Seminarul 11 - partea 2

Inegalitatea Markov:

Fie X o v.a. astfel încât $X \geq 0$, adică X ia valori nenegative. Dacă X are medie finită, atunci, pentru $a > 0$, avem

$$P(X \geq a) \leq \frac{M(X)}{a}.$$

Inegalitatea Cebîşev

Fie X o v.a. arbitrară de medie $M(X)$ şi dispersie $\sigma^2(X)$ finite. Atunci:

$$P(|X - M(X)| \geq a) \leq \frac{\sigma^2(X)}{a^2}, \; a > 0.$$

P. propuse

1) $X \sim Bin(n, p)$; $p = \frac{1}{2}$

$$P\left(X \geq \frac{3n}{4}\right) = ?$$

$$X \sim Bin(n,p) \Rightarrow M(X) = n \cdot p \qquad \Rightarrow M(X) = \frac{n}{2}$$

$$\sigma^2(X) = np(1-p) \qquad \sigma^2(X) = \frac{n}{2} \cdot \frac{1}{2} = \frac{n}{4}$$

Markov

$$P\left(X \geq \frac{3n}{4}\right) \leq \frac{M(X)}{\frac{3n}{4}} = \frac{\frac{\cancel{n}}{\cancel{2}}}{\frac{3\cancel{n}}{\cancel{4}_{2}}} = \frac{2}{3}$$

Cebîşev

$$P\left(X \geq \frac{3n}{4}\right) = P\left(X - \frac{n}{2} \geq \frac{n}{4}\right) = P\left(\left|X - \frac{n}{2}\right| \geq \frac{n}{4}\right) \leq$$

$$\leq \frac{\frac{n}{4}}{\left(\frac{n}{4}\right)^2} = \frac{4}{n} \Rightarrow \text{Cebîşev oferă o margine superioară mai bună}$$

P. propuse

2) $X \sim Bin(n,p) \Rightarrow M(X) = np$

$\sigma^2(X) = np(1-p)$

$P(X \geq \alpha n)$

$P(X \geq a) = \frac{M(X)}{a}$

$\Rightarrow P(X \geq \alpha n) = \frac{M(X)}{\alpha n} = \frac{\cancel{n}p}{\alpha \cancel{n}} = \frac{p}{\alpha}$

$P(|X - M(X)| \geq a) = \frac{\sigma^2(X)}{a^2}$

$P(|X - np| \geq \alpha n - np) = P(|X - np| \geq n(\alpha - p)) = \frac{\sigma^2(X)}{n^2(\alpha-p)^2} =$

$= \frac{\cancel{n}p(1-p)}{n^{\cancel{2}}(\alpha-p)^2} = \frac{p(1-p)}{n(\alpha-p)^2} \xrightarrow{n \to \infty} 0$ $\Rightarrow$ e mai bună

3) X_i , $i = 1, 2, 3$

$X_i \sim Bin(n, p_i)$

$P(Z \geq \alpha n)$; $p < \alpha < 1$; $Z = \sum_{i=1}^{3} X_i$

$Z = X_1 + X_2 + X_3$

$M(Z) = M(X_1 + X_2 + X_3) = n \cdot p_1 + n \cdot p_2 + n \cdot p_3$

$P(Z \geq \alpha n) = \frac{M(Z)}{\alpha n} = \frac{p_1 + p_2 + p_3}{\alpha}$

$p_i = p$, $\alpha = 2p$ $\Rightarrow P(z \geq \alpha m) = \frac{3\cancel{m}\cancel{p}}{2\cancel{p}\cancel{m}} = \frac{3}{2}$

4) $X \sim Exp(\theta)$, $M(x) = \theta$, $\sigma^2(x) = \theta^2$

$P(x \geq a)$, $a > 0$

$P(x \geq a) = \frac{\theta}{a}$

5) $X \sim Exp(\theta)$ $\Rightarrow M(x) = \theta$

$\sigma^2(x) = \theta^2$

$P(|x - \theta| \geq a) = \frac{\theta^2}{a^2}$

6) $M(x) = 25 \cdot 10^3$

$P(x > 5 \cdot 10^4) = 0{,}01$

$\sigma = ?$

$P(x - 25 \cdot 10^3 > 5 \cdot 10^3 \cdot 5 \cdot 2 - 25 \cdot 10^3) = P(x - 25 \cdot 10^3 > 25 \cdot 10^3) =$

$= P(|x - 25 \cdot 10^3| \geq 25 \cdot 10^3 + 1) \leq \frac{\sigma^2(x)}{(25 \cdot 10^3 + 1)^2}$

$\frac{\sigma^2(x)}{(25 \cdot 10^3 + 1)^2} \geq 0{,}01$ $\Rightarrow \sigma^2(x) \geq (25001)^2 \cdot 0{,}01$

$\sigma(x) \geq 25001 \cdot 0{,}1$

$\sigma(x) \geq 2500{,}1$