Chapter 2 Geometrical Optics

Introduction

- Geometrical Optics treats light as a collection of rays that travel in straight lines and bend when they pass through or reflect from surfaces.
  - It deals with situations where *Wavelength is negligible compared with the dimension of the relevant component of optical systems*

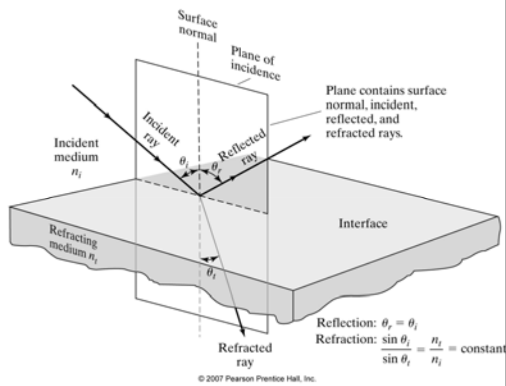
- Physical optics is a more comprehensive model of light, which includes wave effects such as diffraction and interference that cannot be accounted for in geometric optics.
  - It deals with situations where *the non-zero wavelength of light must be reckoned with.*
  - Analogously, when the de Broglie wavelength of a material object is negligible we have Classical Mechanics, when it is not, we have the domain of Quantum Mechanics.

2.1 Huygens' Principle

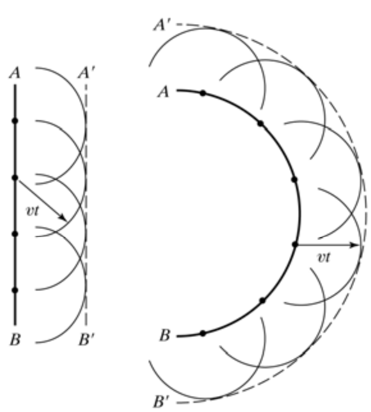
Law of reflection: $\boxed{\theta_i = \theta_r}$

Law of refraction (Snell's Law):

$$\boxed{n_i \cdot \sin\theta_i = n_t \cdot \sin\theta_t}$$



2.1 Huygens' Principle



(a) **Plane waves** © 2007 Pearson Prentice Hall, Inc. (b) **Spherical waves**

- Wavefront : The surfaces joining all points of equal phase are known as wavefronts.
- Each wavefront is surface of constant phase
- Huygens's Principle : every point on a propagating wavefront serves as the source of spherical secondary wavelets, such that the wavefront at some later time is the envelope of these wavelets.

2.1 Huygens' Principle

Reflection

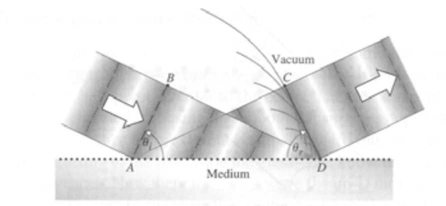


Figure 4.16 Plane waves enter from the left and are reflected off to the right. The reflected wavefront $\overline{CD}$ is formed of waves scattered by the atoms on the surface from *A* to *D*. Just as the first wavelet arrives at *C* from *A*, the atom at *D* emits, and the wavefront along $\overline{CD}$ is completed.

- Optical Path Length (*OPL*) = *n***L*
- For wavefronts AB and CD, the travel time of light from A → C is equal to that from B → D

$$\boxed{t = \frac{d}{\upsilon} = \frac{d}{c/n} = n \cdot d / c = OPL / c}$$

- *OPL* of AC must be equal to *OPL* of BD

$$\boxed{OPL(\overline{AC}) = OPL(\overline{BD})} \longrightarrow \boxed{n \cdot \overline{AD} \sin\theta_i = n \cdot \overline{AD} \sin\theta_r}$$

$\longrightarrow$ Law of reflection: $\boxed{\theta_i = \theta_r}$

2.1 Huygens' Principle

Refraction

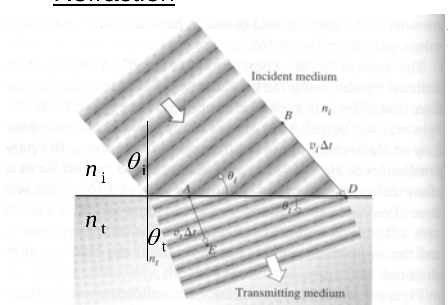


Figure 4.19 The refraction of waves. The atoms in the region of the surface of the transmitting medium reradiate wavelets that combine constructively to form a refracted beam.

- Wavefronts "bend" as they cross the boundary because of the speed change
- For wavefronts AB and DE, *OPL* of AE must be equal to *OPL* of BD

$$\boxed{OPL(\overline{BD}) = OPL(\overline{AE})}$$

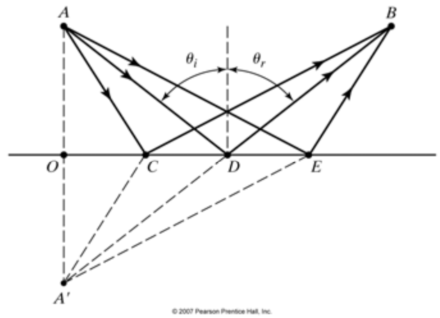
$$\boxed{n_i \cdot \overline{AD} \sin\theta_i = n_t \cdot \overline{AD} \sin\theta_t}$$

Law of refraction (Snell's Law): $\boxed{n_i \cdot \sin\theta_i = n_t \cdot \sin\theta_t}$

2.2 Fermat's Principle

- Principle of Least Time: The actual path between two points taken by a beam of light is the one that is traversed in the least time
- Re-state Fermat's Principle : $\boxed{t = OPL / c}$
  Light, in going from point S to P, traverses the route having the smallest *OPL*
- Fermats' Principle encompass both Reflection and Refraction

Applied to Reflection ⇨



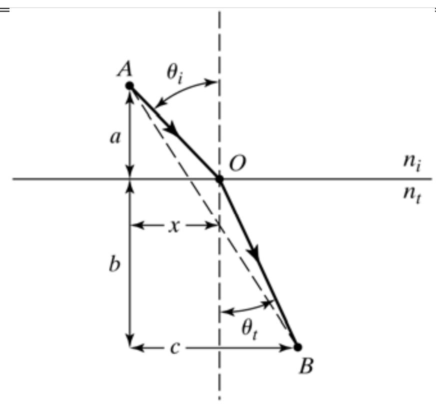
We can derive: $\boxed{\theta_i = \theta_r}$

2.2 Fermat's Principle

Applied to Refraction:

We can derive:

$$\boxed{n_t \cdot \sin\theta_t = n_i \cdot \sin\theta_i}$$



2.3 Principle of Reversibility

- Any actual ray of light in an optical system, if reversed in direction, will retrace the same path backward.

2.4 Reflection in Plane Mirrors

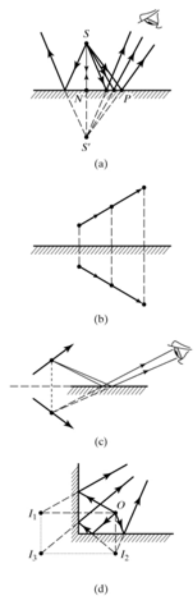


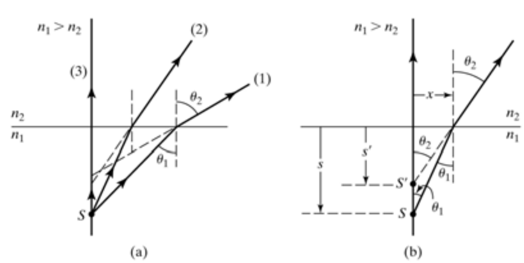
Fig. 2.8 Image formation in a plane mirror

Virtual image

Magnification: unity

Multiple images

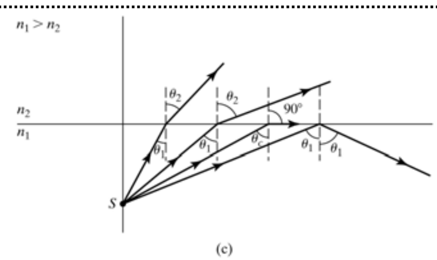
2.5 Refraction through Plane Surfaces



- Paraxial rays: $\boxed{\sin\theta \cong \tan\theta \cong \theta}$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$$\Rightarrow \boxed{s' = \left(\frac{n_2}{n_1}\right) s}$$



- Total Internal Reflection:

$$\boxed{n_1 > n_2}$$

Critical angle: $\boxed{\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$

is the incident angle when refraction angle is equal to 90°

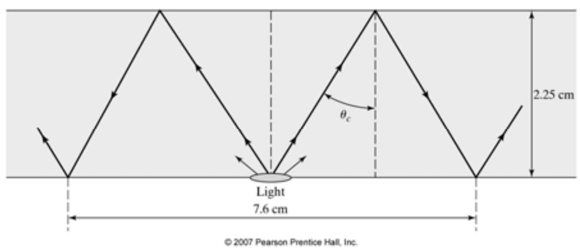
2.5 Refraction through Plane Surfaces

Example

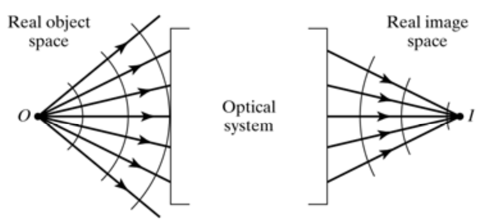
Prove that to some looking straight down into a swimming pool, any object in the water will appear to be ¾ of its true depth.

Problems 2-7

A small source of light at the bottom face of a rectangular glass slab 2.25 cm thick is viewed from above. Rays of light totally internally reflected at the top surface outline a circle of 7.6 cm in diameter on the bottom surface. Determine the refractive index of the glass



2.6 Imaging by an optical system



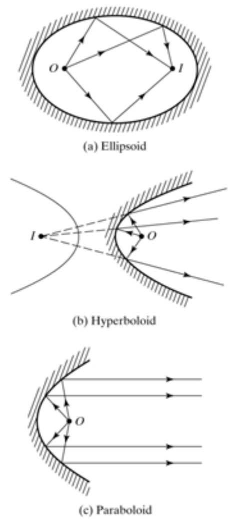
- O and I are conjugate points – any pair of object-image points …. by the principle of reversibility can be interchanged

- Nonideal images due to: Light Scattering, aberrations, and diffractions

2.6 Imaging by an optical system

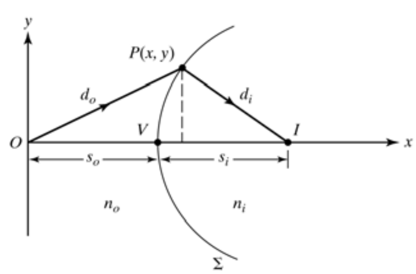
- Cartesian surface: Reflecting or refracting surfaces that form perfect images are called Cartesian surface

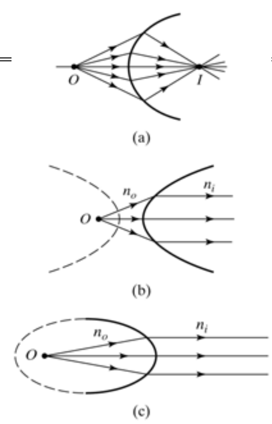
- Cartesian reflecting surfaces (E.g. ellipsoid and hyperboloid) show conjugate object and image points



2.6 Imaging by an optical system

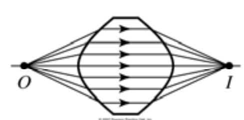
- Cartesian refracting surface:





$$\boxed{n_o d_o + n_i d_i = n_o s_o + n_i s_i = \text{ contant}}$$

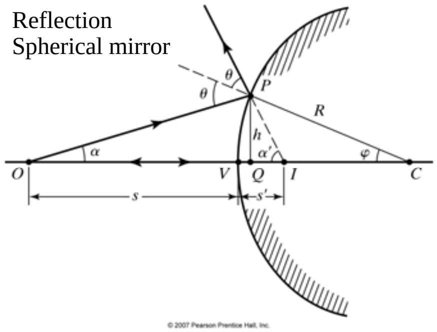
$$\boxed{n_o\sqrt{x^2 + y^2} + n_i\sqrt{y^2 + \left(s_o + s_i - x\right)^2} = \text{ contant}}$$



Aberration-free imaging of point object O by a double hyperbolic lens

2.7 Reflection at a spherical surface

Reflection
Spherical mirror



- Vertex (V)
- Optical axis

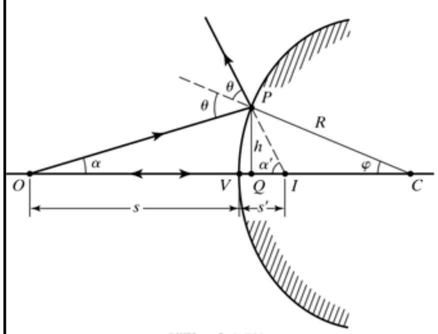
Length related:

- Radius of spherical surface: R
- Object distance: s
- Image distance: s'
- Focal length of the mirror: f

➢ Paraxial rays approximation (first-order optics):

α, α', and φ are very small ⇨ $\boxed{\sin\varphi \cong \tan\varphi \cong \varphi, \quad \cos\varphi \cong 1}$

2.7 Reflection at a spherical surface



➢ Angle relations: $\boxed{\alpha - \alpha' = -2\varphi}$

➢ Mirror equation: $\boxed{\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}}$

with $\boxed{f = -\dfrac{R}{2}}$

Convex mirror: $f < 0$
Concave mirror: $f > 0$

Sign convention		
s	+	O is to the left of V
s'	+	I is to the left of V
R	+	C is to the right of V
f	+	F is to the left of V

2.7 Reflection at a spherical surface

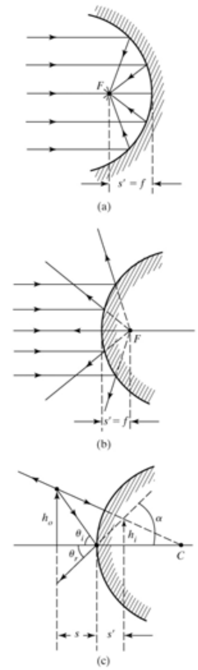
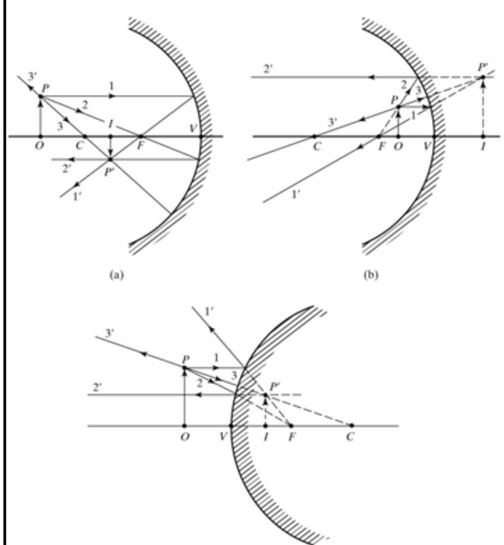


Fig. 2-16(a), (b): Location of focal points
Fig. 2-16(c): Construction to determine magnification

➢ Lateral magnification: $\boxed{m = \frac{h_i}{h_o} = -\frac{s'}{s}}$

Meanings associated with the sign		
Quantity	Sign	
	+	-
s	real object	virtual object
s'	real image	virtual image
f	concave mirror	convex mirror
m	erect image	inverted image

2.7 Reflection at a spherical surface



(a) (b) (c)

Rules for ray tracing

- Ray 1 parallel to central axis will strike the mirror, reflect and pass through the focal point as labeled Ray 1'
- Ray 2 passing through the focal point will emerge from the mirror parallel to central axis as labeled Ray 2'
- Ray 3 passing through the center of the mirror reflects along itself as labeled Ray 3'

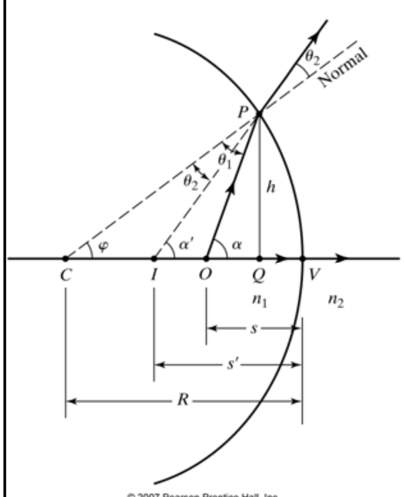
2.7 Reflection at a spherical surface

- How do we describe an image?
  - Type: Virtual or real
  - Orientation: erect or inverted
  - Relative size

Example 2-1

An object 3 cm high is placed 20 cm from (a) a convex and (b) a concave spherical mirror, each of 10-cm focal length. Determine the position and nature of the image in each case.

2.8 Refraction at a spherical surface



- Refraction equation: $\boxed{\dfrac{n_1}{s} + \dfrac{n_2}{s'} = \dfrac{n_2 - n_1}{R}}$

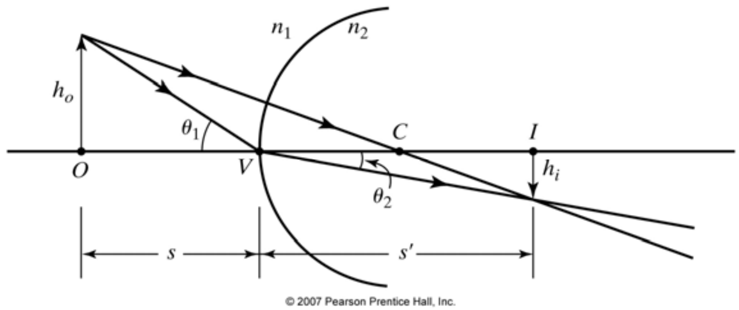
- Special case: plane refracting surface

$\boxed{R \to \infty} \Rightarrow \boxed{s' = -\left(\dfrac{n_2}{n_1}\right)s}$

s': apparent depth

Sign convention		
s	+	O is to the left of V
s'	+	I is to the right of V
R	+	C is to the right of V

2.8 Refraction at a spherical surface



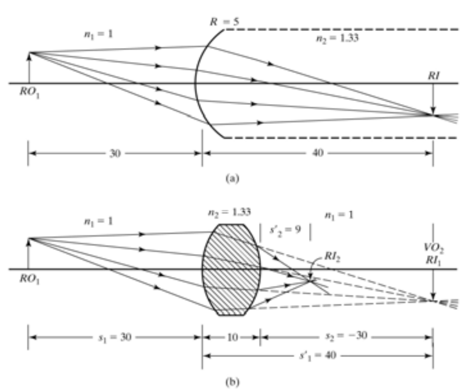
- Lateral magnification:

$$\boxed{m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}}$$

Meanings associated with the sign		
Quantity	Sign	
	+	-
s	real object	virtual object
s'	real image	virtual image
m	erect image	inverted image

Example 2-2

As an extended example of refraction by spherical surfaces, refer to Fig. 2-20.
In (a), a real object is positioned in air, 30 cm from a convex spherical surface of radius 5 cm. To the right of the interface, the refractive index is that of water. Find the image distance and lateral magnification of the image.
In (b), suppose the second medium is only 10 cm thick, forming a thick lens, with a second concave spherical surface, also of radius of 5cm. Find the final image distance, magnification, and describe the final image.



$$\boxed{s_2 = t - s_1'}$$

2.9 Thin Lenses

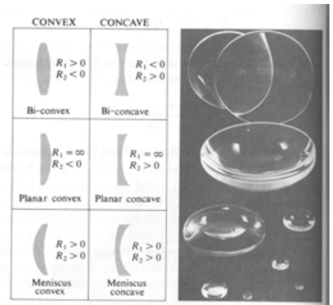
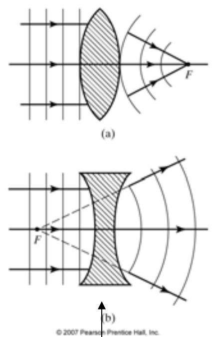


Figure 5.12 Cross sections of various centered spherical simple lenses. The surface on the left is #1 since it is encountered first. Its radius is R_1. (Photo courtesy of Melles Griot.)

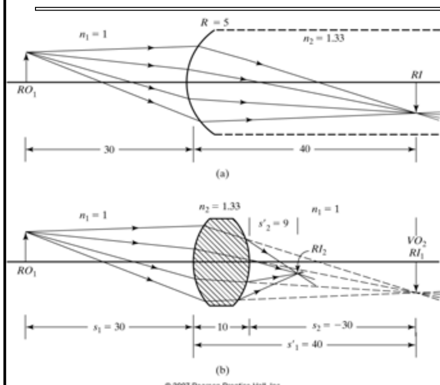


Converging lens (positive lens)
$f > 0$

Diverging lens (negative lens)
$f < 0$

thickness $\cong 0$

2.9 Thin Lenses



$\boxed{s_2 = t - s_1'}$ Thin lens (t $\cong$ 0): $\boxed{s_2 = -s_1'}$

- Thin-lens equation: $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$

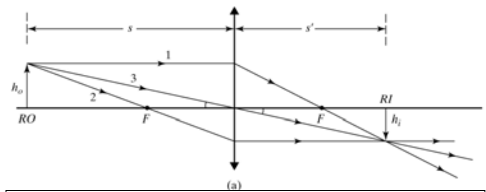
Or using $\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$

- Focal length f :
  image distance for an object at infinity or *object distance for an image at infinity*

- Lensmaker's equation: $\boxed{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

2.9 Thin Lenses

- Lateral magnification:



$$\boxed{m = \frac{h_i}{h_o} = -\frac{s'}{s}}$$

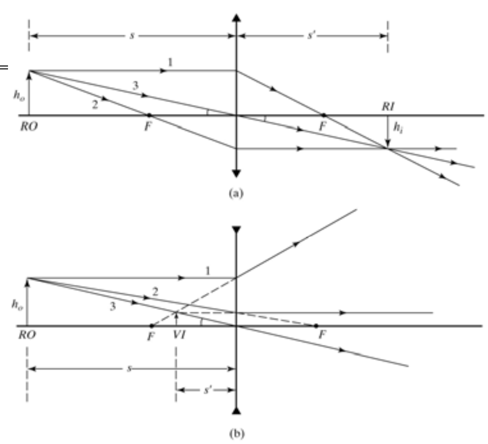
$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

$$\boxed{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Meanings associated with the sign		
Quantity	Sign	
	+	-
s	real object	virtual object
s'	real image	virtual image
m	erect image	inverted image

Sign convention		
s	+	O is to the left of V
s'	+	I is to the <u>right</u> of V
$R_{1,2}$	+	C is to the right of V

2.9 Thin Lenses



<u>Rules for ray tracing</u>

- Ray 1 parallel to central axis will pass through the <u>right</u> focal point of a <u>converging</u> lens (or the <u>left</u> focal point of the <u>diverging</u> lens)
- Ray 2 passing through the <u>left</u> focal point of a <u>converging</u> lens (or the <u>right</u> focal point of the <u>diverging</u> lens) will emerge from lens parallel to central axis
- Ray 3 is the undeviated ray through center of lens (Ray 3 is also called central ray)

2.9 Thin Lenses

TABLE 2-1 SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

	Spherical surface	Plane surface
Reflection	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, f = -\frac{R}{2}$	$s' = -s$
	$m = -\frac{s'}{s}$	$m = +1$
	Concave: $f > 0, R < 0$	
	Convex : $f < 0, R > 0$	
Refraction Single surface	$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$	$s' = -\frac{n_2}{n_1}s$
	$m = -\frac{n_1 s'}{n_2 s}$	$m = +1$
	Concave: $R < 0$	
	Convex : $R > 0$	
Refraction Thin lens	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$	
	$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	
	$m = -\frac{s'}{s}$	
	Concave: $f < 0$	
	Convex : $f > 0$	

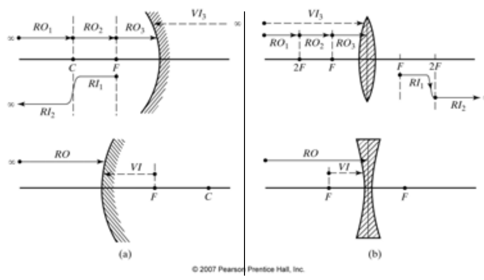
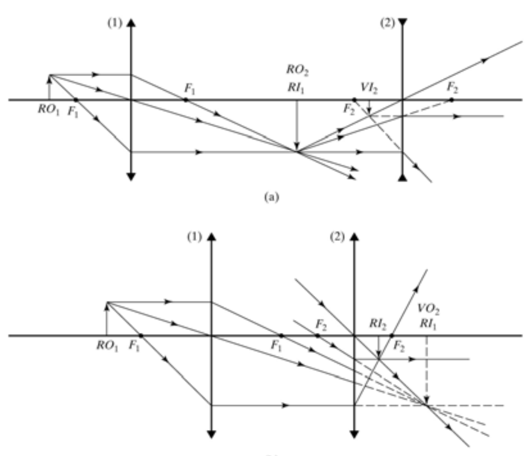


Fig.2-24 Summary of image formation by (a) spherical mirror and (b) thin lens

2.9 Thin Lenses

Example 2-3

Find and describe the intermediate and final images produced by a two-lens system such as the one sketched in Fig. 2-23a. Let f_1 = 15 cm, f_2 = 15 cm, and their separation be 60 cm. Let the object be 25 cm from the first lens, as shown.



2.9 Thin Lenses

Problems 2-18

One side of a fish tank is built using a large-aperture thin lens make of glass (n = 1.5). The lens is equiconvex, with radii of curvature 30 cm. A small fish in the tank is 20 cm from the lens. Where does the fish appear when viewed through the lens? What is its magnification?

2.10 Vergence and Refractive Power

$$\boxed{\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\boxed{V+V'=P}$$

P : refractive power

Unit: Diopter (D), 1/m

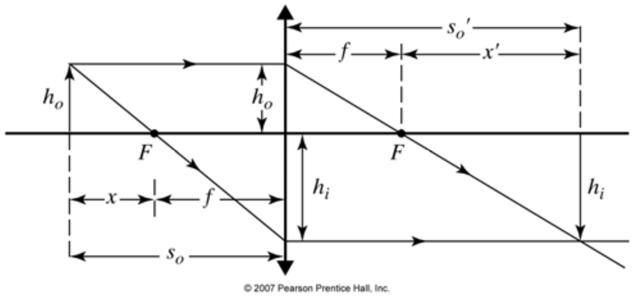
Lenses in contact:

- Effective focal length of thin-lens combinations with N lenses in contact:

$$\boxed{\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\cdots+\frac{1}{f_N}}$$

- Refractive powers: $\boxed{P=P_1+P_2+\cdots+P_N}$

2.11 Newtonian Equation for the Thin Lens

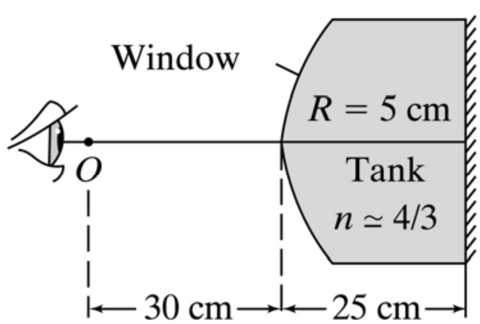


- Newtonian form of lens equation: $\boxed{xx' = f^2}$
- Lateral magnification: $\boxed{m = \frac{h_i}{h_o} = -\frac{s'}{s} = -\frac{x'}{f} = -\frac{f}{x}}$

Problems

Problems 2-15

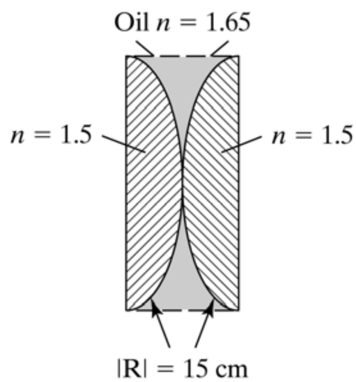
A small object faces the convex spherical glass window of a small water tank. The radius of curvature of the window is 5 cm. The inner back side of the tank is a plane mirror, 25 cm from the window. If the object is 30 cm outside the window, determine the nature of its final image, neglecting any refraction due to the thin glass window itself.



Problems

Problems 2-20

Two identical, thin, plano-convex lenses with radii of curvature of 15 cm are situated with their curved surfaces in contact at their centers. The intervening space is filled with oil of refractive index 1.65. The index of the glass is 1.5. Determine the focal length of the combination. (Hint: think of the oil layer as an intermediate thin lens.)



Problems

Problems 2-22

A diverging thin lens and a concave mirror have focal lengths of equal magnitude. An object is placed (3/2)*f* from the diverging lens, and the mirror is placed a distance 3*f* on the other side of the lens. Using Gaussian optics, determine the final image of the system, after two refractions (a) by a three-ray diagram and (b) by calculation.