Final Exam Review

Wednesday, December 2, 2020 5:04 PM

1. Find the mean, median, and mode of the data, if possible. If any of these measures cannot be found or a measure does not represent the center of the data, explain why.

The durations (in minutes) of power failures at a residence in the last 5 years are listed below.

21 111 19 35 19 86 43 66 41 21

SOLUTION: Put the data in ascending order

$X_i = 19 \ 19 \ 21 \ 21 \ 35 \ 41 \ 43 \ 66 \ 86 \ 111$

Mean $\bar{X} = \frac{\Sigma X_i}{10} = \frac{462}{10} = 46.2$

Median $= \frac{35+41}{2} = \frac{76}{2} = 38$

Mode $= 19, 21$

Note: Both the mean and the medians are measurements of the center of the data. The mode might be a measurement of the center, but in this example the mode values represent the smallest data values.

2. (a) Find the five-number summary, and (b) draw a box-and-whisker plot of the data.

4 8 8 6 2 9 8 7 9 6 9 5 2 6 2 9 8 7 7 9

Solution. Put the data ascending order

$$\begin{array}{cccccccccccccccccccc} & & & & & Q_1 & & & & & Q_2 & & & & & Q_3 & & & & \\ 2 & 2 & 2 & 4 & \underline{5} & \underline{6} & 6 & 6 & 7 & \underline{7} & \underline{7} & 8 & 8 & 8 & \underline{8} & \underline{9} & 9 & 9 & 9 & 9 \end{array} \rightarrow 20 \text{ values}$$

Median $= Q_2 = \frac{7+7}{2} = 7$

$Q_1 = \frac{5+6}{2} = 5.5$

$Q_3 = \frac{8+9}{2} = 8.5$

Five Number Summary

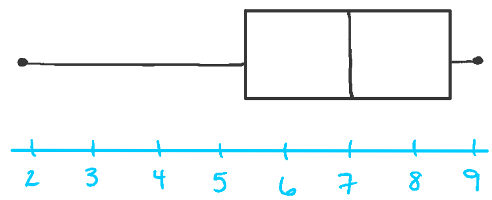
1.) Min = 2

2) $Q_1 = 5.5$

3) $Q_2 = 7$

4) $Q_3 = 8.5$

5.) Max = 9



(3) Outside a home, there is a 4-key keypad with letters A, B, C, and D that can be used to open the garage if the correct four-letter code is entered. Each key may be used only once. How many codes are possible?

Solution $\underset{\text{Pos 1}}{\underline{4}} \cdot \underset{\text{Pos 2}}{\underline{3}} \cdot \underset{\text{Pos 3}}{\underline{2}} \cdot \underset{\text{Pos 4}}{\underline{1}}$

= 24 possible codes

(6) Use the Central Limit Theorem to find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution.

The mean price of photo printers on a website is \$223 with a standard deviation of \$56. Random samples of size 30 are drawn from this population and the mean of each sample is determined.

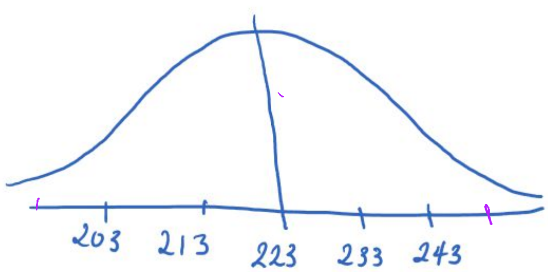
The mean of the distribution of sample means is 223.

The standard deviation of the distribution of sample means is 10.224. $\longrightarrow$ $S.D. = \frac{\sigma}{\sqrt{n}}$

(Type an integer or decimal rounded to three decimal places as needed.)

$$= \frac{56}{\sqrt{30}}$$

$$= 10.224$$



(7)

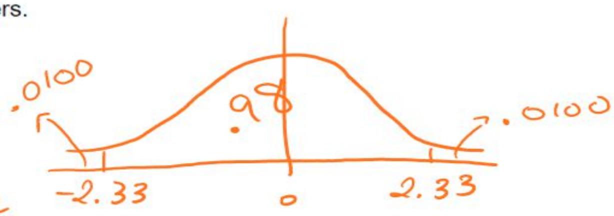
Find the minimum sample size n needed to estimate μ for the given values of c, σ, and E.

$c = 0.98$, $\sigma = 6.5$, and $E = 1$

Assume that a preliminary sample has at least 30 members.

SOLUTION:

$$n = \left(\frac{z\sigma}{E}\right)^2$$



$$n = \left(\frac{2.33(6.5)}{1}\right)^2$$

$$n = 229$$

(8)

You are given the sample mean and the population standard deviation. Use this information to construct the 90% and 95% confidence intervals for the population mean. Interpret the results and compare the widths of the confidence intervals. If convenient, use technology to construct the confidence intervals.

A random sample of 60 home theater systems has a mean price of \$116.00. Assume the population standard deviation is \$17.60.

SOLUTION: $n = 60$, $\bar{x} = 116$, $\sigma = 17.60$

90%: $116 - 1.645\left(\frac{17.60}{\sqrt{60}}\right) < \mu < 116 + 1.645\left(\frac{17.60}{\sqrt{60}}\right)$

$116 - 3.74 < \mu < 116 + 3.74$

$112.26 < \mu < 119.74$

Interval: $(112.26, 119.74)$

95%: $116 - 1.96\left(\frac{17.60}{\sqrt{60}}\right) < \mu < 116 + 1.96\left(\frac{17.60}{\sqrt{60}}\right)$

$116 - 4.45 < \mu < 116 + 4.45$

$111.55 < \mu < 120.45$

Interval: $(111.55, 120.45)$

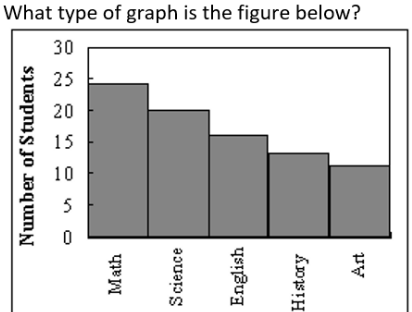
The 95% confidence interval is larger than the 90% confidence interval. The greater the confidence, the larger the interval will be.

9. Which type of graph represents the data by using vertical bars of various heights to indicate frequencies?
A) ogive
B) frequency polygon
C) histogram
D) cumulative frequency

10. If two classes are 9 - 16 and 17 - 24, then the upper class boundary of 9 - 16 is

16.5

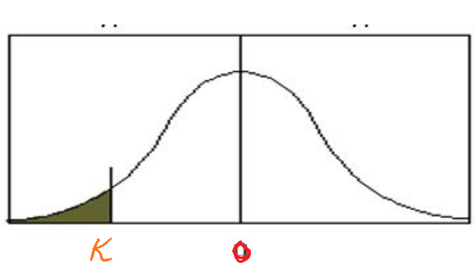
11. What type of graph is the figure below?



A) Pareto chart
B) pictograph
C) ogive
D) pie graph

12. In a study of size 10 where the variance is unknown, the distribution that should be used to calculate confidence intervals is
A) a normal distribution
B) a t distribution with 9 degrees of freedom
C) a t distribution with 10 degrees of freedom
D) a t distribution with 11 degrees of freedom

13. State if the following graph represents a left-tailed, a right-tailed or a two-tailed test. State the null and alternative hypotheses.



Left-tailed test

$H_0: \mu \geq K$

$H_A: \mu < K$

14. A garbage collector believes that he averages picking up more than four tons of garbage per day. What is the null hypothesis for his statement?

$H_0: \mu \leq 4$

$H_A: \mu > 4$

(15) At a certain college, there were 700 science majors, 100 engineering majors, and 800 business majors. If one student was selected at random, the probability that they are an engineering major is

$$\frac{100}{700+100+800} = \frac{100}{1600} = \boxed{\frac{1}{16}}$$

(16) The average length of crocodiles in a swamp is 12 feet (μ). If the lengths are normally distributed with a standard deviation of 1.9 (σ), find the probability that a crocodile is more than 11.5 feet long.

$$P(X > 11.5) \Rightarrow z = \frac{11.5 - 12}{1.9} = -0.26$$

$$P(z > -0.26) = 1 - P(z < -0.26)$$
$$= 1 - 0.3974$$
$$= 0.6026 \longrightarrow 60.26\%$$

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602

(17) Each year, 33 race cars start the Indianapolis 500. How many ways can the cars finish first, second, and third?

Solution: Order matters so this is a permutation: $_{33}P_3 = \frac{33!}{(33-3)!}$

$$= \frac{33!}{30!}$$

$$= \frac{33 \cdot 32 \cdots 2 \cdot 1}{\cancel{30 \cdot 29 \cdots 2 \cdot 1}}$$

$$= 33 \cdot 32 \cdot 31$$

$$= 32,726$$

OR

$\underset{1ST}{\underline{33}} \cdot \underset{2ND}{\underline{32}} \cdot \underset{3RD}{\underline{31}}$

(18) The manager of an accounting department wants to form a three-person advisory committee from the 20 employees in the department. In how many ways can the manager form this committee?

Solution: Order does not matter so this is a combination: $C = 20!$

Solution: Order does not matter so this is a combination: ${}_{20}C_3 = \frac{20!}{(20-3)!\,3!}$

$$= \frac{20!}{\cancel{17!}\,3!}$$

$$= \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1}$$

$= 1{,}140$ different committees

(19) If a single card is drawn from an ordinary deck of cards, what is the probability of drawing either a heart or a 6?

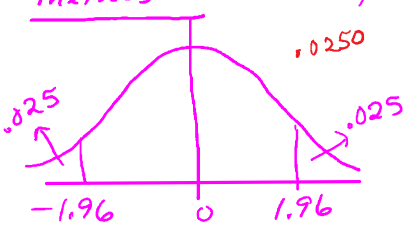
Solution: $\frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

(20) Dr. Christina Cuttleman, a nutritionist, claims that the average number of calories in a serving of popcorn is 75 (μ) with a standard deviation of 7 (σ). A sample of 50 (n) servings of popcorn yields an average of 78 ($\bar{x}$) calories. Check Dr. Cuttleman's claim at $\alpha = 0.05$.

Solution Method 1: $H_0: \mu = 75$ (claim) $\quad H_A: \mu \neq 75$



$$z = \frac{78-75}{7/\sqrt{50}} \qquad z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$z = 3.03$$

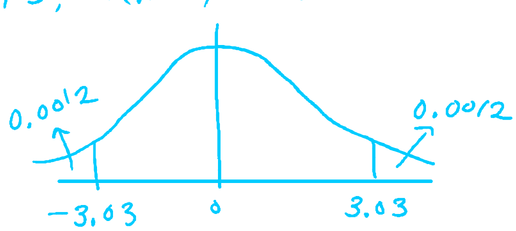
Since $3.03 > 1.96$, 3.03 is in the reject region so we reject the hypothesis that an average serving of popcorn is 75 calories.

Solution: Method 2

$H_0: \mu = 75, \quad H_A = \mu \neq 75$

$$z = \frac{78-75}{7/\sqrt{50}}$$

$$z = 3.03$$



$P\text{-value} = 2(0.0012)$
$= 0.0024$

Since the P-value $< \alpha$, $0.0024 < 0.05$ then reject Ho.

(25) Reginald Brown, an inspector from the Department of Weights and Measures, weighs 15 eighteen-ounce cereal boxes of corn flakes. He finds their mean weight to be 17.78 ounces with a standard deviation of 0.4 ounces. Are the cereal boxes lighter than they should be? Let $\alpha = 0.01$.

$n = 15$
$d.f. = 14$

Solution: $H_o: \mu \geq 18$, $H_a: \mu < 18$

d.f.	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947

This is a left-tailed test so $t_c = -2.624$

$$t = \frac{17.78 - 18}{0.4/\sqrt{15}} \qquad t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$t = -2.130$

Since $-2.130 > -2.624$, $t = -2.130$ does not fall in the critical region

Then, we fail to reject Ho.

Therefore, there is not enough evidence to support the claim that the cereal boxes are lighter than they should be.