Chapter 9. Coherence

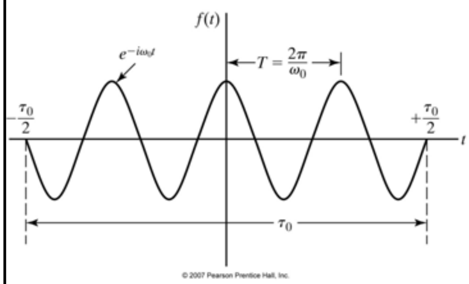
Suggested problems: p241{8, 9, 12, 16, 17}

Introduction

- The term Coherence is used to describe the correlation between phases of monochromatic radiations
- **Coherence** is an ideal property of waves that enables stationary (i.e. temporally and spatially constant) interference
- Temporal coherence is related to *the frequency spread of the source*. Temporal coherence tells us how monochromatic a source is. Temporal coherence is a measure of the correlation relationship between waves observed at different moments in time
- Spatial coherence is related to *the size of the source*. Spatial coherence is a measure of the correlation between waves at different points in space

Fourier Analysis of a Finite Harmonic Wave Train

- A finite harmonic wave train with a frequency ω_0 and a lifetime τ_0:



$$f(t) = \begin{cases} e^{-i\omega_o t}, & -\tau_0/2 < t < \tau_0/2 \\ 0, & \text{elsewhere} \end{cases}$$

- Find the frequency spectrum from Fourier integral transforms:

$$f(t) = \int_{-\infty}^{\infty} g(\omega) e^{-i\omega t} d\omega \qquad g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

$$\Rightarrow \quad g(\omega) = \frac{\tau_0}{2\pi}\frac{\sin\left[(\omega-\omega_0)\tau_0/2\right]}{\left[(\omega-\omega_0)\tau_0/2\right]} = \frac{\tau_0}{2\pi}\frac{\sin u}{u} = \frac{\tau_0}{2\pi}\operatorname{sinc} u$$

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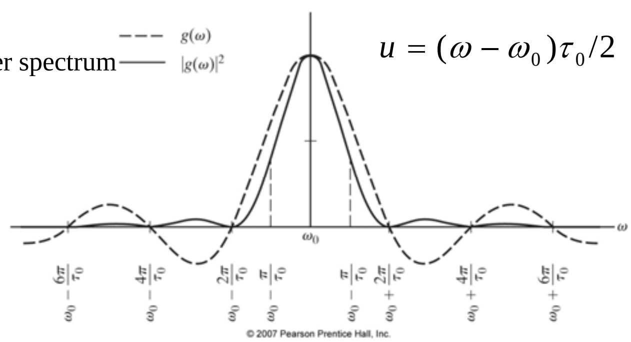
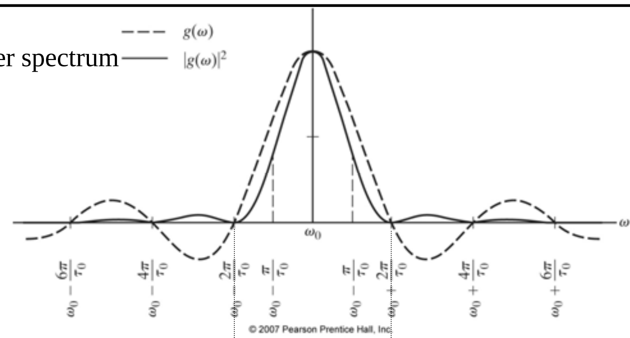
Power spectrum



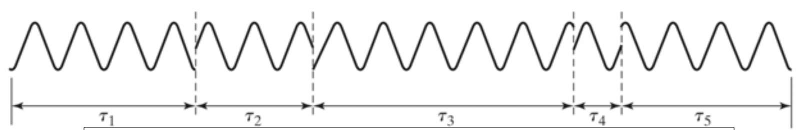
$$u = (\omega - \omega_0)\tau_0/2$$

- At center, $\omega = \omega_0$, i.e., $u = 0$, gives sinc $u = 1$, i.e., max $g(\omega)$
- Zero amplitude points: $u = n\pi$ ($n = \pm1, \pm2, \pm3, \ldots\ldots$)



- Width of the central maximum : ← $4\pi / \tau_0$ →
- The shorter the wave train (i.,e, the lifetime τ_0), the wider the central maximum, i.e, the frequency bandwidth
- For a wave train with a lifetime τ_0, a frequency bandwidth $\Delta\omega$ required to represent the finite harmonic wave train is: $\boxed{\Delta\omega = \frac{2\pi}{\tau_0}}$ or $\boxed{\Delta\nu = \frac{1}{\tau_0}}$
- If $\tau_0 \rightarrow \infty$ (infinite wave train), then $\Delta\omega = 0$, a single frequency ω_0 suffices to represents the wave train
- If $\tau_0 \rightarrow 0$ (a pulse), then $\Delta\omega = \infty$ (the shorter the pulse, the wider the spectrum)

Temporal Coherence and Line Width



Harmonic wave trains of varying finite length or lifetime τ

Parameters:

- Coherence time (average wave train lifetime) τ_0
- Coherence length l_t
- Frequency band $\Delta\nu$
- Spectral line width $\Delta\lambda$

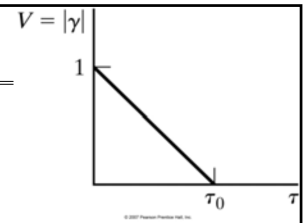
$$\boxed{l_t = c\tau_0}$$

$$\boxed{\Delta\nu = \frac{1}{\tau_0}} \quad \boxed{\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda} \quad \boxed{\Delta\lambda = \frac{\lambda^2}{l_t}}$$

Example: White light has a “line width” $\Delta\lambda$ of around 300 nm (extending roughly from 400 to 700 nm). What is the coherence length?

- Line width of the light sources: Mercury gas-discharge lamp at 546 nm is 0.025 nm; laser radiation, CO_2 laser’s line width is 10^{-5} nm
- To observe interference phenomenon, the path difference has to be smaller than the coherence length of the light source

Partial Coherence



- Interference fringe visibility V: $\boxed{V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}}$
- Interference fringe visibility can be expressed as:

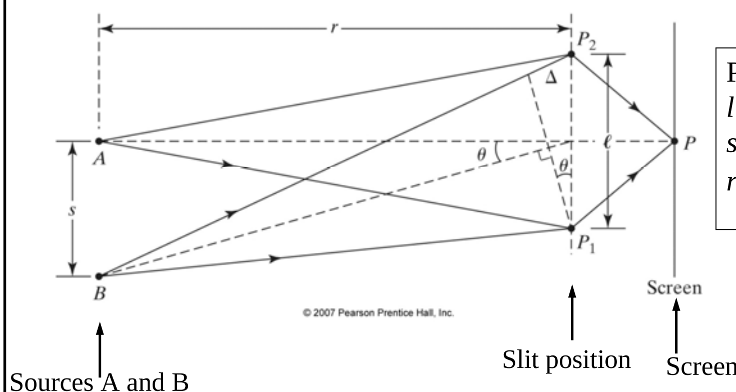
$$\boxed{V = 1 - \frac{\tau}{\tau_0} = 1 - \frac{\Delta}{l_t} = 1 - \frac{\Delta \cdot (\Delta\lambda)}{\lambda^2}}$$

τ : time delay between two interference beams
Δ : optical path difference between the 2 beams
τ_0 : source lifetime
l_t : source coherence length

1. Complete incoherence $\tau \geq \tau_0$ or $\Delta \geq l_t$
2. Complete coherence $\tau \approx 0$ or $\Delta \approx 0$
3. Partial coherence $0 < \tau < \tau_0$ or $0 < \Delta < l_t$

Example 9-1: In an interference experiment, a light beam is split into two equal-amplitude parts. The two parts are superimposed again after traveling along different paths. The light is of wavelength 541 nm with a line width of 0.1 nm, and the path difference is 1.6 mm. Determine the visibility of the interference fringes. How is the visibility modified if the path difference is doubled?

Spatial Coherence and Spatial Coherence Width

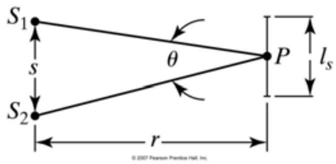


Parameters:
l : Slit width
s : Source width
r : Distant between source and slits

- BP2 – BP1 $\approx \Delta = l \sin\theta \approx l\, s / r$ $\quad \boxed{l\dfrac{s}{r} = \dfrac{\lambda}{2}}$
- $\Delta = \lambda / 2$, then the fringe will disappear
- Spatial coherence width (or max slit width for two slit interference): $\boxed{l_s < \dfrac{r\lambda}{s} \cong \dfrac{\lambda}{\theta}}$

θ : angle separation of the sources from the plane of the slits

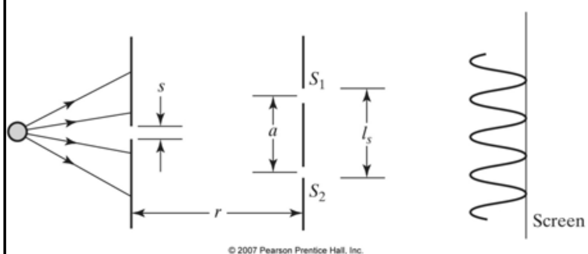
Spatial Coherence and Spatial Coherence Width



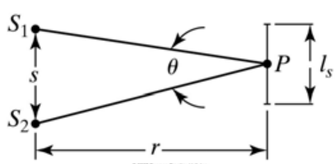
- Spatial coherence width l_s:

$$\boxed{l_s < \frac{r\lambda}{s} \cong \frac{\lambda}{\theta}}$$

Example 9-2: Let the source-to-slit distance be 20 cm, the slit separation 0.1 mm, and the wavelength 546 nm. Determine the maximum width of the primary or single slit.

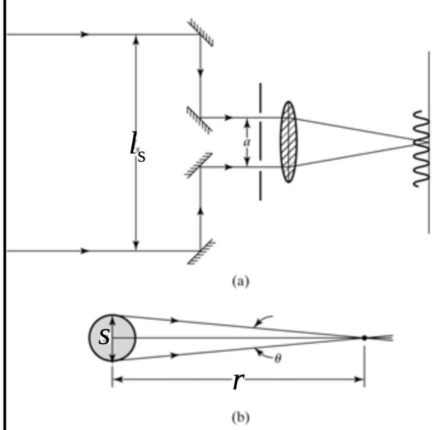


Spatial Coherence and Spatial Coherence Width



- Spatial coherence width l_s:

$$\boxed{l_s < \frac{r\lambda}{s} \cong \frac{\lambda}{\theta}}$$



- Measure the angular diameter of stars:

For circular aperture: $\boxed{l_s < \frac{1.22\lambda}{\theta}}$

$$\boxed{\theta \approx \frac{s}{r}}$$

17. A filtered mercury lamp produces green light at 546.1 nm with a linewidth of 0.05 nm. Determine the visibility of the fringes on a screen1 m away, in the vicinity of the fringe of order m = 20. If the discharge lamp is replaced with a white light source and a filter of bandwidth 10 nm at 546 nm, how does the visibility change?