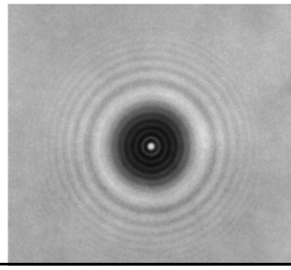
Chapter 13. Fresnel Diffraction

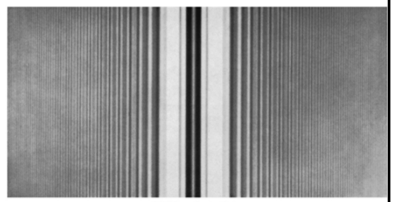
Introduction

- Two types of diffractions:
  - Fraunhofer diffraction, or far-field diffraction: *Infinite observation distance*
  - Fresnel diffraction or near-field diffraction: *Finite observation distance*
- Fraunhofer diffraction: Both the source and observation screen are effectively *far* enough from the diffraction aperture so that wavefronts arriving at the aperture and observation screen may be considered as *planar* wave
- Fresnel diffraction: Both the source and observation screen are *close* enough from the diffraction aperture, so that *wavefront curvature* must be taken into account

Diffraction pattern due to an opaque circular disc

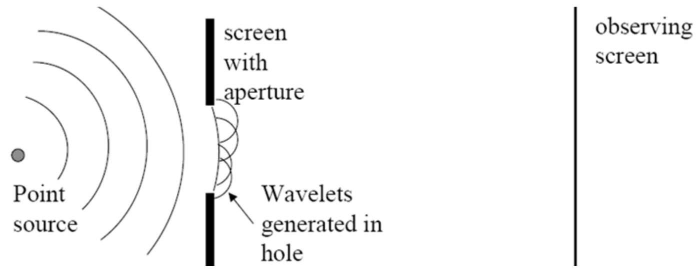


Diffraction pattern for a fine wire



Fresnel-Kirchhoff Diffraction Integral

- Fresnel diffraction: Both the source and observation screen are *close* enough from the diffraction aperture, so that *wavefront curvature* must be taken into account



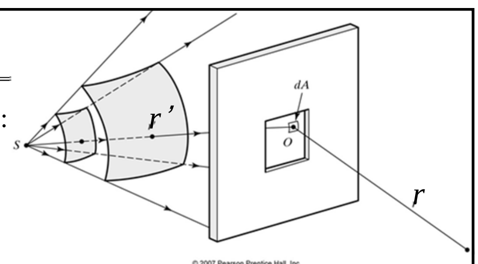
- Huygens-Fresnel Principle:

  1. Each Huygens wavelet illuminates the observing screen
  2. The resultant amplitude at the observing screen is a superposition of all the wavelets at the aperture

Fresnel-Kirchhoff Diffraction Integral

- The electric field at point O in the aperture:

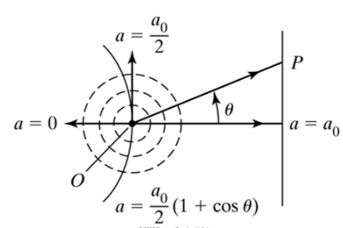
$$\boxed{E_O = \frac{E_S}{r'} e^{i[kr' - \omega t]}}$$



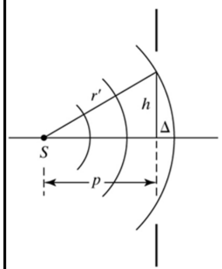
- The resultant field at point P due to an elemental area dA: $\boxed{dE_P = \frac{dE_O}{r} e^{i[kr - \omega t]}}$

- Fresnel-Kirchhoff diffraction formula: $E_p = \frac{-ikE_S}{2\pi} e^{-i\omega t} \iint_{aperture} F(\theta) \frac{e^{ik(r+r')}}{rr'} dA$

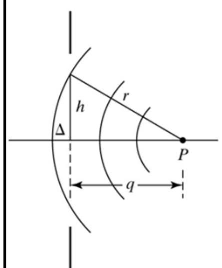
Obliquity Factor : $\boxed{F(\theta) = \frac{1+\cos\theta}{2}}$



Criterion for Fresnel Diffraction



(a)



(b)

Source S and test field at P

Incident wave curvature (a):

- Condition for significant curvature (i.e, near field) is: $\boxed{\Delta > \lambda}$

$$\boxed{\Delta = r' - \sqrt{r'^2 - h^2}} \Longrightarrow \boxed{\Delta \cong \frac{h^2}{2p} > \lambda} \qquad (13\text{-}9)$$

Diffracted wave curvature (b):

- Condition for significant curvature (i.e, near field) is: $\boxed{\Delta > \lambda}$

$$\boxed{\Delta = r' - \sqrt{r'^2 - h^2}} \Longrightarrow \boxed{\Delta \cong \frac{h^2}{2q} > \lambda} \qquad (13\text{-}10)$$

Near field condition:

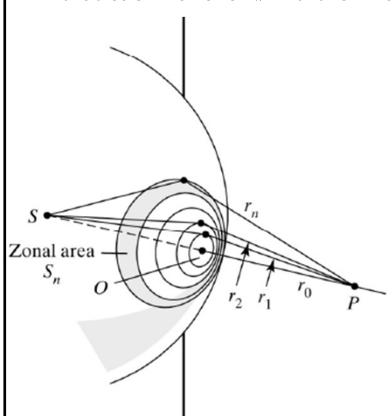
$$\boxed{\frac{1}{2}\left(\frac{1}{p} + \frac{1}{q}\right)h^2 > \lambda} \quad \text{or} \quad \boxed{d < \frac{A}{\lambda}}$$

A : Area of the aperture
d : either p or q

Fresnel Diffraction from Circular Aperture

How to find the field at the observation point P?

- **Fresnel zones (or Half-period Zone)** are concentric rings spaced in such a way that each zone is $\lambda/2$ farther from the field point P than the preceding zone.



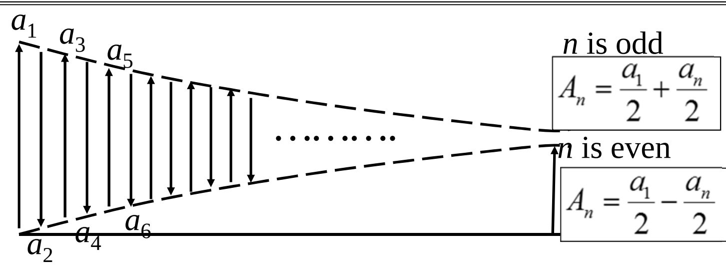
(a)

$$\boxed{r_1 = r_0 + \lambda/2;\ r_2 = r_1 + \lambda/2;\ r_3 = r_2 + \lambda/2 \cdots\cdots}$$

1. Draw circles with radii, $r_o + \lambda/2$, $r_o + \lambda$, $r_o + 3\lambda/2$, $r_o + 2\lambda$, $r_o + 5\lambda/2$, $r_o + 3\lambda$, etc.
2. Each zone has nearly same area
3. Wavelets from <u>any one zone</u> are in phase at P
4. **Each successive zone's contribution is exactly out of phase with that of the preceding zone**
5. Field at P due to 1st zone is a_1, 2nd zone is a_2, etc.
6. The resultant wave amplitudes at P from n half-period zones can be expressed as :

$$\boxed{A_n = a_1 + a_2 e^{i\pi} + a_3 e^{i2\pi} + a_4 e^{i3\pi} + \cdots a_n e^{in\pi} = a_1 - a_2 + a_3 - a_4 + \cdots a_n}$$

Fresnel Diffraction from Circular Aperture



- Each succeeding amplitude is slightly smaller than the preceding one ($a_2 < a_1$):
  - A gradual increase due to slightly increasing zone areas , a gradual decrease due to inverse-square law, and a gradual decrease due to obliquity factor.
- The resultant wave amplitudes at P: $\boxed{A_n = a_1 - a_2 + a_3 - a_4 + \cdots a_n}$
- For N zone: if n is odd, $\boxed{A_n = \frac{a_1}{2} + \frac{a_n}{2}}$; if n is even, $\boxed{A_n = \frac{a_1}{2} - \frac{a_n}{2}}$

Fresnel Diffraction from Circular Aperture

The resultant wave amplitudes at P: $\boxed{A_n = a_1 - a_2 + a_3 - a_4 + \cdots a_n}$

- For N zone: if n is odd, $\boxed{A_n = \frac{a_1}{2} + \frac{a_n}{2}}$; if n is even, $\boxed{A_n = \frac{a_1}{2} - \frac{a_n}{2}}$

If *N* is small ($a_1 = a_N$):

For odd N, the resultant amplitude A = a_1, that of the 1^{st} zone alone;

For even N, the resultant amplitude A is near zero

If *N* is large ($a_N = 0$):

For either odd or even N, the resultant amplitude is $\boxed{A_n = \frac{a_1}{2}}$, *half that of the 1^{st} zone alone.*

1. For example, if removing the opaque shield, then all zones of an unobstructed wavefront contribute . $\boxed{a_n \approx 0}$ $\boxed{A_n = \frac{a_1}{2}}$
2. The unobstructed irradiance at P is only ¼ that due to the 1^{st}-zone aperture alone

Fresnel Diffraction from Circular Aperture

Observations from the experiment :

1. A circular aperture coincides with the 1st Fresnel zone: The amplitude at P, $A_p = a_1$
2. The circular aperture only admits 2 Fresnel zone, $A_p = 0$ (dark spot at the center P)
3. Remove the opaque shield, $A = a_1/2$. The unobstructed irradiance (A^2) at P is only ¼ that due to the 1st-zero alone
4. When light shines on a round obstacle (just coving 1st zone), a bright spot, a Poisson spot or an Arago spot appears at the center

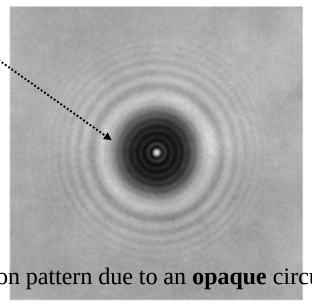


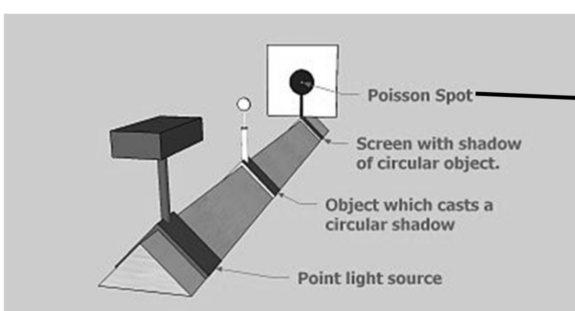
Fig. 13-6

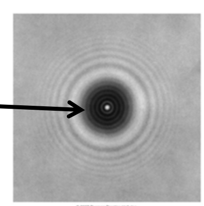
Diffraction pattern due to an **opaque** circular disc

Fresnel Diffraction from Circular Aperture

Light in the Shadows

Poisson, Fresnel, or Arago's Bright Spot is a bright spot that appears at the centre of a circular object's shadow due to Fresnel diffraction. It was deduced by **Poisson** from **Fresnel**'s theory, and experimentally tested by **Arago**.





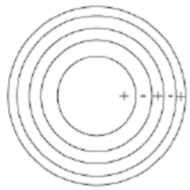
The Fresnel Zone Plate

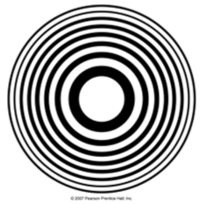
- **A Fresnel Zone plate** is an aperture that **block alternates** half-period zones
- A **Fresnel Zone plate** is device used for focusing light or exhibiting wave characte

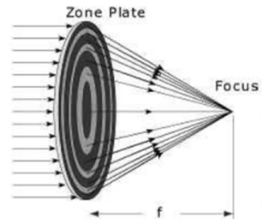
Fresnel Zone

Fresnel Zone plate
(**block the even ones**)

Focusing light

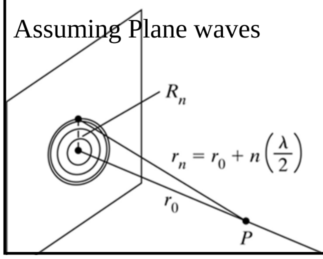






Resultant amplitude: $A_n = a_1 + a_3 + a_5 + a_7 + a_9 \cdots\cdots$

Assuming Plane waves

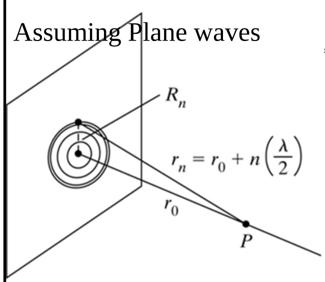


- **Radius** of the n^{th} zone: $R_n^2 = \left(r_0 + \frac{n\lambda}{2}\right)^2 - r_0^2$

$R_n = \sqrt{n r_o \lambda}$ (For $n\lambda / r_o << 1$)

- The distance and the wavelength determine the radius of the 1st zone plate

Assuming Plane waves



The Fresnel Zone Plate

Radius of n^{th} zone: $R_n = \sqrt{n r_o \lambda}$

- Incident wave brought to focus at P, acting like a **<u>lens</u>**

First focal length f_1 : $f_1 = \frac{R_1^2}{\lambda} = \frac{R_n^2}{n\lambda}$

<u>For a fixed zone plate, we have:</u>

1. When moving P toward the zone plate (i.e, r_o decreases), n the number of Fresnel zone increases.
2. If moving toward the plate to a half distance of r_o, corresponding to the case where each Fresnel zone splits into 2 zones. The two zones are out of phase, so no light is focused at $r_o/2$
3. If moving toward the plate to $r_o/3$, each Fresnel zone splits into 3 zones (Two cancel due to out of phase, only one remains), and the irradiance at P is 1/9 that at r_o
4. So, the focal points (maxima irradiance) are located at: $f_n = \frac{R_1^2}{n\lambda} = \frac{f_1}{n}$ (n is odd)

   Minima irradiance are located at: When n is **even**

Problem 13.3 (331)

A distant source of sodium light (589.3 nm) illustrates a circular hole. As the hole increases in diameter, the irradiance at an axial point 1.5 m from the hole passes alternately through maxima and minima. What are the diameters of the holes that produce (a) the first two maxima and (b) the first two minima?

Problems 13.5: The zone plate radii given by Eq. (13-20) were derived for the case of plane waves incident on the aperture. If instead the incident waves are spherical, from an axial point source at distance p from the aperture, show that the necessary modification yields $R_n = \sqrt{nL\lambda}$

Where q is the distance from aperture to the axial point of detection and L is defined by $\frac{1}{L} = \frac{1}{p} + \frac{1}{q}$