Chapter 8 Optical Interferometry

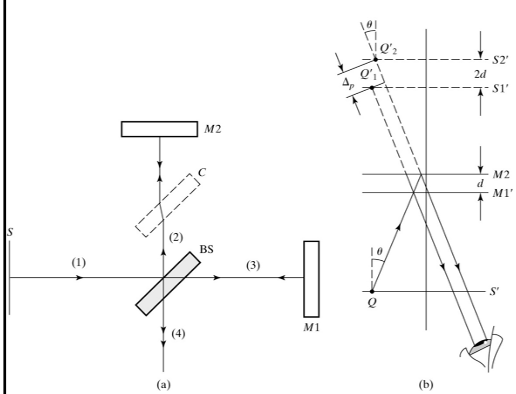
8.1 The Michelson Interferometers

- Amplitude-division interferometer:

  Michelson Interferometer: *Make use of two beam interference*
  Fabry-Perot (FP) Interferometer: *Make use of multiple beam interference*

- Applications : measure wavelength, measure thickness, flatness, etc.

  Michelson Interferometers (using 2 Mirrors and 1 Beam splitter)



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- Consider M_1 ll M_2 : Fringe of equal inclination (rings)
- OPD: $\boxed{\Delta_p = 2d\cos\theta}$

  θ:: the inclination of the beam relative to the optical axis

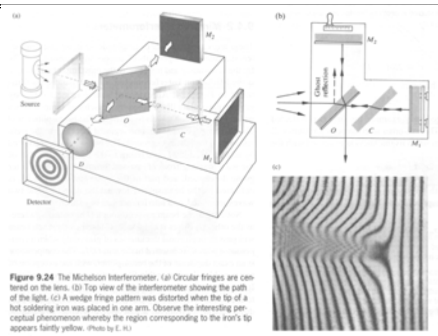
- π phase shift due to reflection: $\boxed{\Delta_r = \lambda/2}$

  One beam is external reflected and the other is internal reflected.

- Dark fringes: $\boxed{2d\cos\theta = m\lambda}$

Compensate plate C: to compensate pass difference between two paths

8.1 The Michelson Interferometers



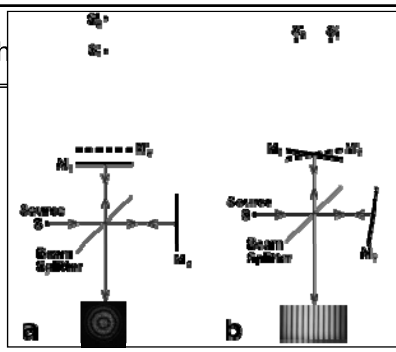
- Dark fringes: $\boxed{2d\cos\theta = m\lambda}$

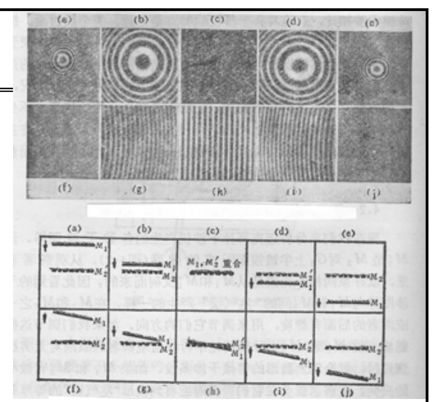
  θ: angular radius

- The center ring is dark with highest order *m*: $\boxed{m_{\max} = \dfrac{2d}{\lambda}}$

- Angular separation : $\boxed{\left|\triangle\theta\right| = \dfrac{\lambda_0 \triangle m}{2d\sin\theta}}$

8.1 Th





Q: do you think the circular fringes have uniform angular separation?

Q: When M_2 is moved toward M_1, what happen to the fringe ?

1. The rings shrink toward the center with the highest-order disappearing whenever *d* decrease by $\lambda / 2$
2. Each remaining ring broadens as more and more fringes vanish at the center until only a few fill the whole screen, which can be explained by the <u>angular separation</u> of the ring.

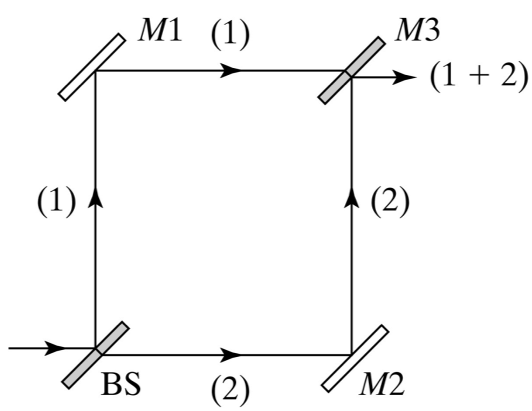
8.1 The Michelson Interferometers

Example 8-1

Fringes are observed due to monochromatic light in a Michelson interferometer. When the movable mirror is translated by 0.073 mm, a shift of 300 fringes is observed. What is the wavelength of the light? What displacement of the fringe system takes place when a flake of glass of index 1.51 and 0.005 mm thickness is placed in one arm of the interferometer? (Assume that the light beam is normal to the glass surface)

8.3 Variation of The Michelson Interferometers **

Mach-Zehnder Inteferometer



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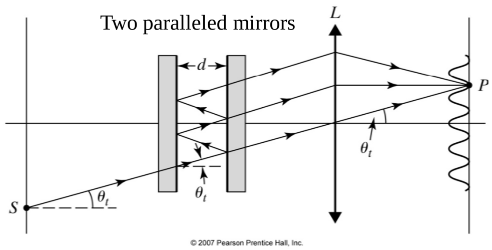
8.4 The Fabry-Perot Interferometers

- Fabry-Perot interferometer : Formed by 2 parallel mirrors (r_1, r_2), separated by a distance d



- Condition for brightness :

$$\boxed{2n_f d\cos\theta_t = m\lambda}$$

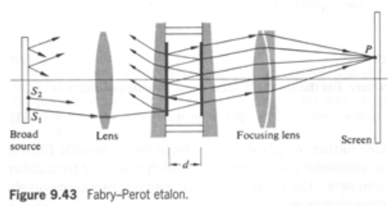


Figure 9.43 Fabry–Perot etalon.

$\Rightarrow$

Fringe of equal inclination

8.5 FP Transmission: The Airy function

See p186

$$\boxed{\cos\delta = 1 - 2\sin^2\frac{\delta}{2}}$$

- For lossless mirrors, the <u>transmittance</u> of FP cavity:

$$\boxed{T = \frac{(1-r^2)^2}{(1+r^4)-2r^2\cos\delta} = \frac{1}{1+\left[\frac{4r^2}{(1-r^2)^2}\right]\sin^2(\delta/2)}}$$

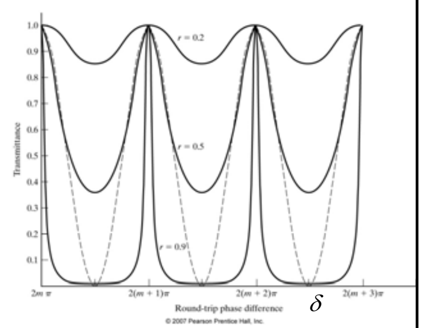
Airy function

- Round-trip phase shift: $\boxed{\delta = 2kd}$

- Parameters:
  - Coefficient of finesses F:
  - Half width $\delta_{1/2}$
  - Finesse $\mathscr{F}$

- Coefficient of finesses:

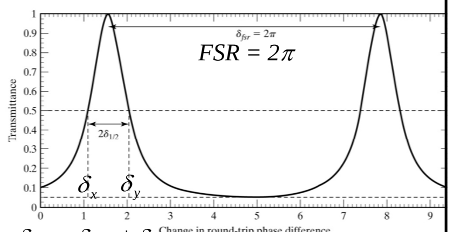
$$\boxed{F = \frac{4r^2}{(1-r^2)^2}} \Rightarrow \boxed{T = \frac{1}{1+F\sin^2(\delta/2)}}$$



F is a measure of fringe contrast : $\boxed{\text{Fringe Contrast} = \frac{T_{max} - T_{min}}{T_{min}} = F}$

8.5 FP Transmission: The Airy function

- Half width $\delta_{1/2}$: *Half-width at half max.* (HWHM)
  Fall width $2\delta_{1/2}$: *Fall-width at half max.* (FWHM)



$T_{\max} = 1$; *Half-max:* $T = 1/2$

$$\boxed{T = \frac{1}{1 + F\sin^2\frac{\delta}{2}}} \Rightarrow \boxed{F\sin^2\frac{\delta}{2} = 1}$$

$$\delta_{x,y} = \delta_{\max} \pm \delta_{1/2}$$

Two solutions $\boxed{\delta_{x,y} = 2m\pi \pm 2\sin^{-1}\frac{1}{\sqrt{F}}} \Rightarrow \boxed{\delta_x - \delta_y = 4\sin^{-1}\frac{1}{\sqrt{F}} = 2\delta_{1/2}} \Rightarrow \boxed{\delta_{1/2} = 2\sin^{-1}\frac{1}{\sqrt{F}}}$

- Finesse $\mathscr{F}$: *Ratio of* <u>*phase separation*</u> *between adjacent transmittance peaks to the full width* $2\delta_{1/2}$

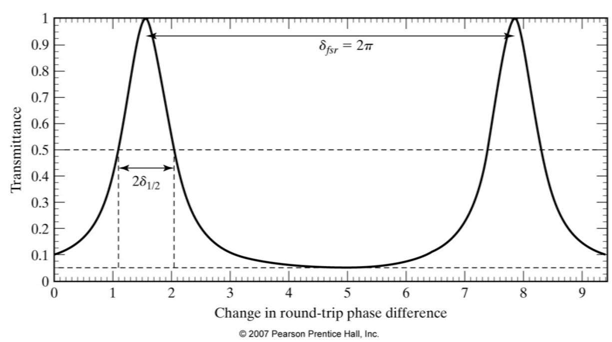
$$\mathscr{F} = 2\pi / (2\delta_{1/2}) = \pi / \delta_{1/2} \Rightarrow \boxed{\mathscr{F} = \frac{\pi}{2\sin^{-1}\frac{1}{\sqrt{F}}}}$$

Considering F is very large number: $\boxed{\mathscr{F} = \pi\sqrt{F}/2 = \frac{\pi r}{1 - r^2}}$ (Eq. 8-29)

8.5 FP Transmission: The Airy function

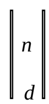
Problem 8-2

Estimate the coefficient of finesse F, the finesse $\mathscr{F}$, and the mirror reflectivity r for a Fabry-Perot cavity with the transmittance curve shown in Fig. 8-10.

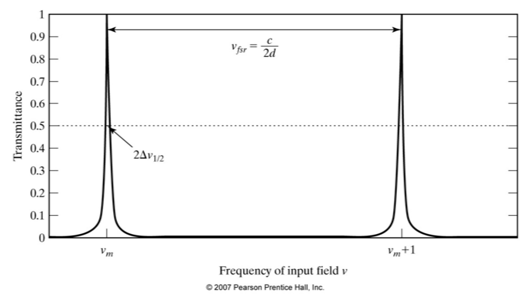


8.7 Variable-Input Frequency FP Interferometers

- When d is fixed, often call it as etalon, or FP etalon



d: cavity thickness
n: cavity index



- Resonance conditions (max condition): $\boxed{\delta = 2kd = 4\pi(\nu / c)nd = 2m\pi}$

$$\boxed{\nu_m = m\frac{c}{2nd}}$$

- Free Spectral Range (FSR) of the FP interferometer: $\boxed{f_{FSR} = \Delta\nu = \frac{c}{2nd}}$

  FSR: Frequency separation between adjacent transmittance peaks

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Problem 8-13

Apply the reasoning used to calculate the finesse of a Fabry-Perot interferometer to the Michelson interferometer. Using the irradiance of Michelson fringes as a function of phase, calculate (a) the fringe separation; (b) the fringe width at half-maximum; (c) their ratio, the finesse.