Since the beams heading towards the detector encounter the 50-50 beam splitter as they enter and exit the respective interferometer arms,

$$P_1 = P_2 = P_0/4$$

and

$$P_{\text{det}} = \frac{P_0}{4} + \frac{P_0}{4} + 2\frac{P_0}{4}\cos\delta = \frac{P_0}{2}(1 + \cos\delta)$$

Note that the detected power varies between zero and the full laser power P_0 as $\cos\delta$ varies from -1 to 1. Since the detected power is to be zero in the absence of a strain caused by a gravitational wave, the phase shift can be written profitably as

$$\delta = \pi + \delta_g$$

where δ_g is the phase shift induced by the gravitational wave. Using this form for the phase difference between the two beams and using common trigonometric identities, the detected power can be written as

$$\begin{aligned} P_{\text{det}} &= \frac{P_0}{2}(1 + \cos\delta) = \frac{P_0}{2}(1 + \cos(\pi + \delta_g)) = \frac{P_0}{2}(1 - \cos(\delta_g)) \\ &= P_0 \sin^2(\delta_g/2) \end{aligned}$$

For small arguments, the sine function can be approximated by its argument so that

$$P_{\text{det}} \approx P_0(\delta_g/2)^2$$

The phase difference induced by the gravitational wave is $\delta_g = k\Delta s$, where Δs is the difference in path lengths traveled by the beams passing through the two arms of the interferometer. This path difference Δs is

$$\Delta s \approx 2 \cdot 50\Delta L = 100hL$$

Here, $\Delta L = hL$ is the difference in the lengths of the interferometer arms (of nominal length L) induced by the gravitational wave. The factor of 50 accounts for the approximately 50 round-trips made by the light in the Fabry-Perot cavities in each interferometer arm, and the factor 2 accounts for the fact that the light traverses the length of an arm twice in one round-trip through the Fabry-Perot cavity in that arm. The phase shift δ_g induced by the gravitational strain is, therefore,

$$\begin{aligned} \delta_g &\approx k\Delta s = \frac{2\pi}{\lambda}(100hL) = \frac{2\pi}{4.88 \cdot 10^{-7}\,\text{m}}(100)(10^{-21})(4000\,\text{m}) \\ &= 5.1510^{-9}\,\text{rad} \end{aligned}$$

Using this in the final expression for the detected power,

$$P_{\text{det}} \approx P_0(\delta_g/2)^2 = (10\,\text{W})\left(\frac{5.15 \cdot 10^{-9}}{2}\right)^2 = 6.63 \cdot 10^{-17}\,\text{W}$$

This power corresponds to about 160 photons/s and, while small, is easily detected. However, even very low level environmental noise processes lead to power signals of this and greater levels. As noted, reliable detection of gravitational waves will require isolating the interferometer from environmental noise and separating the gravitational signal from the remaining environmental noise signals.

PROBLEMS

8-1 When one mirror of a Michelson interferometer is translated by 0.0114 cm, 523 fringes are observed to pass the crosshairs of the viewing telescope. Calculate the wavelength of the light.

8-2 When looking into a Michelson interferometer illuminated by the 546.1-nm light of mercury, one sees a series of straight-line fringes that number 12 per centimeter. Explain their occurrence.

8-3 A thin sheet of fluorite of index 1.434 is inserted normally into one beam of a Michelson interferometer. Using light of wavelength 589 nm, the fringe pattern is found to shift by 35 fringes. What is the thickness of the sheet?

8-4 Looking into a Michelson interferometer, one sees a dark central disk surrounded by concentric bright and dark rings. One arm of the device is 2 cm longer than the other, and the wavelength of the light is 500 nm. Determine (a) the order of the central disc and (b) the order of the sixth dark ring from the center.

8-5 A Michelson interferometer is used to measure the refractive index of a gas. The gas is allowed to flow into an evacuated glass cell of length L placed in one arm of the interferometer. The wavelength is λ.

a. If N fringes are counted as the pressure in the cell changes from vacuum to atmospheric pressure, what is the index of refraction n in terms of N, λ, and L?
b. How many fringes would be counted if the gas were carbon dioxide ($n = 1.00045$) for a 10-cm cell length, using sodium light at 589 nm?

8-6 A Michelson interferometer is used with red light of wavelength 632.8 nm and is adjusted for a path difference of 20 μm. Determine the angular radius of the (a) first (smallest-diameter) ring observed and (b) the tenth ring observed.

8-7 A polished surface is examined using a Michelson interferometer with the polished surface replacing one of the mirrors. A fringe pattern characterizing the surface contour is observed using He-Ne light of wavelength 632.8 nm. Fringe distortion over the surface is found to be less than one-fourth the fringe separation at any point. What is the maximum depth of polishing defects on the surface?

8-8 The plates of a Fabry-Perot interferometer have a reflection coefficient of $r = 0.99$. Calculate the minimum (a) resolving power and (b) plate separation that will accomplish the resolution of the two components of the H-alpha doublet of the hydrogen spectrum, whose separation is 1.360 nm at 656.3 nm.

8-9 A Fabry-Perot interferometer is to be used to resolve the mode structure of a He-Ne laser operating at 632.8 nm. The frequency separation between the modes is 150 MHz. The plates are separated by an air gap and have a reflectance (r^2) of 0.999.

a. What is the coefficient of finesse of the instrument?
b. What is the resolving power required?
c. What plate spacing is required?
d. What is the free spectral range of the instrument under these conditions?
e. What is the minimum resolvable wavelength interval under these conditions?

8-10 A Fabry-Perot etalon is fashioned from a single slab of transparent material having a high refractive index ($n = 4.5$) and a thickness of 2 cm. The uncoated surfaces of the slab have a reflectance (r^2) of 0.90. If the etalon is used in the vicinity of wavelength 546 nm, determine (a) the highest-order fringe in the interference pattern, (b) the ratio T_{max}/T_{min}, and (c) the resolving power.

8-11 The separation of a certain doublet is 0.0055 nm at a wavelength of 490 nm. A variable-spaced Fabry-Perot interferometer is used to examine the doublet. At what spacing does the mth order of one component coincide with the $(m + 1)$th order of the other?

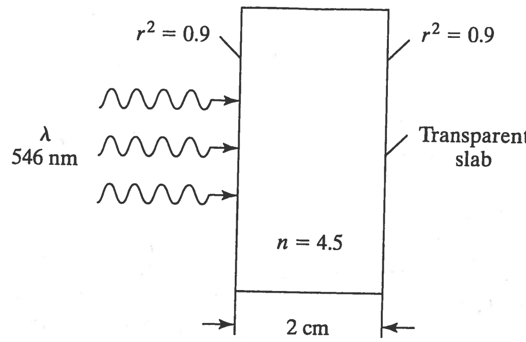


Figure 8-18 Problem 8-10.

8-12 White light is passed through a Fabry-Perot interferometer in the arrangement shown in Figure 8-19, where the detector is a spectroscope. A series of bright bands appear. When mercury light is simultaneously admitted into the spectroscope slit, 150 of the bright bands are seen to fall between the violet and green lines of mercury at 435.8 nm and 546.1 nm, respectively. What is the thickness of the etalon?

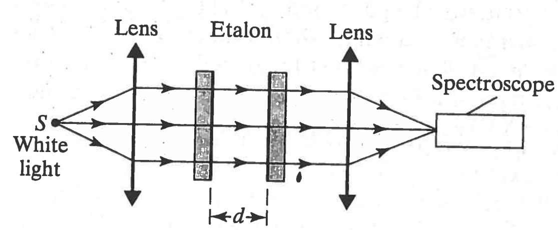


Figure 8-19 Problem 8-12.

8-13 Apply the reasoning used to calculate the finesse of a Fabry-Perot interferometer to the Michelson interferometer. Using the irradiance of Michelson fringes as a function of phase, calculate (a) the fringe separation; (b) the fringe width at half-maximum; (c) their ratio, the finesse.

8-14 Assume that in a Mach-Zehnder interferometer (Figure 8-5), the beam splitter and mirror $M3$ each transmit 80% and reflect 20% of the incident light. Compare the visibility (Eq. (7-17)) when observing the interference of the two emerging beams (shown) with the visibility that results from the two beams emerging from $M3$ along a direction at 90° relative to the first (not shown). For the second case, beam (1) is reflected and beam (2) is transmitted at $M3$.

8-15 Consider the Fabry-Perot cavity shown in Figure 8-8.

a. With the method used in Section 8-5 to derive the Fabry-Perot transmittance, find the reflectance, $R = I_R/I_I$, of a Fabry-Perot cavity. (*Note:* The reflection coefficient for the external surface of the cavity mirror must be $-r$ if that from the internal surface is r and the transmission coefficients t are real.)
b. Using the result from (a) and Eq. (8-24) (or an equivalent form), show that the sum of the irradiances reflected by and transmitted through the Fabry-Perot cavity is

equal to the irradiance in the field incident on the Fabry-Perot. That is, show that $I_R + I_T = I_I$.

8-16 The reflectance R (see Problem 8-15) of a Fabry-Perot etalon is 0.6. Determine the ratio of transmittance of the etalon at maximum to the transmittance at halfway between maxima.

8-17 Find the transmittance, $T = I_T/I_I$, and the reflectance, $R = I_R/I_I$, of a Fabry-Perot cavity with mirrors of (internal) reflection coefficients r_1 and $r_2 \neq r_1$. Take the mirror separation to be d and see the note given in part (a) of Problem 8-15.

8-18 Consider the transmittance of the variable-input-frequency Fabry-Perot cavity shown in Figure 8-15. Assume that the Fabry-Perot cavity used has a length of 10 cm and that the nominal frequency of the laser input is 4.53×10^{14} Hz. Find

a. The finesse, $\mathcal{F}$, of the cavity.
b. The free spectral range, ν_{fsr}, of the transmittance.
c. The FWHM, $2\Delta\nu_{1/2}$, of a transmittance peak.
d. The quality factor, Q, of the cavity.
e. The photon lifetime, τ_p, of the cavity.

8-19 Plot the transmittance, T, as a function of cavity length, d, for a scanning Fabry-Perot interferometer with a monochromatic input of wavelength 632.8 nm if the finesse, $\mathcal{F}$, of the cavity is 15. In the plot let d range from 5 cm to 5.000001 cm.

8-20 Find the values of all the quantities listed in the first column of Table 8-2 for a mirror reflection coefficient of 0.999.

8-21 Consider a light source consisting of two components with different wavelength λ_1 and λ_2. Let light from this source be incident on a scanning Fabry-Perot interferometer of nominal length $d = 5$ cm. Let the scaled transmittance through the Fabry-Perot as a function of the change in the cavity length be as shown in Figure 8-20a and 8-20b. Figure 8-20b shows the first set of dual peaks of Figure 8-20a over a smaller length scale in order to allow a closer examination of the structure of the overlapping peaks.

a. What is the nominal wavelength of the light source?
b. Estimate the difference $\lambda_2 - \lambda_1$ in wavelength of the two components presuming that the overlapping transmittance peaks have the same mode number, $m_2 = m_1 = m$.
c. Estimate the difference $\lambda_2 - \lambda_1$ in wavelength of the two components presuming that the overlapping transmittance peaks have mode numbers that differ by 1, so that $m_2 = m_1 + 1$.

8-22 In this problem we examine experimental absorption spectroscopy data. The output of a variable-frequency diode laser is divided at a beam splitter so that part of the laser beam is incident on a Fabry-Perot cavity of fixed length and part of the laser beam passes through a sample cell containing atmospheric oxygen, as shown in Figure 8-21a. An overlay of the scaled transmittance through the Fabry-Perot cavity (solid curve) and the scaled transmittance through the oxygen cell (curve made with + symbols) as functions of the laser frequency change is shown in Figure 8-21b. The dips in the transmittance through the oxygen cell indicate that the oxygen molecule strongly absorbs these frequencies. The free spectral range of the Fabry-Perot cavity used in the experiment was known to be 11.6 GHz. The free spectral range can be taken to be the distance between the tall transmittance peaks, indicated by the arrows in Figure 8-21b. (A spherical-mirror Fabry-Perot cavity was used in the experiment and so the transmittance includes peaks corresponding to both longitudinal and transverse modes.)

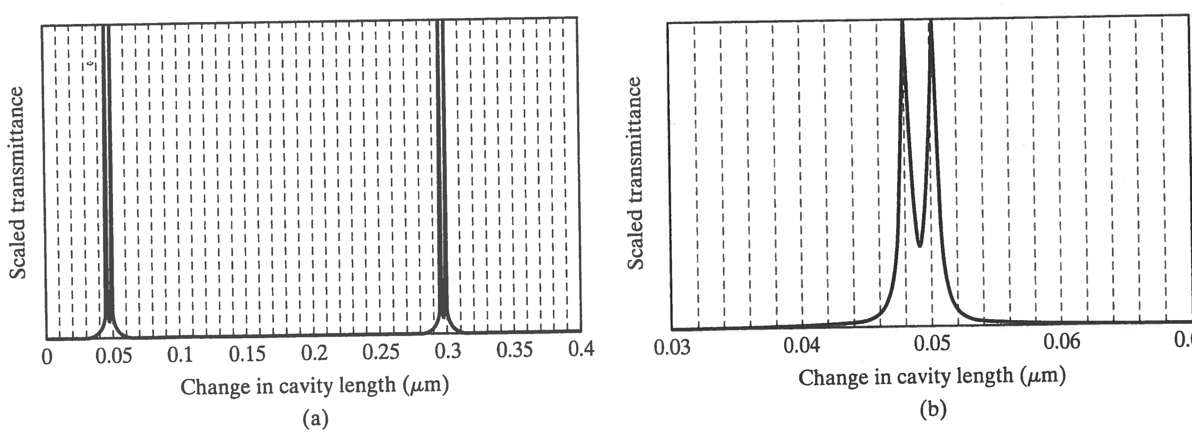
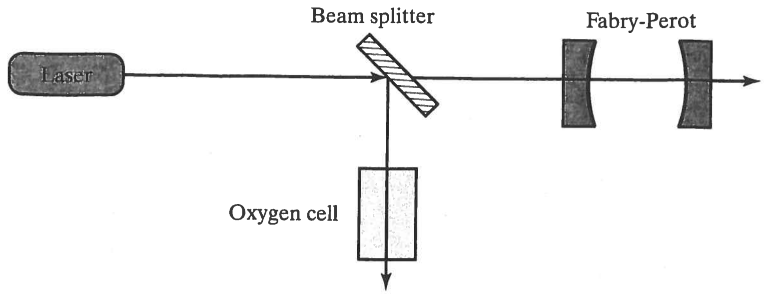
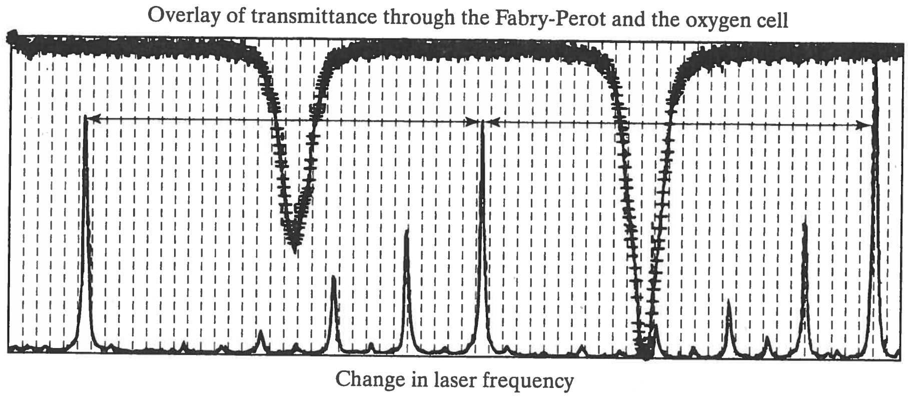


Figure 8-20 Problem 8-21.



(a)



(b)

Figure 8-21 Problem 8-22. (a) Experimental arrangement. (b) Overlay of transmittance curves. (Courtesy of R. J. Brecha, Physics Department, University of Dayton.)

a. Estimate the difference in the frequencies of the two absorption dips shown in Figure 8-21b.

b. Estimate the "full-width-at-half-depth" of each absorption dip.

8-23 Consider the transmittance through a Fabry-Perot interferometer as a function of the variable wavelength λ of its input field. Show that the FWHM of the transmittance peaks is $2\Delta\lambda_{1/2} = \lambda/m\mathcal{F}$ and the separation between transmittance peaks is $\lambda_{fsr} = \lambda/m$. (Here $m = 2d/\lambda$, where d is the length of the Fabry-Perot interferometer.)