

Solution to some exercises with Certainty Factors

Exercise 1

Rule 1: If (A and B and C) then E (CF = 0.8)

Rule 2: If (C or D) then E (CF = 0.9)

Evidences:

CF(A)= 0.4, CF(B)= 0.9, CF(C)=0.7 and CF(D)= 0.8.

The first rule calculates an AND = $\min(0.4, 0.9, 0.7) = 0.4$, then we multiply by the CF of the rule, so $0.4 \cdot 0.8 = 0.32$ which is CF(E)

The second rule calculates an OR = $\max(0.7, 0.8) = 0.8$, multiplied by CF of the rule, $0.8 \cdot 0.9 = 0.72$ is another CF(E)

We must now combine both certainty factors using the parallel combination rule,

$$CF(E) = 0.32 + 0.72(1-0.32) = 0.32 + 0.49 = 0.81$$

The value is higher than the two individual ones, because they are like “accumulating” their individual certainty values.

If we add the rule If E then G (CF= - 1) , then $CF(G) = 0.81 \cdot -1 = -0.81$, which is the opposite than for E. That is, if we believe in E, we do not believe in G.

Is Peter innocent or guilty?

Rule 1: If fingerprints on gun or on place -> guilty. CF=0.75

Rule 2: If has a motive -> guilty. CF= 0.6

Rule 3: If defendant has alibi -> guilty. CF=-0.8

Evidences:

CF(fingerprints on gun) = 0.5

CF(Fingerprints on place)= 0.9

CF(has a motive)= 0.5

CF(has alibi)= 0.95

The first rule gives $CF(\text{guilty}) = \max(0.5, 0.9) \times 0.75 = 0.675$

The second rule gives $CF(\text{guilty}) = 0.5 \times 0.6 = 0.3$

The third rule gives $CF(\text{guilty}) = 0.95 \times -0.8 = -0.76$

We have to combine the results in pairs. We proceed with the two positive ones, which gives a result of $CF(\text{guilty}) = 0.675 + 0.3 \times (1-0.675) = 0.7725$

Next we combine this result with the negative one, so that

$$CF(\text{guilty}) = (0.7725 - 0.76) / (1 - \min(0.7725, 0.76)) = 0.052$$

Holmes

Rule 1: If Watson calls -> alarm sound. CF=0.5

Rule 2: If alarm sound -> there is a burglary. CF= 0.99

Rule 3: If Mrs Gibbons confirms and she is sober -> alarm sound. CF=0.9

Evidences:

CF(Watson calls) = 1.0

CF(Mrs Gibbons confirms)= 1.0

CF(MrsGibbons is sober)= -0.7

The first rule gives CF(alarm sound) = $1.0 \times 0.5 = 0.5$

The third rule gives CF(alarm sound)= $\min(1.0, -0.7) \times 0.9 = -0.63$

Then we combine the two outputs of different sign, we obtain CF(alarm sound) = -0.26

Now, with rule two, we have CF(burglary) = $-0.26 \times 0.99 = -0.257$

Consequently, Holmes will continue drinking at the pub because evidences does not support the claim of a burglary happening at his home.

Virus in my computer

Rule 1: anomalous behaviour -> virus. CF= 0.8

Rule 2: updated antivirus -> virus. CF=-0.95

Rule 3: files couldn't open -> anomalous behaviour. CF=0.8

Rule 4: system works slowly and few programs running -> anomalous behaviour. CF=0.6

I observe the following evidences:

CF few programs = 1.0

CF works too slowly = 0.6

CF files don't open = 0.8

CF updated antivirus = 1.0.

Question 1: According to the previous facts, observed today, is there a virus in my computer?

Rule 3: CF(anomalous behaviour) = $0.8 \times 0.8 = 0.64$

Rule 4: CF(anomalous behaviour) = $\min(0.6, 1.0) \times 0.6 = 0.36$

With the combination rule of two positive CF, we have: $0.64 + (0.36 \times 0.36) = 0.77$

Thus, CF(anomalous behaviour) = 0.77

Rule 1: CF(virus) = $0.77 \times 0.8 = 0.62$

Rule 2: CF(virus) = $1.0 \times -0.95 = -0.95$

With the combination rule of a positive CF with a negative CF, we have: $(0.62 - 0.95) / (1 - \min(0.62, 0.95)) = -0.33 / (1 - 0.62) = -0.87$

Thus, CF(virus) = -0.87

The evidences tell me that there is no virus in my computer with high confidence, 87%

Question 2: What happens if I don't rely too much in the antivirus? More precisely, calculate which is the maximum certainty factor for the "updated antivirus" fact that makes the final value of CF for "computer has virus" of at least 0.3.

Having $CF(\text{anomalous behaviour}) = 0.77$, the first rule is always positive

Having that rule 2 has a negative CF, we have to check $CF(\text{updated antivirus})$:

- If $CF(\text{updated antivirus}) \leq 0$ the final CF of the rule 2 will be positive or zero, and the final $CF(\text{virus})$ will be higher than 0.62
- So, now we check the case of $CF(\text{updated antivirus}) > 0$. Having that rule 2 will give a negative result, we use the combination rule of a positive and negative CF.

We want to find the value a so that: $(0.62 - a * 0.95) / (1 - \min(0.62, a * 0.95)) = 0.3$

so a must be 0.481.

Consequently, if my confidence on "updated antivirus" is larger than 0.481 then the $CF(\text{virus})$ decreases, because I rely on my antivirus. On the contrary, with smaller values than 0.481 the $CF(\text{virus})$ increases because my confidence on the antivirus is not enough.