Übungsaufgaben Analysis 2 für die Übungen am 7.11.24

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Aufgabe 1

a)

Für eine gerade Funktion d.h. $f(x) = f(-x)$ kommen in einer Taylorreihe nur gerade Exponenten vor:

$$\sum_{k=0}^{\infty} a_{2k}x^{2k}$$

$$f(x) = \sum_{k=0}^{\infty} a_k x^k = \sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1}$$
$$f(-x) = \sum_{k=0}^{\infty} a_k (-x)^k = \sum_{k=0}^{\infty} a_{2k}x^{2k} - \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1}$$

$f(x) = f(-x)$ liefert:

$$\sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} = \sum_{k=0}^{\infty} a_{2k}x^{2k} - \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} \qquad | - \sum_{k=0}^{\infty} a_{2k}x^{2k}$$
$$\sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} = -\sum_{k=0}^{\infty} a_{2k+1}x^{2k+1}$$

$\sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} = 0 \Rightarrow \forall k a_{2k+1} = 0$

b)

Für eine gerade Funktion d.h. $f(x) = -f(-x)$ kommen in einer Taylorreihe nur ungerade Exponenten vor:

$$\sum_{k=0}^{\infty} a_{2k+1}x^{2k+1}$$

$$\begin{aligned} f(x) &= \sum_{k=0}^{\infty} a_k x^k = \sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} \\ -f(-x) &= -\sum_{k=0}^{\infty} a_k(-x)^k = -\sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} \end{aligned}$$

$f(x) = -f(-x)$ liefert:

$$\begin{aligned} \sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} &= -\sum_{k=0}^{\infty} a_{2k}x^{2k} + \sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} \qquad \Big| -\sum_{k=0}^{\infty} a_{2k+1}x^{2k+1} \\ \sum_{k=0}^{\infty} a_{2k}x^{2k} &= -\sum_{k=0}^{\infty} a_{2k}x^{2k} \end{aligned}$$

$$\sum_{k=0}^{\infty} a_{2k}x^{2k} = 0 \Rightarrow \forall k a_{2k} = 0$$

Aufgabe 2

arctan ableiten:

$$\begin{aligned} f(x) &= \arctan(x) \qquad |\tan \\ \tan(f(x)) &= x \end{aligned}$$

Einheitskreis:

$$\begin{aligned} \tan(f(x)) = \frac{\sin(f(x))}{\cos(f(x))} &= x \qquad |' \\ \frac{(\cos^2(f(x)) + \sin^2 f(x)) \cdot f'(x)}{\cos^2(f(x))} &= 1 \\ \frac{f'(x)}{\cos^2(f(x))} &= 1 \\ f'(x) &= \cos^2(f(x)) \end{aligned}$$

Pythagoras Identität:

$$\begin{aligned}\cos^2(f(x)) + \sin^2(f(x)) &= 1\\ \cos^2(f(x)) + (x\cos(f(x)))^2 &= 1\\ \cos^2(f(x))(1+x^2) &= 1\\ \cos^2(f(x)) &= \frac{1}{1+x^2}\end{aligned}$$

Ende Einheitskreis:

$$f'(x) = \cos^2(f(x)) = \frac{1}{1+x^2}$$

N-te Ableitung siehe Tabelle Löhmann
Taylorreihe Arcus Tangens

$$T_{\text{arctan},0}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + O(x)$$

$$T_{\text{arctan},0}(x) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \pm \dots$$

Summe

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$

Aprroximation $\tan(\pi/4) = 1$

$$n = 23 : 4T_{\text{arctan},0}(1) = 3.0584$$

$$n = 25 : 4T_{\text{arctan},0}(1) = 3.2184$$

$$\frac{3.0584 + 3.2184}{2} = 3.1384$$

C Code:

```c
#include <stdio.h>
#include <math.h>

static long double calc_taylor_arctan(const int x, const int n)
{
        long double approx = 0;
        for(int k = 0; k <= n; ++k)
        {
                approx += pow(-1, k) * (pow(x, 2*k + 1) / (2*k + 1));
        }

        return approx;
}
```

```c
int main(void)
{
        long double n_24_arctan = 4 * calc_taylor_arctan(1, 24);
        long double n_25_arctan = 4 * calc_taylor_arctan(1, 25);
        printf("4 * taylor arctan, n = 24 at x = 1: %Lf\n", n_24_arctan);
        printf("4 * taylor arctan, n = 25 at x = 1: %Lf\n", n_25_arctan);
        printf("Mean: %Lf\n", (n_24_arctan + n_25_arctan) / 2);
        printf("PI: %f\n", M_PI);

        return 0;
}
```

Aufgabe 3

Siehe Lösung Löhmann

Aufgabe 4

Siehe Lösung Löhmann