PC237 Optics

Final exam aid allowed: Calculator, and 2 single-sided 8.5x11" formula sheets
Length of examination: 2 hours

The final exam will cover the following chapters:

- **Interference of Light (Chapter 7)**
- **Optical Interferometry (Chapter 8)**
- **Coherence (Chapter 9)**
- **Fraunhofer Diffraction (Chapter 11)**
- **The Diffraction Grating (Chapter 12)**
- **Matrix Treatment of Polarization (Chapter 14)**

Interference of Light

- Two-beam Interference:

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta} \quad \text{with} \quad \boxed{\delta = k(s_2 - s_1)}$$

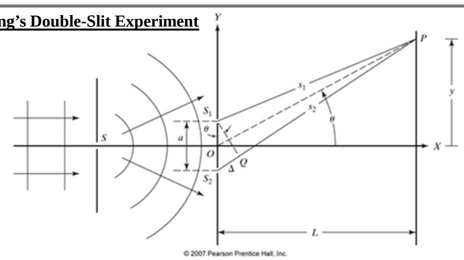
1. Constructive interference: $\boxed{I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}}$ $\boxed{\delta = 2m\pi}$
2. Destructive interference: $\boxed{I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}}$ $\boxed{\delta = (2m+1)\pi}$

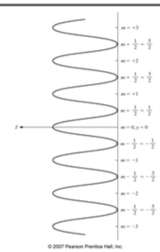
   - Special case: $\boxed{I_1 = I_2 = I_0}$ $\Rightarrow$ $\boxed{I = 4I_o\cos^2\frac{\delta}{2}}$
   - Visibility, a measure of fringe contrast is defined as

$$\boxed{\text{visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}}$$

Interference of Light

Young's Double-Slit Experiment





Optical path difference: $\boxed{\Delta = s_2 - s_1 = a\sin\theta}$ $\boxed{\sin\theta \simeq \tan\theta = y/L}$

Phase difference: $\boxed{\delta = k(s_2 - s_1) = \frac{2\pi}{\lambda}\Delta}$

Irradiance at screen: $\boxed{I = 4I_o\cos^2\frac{\delta}{2} = 4I_o\cos^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}$

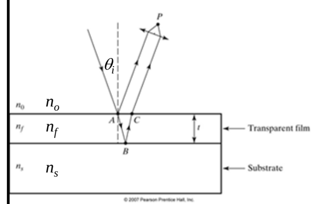
$\Rightarrow$ Bright fringe and dark fringe conditions:

$$\boxed{y_m = m\frac{\lambda L}{a} \quad m = 0, \ \pm1, \ \pm2\cdots} \qquad \boxed{y_m = (m+\frac{1}{2})\frac{\lambda L}{a} \quad m = 0, \ \pm1, \ \pm2\cdots}$$

$\Rightarrow$ Fringe separation: $\boxed{\Delta y = y_{m+1} - y_m = \frac{\lambda L}{a}}$

Interference of Light

Two-beam interference for normal incidence:



For normal incidence: $\boxed{\Delta_p = 2n_f t}$

Note: Need to consider additional *phase shift* due to *reflections*

Constructive interference: $\boxed{\Delta_p + \Delta_r = m\lambda}$

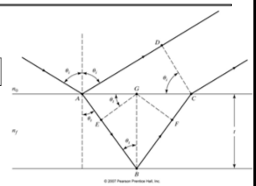
Destructive interference: $\boxed{\Delta_p + \Delta_r = (m+1/2)\lambda}$

- Phase shift due to reflection:

Case 1	two external reflections $n_o < n_f < n_s$ or two internal reflections $n_o > n_f > n_s$	No additional phase shift
Case 2	one internal and one external reflection $n_f > n_o$ and $n_f > n_s$ or $n_f < n_o$ and $n_f < n_s$	Relative phase shift of π

Interference of Light

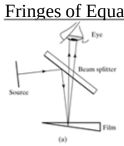
Two-beam interference:



- OPD of the two beams can be obtained: $\boxed{\Delta_p = 2n_f t\cos\theta_t}$
- Conditions for bright and dark fringes:

  Bright: $\boxed{\Delta_p + \Delta_r = m\lambda}$

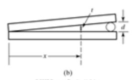
  Dark: $\boxed{\Delta_p + \Delta_r = (m+1/2)\lambda}$
- There are two types of fringes:
  - Fringes of Equal Inclination (θ_t); referred to as Haidinger Fringes
  - Fringes of Equal Thickness (t); referred to as Fizeau Fringes
- Fringes of Equal Thickness:



Normal incidence:

Bright: $\boxed{\Delta_p + \Delta_r = m\lambda}$ with $\boxed{\Delta_p = 2n_f t = 2n_f x\alpha}$

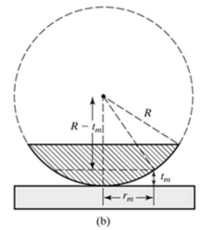
Dark: $\boxed{\Delta_p + \Delta_r = (m+1/2)\lambda}$



- Separation Δx of consecutive fringes: $\boxed{\Delta x = \lambda_f / 2\alpha}$

Interference of Light

- Fringes of Equal Thickness: Newton's Rings



- Find R: $\boxed{R^2 = r_m^2 + (R - t_m)^2}$ ⇨ $\boxed{R = \dfrac{r_m^2 + t_m^2}{2t_m}}$
- If ($n_f < n_2$ and $n_f < n_1$) or ($n_f > n_2$ and $n_f > n_1$), then the center of the fringe appears dark

  Maxima: $\boxed{2n_f t_m = (m - 1/2)\lambda_0}$

  Radius of m^{th} bright ring : $\boxed{r_m = \sqrt{(m-1/2)\lambda_f R}}$

  Minima:

  Radius of m^{th} dark ring : $\boxed{r_m = \sqrt{m\lambda_f R}}$

- Stokes Relations:

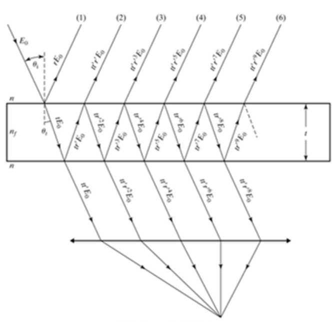
Stokes relations
$r = -r'$
$tt' = 1 - r^2$

r , t : *reflection coefficient* and *transmission coefficient* (from n_1 to n_2)

r' , t' : *reflection coefficient* and *transmission coefficient* (from n_2 to n_1)

Interference of Light

- Multiple-beam Interference in a Parallel Plate :



$$\boxed{\delta = k\Delta, \quad \Delta = 2n_f t\cos\theta_t}$$

$$E_1 = E_0 r e^{i\omega t}$$
$$E_2 = E_0 t r' t' e^{i(\omega t - \delta)}$$
$$E_3 = E_0 t (r')^3 t' e^{i(\omega t - 2\delta)}$$

$$\boxed{E_N = E_0 t (r')^{2N-3} t' e^{i[\omega t - (N-1)\delta]}}$$

$$\Rightarrow E_R = \sum_{N=1}^{\infty} E_N = E_0 e^{i\omega t}\left[\frac{r(1-e^{-i\delta})}{1 - r^2 e^{-i\delta}}\right]$$

Irradiance of reflected beam :

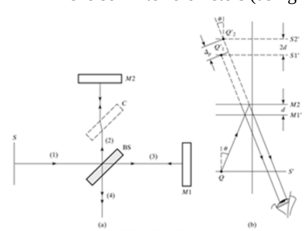
$$I_R = E_R \cdot E_R^* = I_i \frac{2r^2(1-\cos\delta)}{(1+r^4) - 2r^2\cos\delta}$$

Irradiance of transmitted beam :

$$I_T = I_i - I_R = I_i\left\{\frac{(1-r^2)^2}{(1+r^4) - 2r^2\cos\delta}\right\}$$

Optical Interferometry

- Michelson Interferometers (using 2 Mirrors and 1 Beam splitter)



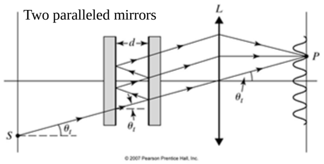
- OPD: $\boxed{\Delta_p = 2d\cos\theta}$
- π phase shift due to reflection: $\boxed{\Delta_r = \lambda / 2}$
- Dark fringes: $\boxed{2d\cos\theta = m\lambda}$

When M_2 is moved toward M_1, what happen to the fringe ?

1. The rings shrink toward the center with the highest-order disappearing whenever d decrease by λ / 2
2. Each remaining ring broadens as more and more fringes vanish at the center until only a few fill the whole screen, which can be explained by the angular separation of the ring.

Optical Interferometry

- Fabry-Perot interferometer : Formed by 2 parallel mirrors (r_1, r_2), separated by a distance d



For lossless mirrors, transmittance of FP cavity:

$$T = \frac{(1-r^2)^2}{(1+r^4)-2r^2\cos\delta} = \frac{1}{1+\left[\frac{4r^2}{(1-r^2)^2}\right]\sin^2(\delta/2)}$$

Round-trip phase shift: $\delta = 2kd$

- Coefficient of finesses: $F = \frac{4r^2}{(1-r^2)^2}$ ⟹ $T = \frac{1}{1+F\sin^2(\delta/2)}$

F is a measure of fringe contrast : Fringe Contrast $= \frac{T_{max}-T_{min}}{T_{min}} = F$

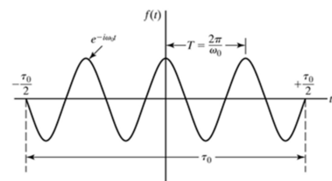
- Half width $\delta_{1/2}$: *Half-width at half max.* (HWHM)
- Finesse $\mathscr{F}$: *Ratio of phase separation between adjacent transmittance peaks to the full width* $2\delta_{1/2}$

$\mathscr{F} = 2\pi / (2\delta_{1/2}) = \pi / \delta_{1/2}$ ⟹ $\boxed{\mathscr{F} = \frac{\pi}{2\sin^{-1}\frac{1}{\sqrt{F}}}}$

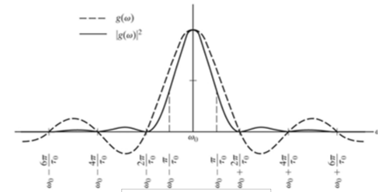
If F is very large number: $\boxed{\mathscr{F} = \pi\sqrt{F}/2 = \frac{\pi r}{1-r^2}}$

Coherence

- **Coherence** is an ideal property of waves that enables stationary (i.e. temporally and spatially constant) interference
- **Temporal coherence** is related to the frequency spread of the source. Temporal coherence tells us how monochromatic a source is.
- **Spatial coherence** is related to the size of the source. Spatial coherence is a measure of the correlation between waves at different points in space



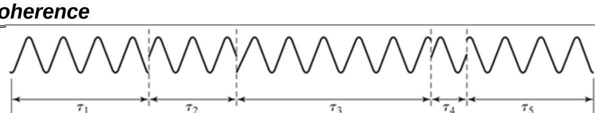
A finite wave train with lifetime τ_0



Power spectrum

- For a wave train with a lifetime τ_0, a frequency bandwidth $\Delta\omega$ required to represent the finite harmonic wave train is: $\boxed{\Delta\omega = \frac{2\pi}{\tau_0}}$ or $\boxed{\Delta\nu = \frac{1}{\tau_0}}$

Coherence



Harmonic wave trains of varying finite length or lifetime τ

Parameters:

- Coherent time τ_0 and Coherent length l_t of a light source : $\boxed{l_t = c\tau_0}$
- $\Delta\lambda$: line width of light source
- λ: center wavelength light source
- $\Delta\nu$: frequency range of light source

$\boxed{\Delta\nu = \frac{1}{\tau_0}}$ $\boxed{\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda}$ $\boxed{\Delta\lambda = \frac{\lambda^2}{l_t}}$

Interference fringe visibility:

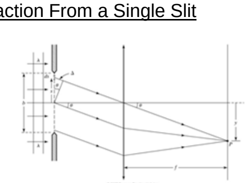
$$\boxed{V = 1-\frac{\tau}{\tau_0} = 1-\frac{\Delta}{l_t} = 1-\frac{\Delta\cdot(\Delta\lambda)}{\lambda^2}}$$

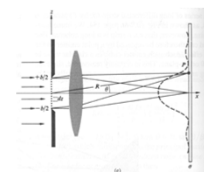
τ : time delay between two interference beams
Δ : optical path difference between the 2 beams
τ_0 : source lifetime
l_t : source coherence length

1. Complete incoherence $\tau \ge \tau_0$ or $\Delta \ge l_t$
2. Complete coherence $\tau \approx 0$ or $\Delta \approx 0$
3. Partial coherence $0 < \tau < \tau_0$ or $0 < \Delta < l_t$

Fraunhofer Diffraction

Diffraction From a Single Slit





- At any angle θ, the irradiance is: $I = \left(\frac{\varepsilon_0 c}{2}\right)E_0^2 = I_0\frac{\sin^2\beta}{\beta^2}$ **with** $\boxed{\beta = \frac{1}{2}kb\sin\theta}$
- Consider: $\boxed{\theta = 0,\quad \sin\beta/\beta = 1}$ ⟹ $\boxed{I = I_0}$ **I_0 : principle maximum**
- Angular positions of m-th order minimum : $\boxed{b\sin\theta_m = m\lambda}$
- Vertical displacement y for zero irradiance : $\boxed{y_m \cong f\sin\theta_m}$ ⟹ $\boxed{y_m \cong \frac{m\lambda f}{b}}$
- The secondary maxima are located at : $\tan\beta = \beta$, $\beta = \pm1.4303\pi, \pm2.459\pi, \pm3.4707$

Fraunhofer Diffraction

Diffraction From a Rectangular Slit with slit width a and b:

- At any angle θ, the irradiance is: $\boxed{I(\theta)=I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2\beta}{\beta^2}\right)=I_0\operatorname{sinc}^2\alpha\operatorname{sinc}^2\beta}$

  with $\alpha=\frac{1}{2}ka\sin\theta,\quad \beta=\frac{1}{2}kb\sin\theta$

Diffraction From a Circular aperture with diameter of D:

- At any angle θ, the irradiance is: $\boxed{I=I_0\left(\frac{2J_1(\gamma)}{\gamma}\right)^2}$ with $\gamma=\frac{1}{2}kD\sin\theta$
- Maximum irradiance at center (θ = 0): $\boxed{I(0)=I_0}$
- 1st zero irradiance: $\boxed{\gamma=3.832}$ $\Rightarrow$ $\boxed{D\sin\theta=1.22\lambda}$

Resolutions:

- Rayleigh's criterion: the max. of one pattern falls directly over the 1st min. of the other
- Limit of resolution: $\boxed{(\Delta\theta)_{\min}=\frac{1.22\lambda}{D}}$

Fraunhofer Diffraction

Double Slit Diffraction:

- The irradiance can be written as: $\boxed{I=4I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\alpha}$ with $\boxed{\begin{array}{l}\alpha=(ka/2)\sin\theta\\ \beta=(kb/2)\sin\theta\end{array}}$

  I_0: flux-density contribution from either slit **Diffraction** **Interference**

- Angular position of Interference maximum: $\boxed{\alpha=(ka/2)\sin\theta=p\pi}$ **or** $\boxed{a\sin\theta=p\lambda}$
- Angular position for Diffraction minima: $\boxed{\beta=(kb/2)\sin\theta=m\pi}$ **or** $\boxed{b\sin\theta=m\lambda}$
- Condition for missing orders: $\boxed{\frac{a}{b}=\frac{p}{m}}$ Considering 1st-order diffraction min: $m = 1$, $p = a / b$

Diffraction from N-slits:

- Irradiance for N-slit diffraction: $\boxed{I(\theta)=I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2}$

  With $\boxed{\beta=(kb/2)\sin\theta}$ $\boxed{\alpha=(ka/2)\sin\theta}$ **N: number of slit** **Diffraction** **Interference**

- **Principle maximum conditions**: $\boxed{\alpha=\frac{p\pi}{N}\quad (p=0,\pm1,\pm2\ldots\pm N\ldots\pm2N)}$

  Principle maxima occur for: $p = 0, \pm N, \pm 2N$
  Secondary minima occur for: p = all other integer values

The Diffraction Gratings

- A grating is a periodic, multiple-slit device designed to take advantage of the sensitivity of its diffraction pattern to the wavelength of the incident light. The directions of these beams depend on the *spacing of the grating* and the wavelength of the light so that the grating acts as a *dispersive* element. It is very useful in wavelength measurement and spectral analysis

- General Grating equations: $\boxed{a(\sin\theta_i+\sin\theta_m)=m\lambda}$

  If $\theta_i = 0$, grating equation: $a\sin\theta_m=m\lambda$

θ_i:	**incident angle**
θ_m:	**diffraction angle**
m:	**diffraction order**

- Resolution of a grating:

  Resolving Power $\mathscr{R}\equiv\frac{\lambda}{(\Delta\lambda)_{\min}}=mN$

N :	number of grating lines
m:	order of principle maximum
λ :	mean wavelength.

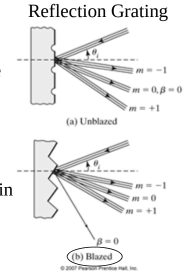
  $\Delta\lambda_{\min}$: is the least resolvable wavelength difference, which is determined by Rayleigh's Criterion

The Diffraction Gratings

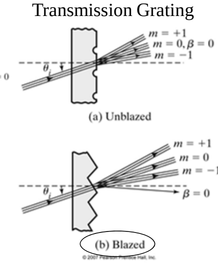
- **For unblazed grating:**

1. The max. of the single-slit diffraction envelope coincides with the most intense interference max; however, there is no dispersion at zeroth-diffraction principle max., which results in a waste of light energy
2. For *transmitted light*, diffraction peak is in the direction of incident beam; for *reflected light*, it is in the direction of the spectrally reflected beam

Reflection Grating



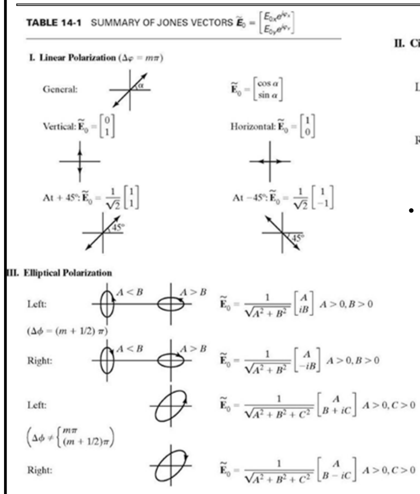
Transmission Grating

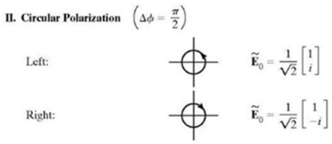


Blazed Gratings:

1. The technique of shaping individual grooves so that the diffraction envelope maximum shifts into another order is called blazing the grating
2. By introducing the blazed angle, for transmission blazed grating **the zero path difference is shifted into the direction of the refracted beam** while for reflection blazed grating, it is shifted into the new reflected beam (to the normal line N', instead of N)

Mathmetical Representation of Polarized Light: Jones Vector





- <u>Elliptically</u> polarized light oriented at an *inclination angle* α relative to the *x*-axis

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$$

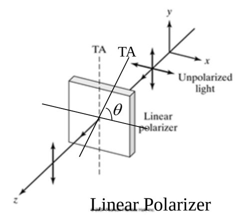
with $E_{0x} = A$, $E_{0y} = \sqrt{B^2 + C^2}$ $\quad \varepsilon = \tan^{-1}\left(\frac{C}{B}\right)$

Rotation direction:

- Adopt the convention that *A* is positive
- If *C* is positive, then it is counter-clockwise rotation
- If *C* is negative, then it is clockwise rotation

Mathmetical Representation of Polarizers: Jones Matrices

- Various optical devices (such as <u>Polarizer</u>, <u>Phase Retarder</u> and <u>Rotator</u>) may modify the state of polarization, which can be described by 2×2 Jones matrices
- **<u>Linear Polarizer</u>**: is a device that converts an unpolarized or mixed-polarization beam of light into a beam with a single linear polarization state. Polarizers are used in many optical techniques and instruments, and polarization filters find applications in photography and liquid crystal display technology.
- General matrix for **<u>linear polarizer</u>** with TA at θ (relative to *x*-axis):



Linear Polarizer

$$M = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

If θ = 90°, then $M = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

Mathmetical Representation of Polarizers: Jones Matrices

- **<u>Phase Retarder</u>**: A retarder or wave plate is an optical device that <u>alters</u> the polarization state of a light wave travelling through it. A wave plate works by <u>shifting</u> the <u>phase</u> between <u>two perpendicular polarization components</u> of the light wave. A typical wave plate is simply a birefringent crystal with a carefully chosen orientation and thickness.

1. Quarter-wave plate (QWP): *OPD* = (4*m*+1)λ / 4 → Δ φ = ± π / 2
2. Half-wave plate (HWP): *OPD* = (4*m*+1)λ / 2 → Δ φ = ± π
3. Full-wave plate (FWP): *OPD* = (4*m*+1)λ → Δ φ = ± 2π

General matrix for phase retarder: $M = \begin{pmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{pmatrix}$

- **<u>Rotator</u>**: The rotator has the effect of rotating the direction of linearly polarized light incident on it by some particular angle

General matrix for rotator: $M = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$

Mathmetical Representation of Polarizers: Jones Matrices

TABLE 14-2 SUMMARY OF JONES MATRICES

I. Linear polarizers					
TA horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	TA vertical	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	TA at 45° to horizontal	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
II. Phase retarders					
		General	$\begin{bmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{bmatrix}$		
QWP, SA vertical	$e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$			QWP, SA horizontal	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
HWP, SA vertical	$e^{-i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$			HWP, SA horizontal	$e^{i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
III. Rotator					
Rotator	$(\theta \to \theta + \beta)$		$\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$		

- If light represented by Jones vector V passes sequentially through a series of polarizing elements (M_1, M_2, M_3….M_m), then **<u>system matrix</u>** is given by:

$$(M_m..M_3M_2M_1)V = M_sV \Rightarrow M_s = M_m..M_3M_2M_1$$