Chapter 7 Interference of Light

7.1 Two-Beam Interference

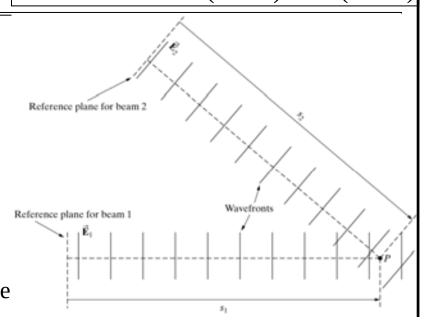
$$\boxed{2\cos A\cos B = \cos(A+B)+\cos(A-B)}$$

- Superposition of the two waves at P:

$$\boxed{\vec{E}_P = \vec{E}_1 + \vec{E}_2}$$

with

$$\boxed{\begin{aligned}\vec{E}_1 &= \vec{E}_{o1}\cos\left(ks_1 - \omega t + \phi_1\right)\\ \vec{E}_2 &= \vec{E}_{o2}\cos\left(ks_2 - \omega t + \phi_2\right)\end{aligned}}$$



- Another symbol I, for Irradiance, E_e (W/m^2):
  Time average of the square of the wave amplitude

$$\boxed{I = \varepsilon_0 c\left\langle \vec{E}\cdot\vec{E}\right\rangle} \qquad \boxed{I = \varepsilon_0 c\left\langle \vec{E}_P\cdot\vec{E}_P\right\rangle = \varepsilon_0 c\left\langle \vec{E}_1\cdot\vec{E}_1 + \vec{E}_2\cdot\vec{E}_2 + 2\vec{E}_1\cdot\vec{E}_2\right\rangle}$$

1st and 2nd term:

$$\boxed{I_1 = \varepsilon_0 c\left\langle \vec{E}_1\cdot\vec{E}_1\right\rangle = \frac{1}{2}\varepsilon_0 c E_{01}^2,\ I_2 = \varepsilon_0 c\left\langle \vec{E}_2\cdot\vec{E}_2\right\rangle = \frac{1}{2}\varepsilon_0 c E_{02}^2}$$

3rd term:

$$\boxed{2\varepsilon_0 c\left\langle \vec{E}_1\cdot\vec{E}_2\right\rangle = ?}$$

- Phase difference between E_1 and E_2: $\boxed{\delta = k(s_2 - s_1) + (\phi_2 - \phi_1)}$

3rd term: $$\boxed{2\varepsilon_0 c\left\langle \vec{E}_1\cdot\vec{E}_2\right\rangle = \varepsilon_0 c E_{01}E_{02}\left\langle\cos\delta\right\rangle = 2\sqrt{I_1 I_2}\left\langle\cos\delta\right\rangle}$$

Assumption: E fields are parallel

7.1 Two-Beam Interference

The irradiance of the combined fields: $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle\cos\delta\rangle}$

Interference term

Sources of the waves:

- If light beams from *independent sources* (i.e, the sources are *mutually incoherent*), then there is no interference term

$$\boxed{2\sqrt{I_1 I_2}\left\langle\cos\left(k(s_2 - s_1) + \phi_2(t) - \phi_1(t)\right)\right\rangle = 0} \Rightarrow \boxed{I = I_1 + I_2}$$

- If the sources are *mutually coherent* (e.g., light from the same laser source (*monochromatic*) is split and recombined at a detector), then

$$\boxed{2\sqrt{I_1 I_2}\left\langle\cos\left(k(s_2 - s_1) + \phi_2(t) - \phi_1(t)\right)\right\rangle = 2\sqrt{I_1 I_2}\cos k(s_2 - s_1) = 2\sqrt{I_1 I_2}\cos\delta}$$

$$\Rightarrow \boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

7.1 Two-Beam Interference

- To tell how monochromatic a source is, coherent time τ_0 or coherent length l_0 of a light source is used
- Coherent length l_0: the propagation distance over which a coherent wave (e.g. an electromagnetic wave) maintains a specified degree of coherence
- Coherent time τ_0: the time interval within which the phase is on average predictable
- Coherent time τ_0 and Coherent length l_0 of a light source :

$$\boxed{\tau_0 = \frac{1}{\Delta\nu}} \qquad \boxed{l_0 = \frac{\lambda^2}{\Delta\lambda}}$$

$\Delta\lambda$: line width of light source
λ: center wavelength light source
$\Delta\nu$: frequency range of light source

- For example, white light has a line width of 300 nm, taking the average wavelength λ at 550 nm, from about equation, we can find coherence length l_0 is about 1000 nm (corresponding 2λ)

7.1 Two-Beam Interference

Q: Two waves arrive at *P* with different time. What about the interference term?

Phase difference due to: $\boxed{\phi_2(t)-\phi_1(t+\delta t)}$

Two-beam Interference:

- For phase difference $\boxed{\phi_2(t)-\phi_1(t+\delta t)}$, if $\boxed{\delta t < \tau_0}$, then $\boxed{\phi_2(t)-\phi_1(t+\delta t) \cong 0}$

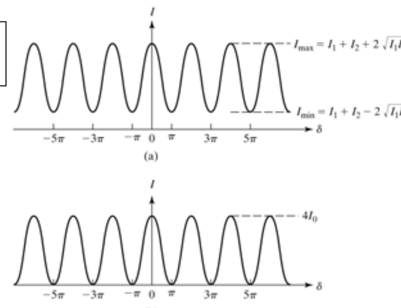
$\Longrightarrow$ $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$ with $\boxed{\delta = k(s_2 - s_1)}$

1. Constructive interference: $\boxed{I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}}$ $\boxed{\delta = 2m\pi}$
2. Destructive interference: $\boxed{I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}}$ $\boxed{\delta = (2m+1)\pi}$

- Special case: $\boxed{I_1 = I_2 = I_0}$ $\Longrightarrow$ $\boxed{I = 4I_o \cos^2\frac{\delta}{2}}$

- Visibility, a measure of fringe contrast is defined as $\boxed{\text{visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}}$

Range of visibility?



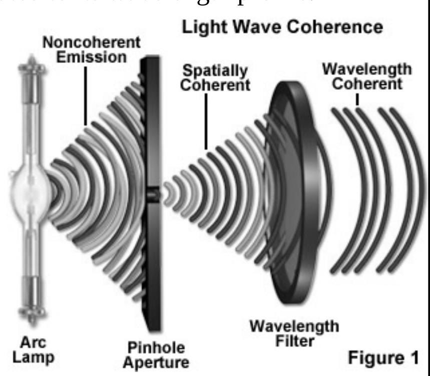
Illumination Sources*

- Most light sources, in fact, exhibit both spatial coherence related to the angular size of the source and temporal coherence related to its wavelength profile.

-- *Arc lamp*
-- *Lasers*

- Arc Lamps: is a class of lamps that produce light by an electric arc

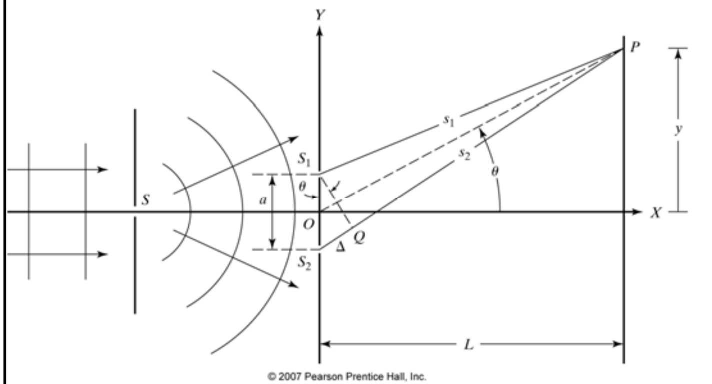
- Laser light is in general said to be monochromatic, directional, and coherent.

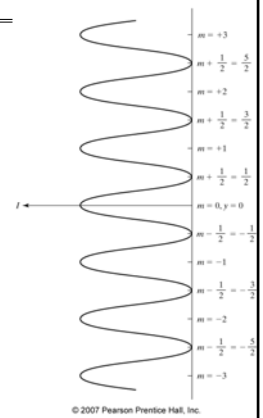


- However, in all practical cases, the laser light is not truly monochromatic. A truly monochromatic wave requires a wave train of infinite duration.

- Ordinary light is not coherent because it comes from independent atoms, which emit on time scales of about 10^{-8} seconds.

7.2 Young’s Double-Slit Experiment

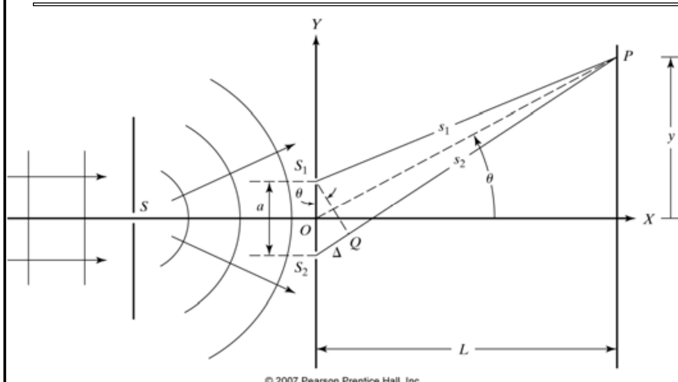




- Dr. Thomas Young succeeded in producing a system of alternating bright and dark bands – Interference fringes

Young's Double Slit Interference: http://vsg.quasihome.com/interfer.htm

7.2 Young’s Double-Slit Experiment



Optical path difference:

$$\boxed{\Delta = s_2 - s_1 = a\sin\theta}$$

Phase difference:

$$\boxed{\delta = k(s_2 - s_1) = \frac{2\pi}{\lambda}\Delta}$$

Irradiance at screen:

$$\boxed{\begin{aligned} I &= 4I_o\cos^2\frac{\delta}{2} \\ &= 4I_o\cos^2\left(\frac{\pi a\sin\theta}{\lambda}\right)\end{aligned}}$$

- Bright fringe and dark fringe conditions:

$$\boxed{y_m = m\frac{\lambda L}{a} \quad m = 0,\ \pm 1,\ \pm 2\cdots} \qquad \boxed{y_m = (m+\frac{1}{2})\frac{\lambda L}{a} \quad m = 0,\ \pm 1,\ \pm 2\cdots}$$

- Fringe separation: $\boxed{\Delta y = y_{m+1} - y_m = \frac{\lambda L}{a}}$

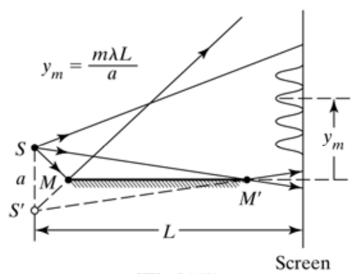
7.2 Young's Double-Slit Experiment

Example 7-2

Laser light passes through two identical and parallel slits, 0.2 mm apart. Interference fringes are seen on a screen 1 m away. Interference maxima are separated by 3.29 mm. What is the wavelength of the light? How does the irradiance at the screen vary, if the contribution of one slit alone is I_0?

7.3 Double-Slit Interference with Virtual Sources

- Lloyd's Mirrors



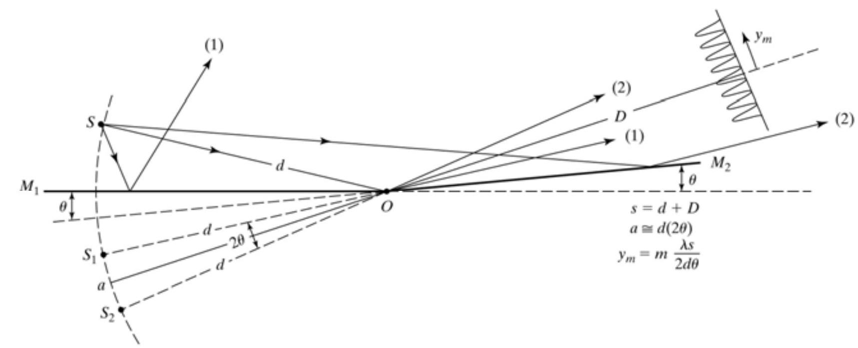
- The distinguishing feature of this device is that at glancing incidence ($\theta_i = \pi / 2$), the reflected beam undergoes a π phase shift (i.e., OPD = $\lambda / 2$).
- Therefore, center of the screen will be the dark fringe instead of bright in Young's double slits experiment.

Min: $\boxed{s_2 - s_1 + \lambda/2 = a\frac{y_m}{L} + \lambda/2 = (m+1/2)\lambda} \longrightarrow \boxed{m=0, \quad y_0 = 0}$

Max: $\boxed{s_2 - s_1 + \lambda/2 = a\frac{y_m}{L} + \lambda/2 = m\lambda}$ Note: mirror is on top in this case, using $+\lambda/2$.

7.3 Double-Slit Interference with Virtual Sources

- Fresnel's Mirrors



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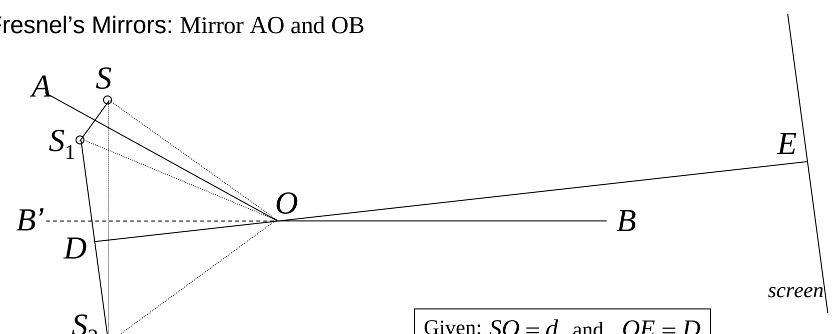
7.3 Double-Slit Interference with Virtual Sources

- Fresnel's Mirrors: Mirror AO and OB



$\angle AOB' = \theta$

$\angle SOA = \angle S_1OA = \alpha$

$\angle S_1OB' = \theta - \alpha$

$\angle B'OS_2 = \angle B'OS = \theta + \alpha$

$\angle S_1OS_2 = 2\theta$

$\angle S_1OD = \angle S_2OD = \theta$

Given: $SO = d$ and $OE = D$

Slit separation S_1S_2:

$a = S_1S_2 = 2d\sin\theta$

$L = DO + OE = d\cos\theta + D$

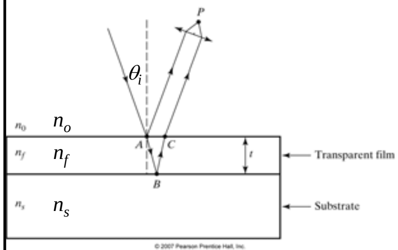
When θ is very small, we have:

$a = 2d\theta$ and $L = d + D$

$$y_m = m\frac{\lambda L}{2d\theta}$$

7.4 Interference in Dielectric Films

- Interference: Wavefront-splitting interference (such as Young's interference)
  Amplitude-splitting interference (such as interference in thin-film)



Two-beam interference:

- Parameters: $\boxed{n_o,\ n_f,\ n_s,\ t}$
- Considering <u>normal incidence</u> $\theta_i = 0$, the *OPD*:

$$\boxed{\Delta = n_f(AB + BC) = 2n_f t}$$

Note: Need to consider additional <u>*phase shift*</u> due to <u>*reflections*</u>

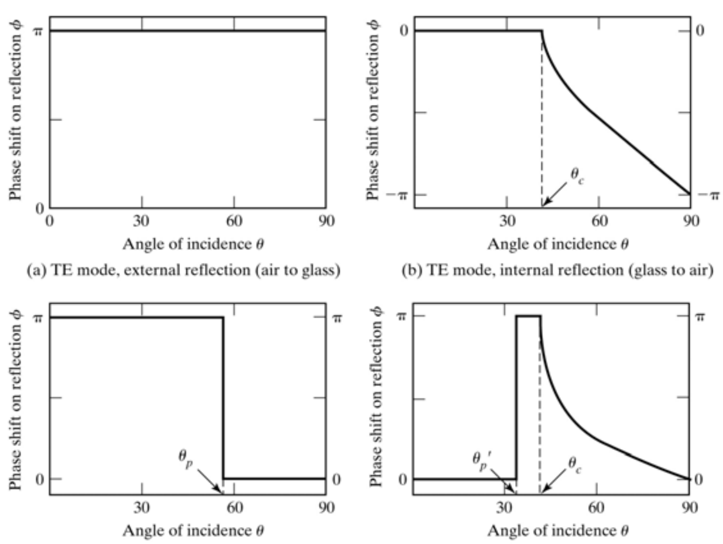
- External reflection: light going from a <u>low</u> index to a <u>high</u> index (such as, from air to glass)
- Internal reflection: light going from a <u>high</u> index to a <u>low</u> index (such as, from glass to air)
- Phase shift due to reflection:

Case 1	two external reflections $\boxed{n_o < n_f < n_s}$ or two internal reflections $\boxed{n_o > n_f > n_s}$	No additional phase shift
Case 2	one internal and one external reflection $\boxed{n_f > n_o \text{ and } n_f > n_s}$ or $\boxed{n_f < n_o \text{ and } n_f < n_s}$	Relative phase shift of π

7.4 Interference in Dielectric Films

See Chapter 23, p500

- Explanation on Phase shift due to reflection:



(a) TE mode, external reflection (air to glass)
(b) TE mode, internal reflection (glass to air)
(c) TM mode, external reflection (air-to-glass)
(d) TM mode, internal reflection (glass-to-air)

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7.4 Interference in Dielectric Films

- Constructive interference: $\boxed{\Delta_p + \Delta_r = m\lambda}$
- Destructive interference: $\boxed{\Delta_p + \Delta_r = (m + 1/2)\lambda}$

$\boxed{\Delta_p = 2n_f t}$

Δ_p: Optical path difference (OPD)
Δ_r: Equivalent OPD arising from phase shifts on reflection
(Phase shift π corresponding OPD of ½ wavelength)

- **<u>Application</u>** of single-layer films: <u>Anti-reflecting coating</u>

1. If $\boxed{t = \frac{\lambda_f}{4} = \frac{\lambda_o}{4n_f}}$ ⇨ $\boxed{\Delta_p = 2n_f t = \frac{\lambda_o}{2}}$ ⇨ Destructive interference

   Quarter wavelength — Considering $\Delta_r = 0$

2. Visibility (or contrast or extinction) reaches max if two waves have the same amplitude

   It can be found that when $\boxed{n_f = \sqrt{n_o n_s}}$, there is a max.transmission and a <u>min. reflection</u> (so it can be used for anti-reflection coating)

2. In case of normal incidence, reflection coefficient: $\boxed{r = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - n_2/n_1}{1 + n_2/n_1}}$

   $\boxed{r = \frac{E_r}{E_i}}$

7.4 Interference in Dielectric Films

- Interference: Wavefront-splitting interference (such as Young's interference)
  Amplitude-splitting interference (such as interference in thin-film)

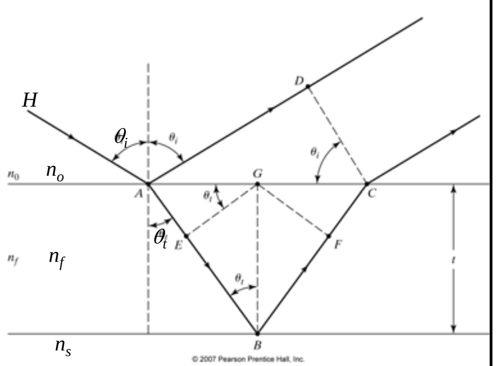
<u>Two-beam Interference in thin-film:</u>

- Two reflections are :
  H → A → D
  H → A → B → C

- Parameters: $\boxed{n_o,\ n_f,\ n_s,\ t,\ \theta_t}$

- OPD of the two beams can be obtained:

$$\boxed{\Lambda = 2n_f t \cos\theta_t}$$



- However, we have to consider if there is an additional phase shift arising from the two reflections themselves.

7.4 Interference in Dielectric Films

- Conditions for bright and dark fringes:

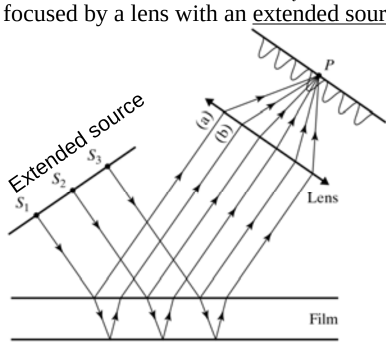
Bright: $\Delta_p + \Delta_r = m\lambda$

Dark: $\Delta_p + \Delta_r = (m + 1/2)\lambda$

with $\Delta_p = 2n_f t \cos\theta_t$

- There are two types of fringes:
  - Fringes of Equal Inclination (θ_t); referred to as Haidinger Fringes
  - Fringes of Equal Thickness (t); referred to as Fizeau Fringes

Fringes of Equal Inclination θ_t
focused by a lens with an extended source



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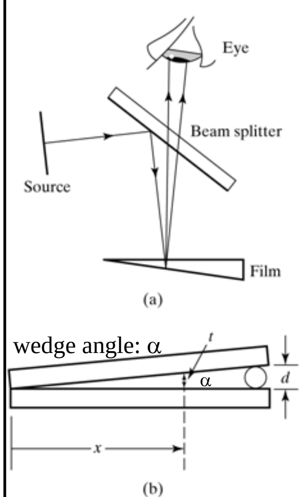
- Different fringes are produced from different inclination angle θ_t
- The same-order fringe is from the same (equal) inclination angle

7.5 Fringes of Equal Thickness

- Fringes of Equal Thickness t, referred to as Fizeau Fringes



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- Conditions for bright and dark fringe:

Bright: $\Delta_p + \Delta_r = m\lambda$

Dark: $\Delta_p + \Delta_r = (m + 1/2)\lambda$

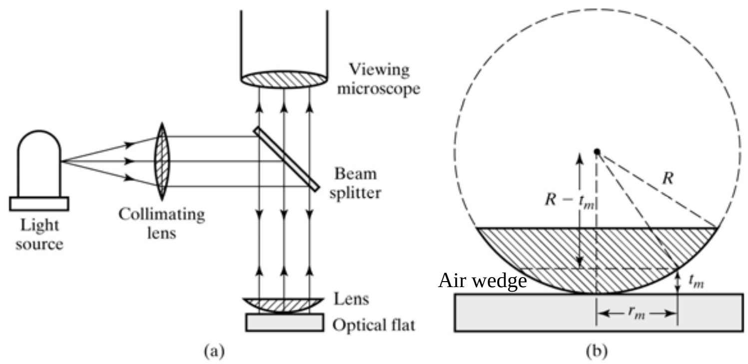
with $\Delta_p = 2n_f t = 2n_f x\alpha$
(Normal incidence)

- Need to consider if there is an additional phase shift arising from the two reflections themselves.
- Separation Δx of consecutive fringes: $\Delta x = \lambda_f / 2\alpha$
- Different fringes are produced from different thickness t
- The same-order fringe is from the same (equal) thickness t

7.6 Newton's Rings

Fringes of Equal Thickness: Newton's Rings

- Newton's Rings are formed between a spherical lens surface and an optical flat.
- Since Fizeau fringes are fringes of equal thickness, their contours directly reveal any non-uniformities in the thickness of the film. So this property can be used to determine the quality of the spherical surface of a lens.



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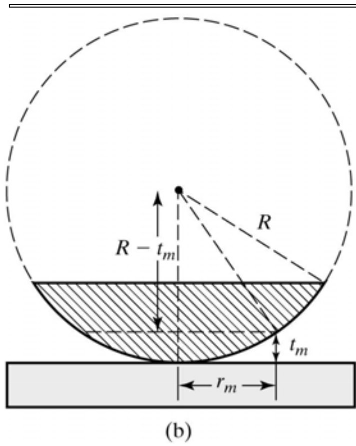
7.6 Newton's Rings



- Parameters: R, t_m, r_m
- Find R: $\boxed{R^2 = r_m^2 + (R - t_m)^2}$

$$\Rightarrow \boxed{R = \frac{r_m^2 + t_m^2}{2t_m}}$$

- If ($n_f < n_2$ and $n_f < n_1$) or ($n_f > n_2$ and $n_f > n_1$), then the center of the fringe appears dark

Maxima: $\boxed{2n_f t_m = (m - 1/2)\lambda_0}$

Radius of m^{th} bright ring : $\boxed{r_m = \sqrt{(m - 1/2)\lambda_f R}}$

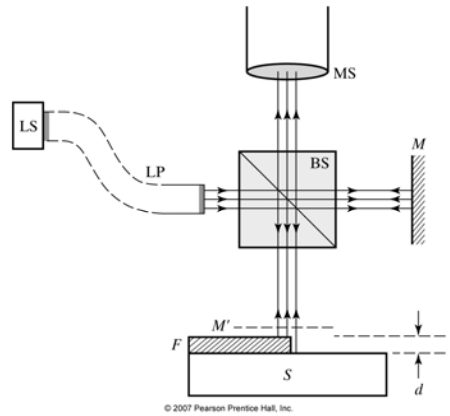
Minima:

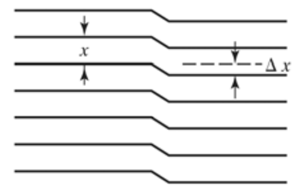
Radius of m^{th} dark ring : $\boxed{r_m = \sqrt{m\lambda_f R}}$

Example 7-3

A plano-convex lens (n =1.523) of 1/8 diopter power is placed, convex surface down, on an optically flat surface as shown in Fig. 7-17a. Using a traveling microscope and sodium light (λ = 589.3 nm), interference fringes are observed. Determine the radii of the first and tenth dark rings.

7.7 Film-Thickness Measurement by Interference



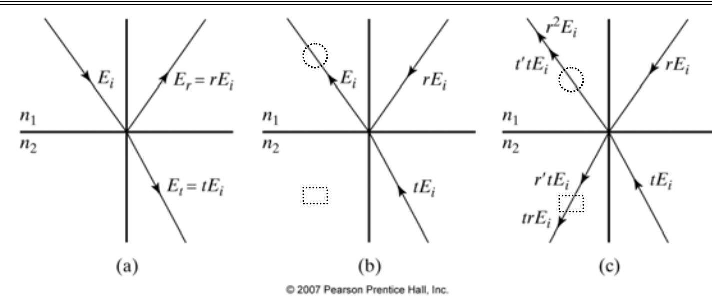


(b)

x: Fringe spacing
Δx: Fringe shift

- Normal incidence: $\boxed{\Delta_p + \Delta_r = 2nt + \Delta_r = m\lambda}$
- Film thickness; $\boxed{d = (\Delta x / x)(\lambda / 2)}$

7.8 Stokes Relations



- Parameters:
  r , t : *reflection coefficient* and *transmission coefficient* (from n_1 to n_2)
  r' , t' : *reflection coefficient* and *transmission coefficient* (from n_2 to n_1)

  $$\boxed{r = \frac{E_r}{E_i}, \quad t = \frac{E_t}{E_i}}$$

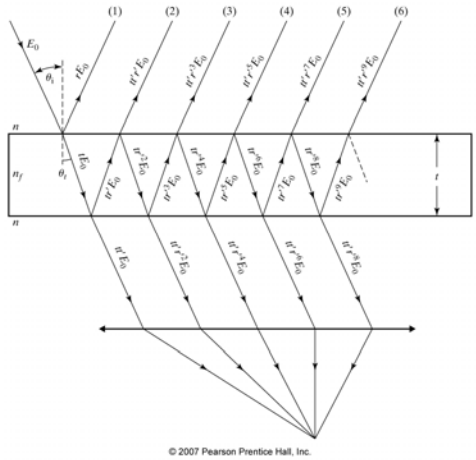
- Fig. (b) : use Principle of Reversibility (Any actual ray of light in an optical system, if reversed in direction, will retrace the same path backward.)

- Stokes relations:

			Stokes relations
See rectangular:	$(E_{oi}t)r' + (E_{oi}r)t = 0$	⇒	$r = -r'$
See circle:	$(E_{oi}r)r + (E_{oi}t)t' = E_{oi}$	⇒	$tt' = 1 - r^2$

(Stokes relations will be used in multi-beam interference)

7.9 Multiple-beam Interference in a Parallel Plate



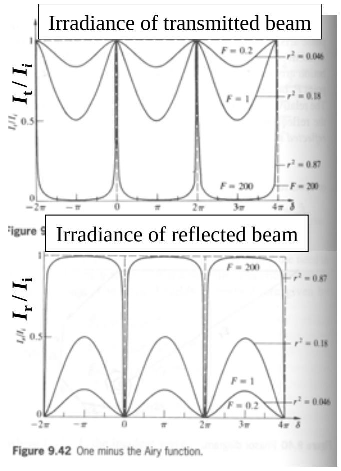


Figure 9.42 One minus the Airy function.

- For multiple beam interference, how can we find the irradiances of the transmitted or reflected beams ? $\boxed{\text{Complex method}}$

- OPD or phase difference δ between adjacent rays: $\boxed{\delta = k\Delta, \quad \Delta = 2n_f t\cos\theta_t}$

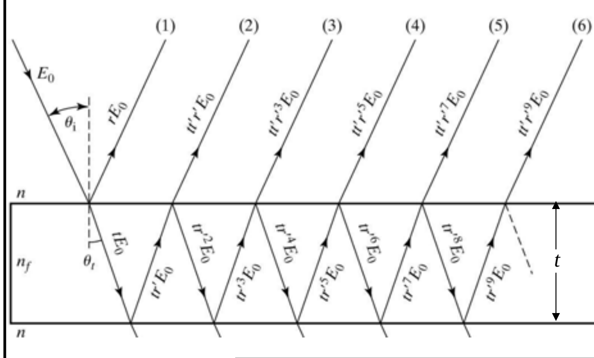
7.9 Multiple-beam Interference in a Parallel Plate

- Parameters : r , r', t, t', δ

  r , r': reflection coefficients for external reflection and internal reflection, respectively

  t , t': transmission coefficients for entering the film and leaving the film, respectively.

  δ : phase difference between the adjacent beams



$$\delta = k\Delta, \quad \Delta = 2n_f t\cos\theta_t$$

$$E_1 = E_0 r e^{i\omega t}$$

$$E_2 = E_0 t r' t' e^{i(\omega t - \delta)}$$

$$E_3 = E_0 t (r')^3 t' e^{i(\omega t - 2\delta)}$$

$$E_N = E_0 t (r')^{2N-3} t' e^{i[\omega t - (N-1)\delta]}$$

- Resultant E_R: $\boxed{E_R = \sum_{N=1}^{\infty} E_N = E_o[re^{i\omega t} + \sum_{N=2}^{\infty} t(r')^{2N-3} t' e^{i[\omega t-(N-1)\delta]}]}$

7.9 Multiple-beam Interference in a Parallel Plate

$$E_R = \sum_{N=1}^{\infty} E_N = E_0 e^{i\omega t}\{r + tr't'e^{-i\delta} + t(r')^3 t'e^{-i2\delta} + \ldots + t(r')^{2N-3}t'e^{i[\omega t-(N-1)\delta]}\}$$

$$= E_0 e^{i\omega t}\{r + tr't'e^{-i\delta}[\underbrace{1+(r')^2 e^{-i\delta} + ((r')^2 e^{-i\delta})^2 + \ldots + ((r')^2 e^{-i\delta})^{N-2}}_{\text{Geometric series}}]\}$$

$$= E_0 e^{i\omega t}\left[r + \frac{r'tt'e^{-i\delta}}{1-(r')^2 e^{-i\delta}} \right]$$

Using Stocks relations: $\boxed{r = -r', \quad tt' = 1 - r^2}$

$$E_R = \sum_{N=1}^{\infty} E_N$$

$$\Rightarrow \quad = E_0 e^{i\omega t}\left[r + \frac{tr't'e^{-i\delta}}{1-(r')^2 e^{-i\delta}} \right] = E_0 e^{i\omega t}\left[r - \frac{rtt'e^{-i\delta}}{1-r^2 e^{-i\delta}} \right]$$

$$= E_0 e^{i\omega t}\left[\frac{r - r(r^2+tt')e^{-i\delta}}{1-r^2 e^{-i\delta}} \right] = E_0 e^{i\omega t}\left[\frac{r(1-e^{-i\delta})}{1-r^2 e^{-i\delta}} \right]$$

7.9 Multiple-beam Interference in a Parallel Plate

$\boxed{\cos\delta = 1 - 2\sin^2\frac{\delta}{2}}$

- Find the irradiance of transmitted wave and reflected wave:

Irradiance of reflected beam :

$$I_R = E_R \cdot E_R^* = E_0 e^{i\omega t}\left[\frac{r(1-e^{-i\delta})}{1-r^2e^{-i\delta}}\right] \cdot E_0 e^{-i\omega t}\left[\frac{r(1-e^{i\delta})}{1-r^2e^{i\delta}}\right] = I_i \frac{2r^2(1-\cos\delta)}{(1+r^4)-2r^2\cos\delta}$$

Irradiance of transmitted beam :

$$I_T = I_i - I_R = I_i\left\{1 - \frac{2r^2(1-\cos\delta)}{(1+r^4)-2r^2\cos\delta}\right\} = I_i\left\{\frac{(1-r^2)^2}{(1+r^4)-2r^2\cos\delta}\right\}$$

- Special positions: $\delta = 2m\pi$ and $\delta = (2m+1)\pi$:

1. If $\delta = 2m\pi$, $I_t \rightarrow$ max. and $I_r \rightarrow$ min. $\Rightarrow$ $\boxed{I_T / I_i = 1, \quad I_R / I_i = 0}$

2. If $\delta = (2m+1)\pi$, $I_t \rightarrow$ min. and $I_r \rightarrow$ max. $\Rightarrow$ $\boxed{(I_T)_{\min} = I_i \frac{(1-r^2)^2}{(1+r^2)^2},}$ $\boxed{(I_R)_{\max} = I_i \frac{4r^2}{(1+r^2)^2}}$

Ch7 Interference of Light

Problem 7-6

Two slits are illuminated by light that consists of two wavelengths. One wavelength is known to be 436 nm. On a screen, the fourth minimum of the 436-nm light coincides with the third maximum of the other light. What is the wavelength of the other light?

Problem 7-9

In an interference experiment of the Young type, the distance between slits is 0.5mm, and the wavelength of the light is 600 nm.

(a) If it is desired to have a fringe spacing of 1 mm at the screen, what is the proper screen distance?

(b) If a thin plate of glass (n = 1.50) of thickness 100 microns is placed over one of the slits, what is the lateral fringe displacement at the screen?

(c) What path difference corresponds to a shift in the fringe pattern from a peak maximum to the (same) peak half maximum?

Ch7 Interference of Light

Problem 7-14

Light of continuously variable wavelength illuminates normally a thin oil (index of 1.3) film on a glass surface. Extinction of the reflected light is observed to occur at wavelength of 525 and 675 nm in the visible spectrum. Determine the thickness of the oil film and the orders of the interference.

Problem 7-21

Plane plates of glass are in contact along one side and held apart by a wire 0.05 mm in diameter, parallel to the edge in contact and 20 cm distant. Using filtered green mercury light (λ = 546 nm), directed normally on the air film between plates, interference fringes are seen. Calculate the separation of the dark fringes. How many dark fringes appear between the edge and the wire?