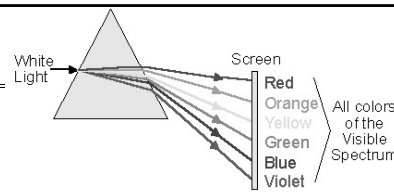
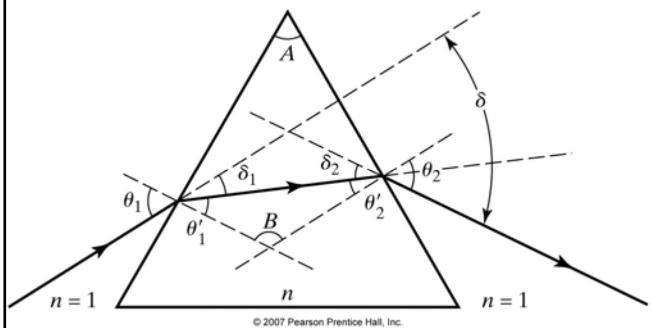
3.3 Prisms



Angular Deviation of a Prism



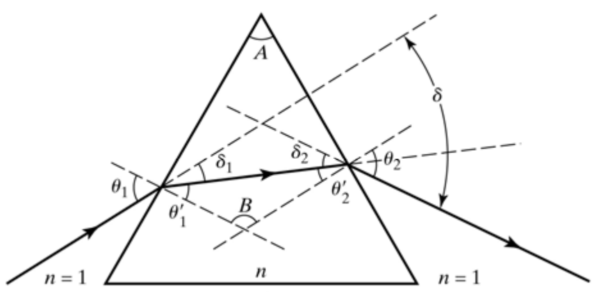
Parameters:

Angular deviation: δ
Apex angle: **A**
Prism index: *n*
Incident angle: $\boldsymbol{\theta_1}$

- The deviation angle is: $\boxed{\delta = \theta_1 + \theta_2 - (\theta'_1 + \theta'_2) = \theta_1 + \theta_2 - A}$

$$\boxed{\delta = \theta_1 + \sin^{-1}\left(\sqrt{n^2 - \sin^2\theta_1}\,\sin A - \sin\theta_1 \cos A\right) - A}$$

3.3 Prisms



- Condition of Minimum deviations: The ray for which the deviation is a minimum traverses the prism symmetrically, that is, parallel to its base.
- Applications : Determination of refractive index *n* of transparent material

$$\boxed{n = \frac{\sin\left[\left(\delta_m + A\right)/2\right]}{\sin\left(A/2\right)}}$$

δ_m : Minimum angular deviation

3.3 Prisms

Dispersion

- Typical normal dispersion curve →

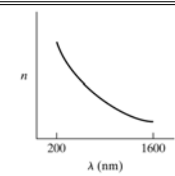


TABLE 3-1 FRAUNHOFER LINES

		n	
λ (nm)	Characterization	Crown glass	Flint glass
486.1	*F*, blue	1.5286	1.7328
589.2	*D*, yellow	1.5230	1.7205
656.3	*C*, red	1.5205	1.7076

- Empirical relation ---Cauchy Relation:

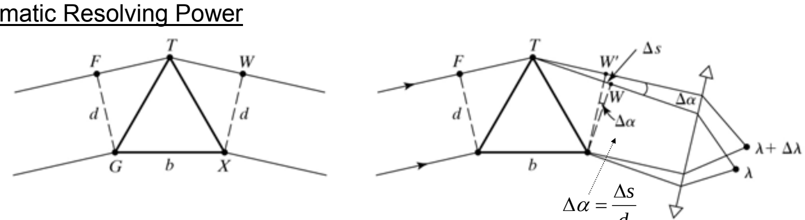
$$\boxed{n_\lambda = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots}$$

A, B, C … are Cauchy constants

- Dispersion: $\boxed{\dfrac{dn}{d\lambda} \simeq -\dfrac{2B}{\lambda^3}}$

3.3 Prisms

Chromatic Resolving Power



(a) Refraction of monochromatic light

(b) Refraction of 2-wavelength components separated by $\Delta\lambda$

Parameters:
d : beam width
b: Prism base
Δs : path difference
$\Delta\alpha$: Angular separation
Δn : $n_{\lambda 1}$- $n_{\lambda 2}$

$$\boxed{\Delta s = b\Delta n, \quad \Delta n = \left(\frac{dn}{d\lambda}\right)\cdot\Delta\lambda}$$

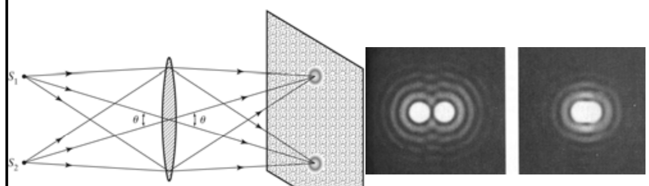
Rayleigh' criterion
Minimum separation: $\Delta\alpha = \dfrac{\lambda}{d}$

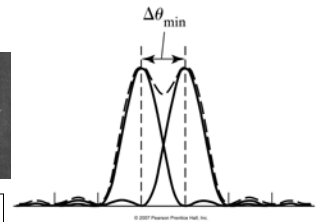
$$\Rightarrow \boxed{\Delta\alpha = \frac{\Delta s}{d} = \left(\frac{b}{d}\right)\left(\frac{dn}{d\lambda}\right)(\Delta\lambda)_{\min} = \frac{\lambda}{d}}$$

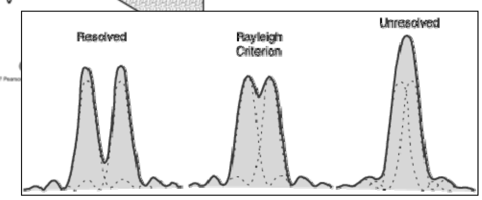
- Resolving power: $\boxed{\Re = \dfrac{\lambda}{(\Delta\lambda)_{\min}} = b\dfrac{dn}{d\lambda}}$

3.3 Prisms

- **Rayleigh's criterion**: Two point sources are regarded as just resolved when the principal diffraction maximum of one image coincides with the first minimum of the other (i.e, the max. of one pattern falls directly over the 1st min. of the other)







Rayleigh's criterion for just resolvable diffraction pattern

- The minimum angular separation: $\boxed{(\Delta\theta)_{\min} = \dfrac{\lambda}{D}}$ $\boxed{(\Delta\theta)_{\min} = \dfrac{1.22\lambda}{D}}$

D: Aperture diameter — Slit Aperture — Circular Aperture

3.3 Prisms

TABLE 3-1 FRAUNHOFER LINES

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656.3	*C*, red	1.5205	1.7076

Example 3-2

Determine the resolving power and minimum resolvable wavelength difference for a prism made from flint glass with a base of 5cm.

Problems 3-7

A parallel beam of white light is refracted by a 60° glass prism in a position of minimum deviation. What is the angular separation of emerging red (n = 1.525) and blue (1.535) light?

3.3 Prisms

Prisms with Special Applications

- Prism can be designed with Constant deviation angles for spectroscopy
  - Constant deviation dispersing prisms, no matter what wavelength.
  - But, rotate the prism, could look at a particular wavelength

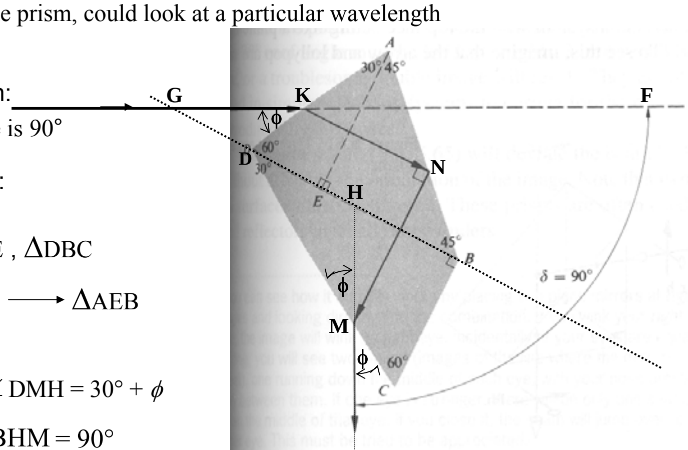
- Pellin-Broca prism:
  The deviation angle is 90°

- Prism consists of :
  - Two 30°-60°-90° ⟶ ΔADE , ΔDBC
  - One 45°-45°-90° ⟶ ΔAEB

$\angle FGB = 60° - \phi$

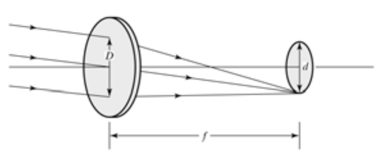
$\angle BHM = \angle HDM + \angle DMH = 30° + \phi$

$\delta_m = \angle FGB + \angle BHM = 90°$



ure 5.58 The Pellin–Broca prism.

3.4 The Camera



E_e: Irradiance, light power at the image plane
D: Diameter of lens
d : image size

$$\boxed{E_e \propto \frac{D^2}{d^2}, \text{ and } d \propto f} \Rightarrow \boxed{E_e \propto \frac{D^2}{f^2}}$$

- f-number: relative aperture of lens $\Rightarrow$ $\boxed{A = \frac{f}{D}}$

- Aperture is usually referred to by photographers as f/A, such as, f/8 (i.e., A=8)

- Total exposure (J/m^2): Irradiance x time
  (time ⟶ Shutter speed)

- Same total exposure:

Shutter speed (s)	Aperture
1/50	f/8
1/100	f/5.6

TABLE 3-2 STANDARD RELATIVE APERTURES AND IRRADIANCE AVAILABLE ON CAMERAS

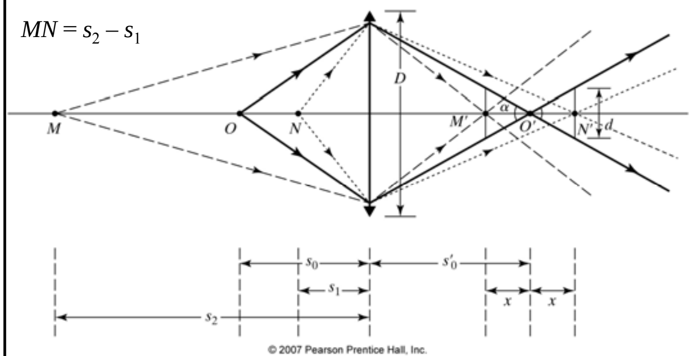
$A = f$-number	$(A = f\text{-number})^2$	Irradiance E_e
1	1	E_0
1.4	2	$E_0/2$
2	4	$E_0/4$
2.8	8	$E_0/8$
4	16	$E_0/16$
5.6	32	$E_0/32$
8	64	$E_0/64$
11	128	$E_0/128$
16	256	$E_0/256$
22	512	$E_0/512$

by a factor of   by a factor of 2

3.4 The Camera

- Depth of field: The interval *MN* in object space conjugated to the interval *M'N'*.

$MN = s_2 - s_1$



d: blurring diameter

Obj. distance	Image dis.
s_o	s_o'
s_1	$s_o' + x$
s_2	$s_o' - x$

$$\frac{D}{s_o'} = \frac{d}{x}$$

$$\Rightarrow \quad s_1 = \frac{s_o f(f + Ad)}{f^2 + Ads_o}, \quad s_2 = \frac{s_o f(f - Ad)}{f^2 - Ads_o} \quad \Rightarrow \quad \text{depth of field} = \frac{2Ads_o(s_o - f)f^2}{f^4 - A^2d^2s_o^2}$$

- The depth of field is greater for smaller aperture (greater *f* number), shorter focal length, longer shorting distance

3.4 The Camera

Example 3-3

A 5 cm focal length lens with an *f* /16 aperture is used to image an object 9 ft away. The blurring diameter in the image is chosen to be d = 0.04 mm. Determine the location of the near point (s_1), far point (s_2), and the depth of the field

3.5 Simple Magnifiers and Eyepieces

- Normal eyes: Eye that is capable of focusing parallel rays on the retina while in a relaxed condition

- Near point :
  The closest point on which the eye can focus.
  For normal eye,
  - teenager → 7 cm;
  - young adult → 12cm;
  - middle-aged → 28~40cm
  - common used number for calculation: **25 cm**

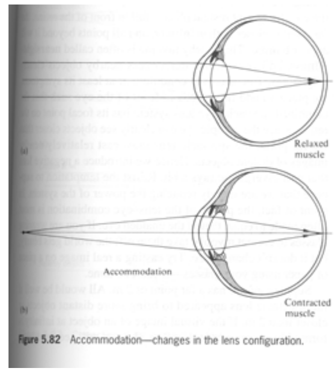
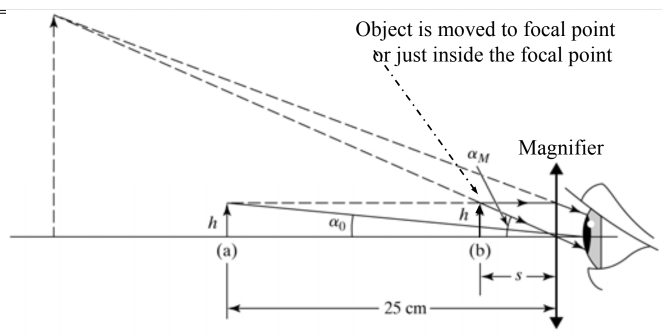


Figure 5.82 Accommodation—changes in the lens configuration.

3.5 Simple Magnifiers and Eyepieces

Simple Magnifier:



- Simple magnifier : *a positive lens, provide an image of a nearly object that is larger than the image seen by the unaided eye.*

- Angular magnification M : *the ratio of the retinal image as seen through the instrument over the size of the retinal image as seen by the unaided eye at normal viewing distance.*

  1. Image viewed at infinity: $\boxed{M = \dfrac{25}{f}}$ (Object is moved to the focal point)
  2. Image viewed at normal near point: $s' = -25$ cm $\boxed{M = \dfrac{25}{f} + 1}$ (Object is just inside the focal point)

3.5 Simple Magnifiers and Eyepieces

- Eyepieces or Oculars: To reduce transverse chromatic aberration, two lens are most often used. The effective focal length f of the two thin lenses separated by a distance L is given:

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}}$$

with

$$\boxed{\begin{aligned} \frac{1}{f_1} &= (n-1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) = (n-1)K_1 \\ \frac{1}{f_2} &= (n-1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right) = (n-1)K_2 \end{aligned}}$$

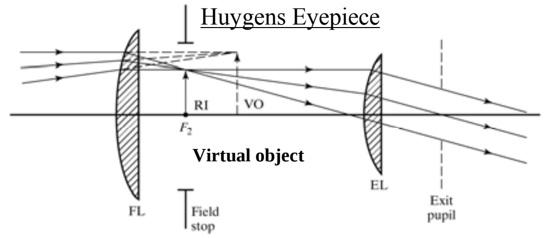
- To correct transverse chromatic aberration: $\boxed{\dfrac{d(1/f)}{dn} = 0}$

⇨ Optimum separation : $\boxed{L = \dfrac{1}{2}(f_1 + f_2)}$

3.5 Simple Magnifiers and Eyepieces

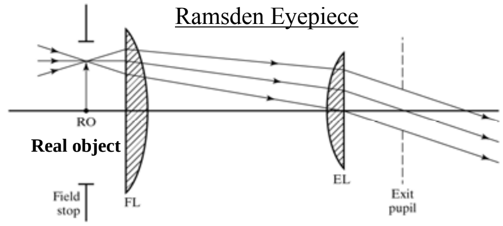
- Eyepieces or Oculars: To reduce transverse chromatic aberration, two lens, *field lens* (FL) and *eye lens* (EL) are most often used.

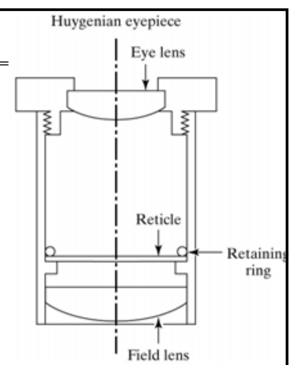
Huygens Eyepiece

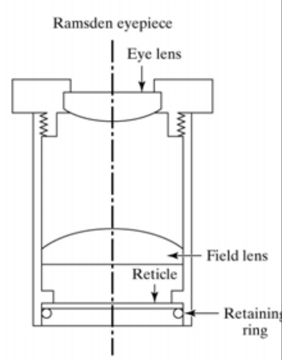


Can we use Huygens eyepieces an ordinary magnifier?

Ramsden Eyepiece







3.5 Simple Magnifiers and Eyepieces

Example 3-4

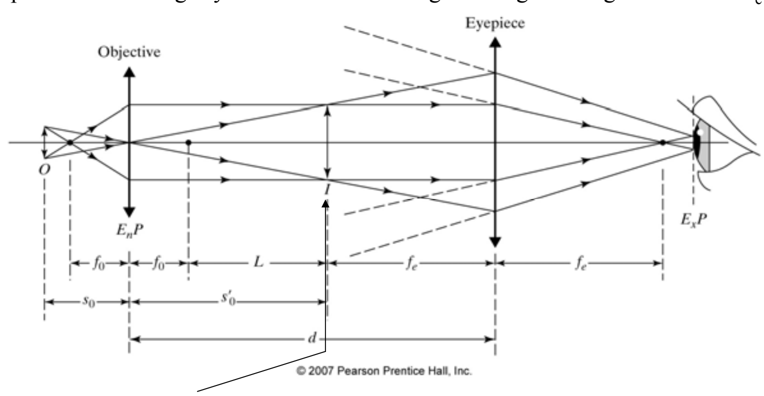
A Huygens eyepiece uses two lenses having focal lengths of 6.25 cm and 2.50 cm, respectively. Determine their optimum separation in reducing chromatic aberration, their equivalent focal length, and their angular magnification when viewing an image at infinity.

3.6 Microscopes

- Compound Microscope : Objective + Eyepiece (it may provide high MP)

Objective: form a real, inverted, magnified image → Lateral Magnification - M_o

Eyepiece: further magnify the intermediate image → Angular Magnification - M_e



- The intermediate image is at or just inside of the 1st focal point f_e of the eyepiece.
- The separation of the two lens: $\boxed{d = f_o + L + f_e}$

3.6 Microscopes

➢ Magnification : $M = M_o M_e = -\left(\frac{L}{f_o}\right)\left(\frac{25}{f_e}\right)$ Unit: cm

or $M = \frac{25}{f_{eff}}$ with $\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

d: Separation of the two lenses
L: Distance between the 2nd focal point and the objective image

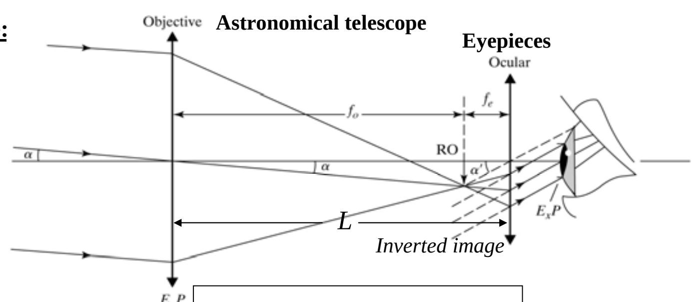
Example 3-5

A microscope has an objective of 3.8 cm focal length and an eyepiece of 5 cm focal length. If the distance between the lenses is 16.4 cm, find the magnification of the microscope.

3.7 Telescopes

➢ Telescopes may be broadly classified as refracting or reflecting, according to whether lenses or mirrors are used to produce the image.

Keplerian telescope:



Length of telescope: $L = f_o + f_e$

➢ Eyepiece is located so that 1st focus overlaps 2nd focus of Objective

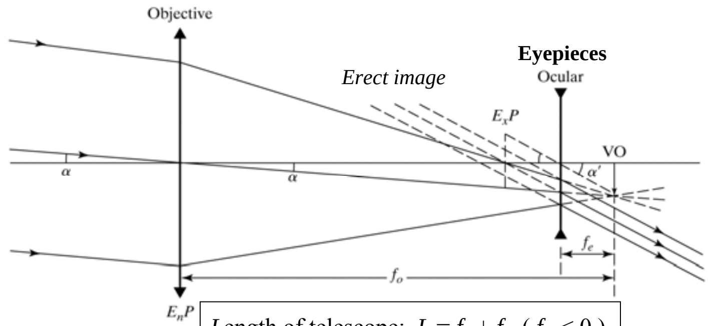
➢ Nearly parallel rays of light from a distant object are collected by a positive lens--Objective, formed a real inverted image in its focal plane

➢ Then the intermediate image, located at or near the focal point of eyepiece, serves as a real object for the Ocular

➢ Angular magnification (when image is viewed at infinity): $M = \frac{\alpha'}{\alpha} = -\frac{f_o}{f_e}$

3.7 Telescopes

Galilean telescope:



Length of telescope: $L = f_o + f_e$ ($f_e < 0$)

- Angular magnification: $M = \dfrac{\alpha'}{\alpha} = -\dfrac{f_o}{f_e}$

- Binocular marked "6×30": M = 6, Diameter of Objective = 30 mm

3.7 Telescopes

Problems 3-28

a) Show that when the final image is not viewed at infinity, the angular magnification of an astronomical telescope may be expressed by

$$M = -\frac{m_{oc} f_{obj}}{s''}$$

where m_{oc} is the linear magnification of the ocular and s" is the distance from the ocular to the final image.

b) For such a telescope using two converging lenses with focal lengths of 30 cm and 4 cm, find the angular magnification when the image is viewed at infinity and when the image is viewed at a near point of 25 cm.