Chapter 12. The Diffraction Grating

12.1 The Grating Equation

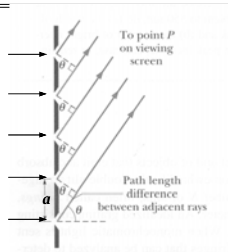
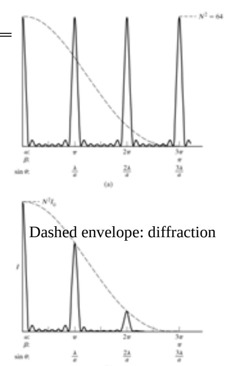
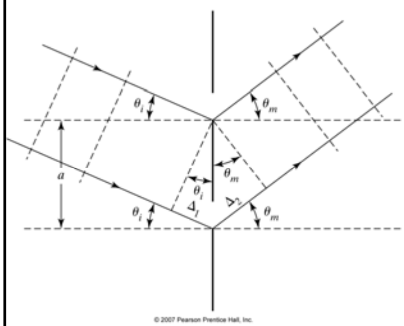


Fig. 37-18 The rays from the rulings in a diffraction grating to a distant point P are approximately parallel. The path length difference between each two adjacent rays is $d \sin\theta$, where θ is measured as shown. (The rulings extend into and out of the page.)



- A grating is a periodic, multiple-slit device designed to take advantage of the sensitivity of its diffraction pattern to the wavelength of the incident light. The directions of these beams depend on the ***spacing of the grating*** and the wavelength of the light so that the grating acts as a ***dispersive*** element. It is very useful in wavelength measurement and spectral analysis
- More common form of diffraction grating is made by ruling or scratching parallel notches into the surface of a flat, a clear glass plate.

12.1 The Grating Equation



- Net path difference for waves from successive slits:

$$\boxed{\Delta = \Delta_1 + \Delta_2 = a\sin\theta_i + a\sin\theta_m}$$

- General Grating equations:

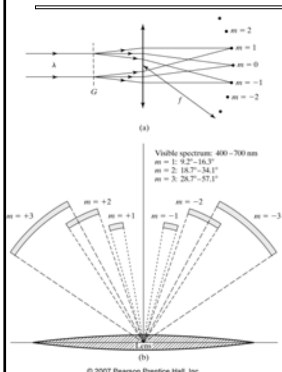
$$\boxed{a(\sin\theta_i + \sin\theta_m) = m\lambda}$$

θ_i: incident angle
θ_m: diffraction angle
m: diffraction order

- Light of all wavelengths appears in the *zeroth-order* peak of the diffraction pattern (i.e., no dispersion in the zeroth-order peak)
- For fixed θ_i, the direction of θ_m, of each *principle maximum* varies with *wavelength*

Grating parameters: N grooves/mm, such as 1000 grooves/mm

12.2 Free Spectral Range of a Grating



$$\boxed{a(\sin\theta_i + \sin\theta_m) = m\lambda}$$

- Free spectral range of a grating : non-overlapping wavelength range in a particular order.

$$\boxed{m\lambda_2 = (m+1)\lambda_1}$$

λ_1: shortest wavelength

- Free spectral range for order m is:

$$\boxed{\lambda_{fsr} = \lambda_2 - \lambda_1 = \frac{\lambda_1}{m}}$$

12.3 Dispersion of a Grating

- Angular dispersion: $\boxed{\mathcal{D} \equiv \frac{d\theta_m}{d\lambda}}$ $\quad \boxed{a(\sin\theta_i + \sin\theta_m) = m\lambda} \longrightarrow \quad \boxed{\mathcal{D} = \frac{m}{a\cos\theta_m}}$

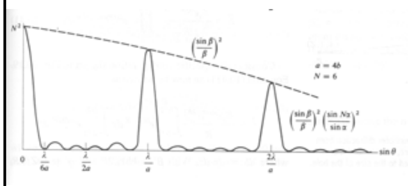
- Linear dispersion: $\boxed{= \frac{dy}{d\lambda} = f\,\frac{d\theta_m}{d\lambda} = f\,\mathcal{D}}$

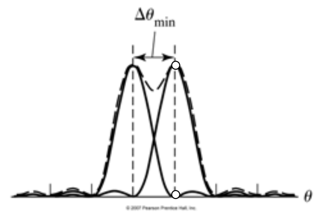
12.4 Resolution of a Grating

- Resolving Power is defined as: $\boxed{\mathscr{R} \equiv \dfrac{\lambda}{(\Delta\lambda)_{\min}}}$ λ : mean wavelength.

$\Delta\lambda_{\min}$: is the least resolvable wavelength difference, which is determined by Rayleigh's Criterion

- **Rayleigh's criterion**: The *m*-th order principle max. for wavelength $\lambda + d\lambda$ must coincide (same θ) with the 1st min. of the neighboring wavelength for wavelength λ in the same order





$$\boxed{a\sin\theta_m = \left(m + \frac{1}{N}\right)\lambda}$$

For min. see green dot at bottom

$$\boxed{a\sin\theta_m = m(\lambda + d\lambda)}$$

For max. see red dot at top

$\Downarrow$

- Resolving Power is given by: $\mathscr{R} = mN$

***N*: number of grating lines**
***m*: order of principle maximum**

12.5 Types of Gratings

Transmission grating

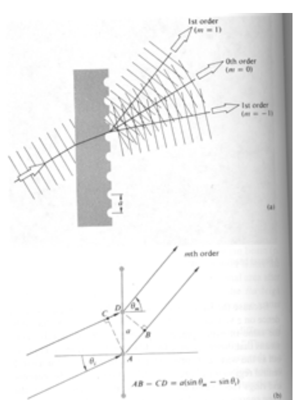


Figure 10.28 A transmission grating.

Reflection grating

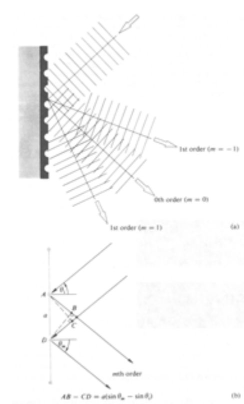
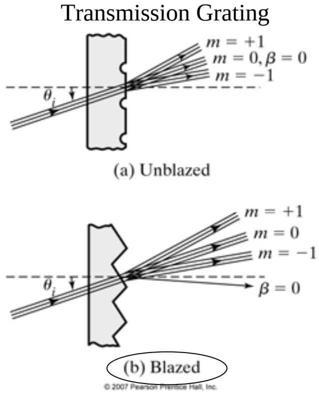


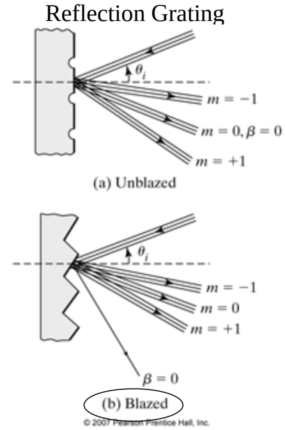
Figure 10.29 A reflection grating.

12.6 Blazed Gratings

Transmission Grating



Reflection Grating

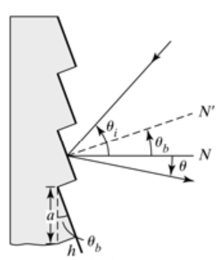


<u>For unblazed grating:</u>

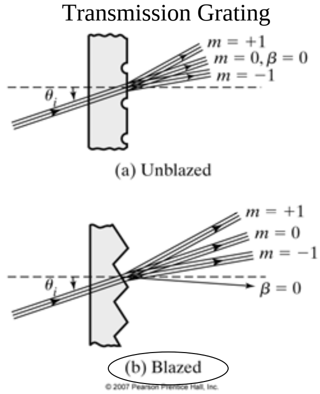
- The max. of the single-slit diffraction envelope coincides with the most intense interference max; however, **there is no dispersion at zeroth-diffraction principle max**., which results in a waste of light energy
- For ***<u>transmitted light</u>***, diffraction peak is in the direction of incident beam; for ***<u>reflected light</u>***, it is in the direction of the spectrally reflected beam

12.6 Blazed Grating

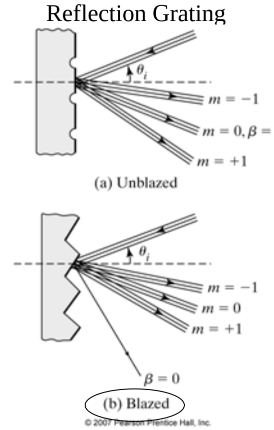
Blazed reflection grating
With blazed angle θ_b



Transmission Grating



Reflection Grating



<u>Blazed grating:</u>

- The technique of shaping individual grooves so that the diffraction envelope maximum shifts into another order is called blazing the grating
- By introducing the blazed angle, for transmission blazed grating **the zero path difference is shifted into the direction of the refracted beam** while for reflection blazed grating, it is shifted into the new reflected beam (to the normal line *N '*, instead of *N*)

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Example 12-3 (p305)

a) What is the angular separation between the 2^{nd}-order principal maximum and the neighboring minimum on either side for the Fraunhofer pattern of a 24-groove grating having a groove separation of 10^{-3} cm and illuminated by light of 600 nm?
b) What slightly longer (slightly shorter) wavelength would have its 2^{nd}-order maximum on top of the minimum adjacent to the 2^{nd}-order maximum of 600-nm light?
c) From your results in parts (a) and (b), calculate the resolving power in second order. Compare this with the resolving power obtained from the theoretical grating resolving power formula, Eq. (12-11).

Example 12-4

How many lines must be ruled on a transmission grating so that it is just capable of resolving the sodium doublet (λ_1=589.592 nm, λ_2 = 588.995 nm) in the first- and second-order spectra?

Chapter 12. The Diffraction Grating

Example 12-7

The two sodium D lines at 5893 Å and 6 Å apart. If a grating with only 400 grooves is available, what is the lowest order possible in which the D lines are resolved?

Example 12-8

A multiple-slit aperture has (a) N = 2, (2) N =10, and (3) N =15,000 slits. The aperture is placed directly in front of a lens of focal length 2 m. The distance between slits is 0.005 mm and the slit width is 0.001 mm for each case. The incident plane wavefronts of light are of wavelength 546 nm. Find, for each case, (a) the separation on the screen between the zeroth- and first-order maxima; (b) the number of bright fringes (principle maxima) that fall under the central diffraction envelope; (c) the width on the screen of the central interference fringe.