

# UP ACM Algolympics 2017





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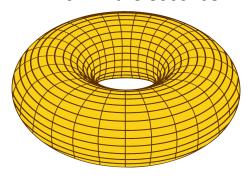
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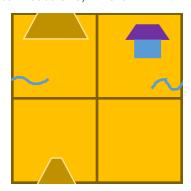


# Problem A Donut Cross

Time Limit: 3 seconds



Michael and Pichael are ants and they are currently in a donut. This donut is divided into MN different sections. In fact, the donut can be represented by an  $N \times M$  grid where the leftmost edge and rightmost edge represent the same part of the donut, and the top edge and the bottom edge represent the same part of the donut. Below is the grid-representation of a donut which is divided into 4 sections, where M = N = 2.



Michael and Pichael live in the same donut section. One day, they wore bored and decided that they wanted to put a sign on the bottom boundary and the right boundary of each section. This sign will either be "Michael cannot cross this boundary" or "Pichael cannot cross this boundary". After all the signs are placed, they are to follow them at all costs. Note that the signs are on the boundary of two sections and can be read from both sides.

However, they realize that putting signs randomly might lock one of them out of different sections. They do not want that. They want to arrange the signs such that despite following the signs, both of them will still be able to access all *MN* sections of the donut.

Given M and N, determine which sign they should put on the boundary of each section.

#### Input

The first line of input contains T, the number of test cases.





Each test case consists of one line containing two integers, N and M, separated by a space.

#### Output

For each test case, output N lines, each containing M two-character strings. The  $j^{\text{th}}$  string in the  $i^{\text{th}}$  line represents the sign to be placed on the section reached if the ants go i sections down and j sections right starting from their house. The first character of this string represents the sign to be placed on the bottom boundary and the second character of this string represents the sign to be placed on the right boundary. If the sign to be placed is "Michael cannot cross this boundary", represent it by the character M. On the other hand, if the sign to be placed is "Pichael cannot cross this boundary", represent it by the character P.

Note that there may be multiple valid answers.

If it is impossible to find a valid assignment of signs at all, output IMPOSSIBLE on a single line instead.

#### **Constraints**

$$1 \le T \le 500$$
  
$$2 \le N, M \le 100$$

Sample input	Sample Output
1	MP PM MM
2 3	MP MM PP





# Problem B Adding Time

Time Limit: 5 seconds

Everyone knows that if you give John Carlo a number, then it will take John Carlo exactly one minute to add all the digits of this number.

Before going home, Krystelle proposes a game to John Carlo. She gives a number *X* and John Carlo must add the digits of this number. John Carlo must then add the digits of this new number. He must do this again and again until he ends up with a one-digit number. After this, John Carlo goes home and will be away from Krystelle until they meet again tomorrow.

Now, Krystelle has a secret crush on John Carlo. Thus, she wants to spend as much time as possible with him. What is the smallest nonnegative number that she must give so that John Carlo stays with her for at least *N* minutes (since he is busy computing sums)?

#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing two integers, N and M, separated by a space.

#### **Output**

For each test case, output one line containing a single number, the answer to the test case modulo M.

#### **Constraints**

$$1 \le T \le 10^5$$
$$0 \le N \le 6$$
$$1 \le M \le 2 \cdot 10^7$$

#### Sample Input

•	• •
4	10
1 1000000	19
2 1000000	199
3 1000000	49
3 50	





### Problem C Wildcard List

Time Limit: 6 seconds

The reality show Filipino Elder Male Sibling is planning how to conduct their next season. After doing the auditions, they ranked the applicants by the likelihood that they will have a mental breakdown. They then decide to choose the top B applicants and they will be on the initial list of housemates. The plan is, at the end of the first week, there will be an elimination portion. There will be D housemates which will be asked to leave, so they can have a mental breakdown outside the house for a change.

Before the shooting of the show even started, one of the producers suggested to mix things up and add more people in the initial list! This means that instead of having B housemates initially, there might be more! The showrunner then suggested that they can add up to L sets of A housemates to make things more exciting. This means we will add more housemates starting from the applicant ranked B+1, moving down the rankings. However, for every set of A people added to the original plan, an additional C housemates shall be chosen to be eliminated at the end of the first week.

The housemates which are asked to leave at the end of the first week will then be added to the wildcard list. The producers have decided on *A*, *B*, *C* and *D*. But they are still thinking how many sets of *A* people (if any at all) they will add to the initial roster. Given these facts, how many possible wildcard lists can they end up with?

For example, consider L=1, A=3, B=2, C=4, D=1. The initial list may either have B=2 people. Or the producer might have added one set of A=3 people, making it 5. Hence, the initial list may either have 2 or 5 people. In the case that the initial list has 2 people, say housemate 1 and housemate 2. Only D=1 of them needs to be eliminated at the end of the first week. Hence, there are two possible wildcard lists – one containing only housemate 1 and another containing only housemate 2. In the case that the initial list has 5 people (meaning that there were A=3 people added in excess of the original B=2), then D+C=5 people must be eliminated. And so there is only one possible wildcard list. This brings us to a total of three possible wildcard lists.

#### Input

The first line of input contains t, the number of test cases.

Each test case consists of one line containing six integers, A, B, C, D, L and p, separated by spaces.

#### Output

For each test case, output one line containing a single number, the number of possible wildcard lists modulo p.





#### **Constraints**

 $1 \le t \le 280$   $0 \le A, B, C, D \le 5$   $0 \le L \le 10^{12}$  2000<math>p is prime

#### Sample Input

5	3
3 2 4 1 1 2017	540
4 3 2 1 11 2017	3237
4 3 2 1 11 12343	0
1 2 3 4 1000 2017	647
5 3 4 2 12345 2017	





# Problem D Never Forget The C

Time Limit: 7 seconds

Dominic is a big fan of indefinite integrals. For example, he knows that the integral

$$\int 3x^2 + 2x + 1 \ dx$$

is equal to  $x^3 + x^2 + x + C$ . But later on, he realized that he didn't like the part where he had to deal with algebra. He just liked the  $\mathbb{C}$ . This is why he became a sailor. So he can always hang around the sea. On the other hand, his girlfriend liked definite integrals such as

 $\int_{-\infty}^{\infty} e^{-x^2} dx$ 

so much that she realized she wanted to go to a *polar* expedition! But enough about her. Let's focus on Dominic's adventures in the sea.

Sailing around the sea is not as easy as solving indefinite integrals. Surely, it is bound to test your limits. In particular, moving from sea to land takes a whole day of preparation for the whole crew. Similarly, moving from land to sea is not as easy and also takes a day. Dominic hates both these days and he calls them Hell Days.

Given a map, their current location and their target location, how many Hell Days must Dominic endure?

#### Input

The first line of input contains two integers r and c separated by a space.

The following r lines describe the grid. Each such line contains a string of length c consisting of the characters:

- C representing a sea tile.
- # representing a land tile.

The following q lines describe the queries. Each query consists of a single line containing four integers  $i_1, j_1, i_2, j_2$ , separated by single spaces, meaning the starting and ending tiles are  $(i_1, j_1)$  and  $(i_2, j_2)$ , respectively.

They can only move in any of the four cardinal directions.

#### Output

For each query, output one line containing a single number, the answer for that query.





#### **Constraints**

 $1 \le r, c \le 500$ 

 $1 \le q \le 200$ 

 $1 \le i_1, i_2 \le r$ 

 $1 \le j_1, j_2 \le c$ 

#### Sample Input

7 11 5	0
CCC###CCC##	0
CC###CC####	3
C##CCC###CC	3
#CCC##CCCCC	0
CCCC###CCCC	
#CCCC####CC	
###CCCCCC##	
1 9 7 9	
7 9 1 9	
1 1 7 11	
4 1 4 10	
3 7 1 11	





### Problem E MechaMocha

Time Limit: 3 seconds

After achieving 1000000 fans from various fake social media accounts and collecting the seven Dragon Balls, local celebrity turned political analyst Mocha was able to evolve into a mechanical robot! Now, she is a local celebrity turned political analyst turned mechanical robot. The Media Men, her archenemy, needs to defeat this mechanical robot in order to protect the Philippine citizens from her mind control powers.

The motherboard for the Mechanical Mocha robot is a  $200000 \text{ mm} \times 200000 \text{ mm}$  rectangle and is covered by various rectangular shields. Some of these shields may be on top of each other.

Areas which are behind at least one shield are called **sensitive spots**. Once any of these sensitive spots are hit, the inner software in the robot becomes destabilized (much like The Media Men when Mocha attacks) and the robot becomes deactivated.

The Media Men has one single shot left before Mocha unleashes her hypnotic powers. They can throw one dart which can penetrate at most one layer of shield. Hence, they must aim it towards a part of the motherboard which has exactly one layer of shield, which we will call Mocha's vulnerable spots. What is the total area (in square millimeters) of Mocha's vulnerable spots?

#### Input

There is a single test case.

The first line of input contains a single integer, N. The next N lines describe the placement of the shields.

The  $i^{th}$  following line contains four integers,  $x_{1,i}, y_{1,i}, x_{2,i}, y_{2,i}$ , separated by single spaces, denoting the lower-left corner  $(x_{1,i}, y_{1,i})$  and upper-right corner  $(x_{2,i}, y_{2,i})$  of the rectangle. The rectangle's sides are parallel to the coordinate axes. This rectangle represents the area covered by the  $i^{th}$  shield.

#### **Output**

For each test case, output one line containing a single number, the answer for that case.

#### **Constraints**

 $1 \le N \le 200000$   $0 \le x_{1,i} < x_{2,i} \le 200000$  $0 \le y_{1,i} < y_{2,i} \le 200000$ 





#### Sample Input

6	1100
10 0 20 50	
0 10 40 40	
30 20 40 30	
60 0 70 10	
100 100 200 200	
100 100 200 200	





# Problem F Illuminati

Time Limit: 2 seconds

As I entered the strange mansion, I saw N gummy candies lined up underneath the chandelier. It could only mean one thing. Aliens from Mars are about to arrive and this is their way of communicating it with us. To signify we are ready, we have to do the Mars ritual of drawing salt triangles around the candies. For example, if there are N = 11 candies in a line, the end result should look like this:



To begin the ritual, you must turn down the lights. Then close the door. Then decide to start at one of the vertices of the soon-to-be-drawn salt triangles. Then, you must hold down the salt container so the salt pours at a constant rate. As the salt pours, you should start chanting the Mars chant: "there's Pochi on the floor... there's Pochi on the floor.." While chanting, you shall walk towards an adjacent corner to draw one of the triangle edges. After this, you must move towards a different corner. However, the path you take must not have salt in it yet. You shall do this until all the edges of all triangles have been drawn. This tells the aliens that we are ready to communicate with them.

Given that you can start in any of the corners, how many different ways are there to do this Mars ritual?



Above is an example of one way to do the ritual if there are N=2 candies.

#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing a single integer, N.

#### Output

For each test case, output one line containing a single number, the answer for that case modulo  $10^9 + 7$ .





#### **Constraints**

 $1 \le T \le 30000 \\ 1 \le N \le 10^{12}$ 

Sample Input	Sample Output

4	12
2	656
6	99840
11	169216183
24	





# Problem G Superstitious

Time Limit: 2 seconds

Stevie writes his own music but each song he releases should have a lucky *N*-verse, where *N* depends on his mood. A lucky *N*-verse means that there are exactly *N* different segments of the verse which sound exactly the same forwards and backwards. For example, if the notes for a verse is DO-RE-DO, then there are 4 different segments which sound exactly the same forwards and backwards. In particular, the four segments are:

- **DO**, only taking the first note of **DO**-RE-DO.
- **RE**, only taking the second note of DO-**RE**-DO.
- **DO**, only taking the third note of DO-RE-**DO**.
- DO-RE-DO, taking the complete verse DO-RE-DO.

Note that all of these segments sound the same when played forwards and backwards. Moreover, the two DO's are considered different segments because they're from different parts of the verse. Moreover, note that DO-DO is not considered a segment of DO-RE-DO because they're not consecutive.

As Stevie is blind, he asks for help from his wife – his superwoman – on writing the notes of the verses he writes. Have you seen his wife? Neither has he. After writing the notes, the couple will then sign, seal and deliver the music sheets to the producers. Now, Stevie wonders if he could make a song that contains a lucky N-verse with N notes, for some N. Given N, write the notes of any N-verse with exactly N notes. If such a verse does not exist, output HEAVEN HELP US ALL instead.

#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing a single integer, N.

#### Output

For each test case, output one line containing an N-verse with exactly N notes. It shall be represented by a string containing only the capital letters C, D, E, F, G, A, B, which correspond to notes DO, RE, MI, FA, SO, LA, TI. Any valid answer will be accepted. If such an N-verse does not exist, output HEAVEN HELP US ALL instead.

#### **Constraints**

 $1 \le T \le 1000$ 

 $1 \le N \le 1000$ 





Sample Input	Sample Output
2	F
1	AG
2	





## Problem H Spoiled Children

Time Limit: 3 seconds

Tonette is a mother of N children whose ages are 1, 2, 3, ..., N years. Her children, unfortunately, are very spoiled. One of her children told us that "We can say whatever the faffle we want!". This child thought that faffle was a bad word.

This evening, Tonette won W blue waffles from a waffle raffle. These blue waffles look extremely nice and are very delicious that you should only see it live. Searching a photo of it in Google images is not advised as photos cannot capture the essence and beauty of this snack. Do not do this under any circumstance.

She is a kind mother and so she decides to share all W waffles to her N children. However, she knows that if a younger child sees that an older child got more waffles, they will get angry and throw a tantrum. Moreover, she tries very hard not to play favorites and so she tries to spread the waffles as equally as possible. In particular, suppose that she gives  $a_i$  waffles to her i-year old child. Let A be the maximum among the  $a_i$  and let B be the minimum among the  $a_i$ . "As equally as possible" means that she wants to minimize |A - B|.

What should the values of the  $a_i$  be?

#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing two integers, N and W, separated by a space.

#### Output

For each test case, output one line containing N integers separated by single spaces. The  $i^{th}$  integer denotes  $a_i$ , the number of waffles given to her i-year old child.

#### **Constraints**

$$1 \le T \le 10^4$$
  
$$1 \le N, W \le 100$$





#### Sample Input

4	6 5
2 11	1 1 1
3 3	1 1 0
3 2	4 4 3
3 11	





# Problem I Tagolympics

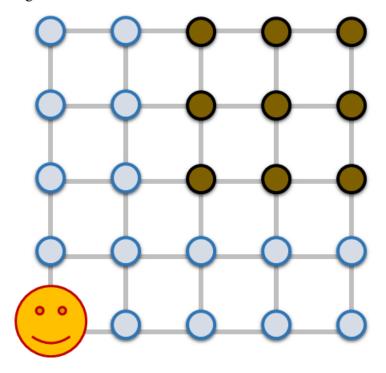
Time Limit: 7 seconds

Following the success of the previous year's GAGOLYMPICS, the organizers have created a different kind of game. It will be a one-versus-one type of competition. One player will be the hider and the other will be the seeker.

The seeker will stand on the southwest of a  $2N \times 2N$  grid. Note that this grid is composed of  $4N^2$  unit squares. The corners of these unit squares (except the corner where the seeker is standing) will each have a post on it. The hider must hide behind one of these posts. The seeker wins if he correctly points which post the hider is hiding.

However, almost all the posts are electric and touching them might kill you. It's a gag for this year's GAGOLYMPICS, bro! We can't have no gags this year because then it will be 0gag. And nobody likes 0gag. So yeah, it's the gag to the hider! So funny right? Hahaha. The only posts which are not electric are those which can be reached by walking at most N units west and at most N units south starting from the northeast corner. Hence, there are  $N^2 + 2N + 1$  posts left! Hence, hiders should only hide behind those posts.

The following image illustrates the case N = 2.



Not only do we have a gag for the seeker but we also have one for the hider! The gag is that not all the poles are visible (nor pointable) from the place where the seeker is standing because some other poles are blocking the way! It's such a funny gag. Hahaha. It's very





funny that I have to laugh again. Hahaha. How many of the  $N^2 + 2N + 1$  posts left should the hider avoid because it's a post visible to the seeker?

*Note:* A post is visible from the seeker if the straight line segment from the seeker's position to the post doesn't contain any other post. In this problem, consider posts and the seeker's position as mathematical points.

#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing a single integer, N.

#### Output

For each test case, output one line containing a single number, the answer for that case.

#### **Constraints**

$$1 \le T \le 5$$
  
$$1 < N < 3 \cdot 10^9$$

Cample Input	Sample Output
Sample Input	Sample Output

5	3
1	4
2	8
3	86
11	609004
1000	

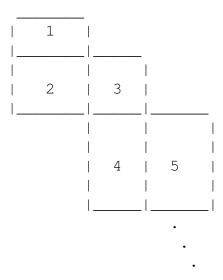




## Problem J Expando

Time Limit: 3 seconds

The EXPANDO is a toy which is composed of N rectangles. In particular, if we index the rectangles from 1 to N, each rectangle shares an edge with the rectangle numbered next to it like so:



The cool thing about it is that you can expand the different rectangles! For example, you can expando rectangles  $r, r+1, \ldots r+k$  together by x. This means that you must increase the lengths of all their sides by x. For example, if you want to expando rectangles 2,3,4 by 5 centimeters, then you should increase each of the following sides by five centimeters:

- the side of rectangle 2 which is not shared with rectangle 3
- the side shared by rectangles 2 and 3
- the side shared by rectangles 3 and 4
- the side of rectangle 4 which is not shared with rectangle 3

Note that whenever you expand a side, you must also expand the opposite side so that the shapes remain rectangles after expanding.

Given the initial dimensions of the N EXPANDO rectangles and how children play with it, determine the total area of a sequence of rectangles.

#### Input

There is a single test case.

The first line of input contains two integers N and Q separated by a space.





The second line contains N+1 integers  $s_0, s_1, \ldots, s_N$ , separated by single spaces. If i is divisible by N, then  $s_i$  represents the side length of the i<sup>th</sup> rectangle which is NOT shared with any other rectangle. Otherwise,  $s_i$  represents the side length of the edge shared by the i<sup>th</sup> rectangle and the (i+1)<sup>th</sup> rectangle.

The following Q lines describe the activities done by the children. Each such line is of the following form:

- P r k x, which means that the children would like to expando rectangles  $r, r + 1, \dots, r + k$  by x centimeters.
- A r k, which means that children would like to know the total area of the rectangles  $r, r+1, \dots, r+k$ .

#### Output

For each line of input that begins with A, output a single line containing a single integer, the answer for that query modulo  $10^9 + 7$ .

#### **Constraints**

$$1 \le N < 10^{5}$$

$$1 \le Q \le 10^{5}$$

$$1 \le x \le 10^{6}$$

$$1 \le s_{i} \le 10^{6}$$

$$1 \le r \le r + k \le N$$

#### Sample Input

•	•
9 7	15
2 7 1 8 2 8 1 8 2 8	16
A 2 1	109
A 5 0	15
A 1 8	18
P 6 1 1	133
A 2 1	
A 5 0	
A 1 8	





# Problem K Planet Link

Time Limit: 3 seconds

A new board game hit the shelves this week! It's called Planet Link. In its board, there are *N* planets and *K* players.

The first phase of the game is determining colors and order. The players choose a color different from gray. No two players may choose the same color. Then, the players randomly draw lots to decide the order of play.

The second phase of the game is the linking phase. There are N gray planets on the board and no **interplanetary bridges**. Two planets P and Q are **linked** if one can follow a series of interplanetary bridges to go from planet P to planet Q. The first player starts by adding an interplanetary bridge between two planets of his choice. The next players take turns adding an interplanetary bridge between two planets of their choice provided that the pair of planets they chose are not yet linked. The linking phase ends as soon as all pairs of planets are linked.

The third phase of the game is the conquering phase. Starting from the first player, one turn goes as follows:

- The player flips a coin. If it is heads, their character dies and is eliminated from the game.
- If the player is still alive at this point and has not conquered a planet before, he conquers any gray planet of his choosing. His character then moves to that newly conquered planet. And the planet becomes its color.
- However, if the player has already conquered a planet before, he may only conquer a gray planet which is reachable by his character using a single interplanetary bridge. He then moves to the newly conquered planet and the planet becomes the character's color.
- If the player cannot conquer a gray planet during that turn, his character dies.

The player who dies last is declared the winner provided that there are no more gray planets. Otherwise, all players lose.

Recall that the board is a bit different every time because the linking phase introduces a different setup of interplanetary bridges every game. Moreover, the third phase depends on how the interplanetary bridges are set up. Suppose it is the start of the third phase. Let P be the number of ways the board may look like when the game ends with at least one winner. Note that P depends on the choices made during the first and second phases. If one tries a different setup, then one might end up with a different value for P. Among these possible values for P, let  $P_{\min}$  be the lowest possible value for P and let  $P_{\max}$  be the highest possible value of P. Find  $P_{\min}$  and  $P_{\max}$ .





#### Input

The first line of input contains T, the number of test cases.

Each test case consists of one line containing two integers, N and K, separated by a space.

#### Output

For each test case, output one line containing a two numbers separated by a space,  $P_{\min}$  and  $P_{\max}$ . Output these numbers modulo  $10^9 + 7$ .

#### **Constraints**

$$1 \le T \le 1500$$

$$N, K \ge 1$$

$$1 \le NK \le 10^9$$

_	Sample Input	Sample Output
	1	6 6
	3 2	





### Problem L Slip'n'Slide

Time Limit: 2 seconds

Slip'n'Slide is one of the planned rides in a water theme park which requires people to go up and start at the very top. This will be called the top chamber. From the top chamber, one can use any of the water slides on that chamber to go down to a different chamber, and so on.

The plan is for the ride to have N different chambers (including the top one). There are two types of themes for the chambers – Hawaiian and Mediterranean. Moreover, each chamber has a positive rating  $R_i$ , determining how beautiful the chamber is.

There are two types of theme park visitors – the variety-seekers and the boring people. If one of the boring people take a slide from a chamber which leads to a chamber of the same theme, then they give that slide a rating of  $X \cdot Y$ , where X is the rating of the chamber from which he originated and Y is the rating of the chamber he ended up in after taking the slide. If they are of a different themes, they rate it 0. Similarly, if one of the variety-seekers take a slide from a chamber which leads to a chamber of a differing theme, then they give that slide a rating of  $X \cdot Y$ , where X is the rating of the chamber from which he originated and Y is the rating of the chamber he ended up in after taking the slide. However, if the two chambers involved are of the same theme, they rate it 0.

The rating a park visitor gives to the ride is the sum of this visitor's ratings for each slide. The park management does not want to make it seem like they favor one the variety-seekers over the boring people or vice-versa. And so, they would like to arrange the ride in such a way that the ride rating given by each type is as close as possible. In particular, if A is the rating that one of the variety-seekers will give to the ride and B is the rating that one of the boring people will give to the ride, then the absolute difference |A - B| must be minimized.

Now, you are given the ratings of each of the chambers and you are in deciding which chamber leads to where. The only constraint is that the ride shall be designed such that there is exactly one set of water slides that leads to each chamber when one comes from the top chamber. You are to arrange the chambers and slides such that |A - B| is minimal. Find |A - B|.

#### Input

The first line of input contains T, the number of test cases.

The first line of each test case contains a single integer, N. The chambers are indexed 1, 2, ..., N.

The next line of each test case contains N strings,  $D_1, D_2, \dots, D_N$ , where  $D_i$  is the description of chamber i. Each description  $D_i$  looks like " $T_iR_i$ " where:





- $T_i$  is the letter 'H' if the chamber is Hawaiian-themed, and 'M' if it is Mediterranean-themed.
- $R_i$  is the rating of chamber i.

#### Output

For each test case, output one line containing a single number, the answer for that case.

#### **Constraints**

$$1 \le T \le 450$$
  
 $2 \le N \le 8$   
 $T_i$  is either 'H' or 'M'  
 $1 < R_i < 10^9$ 

#### Sample Input

#### **Sample Output**

oumpro mput	
3	459
4	79
H30 H11 M50 M51	363
4	
H11 H11 H50 M15	
4	
H11 H11 H11 H11	

#### **Explanation**

In the first case, we can set up three slides connecting the following pairs of slides: (1,4), (2,4) and (3,4). In this setup, a variety-seeker will rate the slides a total of

$$A = 30 \cdot 51 + 11 \cdot 51 + 0 = 2091$$
,

and a boring person will rate the slides a total of

$$B = 0 + 0 + 50 \cdot 51 = 2550.$$

Thus, |A - B| = 459. It can be shown that this is the minimum possible |A - B|.

In the second case, we can set up three slides connecting the following pairs of slides: (1,2), (2,3) and (3,4). In this setup, a variety-seeker will rate the slides a total of

$$A = 0 + 0 + 50 \cdot 15 = 750$$
,

and a boring person will rate the slides a total of

$$B = 11 \cdot 11 + 11 \cdot 50 + 0 = 671.$$

Thus, |A - B| = 79.