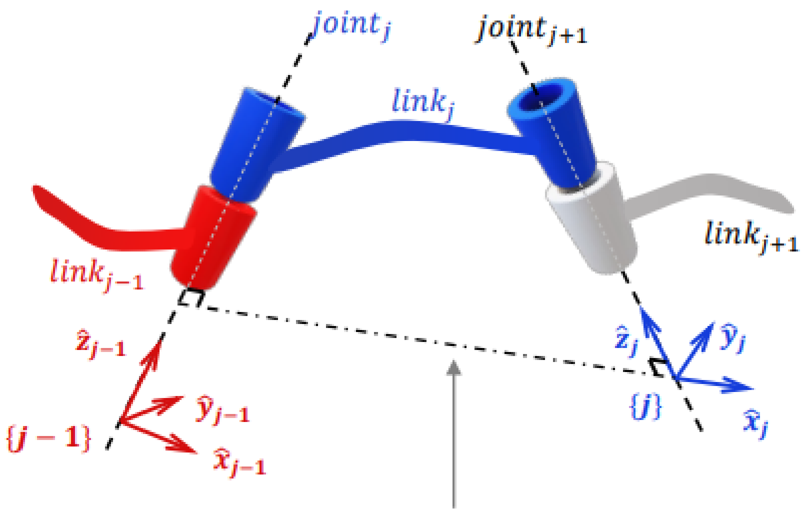
(Denavit-Hartenberg) DH Method

- Alternative to the POE method
- Probably less intuitive
- Frame is located distally to the link and not necessarily at the joint itself
- Only requires 4 parameters (POE requires 6) due to the constraints on how coordinate frames are to be assigned. These are θ_i, d_i, a_i and α_i.

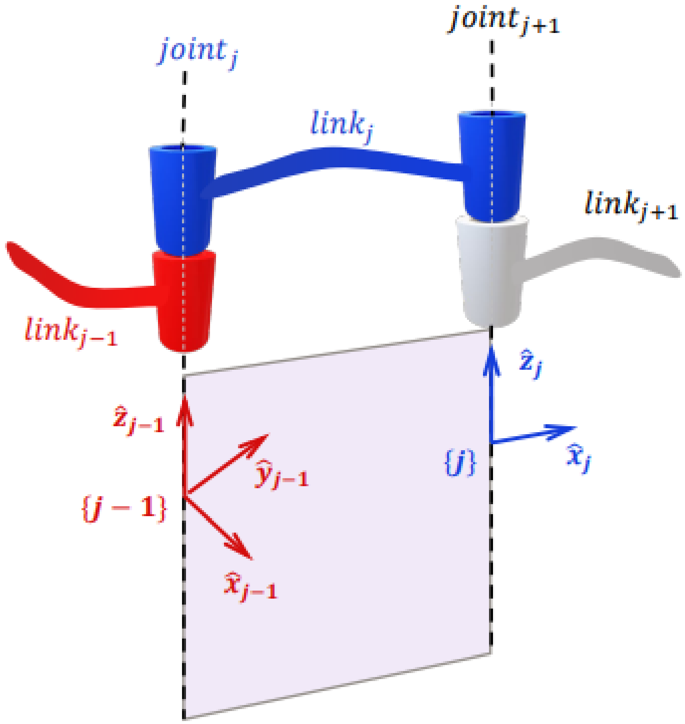
DH classical or **distal** convention - Assigning coordinate frames

$\hat{z}_{j-1}$ and $\hat{z}_j$ are not coplanar



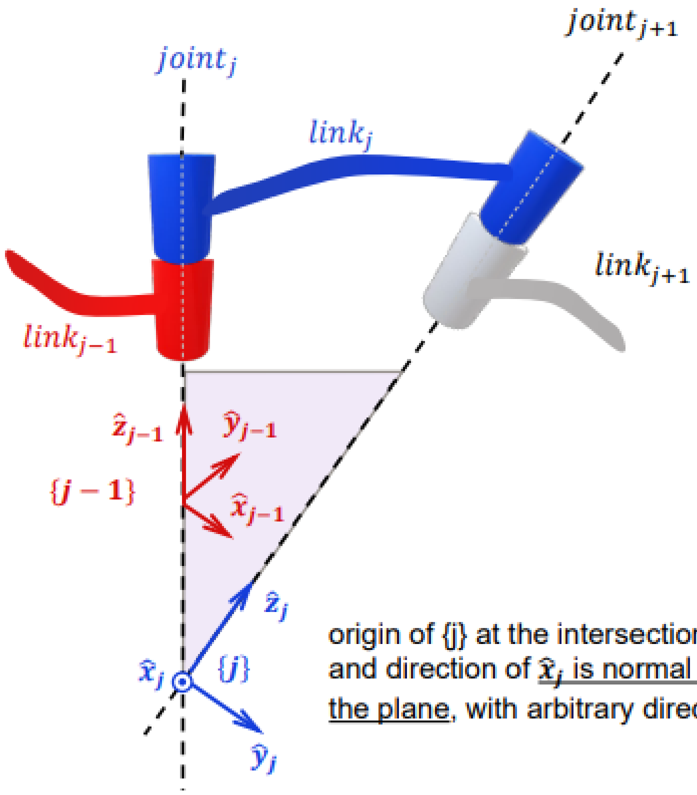
unique shortest line perpendicular to both $\hat{z}_{j-1}$ and $\hat{z}_j$, origin of {j} at the intersection, and $\hat{x}_j$ pointing along the line

$\hat{z}_{j-1}$ and $\hat{z}_j$ are parallel



origin of {j} anywhere along $\hat{z}_j$, and direction of $\hat{x}_j$ can be chosen to point towards or away from $\hat{z}_{j-1}$

$\hat{z}_{j-1}$ and $\hat{z}_j$ intersect

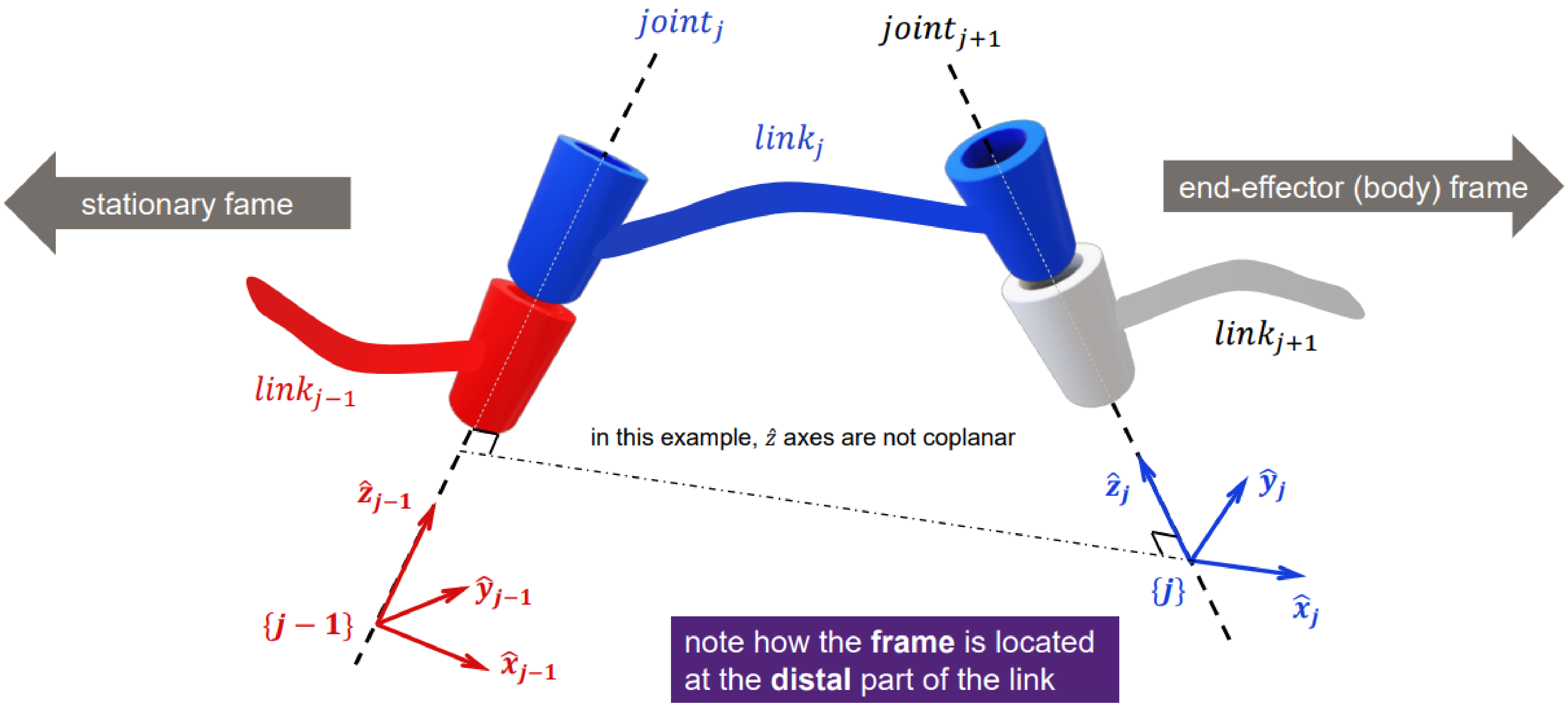


origin of {j} at the intersection, and direction of $\hat{x}_j$ is normal to the plane, with arbitrary direction

Recap:

Joints, j_i and j_{i+1}

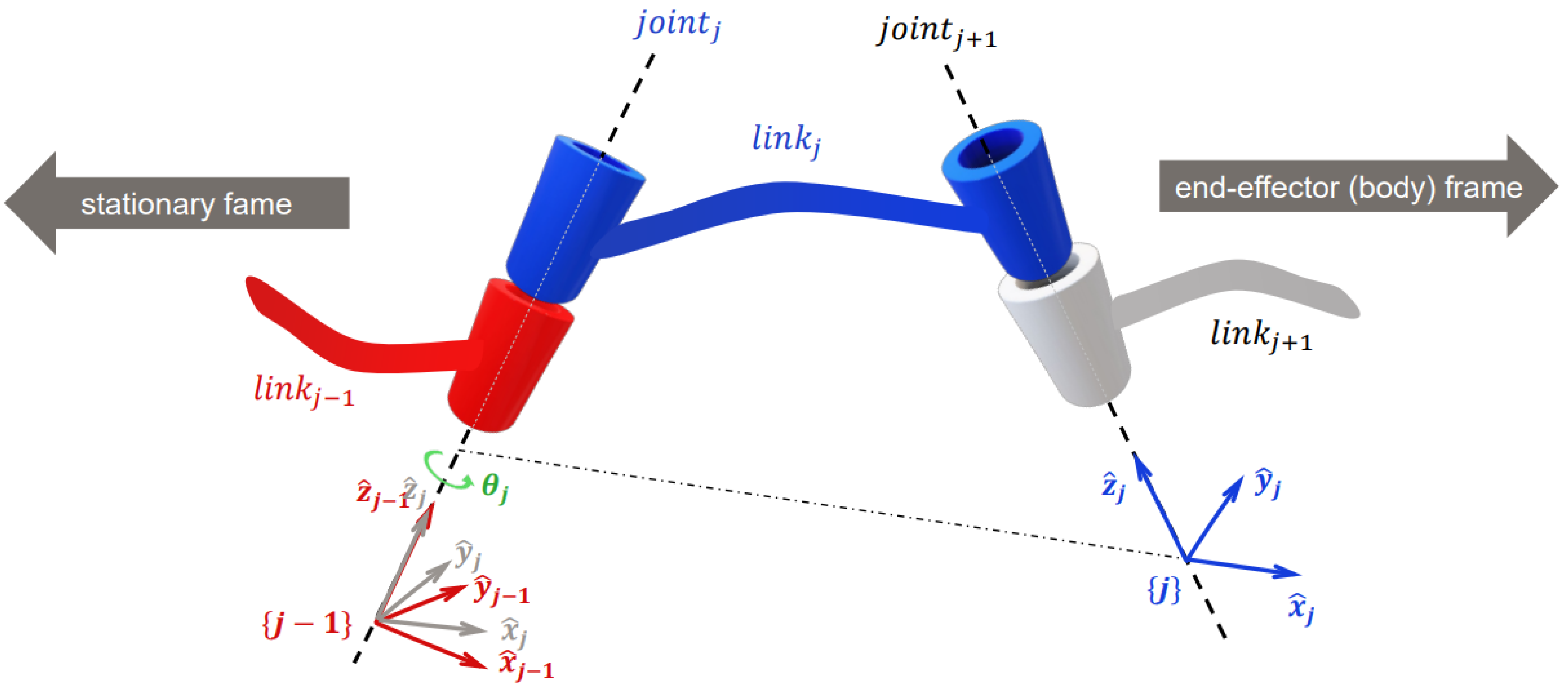
$$\boxed{T_j^{j-1} = A_j = Rot(\hat{z},\theta_j)Trans(\hat{z},d_j)\ Trans(\hat{x},a_j)\ Rot(\hat{x},\alpha_j)}$$



θ_j: Rotation about $\hat{z_{j-1}}$ to align j_{j+1}'s x axis

DH **classical** or **distal** convention

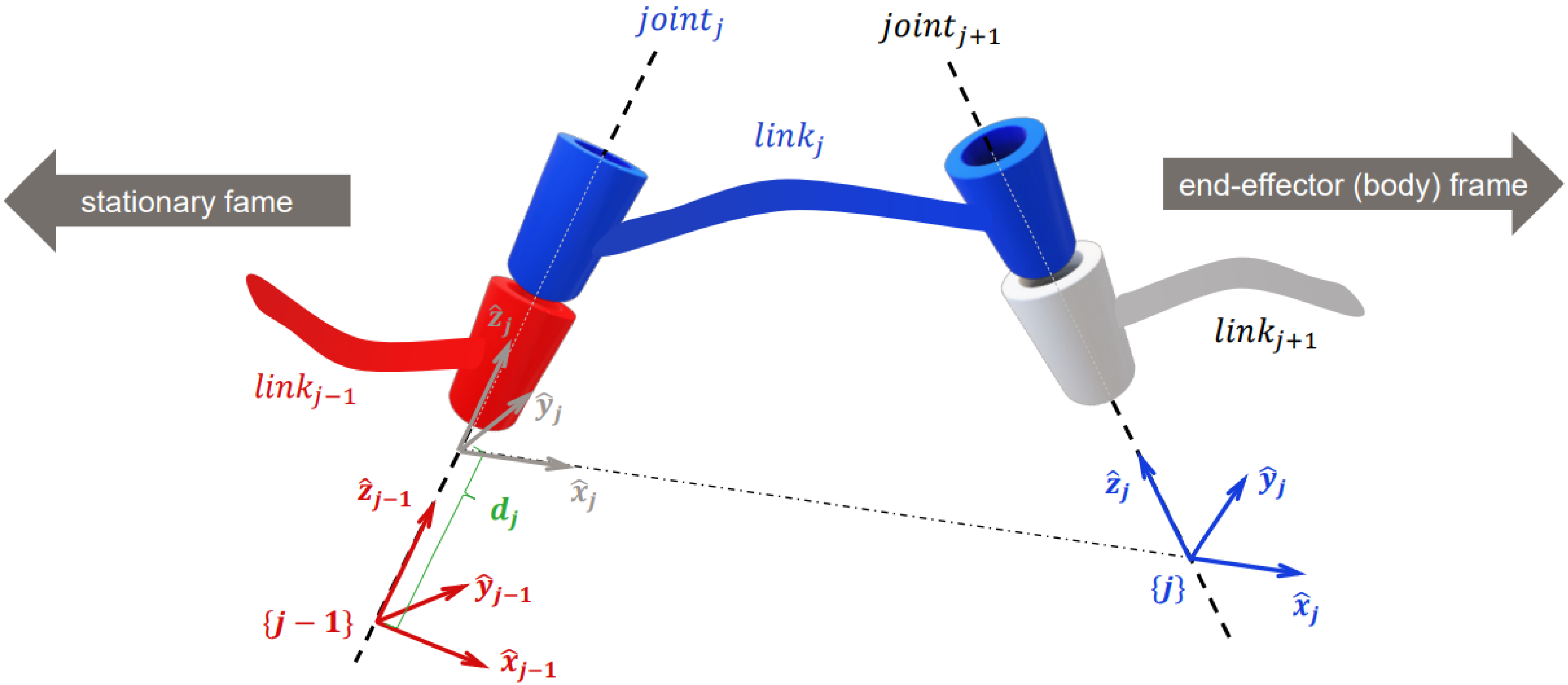
$$T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ Trans(\hat{x}, a_j)\ Rot(\hat{x}, \alpha_j)$$



d_j: Translation along $\hat{z_{j-1}}$ to align j_{j+1}'s x axis

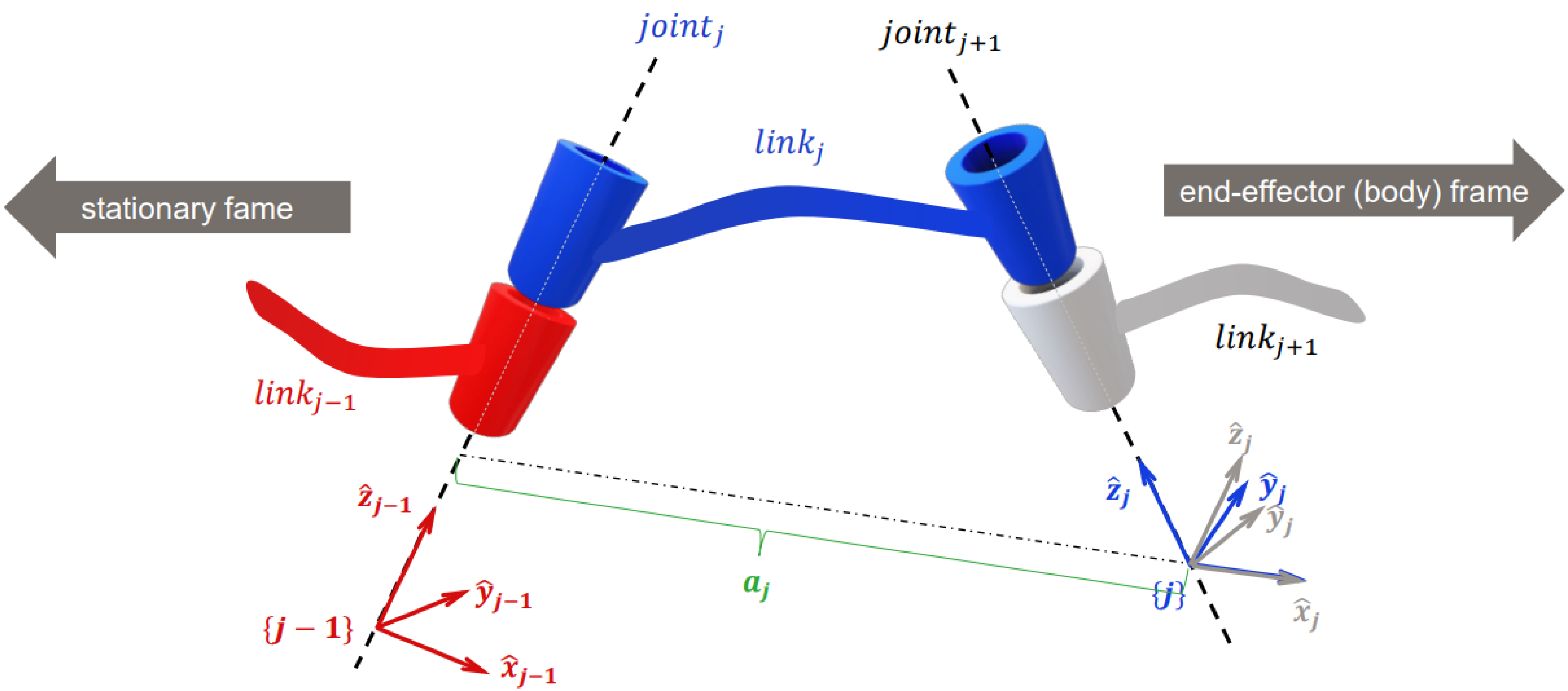
DH **classical** or **distal** convention

$T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ Trans(\hat{x}, a_j)\ Rot(\hat{x}, \alpha_j)$



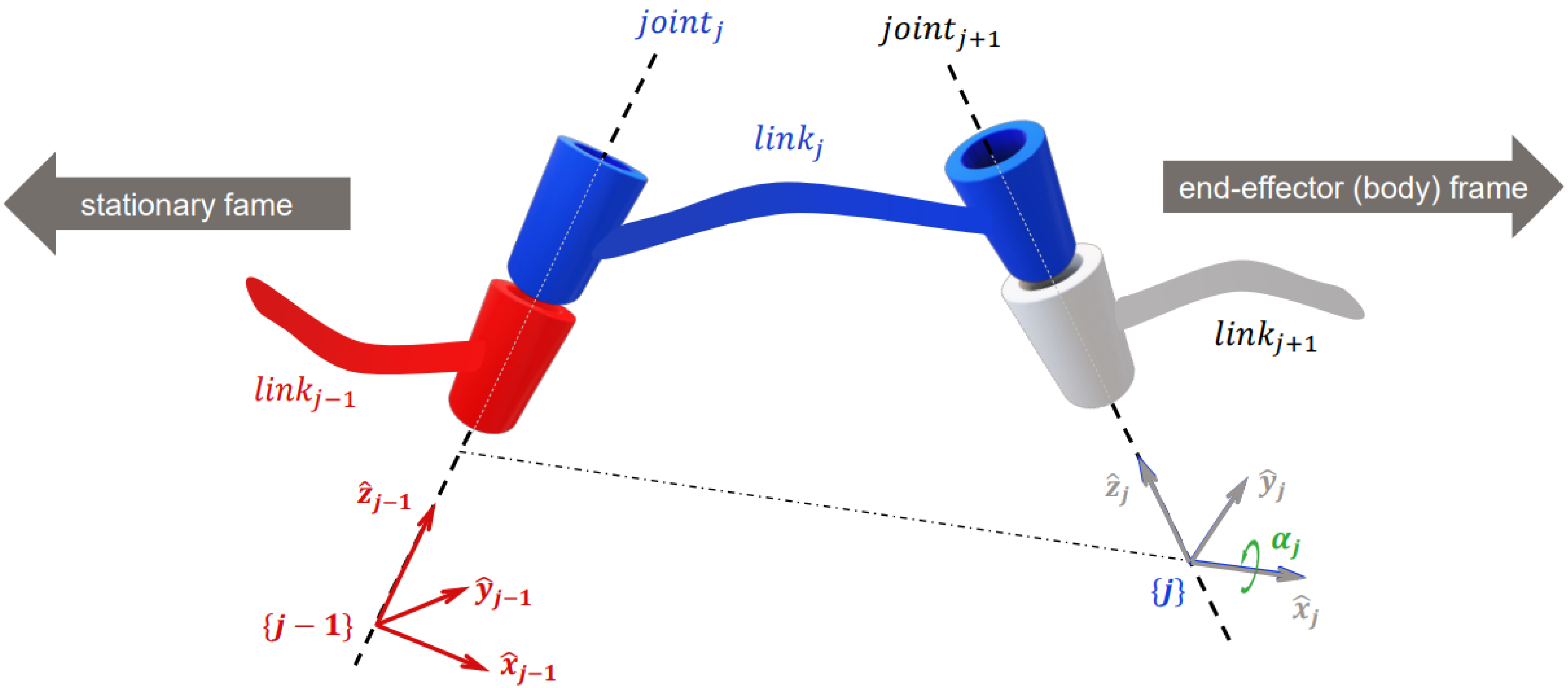
a_j: Translation along $\hat{x}_j$ to align j_{j+1}'s z axis

DH **classical** or **distal** convention $T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ \boldsymbol{Trans(\hat{x}, a_j)}\ Rot(\hat{x}, \alpha_j)$



α_j: Rotation about $\hat{x}_j$ to align j_{j+1}'s z axis

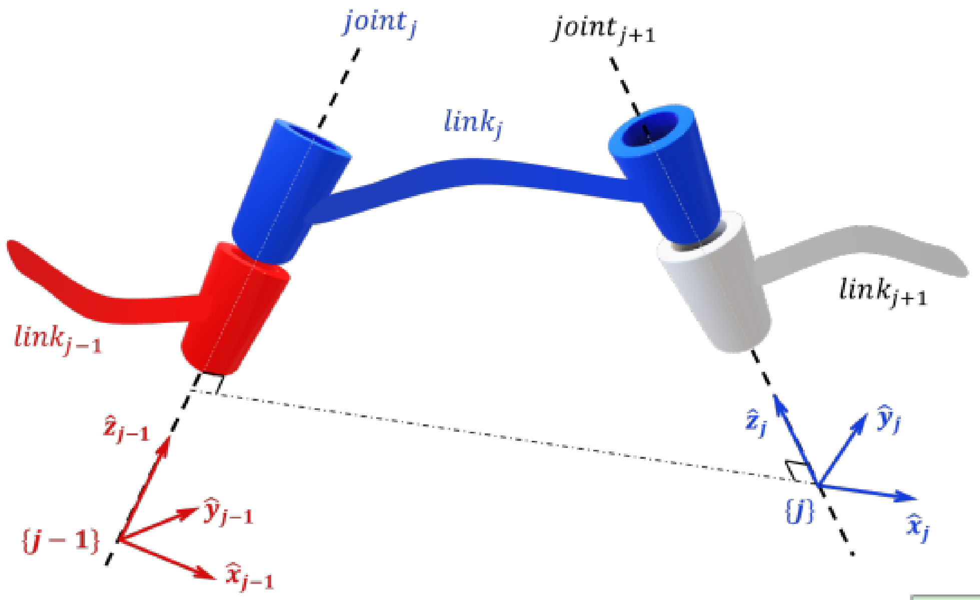
DH **classical** or **distal** convention $T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ Trans(\hat{x}, a_j)\ Rot(\hat{x}, \alpha_j)$



DH Summary

DH **classical** or **distal** convention

$$\boxed{T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ Trans(\hat{x}, a_j)\ Rot(\hat{x}, \alpha_j)}$$



	Name	Measured		About / Along
		From	To	
θ_j	joint angle (revolute joint variable)	x_{j-1}	x_j	z_{j-1}
d_j	link off-set (prismatic joint variable)	x_{j-1}	x_j	z_{j-1}
a_j	link length (constant)	z_{j-1}	z_j	x_j
α_j	twist angle (constant)	z_{j-1}	z_j	x_j

DH Summary: Transformation Matrix

DH **classical** or **distal** convention

$$\boxed{\boldsymbol{T_j^{j-1} = A_j = Rot(\hat{z}, \theta_j) Trans(\hat{z}, d_j)\ Trans(\hat{x}, a_j)\ Rot(\hat{x}, \alpha_j)}}$$

$$\begin{aligned}
A_i &= \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

*note that i=j in this notation

Spong, et al., Robot Modeling and Control

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Then, $T_i^0 = A_0 A_1 \ldots . A_{(i-1)} Ai$

Q2.2 PPPRRR Robot

Derive the **forward kinematic equations** for the following robot configuration using the **classical (distal) DH-convention.**

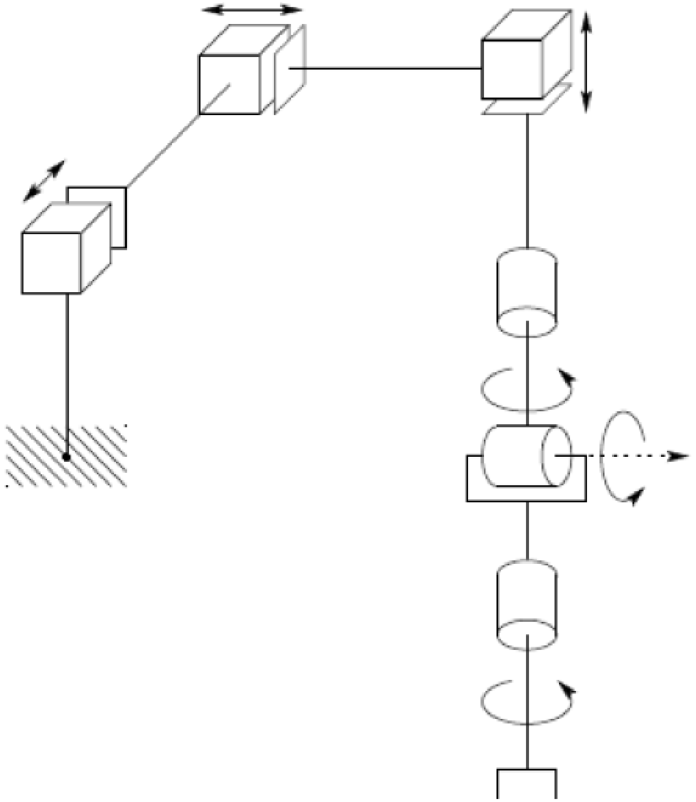

Figure 3 – PPPRRR Robot (exercise 2.1)
[Image source: Spong, et al. , Robot Modeling and Control, 2005]

Q2.3 Quanser QArm robot

Derive the **forward kinematic equations** for the following robot configuration using the **classical (distal) DH-convention.**

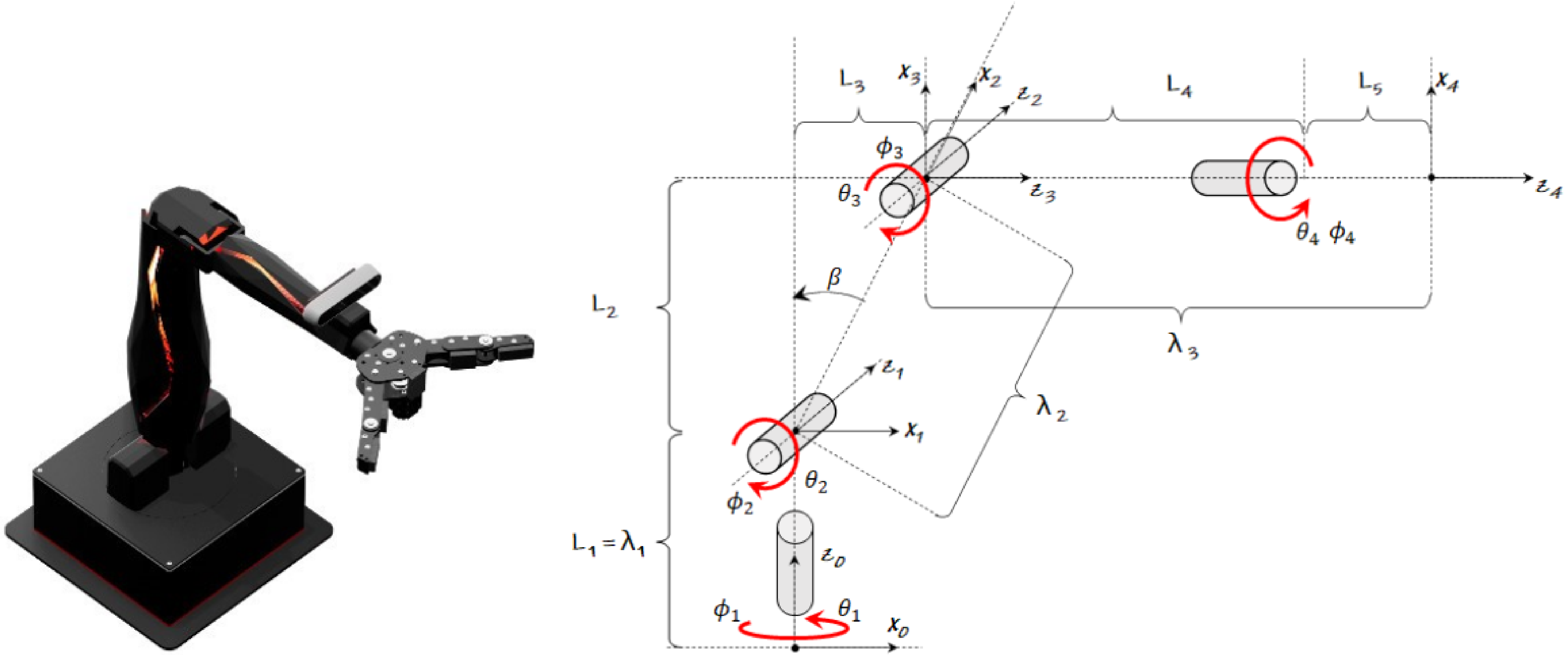

Figure 4 – 4DOF Quanser QArm Robot (exercise 2.3)