Lösungen für Übungsaufgaben Analysis 1 für den 21.06.24

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Aufgabe 1

$x_1 = -3.17663$
$x_2 = -2.58052$
$x_3 = 0.32205$
$x_4 = 2.28381$
$x_5 = -3.15127$

Aufgabe 3

$$\begin{aligned}
\varphi(x) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \\
\text{mit } a &= \frac{1}{\sqrt{2\pi}\sigma} \text{ und } b = \frac{1}{2\sigma^2} \\
\varphi(x) &= a \cdot e^{-b(x-\mu)^2} \\
\varphi'(x) &= -2ab\,(x-\mu)\,\mathrm{e}^{-b(x-\mu)^2} \\
\varphi''(x) &= -2ab\left[1 - 2b(x-\mu)^2\right] e^{-b(x-\mu)^2} \\
\varphi'''(x) &= -4ab^2 e^{-b(x-\mu)^2}(-3 + 2b(x-\mu)^2)(x-\mu)
\end{aligned}$$

Aufgabe 4

$$
\begin{aligned}
f(x) &= \ln(\cos x) \\
f'(x) &= -\frac{\sin(x)}{\cos(x)} \\
f''(x) &= -\frac{\sin^2(x)}{\cos^2(x)} - 1 \\
f'''(x) &= -\frac{2\sin(x)\left(\sin^2(x) + \cos^2(x)\right)}{\cos^3(x)}
\end{aligned}
$$

Aufgabe 5

$$a^{x+y} = (x+y)\ln(a) = e^{x\ln(x)+y\ln(y)} = e^{x\ln(x)}e^{y\ln(y)} = a^x a^y$$

Aufgabe 6

SS. 40 Skript

$$
\begin{aligned}
\sin^2(\alpha) + \cos^2(\alpha) &= 1 \\
\cos^2(\alpha) &= 1 - \sin^2(\alpha) \\
f(x) &= \arcsin(x) \\
\sin(f(x)) &= x \\
\cos(f(x)) \cdot f'(x) &= 1 \\
f'(x) &= \frac{1}{\cos(\arcsin(x))} \\
f'(x) &= \frac{1}{\sqrt{1 - sin^2(\arcsin(x))}} \\
f'(x) &= \frac{1}{\sqrt{1-x^2}}
\end{aligned}
$$

Aufgabe 7

$f(x) = ae^x + \frac{b}{x}$
$f'(x) = ae^x - \frac{b}{x^2}$
$f''(x) = e^x + \frac{2b}{x^3}$

$$f(x) = ae^x + \frac{b}{x}$$
$$f(2) = ae^2 + \frac{b}{2} = 10$$
$$a = \frac{10}{e^2} - \frac{b}{2e^2}$$
$$\boxed{a = \frac{20-b}{2e^2}}$$
$$f(3) = \frac{20-b}{2e^2}e^3 + \frac{b}{3} = 9$$
$$f(3) = 10e - \frac{be}{2} + \frac{b}{3} = 9$$
$$f(3) = \frac{-3be + 2b}{6} = 9 - 10e$$
$$f(3) = b\frac{(-3e+2)}{6} = 9 - 10e$$
$$\boxed{b = \frac{54-60e}{-3e+2}}$$

Aufgabe 8

$$u^1 = e^{-x}\cos(x + \frac{\pi}{4}) \qquad u' = -e^{-x}\cos(x + \frac{\pi}{4}) - e^{-x}\sin(x + \frac{\pi}{4})$$
$$v = x \qquad v' = 1$$

^{1} :

$$u = e^{-x} \qquad u' = -e^{-x}$$
$$v = \cos(x + \frac{\pi}{4}) \qquad v' = -\sin(x + \frac{\pi}{4})$$

$$-e^{-x}\left[\frac{x\cos(x + \frac{\pi}{4}) + x\sin(x + \frac{\pi}{4}) + \cos(x + \frac{\pi}{4})}{x^2}\right]$$
$$-e^{-x}\left[\frac{\cos(x + \frac{\pi}{4})(x+1)}{x^2} + \frac{\sin(x + \frac{\pi}{4})}{x}\right]$$

Aufgabe 9

a)

$$\lim_{x\to 0} \frac{\log_{10} x}{\ln(\sin x)}$$
$$\lim_{x\to 0} \frac{\frac{\ln(x)}{\ln(10)}}{\ln(\sin x)}$$
$$\lim_{x\to 0} \frac{\ln(x)}{\ln(10)\ln(\sin x)} \quad |'$$
$$\frac{1}{\ln(10)} \lim_{x\to 0} \frac{\sin x}{x\cos x} \quad |'$$
$$\frac{1}{\ln(10)} \lim_{x\to 0} \frac{\cos x)}{\cos x + x\sin x} = 1$$
$$\frac{1}{\ln(10)}$$

b)

$$\lim_{x\to 0} \frac{e^x - 5^x}{\sin x}$$
$$\lim_{x\to 0} \frac{e^x - e^{x\ln(5)}}{\sin x} \quad |'$$
$$\lim_{x\to 0} \frac{e^x - \ln(5)e^{x\ln(5)}}{\cos x}$$
$$1 - \ln(5)$$

Aufgabe 10

$S(x) = 2(x^2 + a^2)^{\frac{1}{2}} + b - x$
$S'(x) = \frac{2x}{\sqrt{x^2+a^2}} - 1$
$S''(x) = \frac{2}{\sqrt{x^2+a^2}} - \frac{2x^2}{(x^2+a^2)^{\frac{3}{2}}}$

$$\frac{2x}{\sqrt{x^2+a^2}} - 1 = 0$$
$$\frac{4x^2}{x^2+a^2} = 1$$
$$4x^2 = x^2 + a^2$$
$$3x^2 = a^2$$
$$x^2 = \frac{1}{3}a^2$$
$$x = \pm\frac{1}{\sqrt{3}}a$$

Aufgabe 11

$x = v_0 \cdot t$ Geschw. * Zeit

$H - h = \frac{1}{2}g \cdot t^2$ Freier Fall

$$t^2 = \frac{2(H-h)}{g}$$
$$t = \sqrt{\frac{2(H-h)}{g}}$$
$$x = \sqrt{2gh} \cdot \sqrt{\frac{2(H-h)}{g}}$$
$$x = \sqrt{4h(H-h)}$$
$$4h(H-h) = 0 \quad |'$$
$$4H - 4h - 4h = 0$$
$$H - 2h = 0$$
$$h = \frac{H}{2}$$