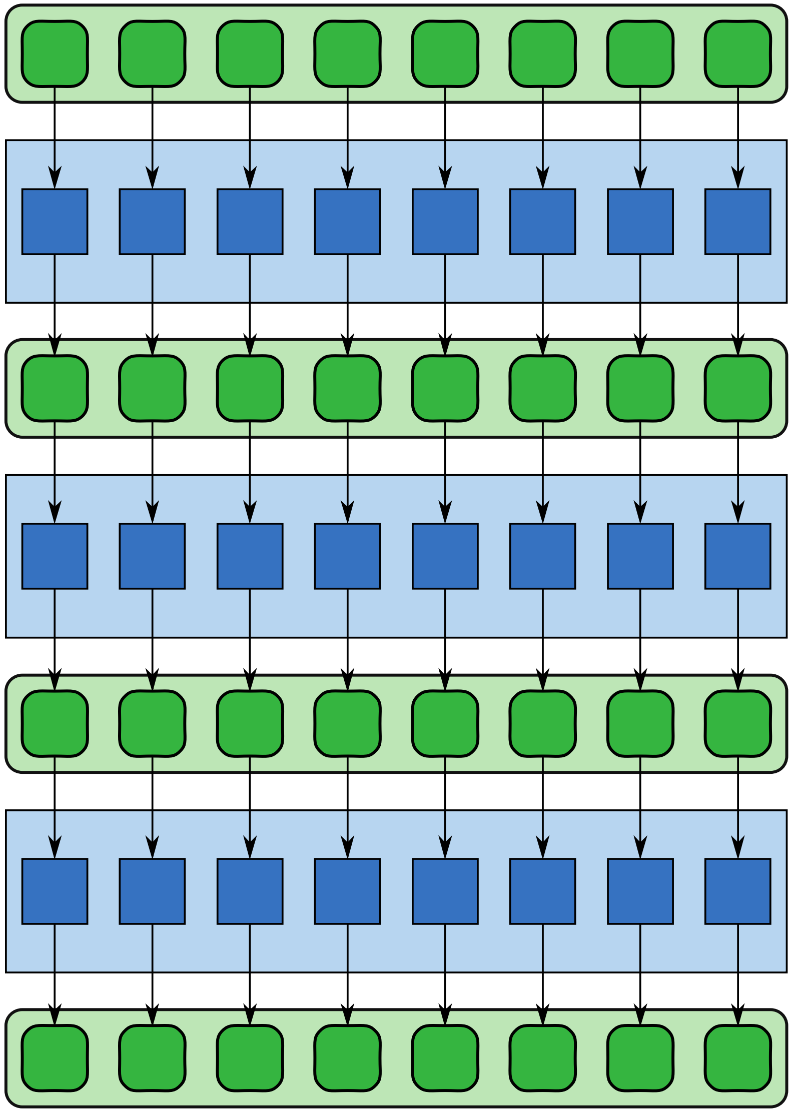
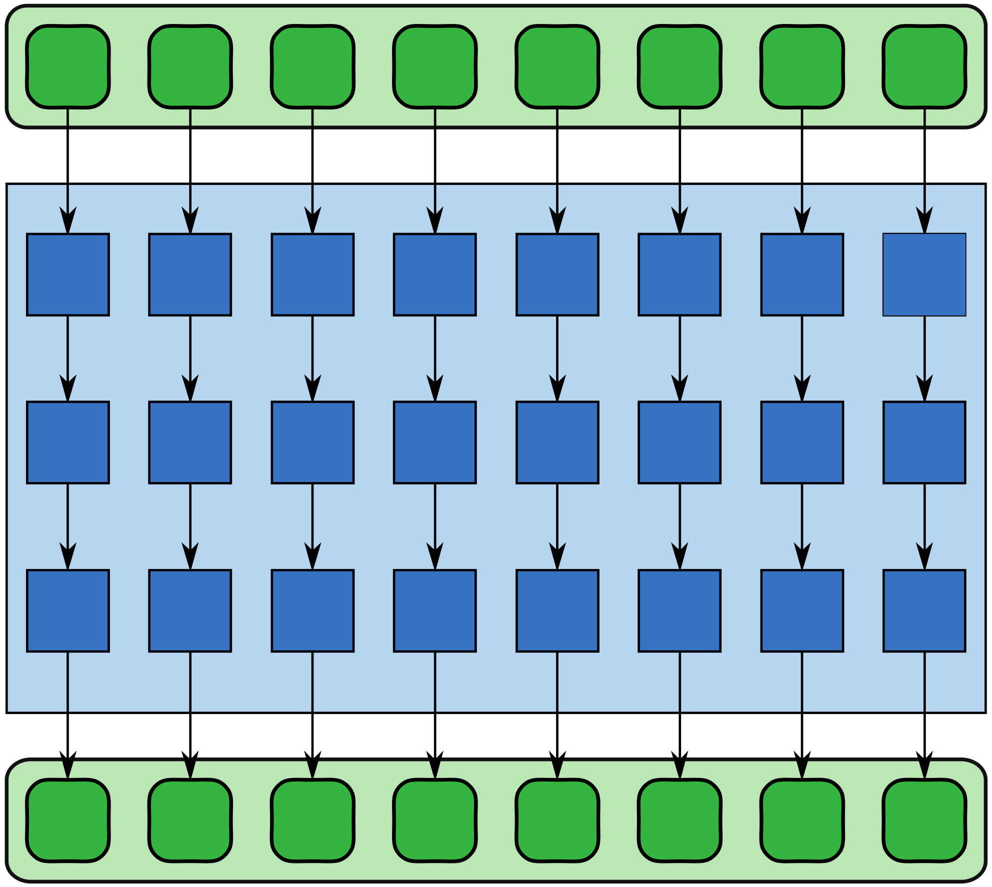
- Example Implementation: Scaled Vector Addition (SAXPY)
  - Problem Description
  - Various Implementations

Sequences of Maps



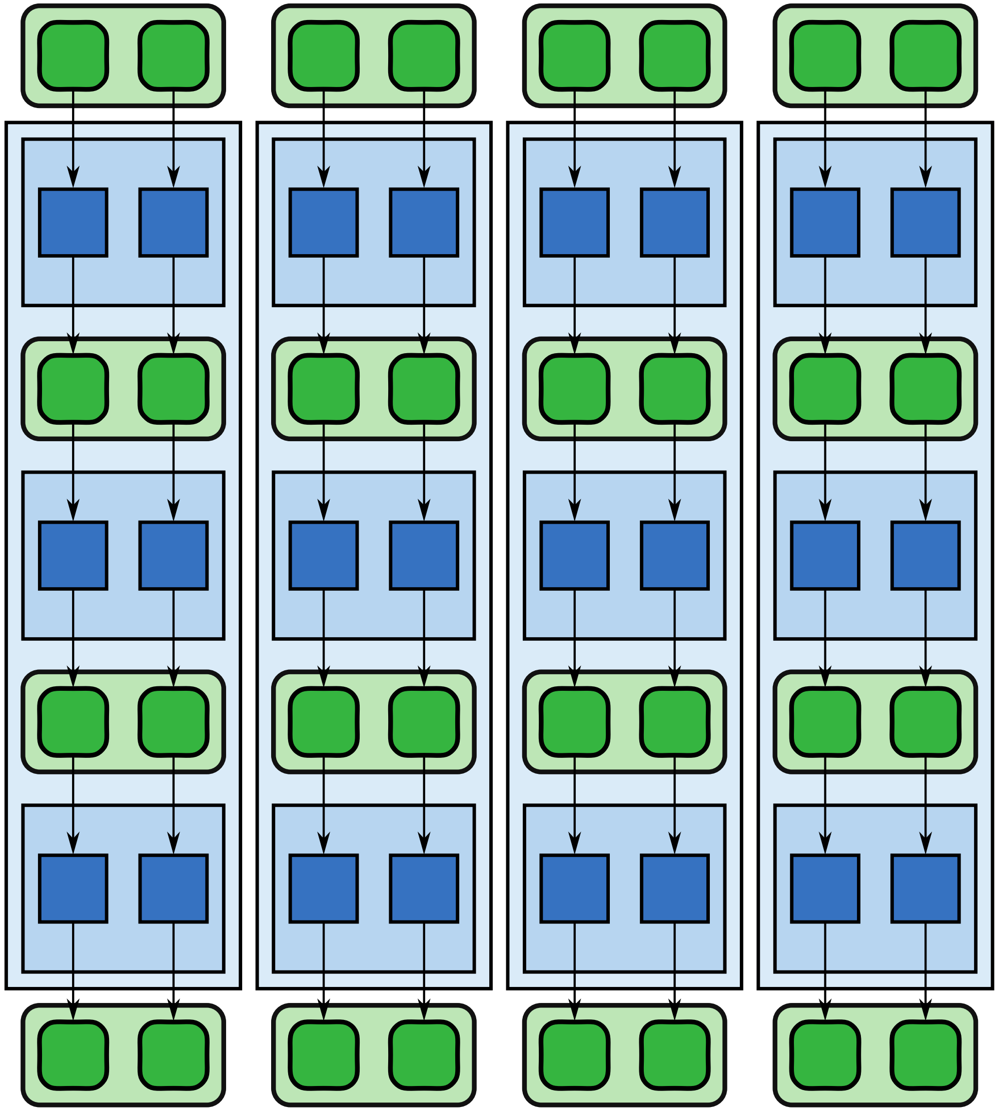
- Often several map operations occur in sequence
  - Vector math consists of many small operations such as additions and multiplications applied as maps
- A naïve implementation may write each intermediate result to memory, wasting memory BW and likely overwhelming the cache

Code Fusion



- Can sometimes "fuse" together the operations to perform them at once
- Adds arithmetic intensity, reduces memory/cache usage
- Ideally, operations can be performed using registers alone

Cache Fusion



- Sometimes impractical to fuse together the map operations
- Can instead break the work into blocks, giving each CPU one block at a time
- Hopefully, operations use cache alone

Related Patterns to Map

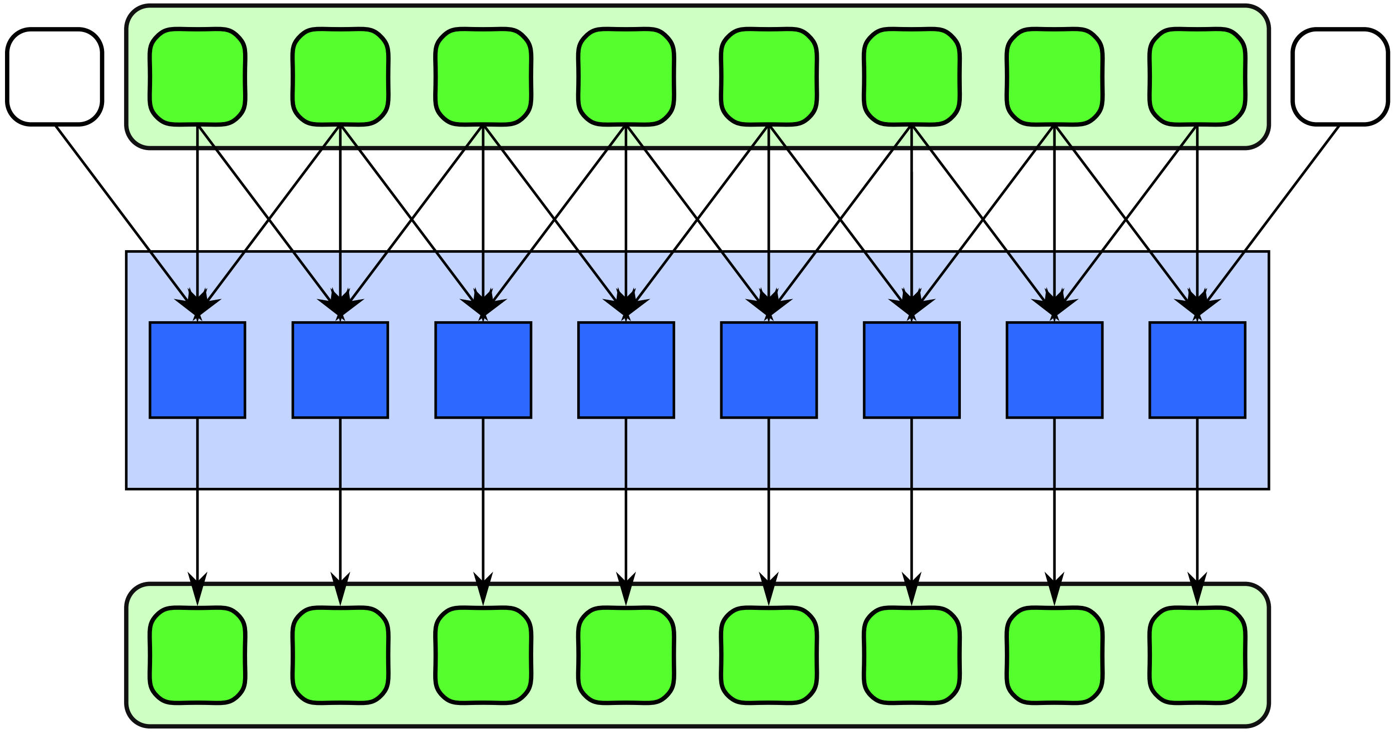
Three patterns related to map are discussed here:

- Stencil
- Workpile
- Divide-and-Conquer

They will be discussed more in detail in a later lecture.

Stencil

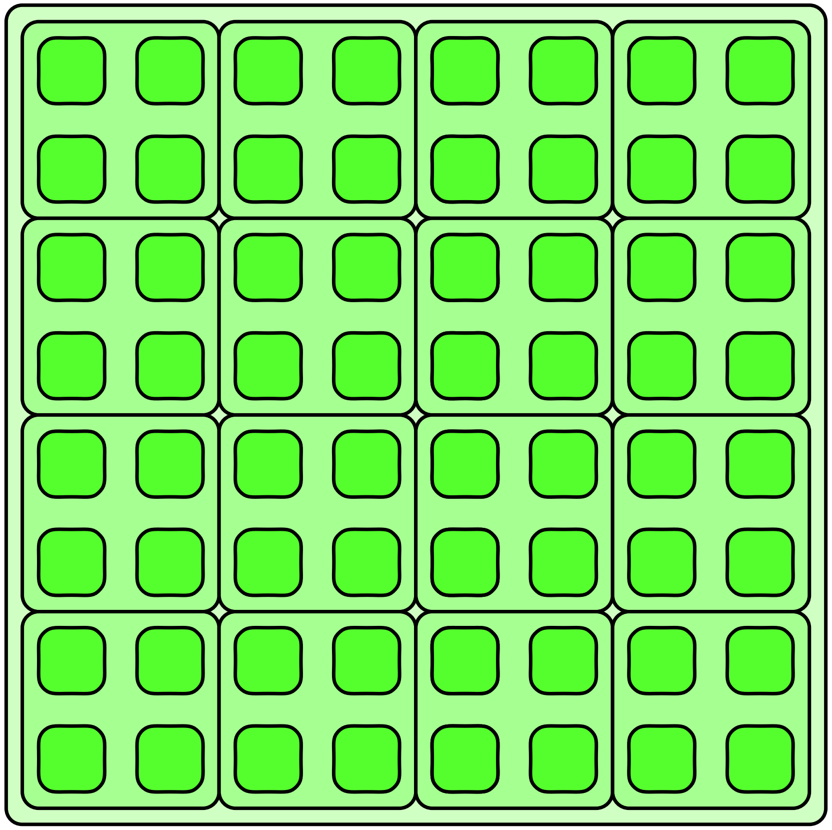
- Each instance of the map function accesses neighbors of its input, offset from its usual input
- Common in imaging and PDE solvers



Workpile

- Work items can be added to the map while it is in progress, from inside map function instances
- The "pile" of work grows and is consumed by the map
- Workpile pattern terminates when no more work is available

Divide-and-Conquer



- Applies if a problem can be divided into smaller subproblems recursively until a base case is reached that can be solved serially

Map Pattern - Overview

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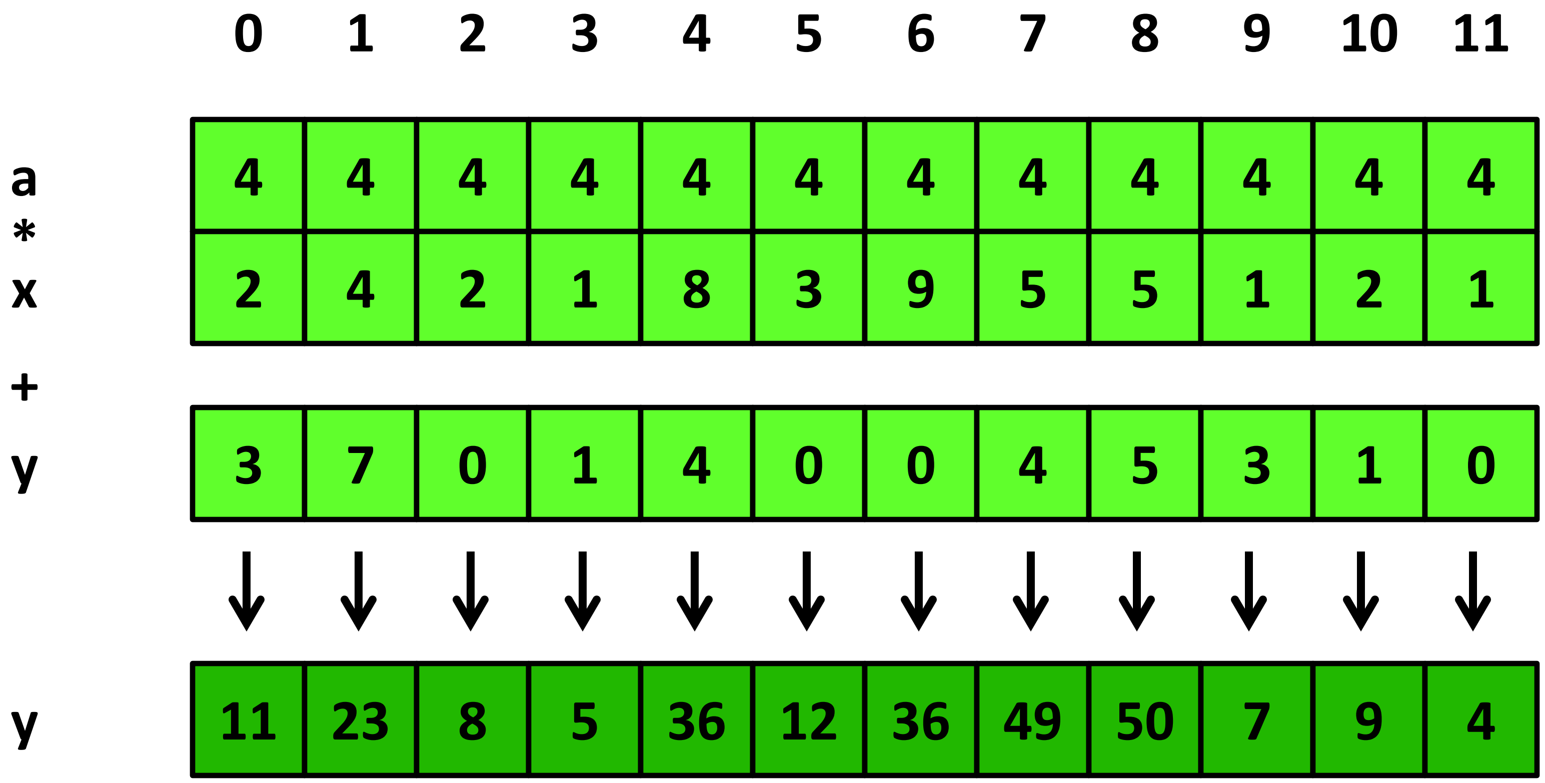
  - Problem Description
  - Various Implementations

Problem Description

- $y \neg \;\; ax + y$
  - Scales vector *x* by *a* and adds it to vector *y*
  - Result is stored in input vector *y*
- Comes from the BLAS (Basic Linear Algebra Subprograms) library
- **Every element in vector *x* and vector *y* are independent**

Visual: $y \leftarrow ax + y$



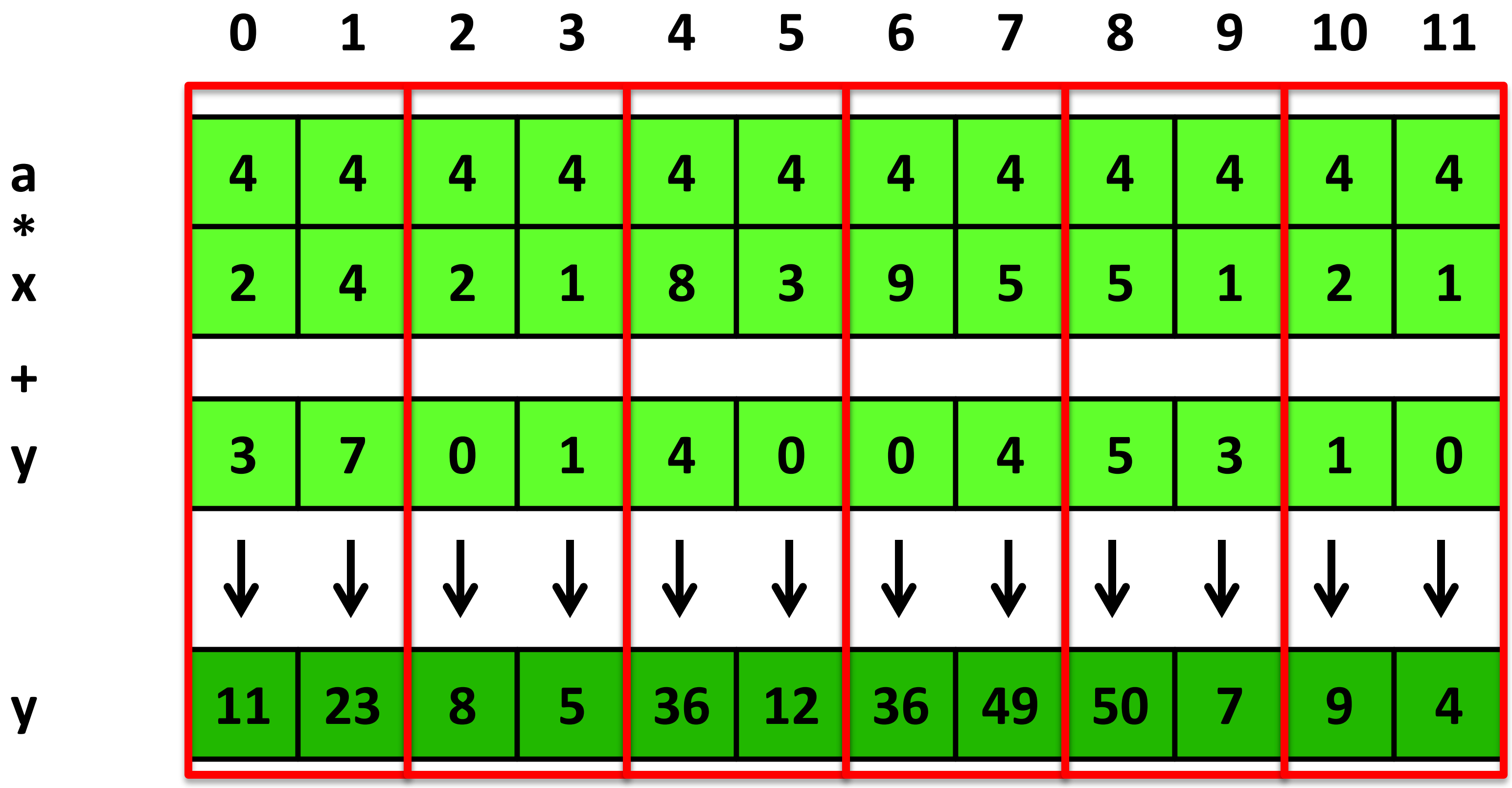
	0	1	2	3	4	5	6	7	8	9	10	11
a	4	4	4	4	4	4	4	4	4	4	4	4
*												
x	2	4	2	1	8	3	9	5	5	1	2	1
+												
y	3	7	0	1	4	0	0	4	5	3	1	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
y	11	23	8	5	36	12	36	49	50	7	9	4

Visual: $y \neg \quad ax + y$

	0	1	2	3	4	5	6	7	8	9	10	11
a	4	4	4	4	4	4	4	4	4	4	4	4
*												
x	2	4	2	1	8	3	9	5	5	1	2	1
+												
y	3	7	0	1	4	0	0	4	5	3	1	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
y	11	23	8	5	36	12	36	49	50	7	9	4

Twelve processors used → one for each element in the vector

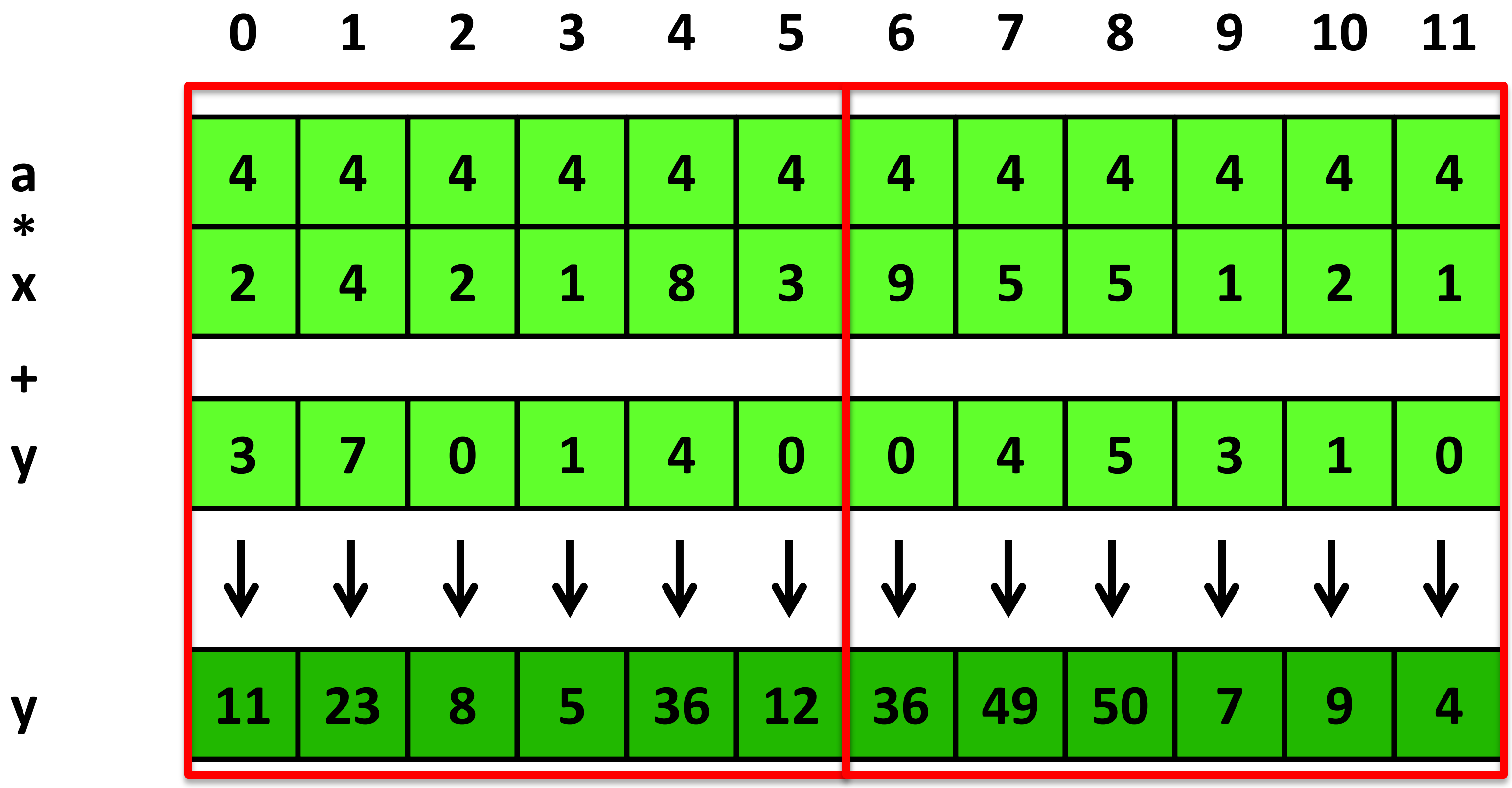
Visual: $y \neg \ ax + y$



	0	1	2	3	4	5	6	7	8	9	10	11
a	4	4	4	4	4	4	4	4	4	4	4	4
*												
x	2	4	2	1	8	3	9	5	5	1	2	1
+												
y	3	7	0	1	4	0	0	4	5	3	1	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
y	11	23	8	5	36	12	36	49	50	7	9	4

Six processors used → one for every two elements in the vector

Visual: $y \neg \quad ax + y$



	0	1	2	3	4	5	6	7	8	9	10	11
a	4	4	4	4	4	4	4	4	4	4	4	4
*												
x	2	4	2	1	8	3	9	5	5	1	2	1
+												
y	3	7	0	1	4	0	0	4	5	3	1	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
y	11	23	8	5	36	12	36	49	50	7	9	4

Two processors used → one for every six elements in the vector

Serial SAXPY Implementation

```
void saxpy_serial(
    size_t n,           // the number of elements in the vectors
    float a,            // scale factor
    const float x[],    // the first input vector
    float y[]           // the output vector and second input vector
) {
    for (size_t i = 0; i < n; ++i)
        y[i] = a * x[i] + y[i];
}
```

TBB SAXPY Implementation

```
void saxpy_tbb(
    int n,          // the number of elements in the vectors
    float a,        // scale factor
    float x[],      // the first input vector
    float y[]       // the output vector and second input vector
) {
    tbb::parallel_for(
        tbb::blocked_range<int>(0, n),
        [&](tbb::blocked_range<int> r) {
            for (size_t i = r.begin(); i != r.end(); ++i)
                y[i] = a * x[i] + y[i];
        }
    );
}
```

Cilk Plus SAXPY Implementation

```
void saxpy_cilk(
    int n,          // the number of elements in the vectors
    float a,        // scale factor
    float x[],      // the first input vector
    float y[]       // the output vector and second input vector
) {
    cilk_for (int i = 0; i < n; ++i)
        y[i] = a * x[i] + y[i];
}
```

OpenMP SAXPY Implentation

```
void saxpy_openmp(
    int n,       // the number of elements in the vectors
    float a,     // scale factor
    float x[],   // the first input vector
    float y[]    // the output vector and second input vector
) {
#pragma omp parallel for
    for (int i = 0; i < n; ++i)
        y[i] = a * x[i] + y[i];
}
```

OpenMP SAXPY Performance