Appendix B: Multimodal Mathematical function to optimize with GA

Choose **only one** function from the two multimodal families of functions (either from *Shubert* or *Shekel* family).

Shekel **family functions**

Definition

Shekel5, Shekel7, Shekel10 functions: $f_{4,m}(x_1,...,x_4) = -\sum_{i=1}^{m}\left(\sum_{j=1}^{4}(x_j - a_{ij})^2 + c_i\right)^{-1}$, $m = 5, 7, 10$.

The constants c_i and a_{ij} are given in Table 1.

Properties

- Global minima: $f_{4,5}(4.00004, 4.00013, 4.00004, 4.00013) = -10.1532;$
  $f_{4,7}(4.00057, 4.00069, 3.99949, 3.99961) = -10.4029;$
  $f_{4,10}(4.00075, 4.00059, 3.99966, 3.99951) = -10.5364;$

Search domain

- $D_4 = \{(x_1,...,x_4) \in R^4 : -10 \le x_i \le 10, i = 1,...,4\}$

Table 1. Data for *Shekel 5,7,10* functions.

i	a_{ij}				c_i
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4
6	2.0	9.0	2.0	9.0	0.6
7	5.0	5.0	3.0	3.0	0.3
8	8.0	1.0	8.0	1.0	0.7
9	6.0	2.0	6.0	2.0	0.5
10	7.0	3.6	7.0	3.6	0.5

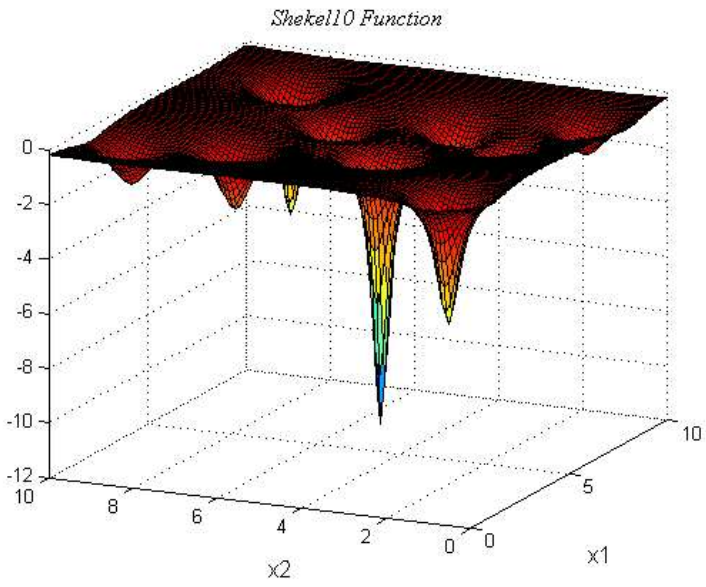

Fig.1. *Shekel10* function.

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***Shubert* family functions**

Definition

- *Shubert1*: $f_1(x_1,...,x_n)=\prod_{i=1}^{n}\left[\sum_{j=1}^{5} j\cos\left((j+1)x_i+j\right)\right]$;
- *Shubert2*: $f_2(x_1,...,x_n)=\prod_{i=1}^{n}\left[\sum_{j=1}^{5} j\cos\left((j+1)x_i+j\right)\right]+\frac{1}{2}\left((x_1+1.42513)^2+(x_2+0.80032)^2\right)$;
- *Shubert3*: $f_3(x_1,...,x_n)=-\sum_{i=1}^{n}\left[\sum_{j=1}^{5} j\sin\left((j+1)x_i+j\right)\right]$.

Properties

- Number of local minima: 760 for f_1, many for f_2, and 400 for f_3.
- Number and value of the global minima:
  - 18 for *Shubert1*: $f_1^*=-186.730909$ (all 2D minimisers are given in Table 2);
  - 1 for *Shubert2*: $f_2^*=-186.730909$;
  - 9 for *Shubert3*: $f_3^*=-24.062499$.

Search domain

- $D_1=\left\{(x_1,x_2)\in R^2:-10\le x_i\le 10, i=1,2\right\}$, (n=2);

Table 2. Argument values for *Shubert1* global minima.

x_1^*	x_2^*
-7.0835	4.8580
-7.0835	-7.7083
-1.4251	-7.0835
5.4828	4.8580
-1.4251	-0.8003
4.8580	5.4828
-7.7083	-7.0835
-7.0835	-1.4251
-7.7083	-0.8003
-7.7083	5.4828
-0.8003	-7.7083
-0.8003	-1.4251
-0.8003	4.8580
-1.4251	5.4828
5.4828	-7.7083
4.8580	-7.0835
5.4828	-1.4251
4.8580	-0.8003

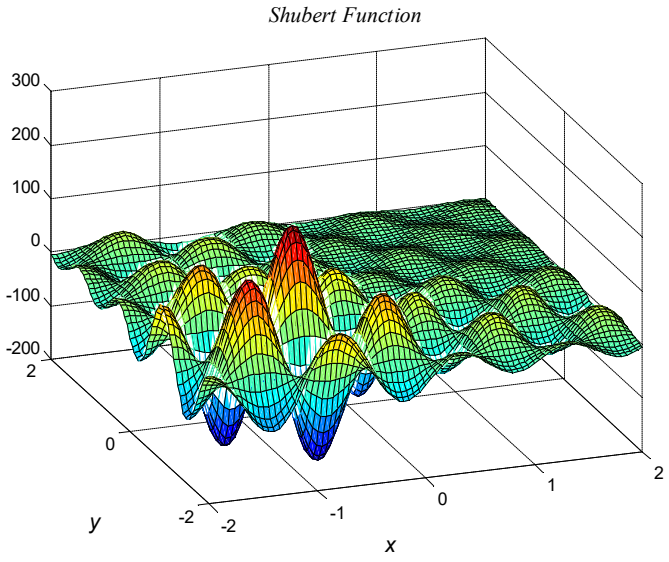

Fig.2. *Shubert* function.

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