Seminarul 3

1) 20 de chip-uri
5 sunt defecte
Se aleg 3 la întâmplare

E_i - al i-lea chip este defect, $i = \overline{1,3}$
$\bar{E}_1$ - primul chip ales e defect
E_2 - al doilea — " — " —
E_3 - al treilea — " — " —

a) prob. să fie toate 3 defecte

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2 | E_1) \cdot P(E_3 | E_1 \cap E_2)$$

Probabilități la ev. dependente

$P(E_1) = \frac{5}{20}$

$P(E_2 | E_1) = \frac{4}{19}$ (am extras deja un chip și el a fost defect)

$P(E_3 | E_1 \cap E_2) = \frac{3}{18}$ — " —

$\Rightarrow P(E_1 \cap E_2 \cap E_3) = \frac{\cancel{5}}{\cancel{20}} \cdot \frac{\cancel{4}}{19} \cdot \frac{\cancel{3}}{\cancel{18}_6} = \frac{1}{6 \cdot 19} = \frac{1}{114}$

b) $P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = P$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(E_1) \cdot P(\bar{E}_2 | E_1) \cdot P(\bar{E}_3 | E_1 \cap \bar{E}_2) = \frac{5}{\cancel{20}} \cdot \frac{\cancel{15}}{19} \cdot \frac{\cancel{14}^7}{\cancel{18}_{6_3}} = \frac{35}{3 \cdot 19}$

($P(E_1) = \frac{5}{20}$, $P(\bar{E}_2 | E_1) = \frac{15}{19}$, $P(\bar{E}_3 | E_1 \cap \bar{E}_2) = \frac{14}{18}$)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = P(\bar{E}_1) \cdot P(E_2 | \bar{E}_1) \cdot P(\bar{E}_3 | \bar{E}_1 \cap E_2) = \frac{35}{3 \cdot 19}$

($P(\bar{E}_1) = \frac{15}{20}$, $P(E_2 | \bar{E}_1) = \frac{6}{19}$, $P(\bar{E}_3 | \bar{E}_1 \cap E_2) = \frac{14}{18}$)

$$P(\overline{E_1} \cap \overline{E_2} \cap E_3) = \frac{35}{3 \cdot 19}$$

$$\Rightarrow P = \frac{8 \cdot 35}{3 \cdot 19} = \frac{35}{19}$$

2) $P(A) = 0,25$
$P(B) = 0,3$
$P(A \cup B) = ?$
$P_B(\overline{A}) = ?$
$P(A \cap \overline{B}) = ?$

$P(\overline{A}) = 1 - P(A)$ $\quad P(\overline{A}) = 0,75$

La evenimente independente

$P(A \cap B) = P(A) \cdot P(B)$

$P_B(A) = \frac{P(A \cap B)}{P(B)} = P(A)$ $\quad P_A(B) = \frac{P(A \cap B)}{P(A)} = P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) = 0,25 + 0,3 - 0,075 = 0,475$

$P_B(\overline{A}) = \frac{P(\overline{A} \cap B)}{P(B)} = \frac{P(\overline{A}) \cdot P(B)}{P(B)} = 0,75$

$P(A \cap \overline{B}) = P(A) \cdot (1 - P(B)) = 0,25 \cdot 0,7 = 0,175$

3) independente
$P(A_1) = 0,9$
$P(A_2) = 0,8$
$P(A_3) = 0,85$

a) $P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = 0,9 \cdot 0,8 \cdot 0,85 = 0,612$

b) $P(A_1 \cap A_2 \cap \overline{A_3}) = 0,9 \cdot 0,8 \cdot 0,15 = 0,108$

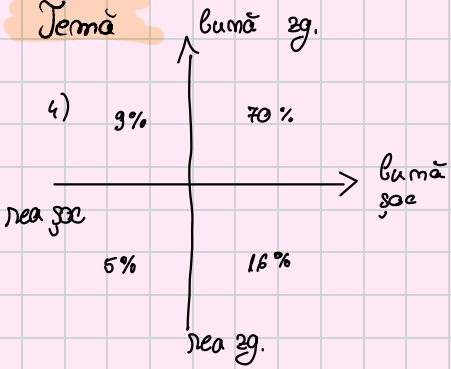
c) $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) +$

$+ P(A_1 \cap A_2 \cap A_3) = 0,9 + 0,8 + 0,85 - 0,72 - 0,68 - 0,765 + 0,612 = 0,997$

sau $1 - P(\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}) = 1 - 0,1 \cdot 0,2 \cdot 0,15 = 0,997$

Temă

4)



$P(A)$ - rezistență bună la soc

$P(B)$ — " — " — " zgârieturi

$P(A) = \frac{70+16}{100} = \frac{86}{100}$

$P(B) = \frac{70+9}{100} = \frac{79}{100}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{70}{79}$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{70}{86}$

$\neq$, $\neq$ $\Rightarrow$ sunt ev. dependente

5) 100 produse, 5 au defecțiuni } ev. dependente

ca la ex 1 avem E_1, E_2, E_3

$$P = P(\overline{E_1} \cap \overline{E_2} \cap E_3) = P(\overline{E_1}) \cdot P(\overline{E_2} \mid E_1) \cdot P(\overline{E_3} \mid \overline{E_1} \cap \overline{E_2}) = \frac{95}{100} \cdot \frac{94}{99} \cdot \frac{93}{98} = 0,86$$

$\overline{E_1}$: $\frac{95}{100}$ — $\overline{E_2}$: $\frac{94}{99}$ — $\overline{E_3}$: $\frac{93}{98}$

6) $P(A \mid B) = 0,4$

$P(B) = 0,5$

$P(A \cap B) = ?$

$P(\overline{A} \cap B) = ?$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(B) \cdot P(A \mid B) = 0,2$$

$$P(\overline{A} \mid B) = 1 - P(A \mid B) = 0,6$$

$$\Rightarrow P(\overline{A} \cap B) = P(B) \cdot P(\overline{A} \mid B) = 0,5 \cdot 0,6 = 0,3$$

7) $P(A \mid B) = 0,2$

$P(B) = 0,8 \Rightarrow P(\overline{B}) = 0,2$

$P(A \mid \overline{B}) = 0,3$

$P(A) = ?$

$P(A \cap B) = 0,16$

$P(A \cap \overline{B}) = 0,2 \cdot 0,3 = 0,06$

$$P(A) = P(B) \cdot P(A \cap B) + P(\overline{B}) \cdot P(A \cap \overline{B})$$

$P(A) = 0,8 \cdot 0,16 + 0,2 \cdot 0,06 = 0,14$

8) $P(E_1) = 0,05$

$P(E_2) = 0,1$

$P(E_3) = 0,3$

a) $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \cdot P(\overline{E_2} \mid \overline{E_1}) = 0,95 \cdot 0,9 = 0,855$

$P(\bar{E}_2 \mid \bar{E}_1) = P(\bar{E}_2) = 0,9$

b) $P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(\bar{E}_1) \cdot P(\bar{E}_2 \mid \bar{E}_1) \cdot P(\bar{E}_3 \mid \bar{E}_1 \cap \bar{E}_2) = 0,95 \cdot 0,9 \cdot 0,7 = 0,59$

$P(\bar{A} \mid B) = 1 - P(A \mid B)$

9) 5 întrebări → 3 răsp/întrebare

a) $P(E_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap \bar{E}_5) + P(\bar{E}_1 \cap E_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap \bar{E}_5) + \ldots + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap E_5)$

$= 5 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^4 = \frac{5 \cdot 16}{243} = \frac{80}{243} = 0,33$

b) $P = 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap \bar{E}_5) = 1 - \left(\frac{2}{3}\right)^5 = 1 - \frac{32}{243} = \frac{211}{243} = 0,87$

c) $P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4 \cap \bar{E}_5) = \left(\frac{2}{3}\right)^5 = \frac{32}{243} = 0,13$

d) $P(0) + P(a) = \frac{80 + 32}{243} = \frac{112}{243} = 0,46$

10) $P(A) = 0,3$

$P(B) = 0,4$

$P(A \cup B) = ?$

$P(A \setminus B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0,7 - 0,12 = 0,58$

$P(A \setminus B) = P(A) - P(A \cap B) = 0,3 - 0,12 = 0,18$