CALCUL D'UN DETERMINANT

EXERCICE 33 :

A. car il y'a une colonne nul.

B. car

C. car $L_2 = L_1 + 2L_3$

EXERCICE 34 :

- $|A| = 1 \times 3 \times 1 \times 6 = 18$
- $|B| = 3 \times 1 \times 5 \times 2 = 30$
- $|C| = 2 \times 2 \times 1 = 4$

EXERCICE 35 :

- $|A| = \begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 6 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 3 & 4 & 5 \\ 1 & 0 & 2 \\ 6 & 0 & 0 \end{vmatrix} = + \begin{vmatrix} 4 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 6 & 0 \end{vmatrix} = - \begin{vmatrix} 4 & 5 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{vmatrix} = -4 \times 2 \times 6 = -48$

- $|B| = \begin{vmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{vmatrix} = + \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 4 & 0 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = -1 \times 2 \times 3 \times 4$

$|B| = -24.$

- $|C| = \begin{vmatrix} 0 & 3 & 2 & 5 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 2 & 5 & 3 & 4 & 1 \\ 0 & 3 & 0 & 1 & 0 \\ 3 & 2 & 7 & 4 & 0 \end{vmatrix} = - \begin{vmatrix} 0 & 3 & 2 & 5 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 3 & 2 & 7 & 4 & 0 \\ 0 & 3 & 0 & 1 & 0 \\ 2 & 5 & 3 & 4 & 1 \end{vmatrix} = + \begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 3 & 2 & 5 & 0 \\ 3 & 2 & 7 & 4 & 0 \\ 0 & 3 & 0 & 1 & 0 \\ 2 & 5 & 3 & 4 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 \\ 2 & 3 & 7 & 4 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 5 & 2 & 3 & 4 & 1 \end{vmatrix}$

$= + \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 \\ 2 & 3 & 7 & 4 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 5 & 2 & 3 & 4 & 1 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 5 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 7 & 4 & 0 \\ 5 & 2 & 3 & 4 & 1 \end{vmatrix} = + \begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 3 & 5 & 2 & 0 & 0 \\ 2 & 4 & 7 & 3 & 0 \\ 5 & 4 & 3 & 2 & 1 \end{vmatrix} = 12$

EXERCICE 36 :

$|A| = \begin{vmatrix} 5 & 4 & 4 \\ 0 & 2 & 0 \\ 50 & 30 & 0 \end{vmatrix} = 10 \begin{vmatrix} 5 & 4 & 4 \\ 0 & 2 & 0 \\ 5 & 3 & 0 \end{vmatrix} = 10 \times 5 \begin{vmatrix} 1 & 4 & 4 \\ 0 & 2 & 0 \\ 1 & 3 & 0 \end{vmatrix} = 10 \times 5 \times 4 \begin{vmatrix} 1 & 4 & 1 \\ 0 & 2 & 0 \\ 1 & 3 & 0 \end{vmatrix}$

$= 10 \times 5 \times 4 \times 2 \begin{vmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 0 \end{vmatrix} = -400 \begin{vmatrix} 4 & 1 & 1 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{vmatrix} = 400 \begin{vmatrix} 1 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{vmatrix} = -400 \begin{vmatrix} 1 & 1 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{vmatrix}$

$= -400 \times (1) \times (1) \times (1) = -400$

$$|B| = \begin{vmatrix} -4 & -2 & 6 & -2 \\ 0 & 0 & -2 & 0 \\ -6 & 0 & 2 & -4 \\ 0 & 0 & -8 & 2 \end{vmatrix} = (-2)(-2)(-2)(-2) \begin{vmatrix} 2 & 1 & -3 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & -1 & 2 \\ 0 & 0 & 4 & -1 \end{vmatrix} = -16 \begin{vmatrix} 2 & 1 & -3 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & -1 & 2 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$= +16 \begin{vmatrix} 1 & 2 & -3 & 1 \\ 3 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & -1 \end{vmatrix} = -16 \begin{vmatrix} 1 & 2 & 1 & -3 \\ 3 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 4 \end{vmatrix} = +16 \begin{vmatrix} 1 & 2 & 1 & -3 \\ 3 & 0 & 2 & -1 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{vmatrix} = -16 \begin{vmatrix} 2 & 1 & 1 & -3 \\ 0 & 3 & 2 & -1 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= -16 \times (2) \times (3) \times (-1) \times 1 = -48$$

$$|C| = \begin{vmatrix} \sqrt{5} & 9\sqrt{5} & 0 & 3\sqrt{5} \\ 2/3 & 2 & 0 & 0 \\ 1/3 & 1/2 & \sqrt{5}/3 & 3/2 \\ 0 & 1/2 & 0 & 0 \end{vmatrix} = \sqrt{5} \begin{vmatrix} 1 & 9 & 0 & 3 \\ 2/3 & 2 & 0 & 0 \\ 1/3 & 1/2 & \sqrt{5}/3 & 3/2 \\ 0 & 1/2 & 0 & 0 \end{vmatrix} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \sqrt{5} \begin{vmatrix} 3 & 18 & 0 & 6 \\ 2 & 4 & 0 & 0 \\ 1 & 1 & \sqrt{5} & 3 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{36} \sqrt{5} \cdot 3\sqrt{5} \begin{vmatrix} 3 & 18 & 0 & 2 \\ 2 & 4 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \frac{5}{12} \cdot 2 \begin{vmatrix} 3 & 18 & 0 & 2 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} =$$

$$= -\frac{5}{6} \begin{vmatrix} 18 & 3 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = \frac{5}{6} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 18 & 3 & 0 & 2 \end{vmatrix} = -\frac{5}{6} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 18 & 3 & 2 & 0 \end{vmatrix} =$$

$$= \frac{5}{6} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 18 & 3 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = \frac{5}{6} \times 1 \times 1 \times 2 \times 1 = \frac{5}{3}$$

EXERCICE 37 :

- $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix} \xleftrightarrow[L_3 \leftarrow L_3 - 4L_1]{L_2 \leftarrow L_2 - 2L_1} \begin{vmatrix} 1 & 1 & 2 \\ 0 & -3 & -2 \\ 0 & -3 & -4 \end{vmatrix} \xleftrightarrow{L_3 \leftarrow L_3 - L_2} \begin{vmatrix} 1 & 1 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & -2 \end{vmatrix} = 1 \times (-3) \times (-2)$

$|A| = 6$

- $|B| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 2 & -3 & -2 \end{vmatrix} \xleftrightarrow[L_3 \leftarrow L_3 - 2L_1]{L_2 \leftarrow L_2 - 3L_1} \begin{vmatrix} 1 & -1 & 2 \\ 0 & 5 & -5 \\ 0 & -1 & -6 \end{vmatrix} \longleftrightarrow 5 \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -6 \end{vmatrix} \xleftrightarrow{L_3 \leftarrow L_3 + L_2}$

$5 \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -7 \end{vmatrix} = 5 \times 1 \times 1 \times (-7) = -35$

- $|C| = \begin{vmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 2 & -1 & 0 & 3 \\ 4 & -1 & 3 & 9 \end{vmatrix} \underset{L_4 \leftarrow L_4 - 4L_1}{\overset{L_3 \leftarrow L_3 - 2L_1}{=}} \begin{vmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -4 & -5 \\ 0 & -1 & -5 & -7 \end{vmatrix} \underset{L_4 \leftarrow L_4 + L_2}{\overset{L_3 \leftarrow L_3 + L_2}{=}} \begin{vmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -2 & -5 \end{vmatrix} \overset{L_4 \leftarrow L_4 - 2L_3}{=} \begin{vmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \times 1 \times (-1) \times 1$

donc $|C| = -1$

- $|D| = \begin{vmatrix} 0 & -3 & -6 & 4 \\ -1 & -2 & -1 & 3 \\ -2 & -3 & 0 & 3 \\ 1 & 4 & 5 & -9 \end{vmatrix} \overset{L_1 \leftrightarrow L_4}{=} - \begin{vmatrix} 1 & 4 & 5 & -9 \\ -1 & -2 & -1 & 3 \\ -2 & -3 & 0 & 3 \\ 0 & -3 & -6 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 5 & -9 \\ 1 & 2 & 1 & -3 \\ -2 & -3 & 0 & 3 \\ 0 & -3 & -6 & 4 \end{vmatrix} \underset{L_3 \leftarrow L_3 + 2L_1}{\overset{L_2 \leftarrow L_2 - L_1}{=}} \begin{vmatrix} 1 & 4 & 5 & -9 \\ 0 & -2 & -4 & 6 \\ 0 & 5 & 10 & -15 \\ 0 & -3 & -6 & 4 \end{vmatrix}$

$|D| = 10 \begin{vmatrix} 1 & 4 & 5 & -9 \\ 0 & -1 & -2 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & -3 & -6 & 4 \end{vmatrix} \underset{L_4 \leftarrow L_4 - 3L_3}{\overset{L_3 \leftarrow L_3 + L_2}{=}} 10 \begin{vmatrix} 1 & 4 & 5 & -9 \\ 0 & -1 & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 \end{vmatrix} = 1 \times (-1) \cdot 0 \cdot (-5) \cdot 10 = 0$

EXERCICE 38 :

$|A| = \begin{vmatrix} 2 & 5 & 1 \\ 3 & 1 & 2 \\ 5 & 4 & 1 \end{vmatrix} = +1 \begin{vmatrix} 3 & 1 \\ 5 & 4 \end{vmatrix} - 2 \begin{vmatrix} 2 & 5 \\ 5 & 4 \end{vmatrix} + 1 \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = 7 - 2(-17) + (-13) = 28$

- $|B| = \begin{vmatrix} 6 & 1 & 4 \\ 2 & 0 & 3 \\ 1 & 2 & 1 \end{vmatrix} = -2 \begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} + 0 - 3 \begin{vmatrix} 6 & 1 \\ 1 & 2 \end{vmatrix} = -2 \cdot (-7) - 3 \cdot 11 = -19$

$$|D| = \begin{vmatrix} 4 & 1 & 3 \\ 2 & 0 & 0 \\ 0 & 1 & 2 \end{vmatrix} = -2 \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -2(-1) = 2$$

$$|G| = \begin{vmatrix} 2 & 1 & 4 & 5 \\ 0 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 2 \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} - 0 + 1 \cdot \begin{vmatrix} 1 & 4 & 5 \\ 3 & 2 & 5 \\ 0 & 0 & 1 \end{vmatrix} - 0 = 2 \times 3 \times 1 \times 1 + 0 - 0 +$$

$$1 \cdot \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} = 6 - 10 = -4.$$

EXERCICE 39:

$$\begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix} \xrightarrow[L_4 \leftarrow L_4 + L_1]{L_2 \leftarrow L_2 + L_1,\ L_3 \leftarrow L_3 + L_1} \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{vmatrix} = -1 \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix} \xrightarrow{L_3 \leftarrow L_3 - L_2}$$

$$= -1 \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 0 & 2 & -2 \end{vmatrix} = -1\left(-2 \begin{vmatrix} 2 & 2 \\ 2 & -2 \end{vmatrix} + 0\right) = 2(-4-4) = -16.$$

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} \xrightarrow[L_4 \leftarrow L_4 - L_2 - L_3]{L_3 \leftarrow L_3 - 2L_2} \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 0 & -1 & -6 & 1 \\ 0 & -5 & -2 & 1 \end{vmatrix} = 0 - 1 \cdot \overbrace{\begin{vmatrix} 1 & 2 & 3 \\ -1 & -6 & 1 \\ -5 & -2 & 1 \end{vmatrix}}^{L_2 \leftarrow L_2 + L_1,\ L_3 \leftarrow L_3 + 5L_1} + 0 - 0$$

$$= -\begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 4 \\ 0 & 8 & 16 \end{vmatrix} = -\begin{vmatrix} -4 & 4 \\ 8 & 16 \end{vmatrix} = -(64 - 32) = 96$$

$$|C| = \begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[L_3 \leftarrow L_3 - L_2]{} \begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = +\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$|D| = \begin{vmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 \\ 2 & 1 & 0 & 6 \\ 1 & 1 & 1 & 7 \end{vmatrix} \begin{matrix} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_4 - L_1 \end{matrix} \begin{vmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 6 \\ 0 & -1 & 0 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 6 \\ 0 & -1 & 5 \end{vmatrix} = -2 \begin{vmatrix} 1 & 1 \\ -1 & -5 \end{vmatrix} = -12$$

$$|E| = \begin{vmatrix} 10 & 0 & -5 & 15 \\ -2 & 7 & 3 & 0 \\ 8 & 14 & 0 & 2 \\ 0 & -21 & 1 & -1 \end{vmatrix} = 2 \times 5 \begin{vmatrix} 2 & 0 & -1 & 3 \\ -2 & 7 & 3 & 0 \\ 4 & 7 & 0 & 1 \\ 0 & -21 & 1 & -1 \end{vmatrix} = 7 \times 2 \times 10 \begin{vmatrix} 1 & 0 & -1 & 3 \\ -1 & 1 & 3 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & -3 & 1 & -1 \end{vmatrix} \begin{matrix} L_1 \leftarrow L_1 + L_2 \\ L_3 \leftarrow L_3 + 2L_2 \end{matrix}$$

$$= 140 \begin{vmatrix} 0 & 1 & 2 & 3 \\ -1 & 1 & 3 & 0 \\ 0 & 3 & 6 & 1 \\ 0 & -3 & 1 & -1 \end{vmatrix} = -(-1) \times 140 \begin{vmatrix} 1 & 2 & 3 \\ 3 & 6 & 1 \\ -3 & 1 & -1 \end{vmatrix} L_3 \leftarrow L_3 + L_2 \;\; 140 \times \begin{vmatrix} 1 & 2 & 3 \\ 3 & 6 & 1 \\ 0 & 7 & 0 \end{vmatrix}$$

$$= -7 \times 140 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -7 \times (1-9) \times 140 = 7840 .$$

EXERCICE 40 :

$$\Delta = \begin{vmatrix} a+1 & a & a \\ b & b+1 & b \\ c & c & c+1 \end{vmatrix} C_1 - C_2 \begin{vmatrix} 1 & a & a \\ -1 & b+1 & b \\ 0 & c & c+1 \end{vmatrix} C_2 - C_3 \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & b \\ 0 & -1 & c+1 \end{vmatrix} L_1 \leftarrow L_2 + L_1$$

$$\begin{vmatrix} 1 & 0 & a \\ 0 & 1 & a+b \\ 0 & -1 & c+1 \end{vmatrix} L_3 \leftarrow L_3 + L_2 \begin{vmatrix} 1 & 0 & a \\ 0 & 1 & a+b \\ 0 & 0 & a+b+c+1 \end{vmatrix} = a+b+c+1$$

EXERCICE 41 : Soit $a, b \in \mathbb{R}$

$$\begin{vmatrix} a^2 & ab & ab & b^2 \\ ab & a^2 & b^2 & ab \\ ab & b^2 & a^2 & ab \\ b^2 & ab & ab & a^2 \end{vmatrix} L_1 \leftarrow L_1 + L_2 + L_3 + L_4 \begin{vmatrix} (a+b)^2 & (a+b)^2 & (a+b)^2 & (a+b)^2 \\ ab & a^2 & b^2 & ab \\ ab & b^2 & a^2 & ab \\ b^2 & ab & ab & a^2 \end{vmatrix} L_3 \leftarrow L_3 - L_2$$

$$(a+b)^2 \begin{vmatrix} 1 & 1 & 1 & 1 \\ ab & a^2 & b^2 & ab \\ 0 & b^2-a & a^2-b^2 & 0 \\ b^2 & ab & ab & a^2 \end{vmatrix} \underset{\downarrow}{C_2 \leftarrow C_2 + C_3} = (a+b)^2 \begin{vmatrix} 1 & 2 & 1 & 1 \\ ab & a^2+b^2 & b^2 & ab \\ 0 & 0 & a^2-b^2 & 0 \\ b^2 & 2ab & ab & a^2 \end{vmatrix} = (a+b)^2 (a^2-b^2) \begin{vmatrix} 1 & 2 & 1 \\ ab & a^2+b^2 & ab \\ b^2 & 2ab & a^2 \end{vmatrix}$$

$$C_3 = C_3 - C_1 \quad (a+b)^2 (a^2-b^2) \begin{vmatrix} 1 & 2 & 0 \\ ab & a^2+b^2 & 0 \\ b^2 & 2ab & a^2-b^2 \end{vmatrix} = (a+b)^2 (a^2-b^2) \begin{vmatrix} 1 & 2 \\ ab & a^2+b^2 \end{vmatrix}$$

$$= (a+b)^2 (a^2-b^2)^2 (a^2+b^2-2ab) = (a+b)^2 (a^2-b^2)^2 (a-b)^2$$

$$= (a+b)^2 (a-b)^2 (a+b)^2 (a-b)^2 = (a+b)^4 (a-b)^4$$

EXERCICE 42 :

$$D_n = \left.\begin{vmatrix} \overbrace{a & b & \cdots & b}^{n} \\ b & a & \ddots & \vdots \\ \vdots & \ddots & \ddots & b \\ b & \cdots & b & a \end{vmatrix}\right\} n$$

→ À la première colone on ajoute les autres colonnes :

$$D_n = \begin{vmatrix} a+(n-1)b & b & \cdots & b \\ \vdots & a & \ddots & \vdots \\ \vdots & b & \ddots & b \\ a+(n-1)b & b & \cdots\, b & a \end{vmatrix}$$

$$\Rightarrow D_n = a+(n-1)b \begin{vmatrix} 1 & b & \cdots & b \\ \vdots & a & \ddots & \vdots \\ \vdots & b & \ddots & b \\ 1 & b & \cdots\, b & a \end{vmatrix}$$

→ On soustrait la 1ère ligne à toutes les autres

$$\Rightarrow D_n = a+(n-1)b \begin{vmatrix} 1 & b-b & \cdots & b \\ 0 & a-b & 0 & 0 \\ \vdots & 0 & a-b & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots\, 0 & a-b \end{vmatrix} = (a+(n-1)b)(a-b)^{n-1}$$

triangulaire