(a)



(b)

**Figure 2-32** (a) Light rays in a top view of the horizontal (nonfocusing) section of the lens in Figure 2-31. (b) Light rays in a side view of the vertical (focusing) section of the lens in Figure 2-31.

further than $s'$ from the lens, one sees an unfocused blur that has the general shape of the aperture—either of the rectangular cross section of the lens alone or of the lens with an aperture placed against it. Further, it should be evident from Figure 2-31 that, as a screen, initially positioned just behind the lens, moves toward the line image $AB$, the horizontal dimension (width) of the blur increases and its vertical dimension (height) decreases. As the screen moves beyond the line image, its width continues to increase but its height now also increases due to the divergence of the rays after focusing. If an aperture placed in front of the lens is circular, these blur images are elliptical in shape, with changing major and minor axes formed by the width and height of the blur. If the aperture is square, the blurs are rectangular in shape. Widths and heights of the blur pattern can be found at any position of the screen using the geometry apparent in Figure 2-32a and b, respectively. This behavior can be observed easily in the laboratory.

Up to this point we have been dealing with a cylindrical lens whose axis is either horizontal or vertical. Of course, the cylinder axis can be oriented at any angle. An astigmatic eye, for example, while it possesses predominantly spherical optics, might have a cylindrical axis component whose axis could be horizontal, vertical, or some angle in between. To deal with cylindrical lenses and astigmatism in a general way, then, we must be able to determine the effect of combining cylindrical lenses having arbitrary orientations with each other and with spherical lenses. It turns out that two cylindrical lenses can produce the same effect as a sphero-cylindrical lens. Lens prescriptions for vision correction are, in fact, expressed in terms of combinations of spherical and cylindrical lenses. This subject is treated further elsewhere.[7]

## PROBLEMS

**2-1** Derive an expression for the transit time of a ray of light that travels a distance $x_1$ through a medium of index $n_1$, a distance $x_2$ through a medium of index $n_2,\ldots$, and a distance $x_m$ through a medium of index $n_m$. Use a summation to express your result.

**2-2** Deduce the Cartesian oval for perfect imaging by a refracting surface when the object point is on the optical $x$-axis 20 cm from the surface vertex and its conjugate image point lies 10 cm inside the second medium. Assume the refracting medium to have an index of 1.50 and the outer medium to be air. Find the equation of the intersection of the oval with the $xy$-plane, where the origin of the coordinates is at the object point. Generate a table of $(x, y)$-coordinates for the surface and plot, together with sample rays.

**2-3** A double convex lens has a diameter of 5 cm and zero thickness at its edges. A point object on an axis through the center of the lens produces a real image on the opposite side. Both object and image distances are 30 cm, measured from a plane bisecting the lens. The lens has a refractive index of 1.52. Using the equivalence of optical paths through the center and edge of the lens, determine the thickness of the lens at its center.

**2-4** Determine the minimum height of a wall mirror that will permit a 6-ft person to view his or her entire height. Sketch rays from the top and bottom of the person, and determine the proper placement of the mirror such that the full image is seen, regardless of the person's distance from the mirror.

**2-5** A ray of light makes an angle of incidence of 45° at the center of the top surface of a transparent cube of index 1.414. Trace the ray through the cube.

**2-6** To determine the refractive index of a transparent plate of glass, a microscope is first focused on a tiny scratch in the upper surface, and the barrel position is recorded. Upon further lowering the microscope barrel by 1.87 mm, a focused image of the scratch is seen again. The plate thickness is 1.50 mm. What is the reason for the second image, and what is the refractive index of the glass?

**2-7** A small source of light at the bottom face of a rectangular glass slab 2.25 cm thick is viewed from above. Rays of light totally internally reflected at the top surface outline a circle of 7.60 cm in diameter on the bottom surface. Determine the refractive index of the glass.

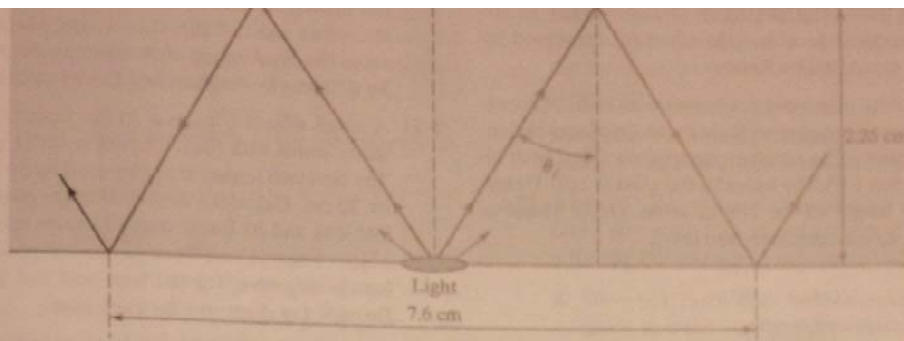[7]See F. L. Pedrotti and L. S. Pedrotti, *Optics and Vision* (Upper Saddle River, N. J.: Prentice Hall, Inc., 1998).

Figure 2-33 Problem 2-7

**2-8** Show that the lateral displacement $s$ of a ray of light penetrating a rectangular plate of thickness $t$ is given by

$$s = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2}$$

where $\theta_1$ and $\theta_2$ are the angles of incidence and refraction, respectively. Find the displacement when $t = 3$ cm, $n = 1.50$, and $\theta_1 = 50°$.

**2-9** A meter stick lies along the optical axis of a convex mirror of focal length 40 cm, with its nearer end 60 cm from the mirror surface. How long is the image of the meter stick?

**2-10** A glass hemisphere is silvered over its curved surface. A small air bubble in the glass is located on the central axis through the hemisphere 5 cm from the plane surface. The radius of curvature of the spherical surface is 7.5 cm, and the glass has an index of 1.50. Looking along the axis into the plane surface, one sees two images of the bubble. How do they arise and where do they appear?
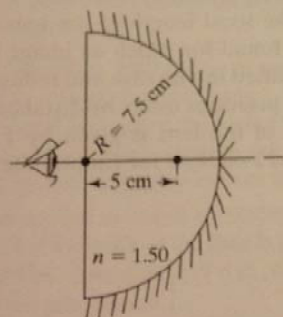
**2-13 a.** At what position in front of a spherical refracting surface must an object be placed so that the refraction produces parallel rays of light? In other words, what is the focal length of a single refracting surface?

**b.** Since real object distances are positive, what does your result imply for the cases $n_2 > n_1$ and $n_2 < n_1$?

**2-14** A small goldfish is viewed through a spherical glass fishbowl 30 cm in diameter. Determine the apparent position and magnification of the fish's eye when its actual position is (a) at the center of the bowl and (b) nearer to the oberver, halfway from center to glass, along the line of sight. Assume that the glass is thin enough so that its effect on the refraction may be neglected.

**2-15** A small object faces the convex spherical glass window of a small water tank. The radius of curvature of the window is 5 cm. The inner back side of the tank is a plane mirror, 25 cm from the window. If the object is 30 cm outside the window, determine the nature of its final image, neglecting any refraction due to the thin glass window itself.
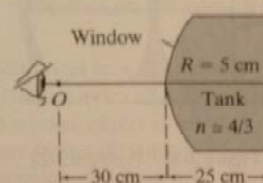


Window

$R = 5$ cm

Tank

$n = 4/3$

$O$

|← 30 cm →|← 25 cm →|

Figure 2-35 Problem 2-15



$R = 7.5$ cm

5 cm

$n = 1.50$

Figure 2-34 Problem 2-10

**2-11** A concave mirror forms an image on a screen twice as large as the object. Both object and screen are then moved to produce an image on the screen that is three times the size of the object. If the screen is moved 75 cm in the process, how far is the object moved? What is the focal length of the mirror?

**2-12** A sphere 5 cm in diameter has a small scratch on its surface. When the scratch is viewed through the glass from a position directly opposite, where does the scratch appear and what is its magnification? Assume $n = 1.50$ for the glass.

**2-16** A plano-convex lens having a focal length of 25.0 cm is to be made with glass of refractive index 1.520. Calculate the radius of curvature of the grinding and polishing tools to be used in making this lens.

**2-17** Calculate the focal length of a thin meniscus lens whose spherical surfaces have radii of curvature of magnitude 5 and 10 cm. The glass is of index 1.50. Sketch both positive and negative versions of the lens.

**2-18** One side of a fish tank is built using a large-aperture thin lens made of glass ($n = 1.50$). The lens is equiconvex, with radii of curvature 30 cm. A small fish in the tank is 20 cm from the lens. Where does the fish appear when viewed through the lens? What is its magnification?

**2-19** Two thin lenses have focal lengths of $-5$ and $+20$ cm. Determine their equivalent focal lengths when (a) cemented together and (b) separated by 10 cm.

**2-20** Two identical, thin, plano-convex lenses with radii of curvature of 15 cm are situated with their curved surfaces in contact at their centers. The intervening space is filled with oil of refractive index 1.65. The index of the glass is 1.50. Determine the focal length of the combination. (*Hint*: Think of the oil layer as an intermediate thin lens.)
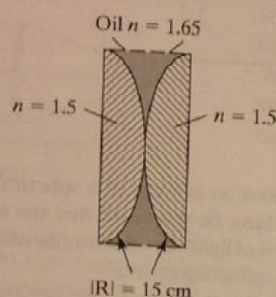


Oil $n = 1.65$

$n = 1.5$    $n = 1.5$

$|R| = 15$ cm

**Figure 2-36**    Problem 2-20

**2-21** An eyepiece is made of two thin lenses each of $+20$-mm focal length, separated by a distance of 16 mm.

    **a.** Where must a small object be positioned so that light from the object is rendered parallel by the combination?
    **b.** Does the eye see an image erect relative to the object? Is it magnified? Use a ray diagram to answer these questions by inspection.

**2-22** A diverging thin lens and a concave mirror have focal lengths of equal magnitude. An object is placed $(3/2)f$ from
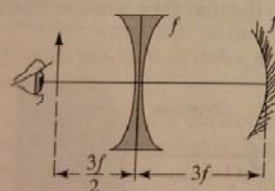


**Figure 2-37**    Problem 2-22

the diverging lens, and the mirror is placed a distance $3f$ on the other side of the lens. Using Gaussian optics, determine the *final* image of the system, after two refractions (a) by a three-ray diagram and (b) by calculation.

**2-23** A small object is placed 20 cm from the first of a train of three lenses with focal lengths, in order, of 10, 15, and 20 cm. The first two lenses are separated by 30 cm and the last two by 20 cm. Calculate the final image position relative to the last lens and its linear magnification relative to the original object when (a) all three lenses are positive, (b) the middle lens is negative, (c) the first and last lenses are negative. Provide ray diagrams for each case.

**2-24** A convex thin lens with refractive index of 1.50 has a focal length of 30 cm in air. When immersed in a certain transparent liquid, it becomes a negative lens with a focal length of 188 cm. Determine the refractive index of the liquid.

**2-25** It is desired to project onto a screen an image that is four times the size of a brightly illuminated object. A plano-convex lens with $n = 1.50$ and $R = 60$ cm is to be used. Employing the Newtonian form of the lens equations, determine the appropriate distance of the object and screen from the lens. Is the image erect or inverted? Check your results using the ordinary lens equations.

**2-26** Three thin lenses of focal lengths 10 cm, 20 cm, and $-40$ cm are placed in contact to form a single compound lens.

    **a.** Determine the powers of the individual lenses and that of the unit, in diopters.
    **b.** Determine the vergence of an object point 12 cm from the unit and that of the resulting image. Convert the result to an image distance in centimeters.

**2-27** A lens is moved along the optical axis between a fixed object and a fixed image screen. The object and image positions are separated by a distance $L$ that is more than four times the focal length of the lens. Two positions of the lens are found for which an image is in focus on the screen, magnified in one case and reduced in the other. If the two lens positions differ by distance $D$, show that the focal length of the lens is given by $f = (L^2 - D^2)/4L$. This is *Bessel's method* for finding the focal length of a lens.



$f$        $f$

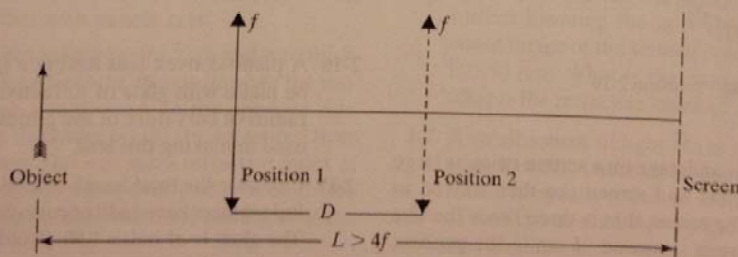Object    Position 1    Position 2    Screen

$D$

$L > 4f$

**Figure 2-38**    Problem 2-27

2-28 An image of an object is formed on a screen by a lens. Leaving the lens fixed, the object is moved to a new position and the image screen moved until it again receives a focused image. If the two object positions are $S_1$ and $S_2$ and if the transverse magnifications of the image are $M_1$ and $M_2$, respectively, show that the focal length of the lens is given by

$$f = \frac{(S_2 - S_1)}{\left(\dfrac{1}{M_1} - \dfrac{1}{M_2}\right)}$$

This is *Abbe's method* for finding the focal length of a lens.

2-29 Derive the law of reflection from Fermat's principle by minimizing the distance of an arbitrary (hypothetical) ray from a given source point to a given receiving point.
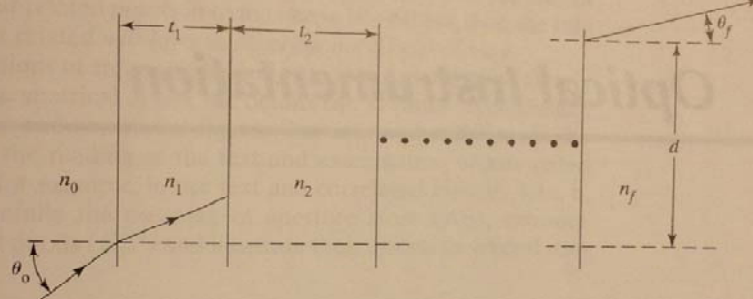
2-30 Determine the ratio of focal lengths for two identical, thin, plano-convex lenses when one is silvered on its flat side and the other on its curved side. Light is incident on the unsilvered side.

2-31 Show that the minimum distance between an object and its image, formed by a thin lens, is $4f$. When does this occur?

2-32 A ray of light traverses successively a series of plane interfaces, all parallel to one another and separating regions of differing thickness and refractive index.

a. Show that Snell's law holds between the first and last regions, as if the intervening regions did not exist.
b. Calculate the net lateral displacement of the ray from point of incidence to point of emergence.



**Figure 2-39**  Problem 2-32

2-33 A parallel beam of light is incident on a plano-convex lens that is 4 cm thick. The radius of curvature of the spherical side is also 4 cm. The lens has a refractive index of 1.50 and is used in air. Determine where the light is focused for light incident on each side.

2-34 A spherical interface, with radius of curvature 10 cm, separates media of refractive index 1 and $\frac{4}{3}$. The center of curvature is located on the side of the higher index. Find the focal lengths for light incident from each side. How do the results differ when the two refractive indices are interchanged?

2-35 An airplane is used in aerial surveying to make a map of ground detail. If the scale of the map is to be 1:50,000 and the camera used has a focal length of 6 in., determine the proper altitude for the photograph.

2-36 Light rays emanating in air from a point object on axis strike a plano-cylindrical lens with its convex surface facing the object. Describe the line image by length and location if the lens has a radius of curvature of 5 cm, a refractive index of 1.60, and an axial length of 7 cm. The point object is 15 cm from the lens.

2-37 A plano-cylindrical lens in air has a curvature of 15 cm and an axial length of 2.5 cm. The refractive index of the lens is 1.52. Find the position and length of the line image formed by the lens for a point object 20 cm from the lens. Light from the object is incident on the convex cylindrical surface of the lens.

2-38 A plano-cylindrical lens in air has a radius of curvature of 10 cm, a refractive index of 1.50, and an axial length of 5 cm. Light from a point object is incident on the concave, cylindrical surface from a distance of 25 cm to the left of the lens. Find the position and length of the image formed by the lens.

2-39 A plano-concave cylindrical lens is used to form an image of a point object 20 cm from the lens. The lens has a refractive index of 1.50, a radius of curvature of 20 cm, and an axial length of 2 cm. Describe as completely as possible the line image of the point.

2-40 Consider the plano-convex cylindrical lens in problem 2-36. If the point object is only 6 cm from the lens, describe the line image.