LECTURE 8: Multiagent Decision Making (II)

Introduction to Multi-Agent Systems (MESIIA, MIA)

URV

Types of Agreement

- **Multiagent encounters (game-like character)**
- **Voting.**
- **Coalition forming.**
- **Auctions (Allocating Scarce Resources)**

Overview

- Allocation of scarce resources amongst a number of agents is central to multiagent systems.

- A resource might be:
  - a physical object
  - the right to use land
  - computational resources (processor, memory, . . .)
  - ..., etc.

- It is a question of supply vs demand
  - If the *resource isn't scarce*..., or if there is *no competition* for the resource...
    - Then there is no trouble allocating it
  - If there is a *greater demand than supply*
    - Then we need to determine how to allocate it

Overview

- In practice, this means we will be talking about auctions.
  - These used to be rare (and not so long ago)
  - However, auctions have grown massively with the Web/Internet

- Now feasible to auction things that weren't previously profitable:
  - eBay
  - Adword auctions

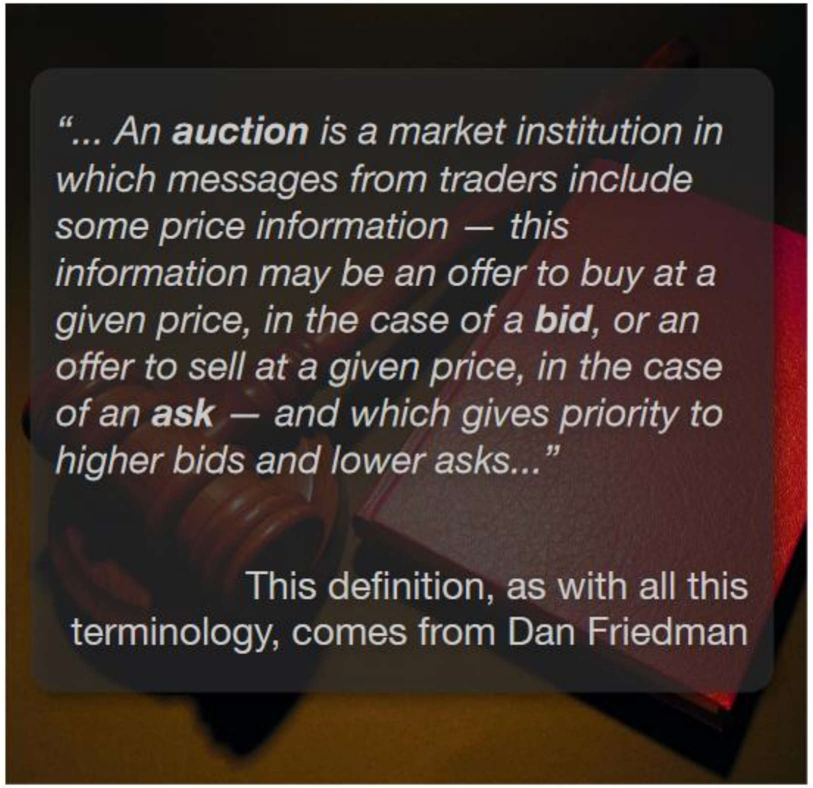
What is an auction

- Auctions are effective in allocating resources efficiently
  - They also can be used to reveal truths about bidders

- Concerned with *traders* and their allocations of:
  - Units of an indivisible *good*; and
  - Money, which is divisible.

- Assume some initial allocation.

- ***Exchange*** is the free alteration of allocations of goods and money between traders



Types of value

- There are several models, embodying different assumptions about the nature of the good.
  - Private Value / Common Value / Correlated Value
    - With a common value, there is a risk that the winner will suffer from the *winner's curse*, where the winning bid in an auction exceeds the intrinsic value or true worth of an item
- Each trader has a value or ***limit price*** that they place on the good.
  - Limit prices have an effect on the behaviour of traders

Private Value

Good has an value to me that is independent of what it is worth to you.

- *For example: John Lennon's last dollar bill.*

Common Value

The good has the same value to all of us, but we have differing estimates of what it is.

- *Winner's curse.*

Correlated Value

Our values are related.

- *The more you're prepared to pay, the more I should be prepared to pay.*

Auction Protocol Dimensions

- ***Winner Determination***
  - Who gets the good, and what do they pay?
    - e.g. first vs second price auctions
- ***Open Cry*** vs ***Sealed-bid***
  - Are the bids public knowledge?
    - Can agents exploit this public knowledge in future bids?
- ***One-shot*** vs ***Iterated Bids***
  - Is there a single bid (i.e. one-shot), after which the good is allocated?
  - If multiple bids are permitted, then:
    - Does the price ascend, or descend?
    - What is the terminating condition?

English Auction

- This is the kind of auction everyone knows.
  - Typical example is sell-side.
- Buyers call out bids, bids increase in price.
  - In some instances the auctioneer may call out prices with buyers indicating they agree to such a price.
- The seller may set a ***reserve price***, the lowest acceptable price.

- Auction ends:
  - at a fixed time (internet auctions); or when there is no more bidding activity.
  - The “last man standing” pays their bid.

English Auction

Classified in the terms we used above:

- ***First-price***
- ***Open-cry***
- ***Ascending***

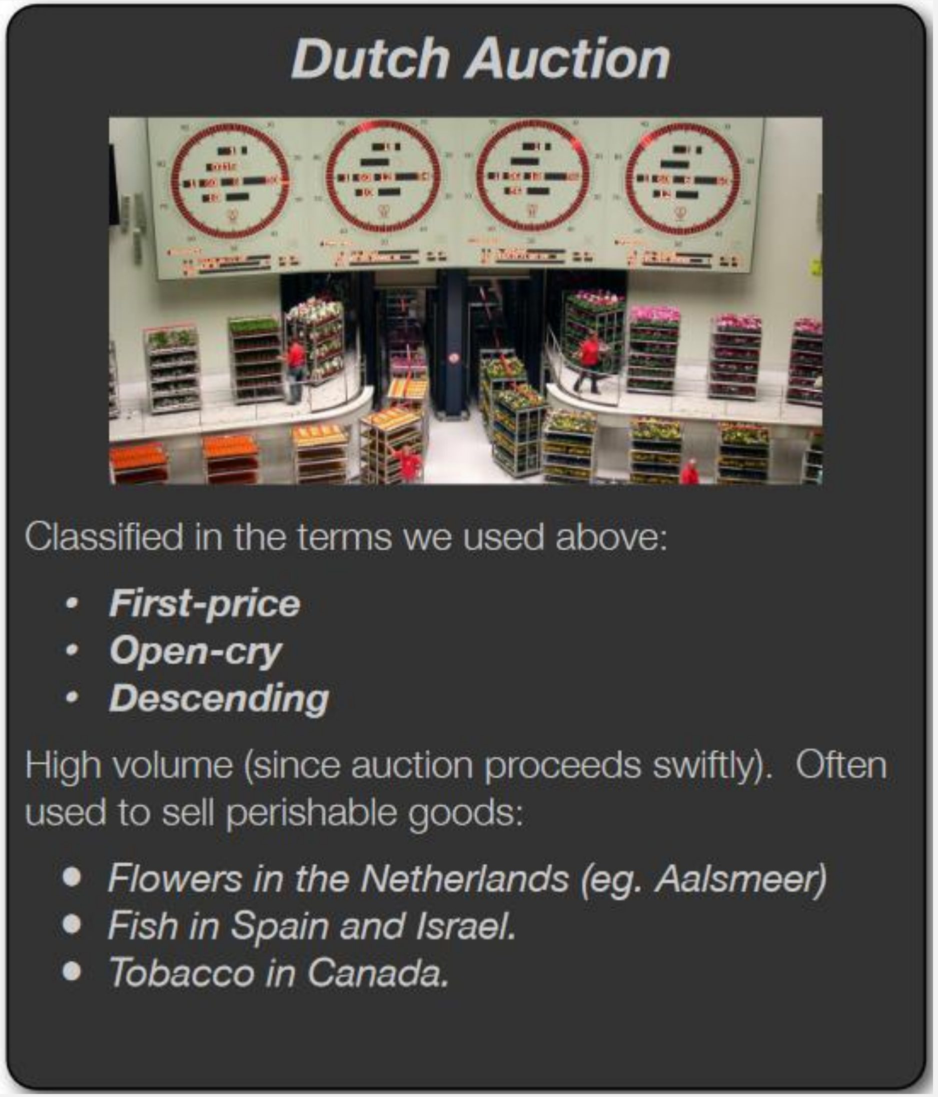
Around 95% of internet auctions are of this kind.
The classic use is the sale of antiques and artwork.

Susceptible to:

- *Winner's curse*
- *Shills*

Dutch Auction

- Also called a “descending clock” auction
  - Some auctions use a clock to display the prices.
- Starts at a ***high price***, and the auctioneer calls out ***descending prices***.
  - One bidder claims the good by indicating the current price is acceptable.
    - ***Ties are broken*** by restarting the descent from a slightly higher price than the tie occurred at.
- The winner pays the price at which they “***stop the clock***”.



First-Price Sealed-Bid Auction

- In an English auction, you get information about how much a good is worth
  - Other people's bids tell you things about the market.
- In a ***sealed bid auction***, none of that happens
  - at most you know the winning price after the auction.
- In the First-Price Sealed-Bid (FPSB) auction the ***highest bid wins as always***.
  - As its name suggests, the winner pays that highest price (which is what they bid).

FPSB

Classified in the terms we used above:

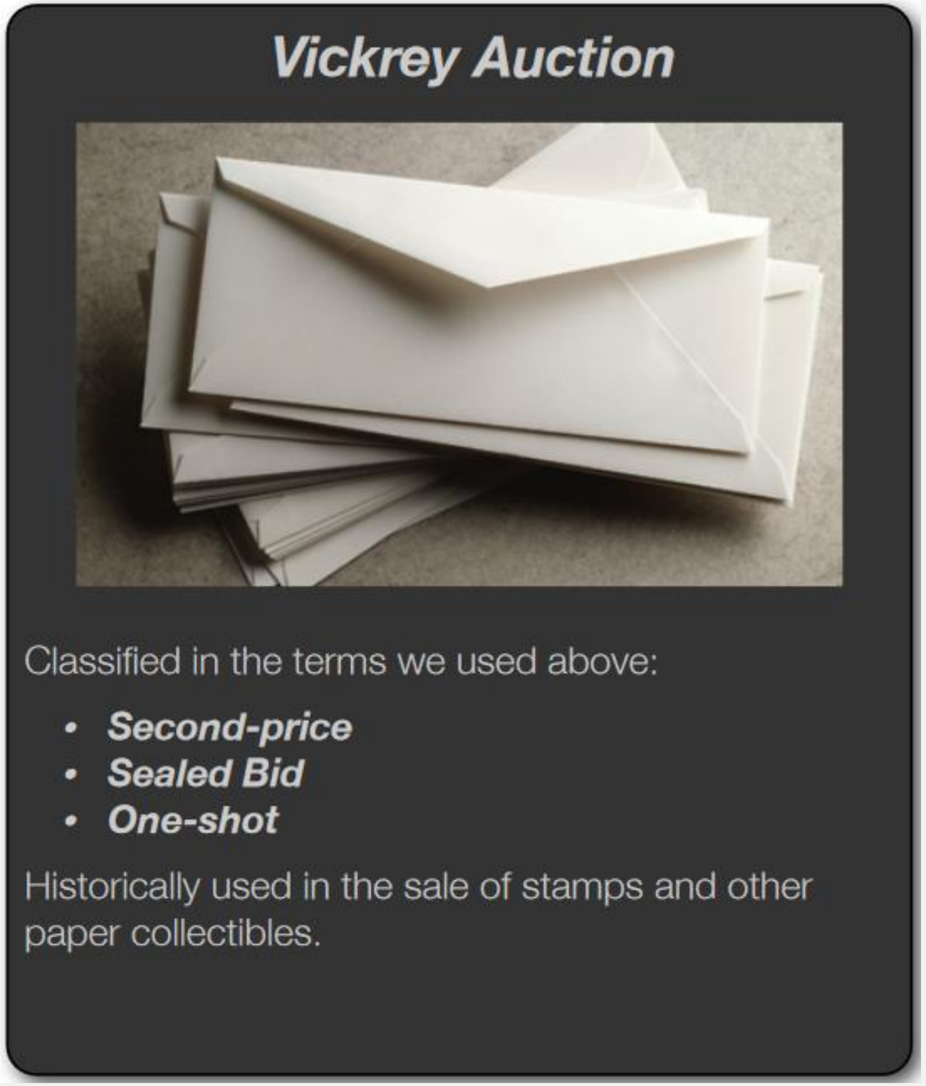
- ***First-price***
- ***Sealed Bid***
- ***One-shot***

Governments often use this mechanism to sell treasury bonds (the UK still does, although the US recently changed to Second-Price sealed Bids).

Property can also be sold this way (as in Scotland).

Vickrey Auction

- The Vickrey auction is a sealed bid auction.
  - The winning bid is the ***highest bid***, but the winning bidder ***pays the amount of the second highest bid***.
- This sounds odd, but it is actually a very smart design.
  - Will talk about why it works later.
- It ***is in the bidders' interest to bid their true value***.
  - ***incentive compatible*** in the usual terminology.
- However, it is not a panacea, as the New Zealand government found out in selling radio spectrum rights
  - Due to interdependencies in the rights, that led to strategic bidding
    - one firm bid NZ$100,000 for a license, and paid the second-highest price of only NZ$6.



Why does the Vickrey auction work?

- Suppose you bid more than your valuation.
  - You may win the good.
  - If you do, you may end up paying more than you think the good is worth.
  - Not so smart.
- Suppose you bid less than your valuation.
  - You stand less chance of winning the good.
  - However, even if you do win it, you will end up paying the same.
  - Not so smart.

Proof of dominance of truthful bidding

- Let v_i be the bidding agent i's value for an item, and b_i be the agent's bid.
  - The payoff for bidder i is:

$$p_i = \begin{cases} v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

- Assume bidder i bids $b_i > v_i$ (i.e. ***overbids***)
  - If $\max_{j \neq i} b_j < v_i$, then the bidder would win whether the bid was truthful. Therefore, the strategies of bidding truthfully and overbidding have equal payoffs
  - If $\max_{j \neq i} b_j > b_i$, then the bidder would loose whether the bid was truthful. Again, both strategies have equal payoffs
  - If $v_i < \max_{j \neq i} b_j < b_i$, then the strategy of overbidding would win the action, but the payoff would be negative (as the bidder will have overpaid). A truthful strategy would pay zero.

Proof of dominance of truthful bidding

- Let v_i be the bidding agent i's value for an item, and b_i be the agent's bid.
  - The payoff for bidder i is:

$$p_i = \begin{cases} v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

- Assume bidder i bids $b_i < v_i$ (i.e. ***underbids***)
  - If $\max_{j \neq i} b_j > v_i$, then the bidder would loose whether the bid was truthful. Therefore, the strategies of bidding truthfully and underbidding have equal payoffs
  - If $\max_{j \neq i} b_j < b_i$, then the bidder would wind whether the bid was truthful. Again, both strategies have equal payoffs
  - If $b_i < \max_{j \neq i} b_j < v_i$, then only the strategy of truth-telling would win the action, with a positive payoff (as the bidder would have). An underbidding strategy would pay zero.

Collusion

- None of the auction types discussed so far are immune to collusion

  - A ***grand coalition of bidders*** can agree beforehand to collude
    - Propose to artificially lower bids for a good
    - Obtain true value for good
    - Share the profit

  - An auctioneer could employ bogus bidders
    - **Shills** could artificially increase bids in open cry auctions
    - Can result in ***winner's curse***

Combinatorial Auctions

- A combinatorial auction is an ***auction for bundles of goods***.
  - A good example of bundles of goods are spectrum licences.
  - For the 1.7 to 1.72 GHz band for Brooklyn to be useful, you need a license for Manhattan, Queens, Staten Island.
  - Most valuable are the licenses for the same bandwidth.
  - But a different bandwidth license is more valuable than no license.
    - a phone will work, but will be more expensive.

Combinatorial Auctions

- Define a set of items to be auctioned as: $\mathcal{Z} = \{z_1, \dots, z_\mathrm{m}\}$

- Given a set of agents $Ag = \{1, \dots, n\}$ the preferences of agent i are given with the ***valuation function***: $v_i: 2^{\mathcal{Z}} \to \mathbb{R}$, meaning that for every possible bundle of goods $\mathcal{Z}' \subseteq \mathcal{Z}$, $v_i(\mathcal{Z}')$ says how much $\mathcal{Z}'$is worth to i.
  - If that sounds to you like it would place a big requirement on agents to specify all those preferences, you would be right.
  - If $v_i(\phi) = 0$, then we say that the valuation function for i is ***normalized***.
    - i.e. Agent i does not get any value from an empty allocation
- Another useful idea is free disposal, $\mathcal{Z}_1 \subseteq \mathcal{Z}_2 \Rightarrow v_i(\mathcal{Z}_1) \leq v_i(\mathcal{Z}_2)$
  - In other words, an agent is never worse off having more stuff

Allocation of Goods

- An outcome is an allocation of goods to the agents.
  - Note that we don't require all off the goods to be allocated
  - Formally an allocation is a list of sets $\mathcal{Z}_1, \ldots, \mathcal{Z}_n$ one for each agent i such that$\mathcal{Z}_i \subseteq \mathcal{Z}$
  - and for all $i, j \in Ag$ such that $i \neq j$, we have $\mathcal{Z}_i \cap \mathcal{Z}_j = \phi$
    - Thus, no good is allocated to more than one agent

- The set of all allocations of $\mathcal{Z}$ to agents Ag is: $alloc(\mathcal{Z}, Ag)$

Maximising Social Welfare

- If we design the auction, we get to say how the allocation is determined.
  - Combinatorial auctions can be viewed as different social choice functions, with different outcomes relating to different allocations of goods
  - A desirable property would be to maximize social welfare
    - i.e. maximise the sum of the utilities of all the agents.

- We can define a social welfare function:

$$sw(\underbrace{Z_1 \dots, Z_n}_{\text{allocations}}, \underbrace{v_1, \dots, v_n}_{\text{valuations}}) = \sum_{i=1}^{n} v_i(Z_i)$$

Defining a Combinatorial Auction

- Given this, we can define a combinatorial auction.
  - Given a set of goods $\mathcal{Z}$ and a collection of valuation functions $v_1, \ldots, v_n$ one for each agent $i \in Ag$, the goal is to find allocation $\mathcal{Z}_1^*, \ldots, \mathcal{Z}_n^*$ that maximses sw:

$$\mathcal{Z}_1^*, \ldots, \mathcal{Z}_n^* = \arg \max_{(\mathcal{Z}_1, \ldots, \mathcal{Z}_n) \in alloc(\mathcal{Z}, Ag)} sw(\mathcal{Z}_1, \ldots, \mathcal{Z}_n, v_1, \ldots, v_n)$$

- Figuring this out, i.e. solving this optimization problem, is called the ***winner determination problem***

Winner Determination

- How do we do this?
- Well, we could get every agent i to declare their valuation :$\hat{v}_i$
  - The hat denotes that this is what the agent says, not what it necessarily is.
  - Remember that the agent may lie! ☺
- Then we just look at all the possible allocations and figure out what the best one is.

- One problem here is representation, valuations are exponential in terms of the number of items: $v_i: 2^Z \rightarrow \mathbb{R}$
  - A naive representation is impractical.
  - In a bandwidth auction with 1122 licenses we would have to specify 2^{1122} values for each bidder.
- Searching through them is computationally intractable

Bidding Languages

- Rather than exhaustive evaluations, allow bidders to construct valuations from the bits they want to mention.
  - An atomic bid β is a pair $(\mathcal{Z}', p)$ where $\mathcal{Z}' \subseteq \mathcal{Z}$
  - A bundle $\mathcal{Z}^\star$ ***satisfies*** a bid $(\mathcal{Z}', p)$ if $\mathcal{Z}' \subseteq \mathcal{Z}^\star$.

- In other words a bundle ***satisfies*** a bid if it contains at least the things in the bid.

- Atomic bids ***define valuations***

$$v_\beta(\mathcal{Z}^\star) = \begin{cases} p & \text{if } \mathcal{Z}^\star \text{ satisfies } (\mathcal{Z}', p) \\ 0 & \text{otherwise} \end{cases}$$

- Atomic bids alone don't allow us to construct very interesting valuations.

XOR Bids

- With XOR bids, ***we pay for at most one***
  - A bid $\beta = (\mathcal{Z}_1, p_1) XOR \ \dots XOR\ (\mathcal{Z}_k, p_k)$ defines a valuation function v_β as follows

$$v_\beta(\mathcal{Z}^\star) = \begin{cases} 0 & \text{if } \mathcal{Z}^\star \text{ does not satisfy any } (\mathcal{Z}_i, p_i) \\ \max\{p_i \mid \mathcal{Z}_i \subseteq \mathcal{Z}^\star\} & \text{otherwise} \end{cases}$$

  - I pay nothing if your allocation $\mathcal{Z}^\star$ doesn't satisfy any of my bids
  - Otherwise, I will pay the largest price of any of the satisfied bids.
- XOR bids are ***fully expressive***, that is they can express any valuation function over a set of goods.
  - To do that, we may need an exponentially large number of atomic bids
    - However, the valuation of a bundle can be computed in polynomial time.

$$B_1 = (\{a, b\}, 3)\ XOR\ (\{c, d\}, 5)$$

"…I would pay 3 for a bundle that contains a and b but not c and d. I will pay 5 for a bundle that contains c and d but not a and b, and I will pay 5 for a bundle that contains a, b, c and d…"

From this we can construct the valuation:

$$\begin{aligned} v_{\beta_1}(\{a\}) &= 0 \\ v_{\beta_1}(\{b\}) &= 0 \\ v_{\beta_1}(\{a, b\}) &= 3 \\ v_{\beta_1}(\{c, d\}) &= 5 \\ v_{\beta_1}(\{a, b, c, d\}) &= 5 \end{aligned}$$

OR Bids

- With OR bids, ***we are prepared to***
- ***pay for more than one bundle***
  - A bid $\beta = (\mathcal{Z}_1, p_1) OR \ \ldots OR \ (\mathcal{Z}_k, p_k)$ defines k valuations for different bundles
  - An allocation of goods $\mathcal{Z}'$ is assigned given a set $\mathcal{W}$ of atomic bids such that:
    - Every bid in $\mathcal{W}$ is satisfied by $\mathcal{Z}'$
    - No goods appear in more than one bundle; i.e. $\mathcal{Z}_i \cap \mathcal{Z}_j = \phi$ for all i, j where $i \neq j$
    - No other subset $\mathcal{W}'$ satisfying the above condition is better:

$$\sum_{(\mathcal{Z}_i, p_i) \in \mathcal{W}'} p_i > \sum_{(\mathcal{Z}_j, p_j) \in \mathcal{W}'} p_j$$

$B_1 = (\{a, b\}, 3) \ OR \ (\{c, d\}, 5)$

"…I would pay 3 for a bundle that contains a and b but not c and d. I will pay 5 for a bundle that contains c and d but not a and b, and I will pay 8 for both bundles that contain a combination of a, b, c and d…"

From this we can construct the valuation:

$$\begin{aligned} v_{\beta_1}(\{a\}) &= 0 \\ v_{\beta_1}(\{b\}) &= 0 \\ v_{\beta_1}(\{a, b\}) &= 3 \\ v_{\beta_1}(\{c, d\}) &= 5 \\ v_{\beta_1}(\{a, b, c, d\}) &= 8 \end{aligned}$$

Note that the **cost of the last bundle is different to that when the XOR bid** was used

OR Bids

- Here is another example!
  - $\beta_3 = OR\big((\{e, f, g\}, 4), (\{f, g\}, 1), (\{e\}, 3), (\{c, d\}, 4)\big)$
  - This gives us:

$$
\begin{aligned}
&v_{\beta_3}(\{e\}) = 3 \\
&v_{\beta_3}(\{e, f\}) = 3 \\
&v_{\beta_3}(\{e, f, g\}) = 4 \\
&v_{\beta_3}(\{b, c, d, f, g\}) = 4 + 1 = 5 \\
&v_{\beta_3}(\{a, b, c, d, e, f, g\}) = 4 + 4 = 8 \\
&v_{\beta_3}(\{c, d, e\}) = 4 + 3 = 7
\end{aligned}
$$

- Remember that if more than one bundle is satisfied, then you pay for each of the bundles satisfied.
  - Also remember free disposal, which is why the bundle $\{e, f\}$ satisfies the bid $(\{e\}, 3)$ as the agent doesn't pay extra for f.

OR Bids

- OR bids are strictly less expressive than XOR bids
  - Some valuation functions cannot be expressed
  - E.g., $v(\{a\}) = 1, v(\{b\}) = 1, v(\{a, b\}) = 1$

- OR bids also suffer from computational complexity
  - Given an OR bid β and a bundle $\mathcal{Z}$, computing $v_\beta(\mathcal{Z})$ is NP-hard.

Winner Determination

- Determining the winner is a combinatorial optimisation problem (NP-hard)
  - But this is a worst case result, so it may be possible to develop approaches that are either ***optimal*** and run well in many cases, or ***approximate*** (within some bounds).
- Typical approach is to code the problem as an ***integer linear program*** and use a standard solver.
  - This is NP-hard in principle, but often provides solutions in reasonable time.
  - Several algorithms exist that are efficient in most cases
- Approximate algorithms have been explored
  - Few solutions have been found with reasonable bounds
- Heuristic solutions based on ***greedy algorithms*** have also been investigated
  - e.g. that try to find the largest bid to satisfy, then the next etc

The VCG Mechanism

- Auctions are easy to strategically manipulate
  - In general ***we don't know*** whether the agents valuations ***are true valuations***.
  - Life would be easier if they were…
  - … so can we make them true valuations?

- Yes!
  - In a generalization of the Vickrey auction.
    - Vickrey/Clarke/Groves Mechanism

- Mechanism is incentive compatible: ***telling the truth is a dominant strategy.***

Recall that we could get every agent i to declare their valuation:

$$\hat{v}_i$$

where the hat denotes that this is what the agent says, not what it necessarily is.

- *The agent may lie!*

The VCG Mechanism

- Need some more notation.
  - ***Indifferent valuation*** function: $v^0(\mathcal{Z}') = 0$ for all $\mathcal{Z}'$
    - i.e. the value for a bid that does not care about the goods
  - sw_{-i} is the ***social welfare function without*** i:

$$sw_{-i}(\mathcal{Z}_1, \dots, \mathcal{Z}_n, v_1, \dots, v_n) = \sum_{j \in Ag, j \neq i} v_j(\mathcal{Z}_j)$$

  - This is how well everyone ***except agent i*** does from $\mathcal{Z}_1, \dots, \mathcal{Z}_n$

- And we can then define the VCG mechanism.

The VCG Mechanism

- Every agent simultaneously declares a valuation $\hat{v}_i$
  - Remember that this not be the actual valuation
- The mechanism computes the allocation $\mathcal{Z}_1^*, \dots, \mathcal{Z}_n^*$:

$$\mathcal{Z}_1^*, \dots, \mathcal{Z}_n^* = \arg \max_{(\mathcal{Z}_1,\dots,\mathcal{Z}_n) \in alloc(\mathcal{Z}, Ag)} sw(\mathcal{Z}_1, \dots, \mathcal{Z}_n, v_1, \dots, v_n)$$

- Each agent i pays p_i
  - This is effectively a ***compensation*** to the other agents for their loss in utility due to i winning an allocation
  - This is the difference in social welfare to agents other than i
  - Between the outcome $\mathcal{Z}_1', \dots, \mathcal{Z}_n'$ when i does not participate
  - And the outcome $\mathcal{Z}_1^*, \dots, \mathcal{Z}_n^*$ when i does participate
  - Formally: $p_i = sw_{-i}(\mathcal{Z}_1', \dots, \mathcal{Z}_n', \hat{v}_1, \dots, v^0, \dots, \hat{v}_n) - sw_{-i}(\mathcal{Z}_1^*, \dots, \mathcal{Z}_n^*, \hat{v}_1, \dots, \hat{v}_i, \dots, \hat{v}_n)$
  - Therefore the mechanism computes, for each agent i the allocation that maximises social welfare were that agent to have declared v^0 to be its valuation:

$$\mathcal{Z}_1^*, \dots, \mathcal{Z}_n^* = \arg \max_{(\mathcal{Z}_1,\dots,\mathcal{Z}_n) \in alloc(\mathcal{Z}, Ag)} sw(\mathcal{Z}_1, \dots, \mathcal{Z}_n, v_1, \dots, v^0, \dots, v_n)$$

The VCG Mechanism

- With the VCG, each agent pays out the cost (to the other agents) of it having participated in the auction.
    - It is incentive compatible for exactly the same reason as the Vickrey auction was before.
        - No agent can benefit by declaring anything other than its true valuation
    - To understand this, think about VCG with a singleton bundle
        - The only agent that pays anything will be the agent i that has the highest bid
        - But if it had not participated, then the agent with the second highest bid would have won
        - Therefore agent i "compensates" the other agents by paying this second highest bid

- Therefore we get a dominant strategy for each agent that guarantees to maximise social welfare.
    - i.e. ***social welfare maximisation can be implemented in dominant strategies***

Summary

- Allocating scarce resources comes down to auctions
- We looked at a range of different simple auction mechanisms.
  - English auction
  - Dutch auction
  - First price sealed bid
  - Vickrey auction
- We looked at the popular field of combinatorial auctions
  - We discussed some of the problems in implementing combinatorial auctions.
- And we talked about the Vickrey/Clarke/Groves mechanism, a rare ray of sunshine on the problems of multiagent interaction

Readings for this week

- Chapters 14 of the book by M.Wooldridge “An introduction to Multi-Agent Systems” (2nd edition).