Advanced Human Language Technologies

Statistical Models of Language

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Outline

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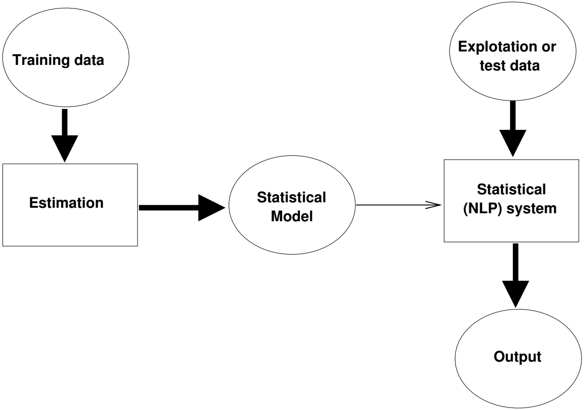
We model to make predictions



Outline

Prediction Models & Similarity Models

- **Prediction Models**: Oriented to *predict* probabilities of future events, knowing past and present.
- **Similarity Models**: Oriented to compute *similarities* between objects (may be used to predict, EBL).

Similarity Models

- Objects represented as feature-vectors, feature-sets, distribution-vectors, ...
- Used to group objects (clustering, data analysis, pattern discovery, ...)
- If classified objects are available, similarity may be used as a prediction (example-based ML techniques).
- Example: Document representation
  - Documents are represented as vectors in a high dimensional $\mathbb{R}^n$ space.
  - Dimensions are word forms, lemmas, NEs, n-grams, ...
  - Values may be either binary or real-valued (count, frequency, ...)
  - Vector-space algebra and metrics can be used

Prediction Models

- Estimation: Using data to infer information about distributions
  - Parametric / non-parametric estimation
  - Finding good estimators: MLE, MEE, ...
  - Explicit / implicit models
- Classification: Predictions based on past behaviour
  - Predict most likely target given classification features (implies independence assumptions!)
  - Granularity of equivalence classes (bins): discrimination power *vs.* statistical reliability
- In general, ML models estimate (i.e. *learn*) conditional probability distributions $\mathrm{P}(\text{target}|\text{features})$
- Many NLP tasks require a posterior search step to find the best combination of predictions.

Prediction Models

Example: Noisy Channel Model (Shannon 48)

Input P(i) → **Channel** P(o|i) → **Output**

NLP Applications

Appl.	Input	Output	$p(i)$	$p(o \mid i)$
MT	L word sequence	M word sequence	$p(L)$	Translation model
OCR	Actual text	Text with mistakes	prob. of language text	model of OCR errors
PoS tagging	PoS tags sequence	word sequence	prob. of PoS sequence	$p(w \mid t)$
Speech recog.	word sequence	speech signal	prob. of word sequence	acoustic model

Given $\mathbf{o}$, we want to find the most likely $\mathbf{i}$

$$\operatorname*{argmax}_{\mathbf{i}} P(\mathbf{i} \mid \mathbf{o}) = \operatorname*{argmax}_{\mathbf{i}} P(\mathbf{o}, \mathbf{i}) = \operatorname*{argmax}_{\mathbf{i}} P(\mathbf{i}) P(\mathbf{o} \mid \mathbf{i})$$

Finding good estimators: MLE

Maximum Likelihood Estimation (MLE)

- Choose the alternative that maximizes the probability of the observed outcome.
- $\bar{\mu}_n$ is a MLE for $E(X)$
- s_n^2 is a MLE for σ^2
- Zipf's Laws. Data sparseness. Smoothing tecnhiques.

$P(a, b)$	dans	en	à	sur	au-cours-de	pendant	selon	
in	0.04	0.10	0.15	0	0.08	0.03	0	0.40
on	0.06	0.25	0.10	0.15	0	0	0.04	0.60
total	0.10	0.35	0.25	0.15	0.08	0.03	0.04	1.0

Finding good estimators: MEE

Maximum Entropy Estimation (MEE)

- Choose the alternative that maximizes the entropy of the obtained distribution, maintaning the observed probabilities.

Observations:

$p(\text{en} \vee \text{à}) = 0.6$; $p((\text{en} \vee \text{à}) \wedge \text{in}) = 0.4$; $p(\text{in}) = 0.5$

$P(a,b)$	dans	en	à	sur	au-cours-de	pendant	selon	
in	0.02	**0.20**	**0.20**	0.02	0.02	0.02	0.02	**0.5**
on	0.06	0.10	0.10	0.06	0.06	0.06	0.06	
total		$\underbrace{}_{\mathbf{0.6}}$						1.0

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Working Example: N-gram models

- Predict the next element in a sequence (e.g. next character, next word, next PoS, next stock value, …), given the *history* of previous elements: $\boxed{P(w_n \mid w_1 \dots w_{n-1})}$
- Markov assumption: Only *local* context (of size $n-1$) is taken into account. $\boxed{P(w_i \mid w_{i-n+1} \dots w_{i-1})}$
- bigrams, trigrams, four-grams ($n = 2, 3, 4$).
  Sue swallowed the large green <?>
- Parameter estimation (number of equivalence classes)
- Parameter reduction: stemming, semantic classes, PoS, …

Model	Parameters
bigram	$20,000^2 = 4 \times 10^8$
trigram	$20,000^3 = 8 \times 10^{12}$
four-gram	$20,000^4 = 1.6 \times 10^{17}$

Language model sizes for a 20,000 words vocabulary

N-gram model estimation

Estimate the probability of the target feature based on observed data. The prediction task can be reduced to having good estimations of the n-gram distribution:

$$\boxed{P(w_n \mid w_1 \dots w_{n-1}) = \frac{P(w_1 \dots w_n)}{P(w_1 \dots w_{n-1})}}$$

- **MLE (Maximum Likelihood Estimation)**

$$\boxed{\begin{array}{l} P_{MLE}(w_1 \dots w_n) = \frac{C(w_1 \dots w_n)}{N} \\ P_{MLE}(w_n \mid w_1 \dots w_{n-1}) = \frac{C(w_1 \dots w_n)}{C(w_1 \dots w_{n-1})} \end{array}}$$

  - No probability mass for unseen events
  - Data sparseness, Zipf's Law
  - Unsuitable for NLP (widely used, though)

Brief Parenthesis: Zipf's Laws

Zipf's Laws (1929)

- Word frequency is inversely proportional to its rank (speaker/hearer minimum effort) $\boxed{f \sim 1/r}$
- Number of senses is proportional to frequency root $\boxed{m \sim \sqrt{f}}$
- Frequency of intervals between repetitions is inversely proportional to the length of the interval $\boxed{F \sim 1/I}$
- Frequency based approaches are hard, since most words are rare
  - Most common 5% words account for about 50% of a text
  - 90% least common words account for less than 10% of the text
  - Almost half of the words in a text occurr only once

Outline

Notation

- $\boxed{C(w_1 \ldots w_n)}$: Observed occurrence count for n-gram $w_1 \ldots w_n$.
- N: Number of observed n-gram occurrences

$$\boxed{N = \sum_{w_1 \ldots w_n} C(w_1 \ldots w_n)}$$

- N_k: Number of classes (n-grams) observed k times.
- B: Number of equivalence classes or bins (number of potentially observable n-grams).

Smoothing 1 - Adding Counts

- **Laplace's Law** (adding one)

  $$P_{LAP}(w_1 \dots w_n) = \frac{C(w_1 \dots w_n) + 1}{N + B}$$

  - For large values of B too much probability mass is assigned to unseen events

- **Lidstone's Law**

  $$P_{LID}(w_1 \dots w_n) = \frac{C(w_1 \dots w_n) + \lambda}{N + B\lambda}$$

  - Usually $\lambda = 0.5$, *Expected Likelihood Estimation*.
  - Equivalent to linear interpolation between MLE and uniform prior, with $\mu = N/(N + B\lambda)$,

    $$P_{LID}(w_1 \dots w_n) = \mu \frac{C(w_1 \dots w_n)}{N} + (1 - \mu) \frac{1}{B}$$

Smoothing 2 - Discounting Counts

- **Absolute Discounting**

$$P_{ABS}(w_1 \ldots w_n) = \begin{cases} \frac{C(w_1 \ldots w_n) - \delta}{N} & \text{if } C(w_1 \ldots w_n) > 0 \\ \frac{(B - N_0)\delta / N_0}{N} & \text{otherwise} \end{cases}$$

- **Linear Discounting**

$$P_{LIN}(w_1 \ldots w_n) = \begin{cases} (1 - \alpha) \frac{C(w_1 \ldots w_n)}{N} & \text{if } C(w_1 \ldots w_n) > 0 \\ \alpha / N_0 & \text{otherwise} \end{cases}$$

Combining Estimators

Simple Linear Interpolation

$$\begin{aligned} P_{LI}(w_n \mid w_{n-2}, w_{n-1}) = &\ \lambda_1 P_1(w_n) \\ &+ \lambda_2 P_2(w_n \mid w_{n-1}) \\ &+ \lambda_3 P_3(w_n \mid w_{n-2}, w_{n-1}) \end{aligned}$$

Backing-off

$$P_{BO}(w_i \mid h) = \begin{cases} (1-\alpha_h)\dfrac{C(h, w_i)}{C(h)} & \text{if } C(h, w_i) > k \\ \delta_{h'} P_{BO}(w_i \mid h') & \text{otherwise} \end{cases}$$

(where $h = w_{i-n+1} \dots w_{i-1}$, $h' = w_{i-n+2} \dots w_{i-1}$)

Different options to determine α_h and $\delta_{h'}$ (e.g. $\alpha_h = \delta_{h'} \ \forall h$)

Outline

Outline

MEM Overview

- Maximum Entropy: alternative estimation technique.
- Able to deal with different kinds of evidence
- ME principle:
  - Do not assume anything about non-observed events.
  - Find the most uniform (maximum entropy, less informed) probability distribution that matches the observations.
- Example:

$p(x,y)$	0	1	
a	?	?	
b	?	?	
total	0.6		1.0

Observations

$p(x,y)$	0	1	
a	0.5	0.1	
b	0.1	0.3	
total	0.6		1.0

One possible $p(x,y)$

$p(x,y)$	0	1	
a	0.3	0.2	
b	0.3	0.2	
total	0.6		1.0

Max.Entropy $p(x,y)$

ME Modeling

- Observed facts are constraints for the desired model p.
- Constraints take the form of feature functions:

$$f_i : \varepsilon \rightarrow \{0, 1\}$$

- The desired model p must satisfy the constraints:
  The expectation predicted by model p for any feature f_i must match the observed expectation for f_i
  i.e.:

$$\begin{aligned} E_p(f_i) &= E_{\tilde{p}}(f_i) \qquad \forall i \\ \sum_{x \in \varepsilon} p(x) f_i(x) &= \sum_{x \in \varepsilon} \tilde{p}(x) f_i(x) \qquad \forall i \end{aligned}$$

Example

- Example:

$$\varepsilon = \{a, b\} \times \{0, 1\}$$

$p(x,y)$	0	1	
a	?	?	
b	?	?	
total	0.6		1.0

- Observed fact: $p(a, 0) + p(b, 0) = 0.6$
- Encoded as a constraint: $E_p(f_1) = 0.6$ where:
  - $f_1(x, y) = \begin{cases} 1 & \text{if } y = 0 \\ 0 & \text{otherwise} \end{cases}$
  - $E_p(f_1) = \sum_{(x,y) \in \{a,b\} \times \{0,1\}} p(x, y) f_1(x, y)$

Outline

Probability Model

- There is an infinite set P of probability models consistent with observations:

$$P = \{p \mid E_p(f_i) = E_{\tilde{p}}(f_i),\ \forall i\}$$

- Maximum entropy model

$$\begin{aligned} p^* &= \operatorname*{argmax}_{p \in P} H(p) \\ &= \operatorname*{argmax}_{p \in P} \left(-\sum_{x \in \varepsilon} p(x) \log p(x) \right) \end{aligned}$$

Conditional Probability Model

- For NLP applications, we are usually interested in conditional distributions $P(Y|X)$, thus, the ME model is

$$p^* = \operatorname*{argmax}_{p \in P} H(p) = \operatorname*{argmax}_{p \in P} H(Y \mid X)$$

where:

$$\begin{aligned} H(Y \mid X) &= \sum_{x \in X} p(x) H(Y \mid X = x) \\ &= -\sum_{x \in X} p(x) \sum_{y \in Y} p(y \mid x) \log p(y \mid x) \\ &= -\sum_{x \in X, y \in Y} p(x, y) \log p(y \mid x) \\ &= -\sum_{x \in X, y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)} \end{aligned}$$

Parameter Estimation

Example: Maximum entropy model for translating *in* to French

- No constraints

$P(x)$	dans	en	à	au-cours-de	pendant	
	0.2	0.2	0.2	0.2	0.2	
total						1.0

- With constraint $p(dans) + p(en) = 0.3$

$P(x)$	dans	en	à	au-cours-de	pendant	
	0.15	0.15	0.233	0.233	0.233	
total	**0.3**					1.0

- With constraints

  $p(dans) + p(en) = 0.3; \quad p(en) + p(\grave{a}) = 0.5$

...Not so easy !

Parameter estimation

- Exponential models

$$p(y \mid x) = \frac{1}{Z(x)} \prod_{j=1}^{k} \alpha_j^{f_j(x,y)} \qquad \alpha_j > 0, \quad Z(x) = \sum_{y} \prod_{i=1}^{k} \alpha_i^{f_i(x,y)}$$

- Can also be formuled as

$$p(y \mid x) = \frac{1}{Z(x)} \exp\left(\sum_{j=1}^{k} \lambda_j f_j(x,y)\right) \qquad (\text{i.e. } \lambda_i = \ln \alpha_i)$$

- Each model parameter weights the influence of a feature.
- Optimal parameters can be computed with:
  - Generalized Iterative Scaling (GIS) [Darroch & Ratcliff 72]
  - Improved Iterative Scaling (IIS) [Della Pietra et al. 96]
  - Limited Memory BFGS (**LM-BFGS**) [Malouf 03]

Example: Text Categorization

- Probabilistic model over $W \times C$ (Words $\times$ Categories). A document is a set of words: $d = (w_1, w_2 \ldots w_N)$. Each combination $w, c \in W \times C$ is a feature:

$$f_{w,c}(d, c') = \begin{cases} \frac{N(w,d)}{N(d)} & \text{if } c = c' \\ 0 & \text{otherwise} \end{cases}$$

- Disambiguation: Select class with highest $P(c \mid d)$

$$P(c \mid d) = \frac{1}{Z(d)} \exp(\sum_i \lambda_i f_i(d, c))$$

MEM Summary

- Advantages
  - Teoretically well founded
  - Enables combination of random context features
  - Better probabilistic models than MLE (no smoothing needed)
  - General approach (features, events and classes)
- Disadvantages
  - Implicit probabilistic model (joint or conditional probability distribution obtained from model parameters).

ME Models are a particular case of **Log-Linear models**

Outline

Log-Linear Models

$$P(y \mid x; \mathbf{w}) = \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{\sum_{y} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}$$

where

- $\mathbf{f}(x, y)$ is a feature vector representing x and y
- $\mathbf{w}$ are the parameters of the model
- $\mathbf{w} \cdot \mathbf{f}(x, y)$ is a score for x and y
- $Z(x) = \sum_{y} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)$ is a normalizer (sums over all possible values y for x); it's sometimes called the *partition function*

Features, Indicator Features

- $\mathbf{f}(x, y)$ is a vector of d features representing x and y

$$(\ \mathbf{f}_1(x, y), \ldots, \mathbf{f}_j(x, y), \ldots, \mathbf{f}_d(x, y) \ )$$

- What's in a feature $\mathbf{f}_j(x, y)$?
  - Anything we can compute using x and y
  - Anything that is informative for (or against) x belonging to class y
  - Indicator features: binary-valued features looking at a single simple property

$$\mathbf{f}_j(c, b) = \begin{cases} 1 & \text{if prefix}(c) = \textit{Mr} \text{ and } b = \text{no} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{f}_k(c, b) = \begin{cases} 1 & \text{if uppercase}(\text{next}(c)) \text{ and } b = \text{yes} \\ 0 & \text{otherwise} \end{cases}$$

Features, Parameters, Inner Products

$$P(y \mid x; \mathbf{w}) = \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{\sum_{y} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}$$

- $\mathbf{f}(x, y) \in \mathbb{R}^{d}$ is a feature vector with d features
- $\mathbf{w} \in \mathbb{R}^{d}$ is a parameter vector, with d parameters

- Inner products (*a.k.a.* dot products)

$$\mathbf{w} \cdot \mathbf{f}(x, y) = \sum_{i=1}^{d} \mathbf{w}_i \ \mathbf{f}_i(x, y)$$

Log-linear Models

$$P(y \mid x; \mathbf{w}) = \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{\sum_{y} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}$$

where

- $\mathbf{f}(x, y)$ is a feature vector representing x and y
  - Arbitrary features of x and y are allowed
  - They are provided for the application in turn
- $\mathbf{w}$ are the parameters of the model
- Two problems:
  - How to make predictions using $P(y \mid x)$
  - How to estimate the parameters $\mathbf{w}$?

Log-linear Models: Name

- Let's take the log of the conditional probability:

$$\begin{aligned}
\log P(y \mid x; \mathbf{w}) &= \log \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{\sum_y \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)} \\
&= \mathbf{w} \cdot \mathbf{f}(x, y) - \log \sum_y \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right) \\
&= \boxed{\mathbf{w} \cdot \mathbf{f}(x, y)} - \log Z(x)
\end{aligned}$$

- Partition function: $Z(x) = \sum_y \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)$
- $\log Z(x)$ is a constant for a fixed x
- In the log space, computations are linear

Log-linear Models: Making Predictions

- Given x, what y in $\{1, \ldots, L\}$ is most appropriate?

$$
\begin{aligned}
\text{best}(x) &= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} P(y \mid x; \mathbf{w}) \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{Z(x)} \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right) \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \mathbf{w} \cdot \mathbf{f}(x, y)
\end{aligned}
$$

Log-linear Models: Making Predictions

- Given x, what y in $\{1, \ldots, L\}$ is most appropriate?

$$
\begin{aligned}
\text{best}(x) &= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} P(y \mid x; \mathbf{w}) \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{Z(x)} \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right) \\
&= \operatorname*{argmax}_{y \in \{1,\ldots,L\}} \mathbf{w} \cdot \mathbf{f}(x, y)
\end{aligned}
$$

- Predictions only require simple inner products (linear)
- No need to exponentiate!

Log-linear Models: Computing Probabilities

$$P(y \mid x; \mathbf{w}) = \frac{\exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)}{Z(x)}$$

- Sometimes we will be interested in computing $P(y \mid x)$
  - It can be used as a measure of confidence, e.g.
    $P(\text{yes} \mid c) = 0.51$ *versus*
    $P(\text{yes} \mid c) = 0.99$
- We need to compute:

$$Z(x) = \sum_{y=\{1,\ldots,L\}} \exp\left(\mathbf{w} \cdot \mathbf{f}(x, y)\right)$$

- Fast as long as L is not too large

Parameter Estimation in Log-linear Models

- How to estimate model parameters $\mathbf{w}$ given a training set:

$$\left\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\right\}$$

- Let's define the conditional log-likelihood of the data:

$$L(\mathbf{w}) = \frac{1}{m} \sum_{k=1}^{m} \log P(y^{(k)} | x^{(k)}; \mathbf{w})$$

- $L(\mathbf{w})$ measures how well $\mathbf{w}$ explains the data. A good value for $\mathbf{w}$ will give a high value for $P(y^{(k)} | x^{(k)}; \mathbf{w})$ for all $k = 1 \ldots m$.
- We want $\mathbf{w}$ that maximizes $L(\mathbf{w})$

Parameter Estimation in Log-Linear Models

- We pose it as an optimization problem
- Find:

$$\mathbf{w}^* = \underset{\mathbf{w} \in \mathbb{R}^d}{\operatorname{argmax}} \, L(\mathbf{w})$$

where

- But low-frequency features may end up having large weights (i.e. overfitting)
- We need a regularization factor that penalizes solutions with a large norm (similar to norm-minimization in SVM):

$$L'(\mathbf{w}) = \frac{1}{m} \sum_{k=1}^{m} \log P(y^{(k)} | x^{(k)}; \mathbf{w}) - \frac{\lambda}{2} ||\mathbf{w}||^2$$

- where λ is a parameter to control the trade-off between fitting the data and model complexity. Tuned experimentally.

Parameter Estimation in Log-Linear Models

- So we want to find:

$$
\begin{aligned}
\mathbf{w}^* &= \operatorname*{argmax}_{\mathbf{w}\in\mathbb{R}^d} L'(\mathbf{w}) \\
&= \operatorname*{argmax}_{\mathbf{w}\in\mathbb{R}^d} \left(\frac{1}{m} \sum_{k=1}^{m} \log P(y^{(k)}|x^{(k)};\mathbf{w}) - \frac{\lambda}{2}||\mathbf{w}||^2 \right)
\end{aligned}
$$

- In general there is no analytical solution to this optimization
- ... but it is a convex function $\Rightarrow$ We use iterative techniques, i.e. gradient-based optimization
- Very fast algorithms exist (e.g. LBFGS)

Parameter Estimation in Log-Linear Models : Gradient step

- Initialize $\mathbf{w} = \mathbf{0}$
- Repeat
  - Compute gradient $\delta = (\delta_1, \ldots, \delta_d)$, where:

$$\delta_j = \frac{\partial L'(\mathbf{w})}{\partial \mathbf{w}_j} \quad \forall j = 1 \ldots d$$

  - Compute step size

$$\beta^* = \operatorname*{argmax}_{\beta \in \mathbb{R}} L'(\mathbf{w} + \beta\delta)$$

  - Move $\mathbf{w}$ in the direction of the gradient

$$\mathbf{w} \leftarrow \mathbf{w} + \beta^*\delta$$

- until convergence ($||\delta|| < \epsilon$)

Log-linear Models: Computing the Gradient

$$\frac{\partial L'(\mathbf{w})}{\partial \mathbf{w}_j} = \frac{1}{m}\sum_{k=1}^{m} \mathbf{f}_j(x^{(k)}, y^{(k)}) - \sum_{k=1}^{m} \sum_{y \in \{1,\ldots,L\}} P(y|x^{(k)}; \mathbf{w})\ \mathbf{f}_j(x^{(k)}, y) - \lambda \mathbf{w}_j$$

- First term: observed mean feature value
- Second term: expected feature value under current $\mathbf{w}$
- In the optimal, observed = expected

Maximum log-likelihood log-linear models correspond to Maximum Entropy models

MEM Overview

ME Models are the **dual formulation** of **Log-Linear models**.

- Maximum Entropy: alternative estimation technique.
- Able to deal with different kinds of evidence
- ME principle:
  - Do not assume anything about non-observed events.
  - Find the most uniform (maximum entropy, less informed) probability distribution that matches the observations.
- Example:

$p(x,y)$	0	1	
a	?	?	
b	?	?	
total	0.6		1.0

Observations

$p(x,y)$	0	1	
a	0.5	0.1	
b	0.1	0.3	
total	0.6		1.0

One possible $p(x,y)$

$p(x,y)$	0	1	
a	0.3	0.2	
b	0.3	0.2	
total	0.6		1.0

Max.Entropy $p(x,y)$

ME Modeling

- Observed facts are constraints for the desired model p.
- Constraints take the form of feature functions:

$$f_i : \varepsilon \to \{0, 1\}$$

- The desired model p must satisfy the constraints:
  The expectation predicted by model p for any feature f_i must match the observed expectation for f_i
  i.e.:

$$\begin{aligned} E_p(f_i) &= E_{\tilde{p}}(f_i) \quad \forall i \\ \sum_{x \in \varepsilon} p(x) f_i(x) &= \sum_{x \in \varepsilon} \tilde{p}(x) f_i(x) \quad \forall i \end{aligned}$$

Probability Model

- There is an infinite set P of probability models consistent with observations:

$$P = \{p \mid E_p(f_i) = E_{\tilde{p}}(f_i),\ \forall i\}$$

- Maximum entropy model

$$\begin{aligned} p^* &= \operatorname*{argmax}_{p \in P} H(p) \\ &= \operatorname*{argmax}_{p \in P} \left(-\sum_{x \in \varepsilon} p(x) \log p(x) \right) \end{aligned}$$

Conditional Probability Model

- For NLP applications, we are usually interested in conditional distributions $P(Y|X)$, thus, the ME model is

$$p^* = \operatorname*{argmax}_{p \in P} H(p) = \operatorname*{argmax}_{p \in P} H(Y \mid X)$$

where:

$$\begin{aligned} H(Y \mid X) &= \sum_{x \in X} p(x) H(Y \mid X = x) \\ &= -\sum_{x \in X} p(x) \sum_{y \in Y} p(y \mid x) \log p(y \mid x) \\ &= -\sum_{x \in X, y \in Y} p(x, y) \log p(y \mid x) \end{aligned}$$

Parameter Estimation

Example: Maximum entropy model for translating *in* to French

- No constraints

$P(x)$	dans	en	à	au-cours-de	pendant	
	0.2	0.2	0.2	0.2	0.2	
total						1.0

- With constraint $p(dans) + p(en) = 0.3$

$P(x)$	dans	en	à	au-cours-de	pendant	
	0.15	0.15	0.233	0.233	0.233	
total	**0.3**					1.0

- With constraints

  $p(dans) + p(en) = 0.3; \quad p(en) + p(\grave{a}) = 0.5$

...Not so easy !

Parameter estimation

- ME models are exponential models, same as log-linear models

$$p(y \mid x) = \frac{1}{Z(x)} \exp\left(\sum_{j=1}^{k} \lambda_j f_j(x, y)\right)$$

$$\text{where } Z(x) = \sum_{y'} \exp\left(\sum_{j=1}^{k} \lambda_j f_j(x, y')\right)$$

- Each model parameter weights the influence of a feature.
- Same convex optimization algorithms are used (e.g. **LM-BFGS** [Malouf 03])

Outline

Example: Text Categorization

- Probabilistic model over $W \times C$ (Words $\times$ Categories). A document is a set of words: $d = (w_1, w_2 \ldots w_N)$. Each combination $w, c \in W \times C$ is a feature:

$$f_{w,c}(d, c') = \begin{cases} \frac{N(w,d)}{N(d)} & \text{if } c = c' \\ 0 & \text{otherwise} \end{cases}$$

- Disambiguation: Select class with highest $P(c \mid d)$

$$P(c \mid d) = \frac{1}{Z(d)} \exp(\sum_i \lambda_i f_i(d, c))$$

Example: Identifying Sentence Boundaries

The president lives in Washington, D.C. The presidents met in Washington D.C. in 2010. Mr. Wayne is young. Mr. Wayne is a Ph.D. I got 98.5%! What?

Goal: given a text, identify tokens that end a sentence

- Candidate characters: .   !   ?
- Candidate tokens: tokens containing candidate characters
- Given a candidate token in a *context* decide whether it ends a sentence or not

Example: Sentence Boundaries

- Candidate: punctuation sign + context

  `c = < sign, prefix, suffix, previous, next >`
- Assume access to *annotated* data:

b	sign	prefix	suffix	prev	next
no	.	D	C.	Washington,	The
yes	.	D.C		Washington,	The
no	.	Mr		2010.	Wayne

- Let's take a probabilistic approach:
  - $\mathrm{P}(\mathtt{yes} \mid \mathrm{c})$: conditional probability of c being end of sentence
  - $\mathrm{P}(\mathtt{no} \mid \mathrm{c})$: conditional probability of c *not* being e.o.s.
  - Obviously, $\mathrm{P}(\mathtt{yes} \mid \mathrm{c}) + \mathrm{P}(\mathtt{no} \mid \mathrm{c}) \; = 1$
  - Predict yes if $\mathrm{P}(\mathtt{yes} \mid \mathrm{c}) > 0.5$
- How to model $\mathrm{P}(\mathtt{yes} \mid \mathrm{c})$ and $\mathrm{P}(\mathtt{no} \mid \mathrm{c})$?

Example system: Identifying Sentence Boundaries

(Reynar and Ratnaparkhi '97)

- Candidate: punctuation sign + context

  `c = < sign, prefix, suffix, previous, next >`

- **Goal**: estimate $\mathrm{P}(\mathtt{yes} \mid \mathtt{c})$ and $\mathrm{P}(\mathtt{no} \mid \mathtt{c})$
- Feature templates:
  1. The `prefix`
  2. The `suffix`
  3. The word `previous`
  4. The word `next`
  5. Whether `prefix` or `suffix` are in ABBREVIATIONS
     - ABBREVIATIONS: list of all training tokens that contain a `.` and are *not* sentence boundaries
  6. Whether `previous` or `next` are in ABBREVIATIONS
- Actual features are generated by applying each template to each training example

Example system: Identifying Sentence Boundaries (Reynar and Ratnaparkhi '97)

Feature Templates

1. The `prefix`
2. The `suffix`
3. The word `previous`
4. The word `next`
5. Whether `prefix` or `suffix` are in Abbreviations
6. Whether `previous` or `next` are in Abbreviations

`< b=no punc=. pref=Mr suff=  prev=2010. next=Wayne >`

Generated Features

$$f_1(c,b) = \begin{cases} 1 & \textbf{if } \text{pref}(c) = \texttt{Mr} \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases} \qquad f_4(c,b) = \begin{cases} 1 & \textbf{if } \text{next}(c) = \texttt{Wayne} \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(c,b) = \begin{cases} 1 & \textbf{if } \text{suff}(c) = \texttt{NULL} \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases} \qquad f_5(c,b) = \begin{cases} 1 & \textbf{if } (\text{abbr}(\text{pref}(c)) \textbf{ or } \text{abbr}(\text{suff}(c))) \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases}$$

$$f_3(c,b) = \begin{cases} 1 & \textbf{if } \text{prev}(c) = \texttt{2010.} \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases} \qquad f_6(c,b) = \begin{cases} 1 & \textbf{if } (\text{abbr}(\text{prev}(c)) \textbf{ or } \text{abbr}(\text{next}(c))) \\ & \quad \textbf{and } b = \texttt{no} \\ 0 & \text{otherwise} \end{cases}$$

Example System: Identifying Sentence Boundaries
(Reynar and Ratnaparkhi '97)

training sentences	test accuracy
500	96.5%
1000	97.3%
2000	97.3%
4000	97.6%
8000	97.6%
16000	97.8%
39441	98.0%

- Corpus: Wall Street Journal, English