

Samples Determinant of the matrix SOLUTIONS

1. Since the matrix contains no zeroes we will expand about the first row.

$$\begin{aligned} & \begin{vmatrix} -9 & -5 & -7 \\ -3 & -8 & -4 \\ 5 & -4 & -3 \end{vmatrix} \\ &= -9 \times \begin{vmatrix} -8 & -4 \\ -4 & -3 \end{vmatrix} - \left(-5 \times \begin{vmatrix} -3 & -4 \\ 5 & -3 \end{vmatrix} \right) - 7 \times \begin{vmatrix} -3 & -8 \\ 5 & -4 \end{vmatrix} \\ &= -9(-8 \times (-3) - (-4 \times (-4))) - (-5(-3 \times (-3) - 5 \times (-4))) - 7(-3 \times (-4) - 5 \times (-8)) \\ &= -9(8) + 5(29) - 7(52) \\ &= -72 + 145 - 364 \\ &= -291 \end{aligned}$$

Since $|A|$ is non-zero the matrix is non-singular and hence invertible.

2. Since the second row contains one zero we will expand about that row.

$$\begin{aligned} & \begin{vmatrix} -2 & 3 & 6 \\ 8 & 0 & 3 \\ -1 & -2 & 9 \end{vmatrix} \\ &= -8 \times \begin{vmatrix} 3 & 6 \\ -2 & 9 \end{vmatrix} + 0 \times \begin{vmatrix} -2 & 6 \\ -1 & 9 \end{vmatrix} - 3 \times \begin{vmatrix} -2 & 3 \\ -1 & -2 \end{vmatrix} \\ &= -8(3 \times 9 - (-2 \times 6)) + 0 - 3(-2 \times (-2) - (-1 \times 3)) \\ &= -8(39) - 3(7) \\ &= -312 - 21 \\ &= -333 \end{aligned}$$

Since $|A|$ is non-zero the matrix is non-singular and hence invertible.

3. Since the third row contains one zero we will expand about that row.

$$\begin{aligned} & \begin{vmatrix} -4 & 5 & -4 \\ 7 & -1 & -7 \\ 5 & 0 & 3 \end{vmatrix} \\ &= 5 \times \begin{vmatrix} -4 & -4 \\ 7 & -7 \end{vmatrix} - 0 \times \begin{vmatrix} -4 & -4 \\ 7 & -7 \end{vmatrix} + 3 \times \begin{vmatrix} -4 & 5 \\ 7 & -1 \end{vmatrix} \\ &= 5(5 \times (-7) - (-1 \times (-4))) - 0 + 3(-4 \times (-1) - 7 \times 5) \\ &= 5(-39) + 3(-31) \\ &= -195 - 93 \\ &= -288 \end{aligned}$$

Since $|A|$ is non-zero the matrix is non-singular and hence invertible.

4. Since the matrix contains no zeroes we will expand about the first row.

$$\begin{aligned}
 & \begin{vmatrix} 8 & -5 & 4 \\ -6 & -8 & 5 \\ 4 & 2 & 9 \end{vmatrix} \\
 &= 8 \times \begin{vmatrix} -8 & 5 \\ 2 & 9 \end{vmatrix} - \left(-5 \times \begin{vmatrix} -6 & 5 \\ 4 & 9 \end{vmatrix} \right) + 4 \times \begin{vmatrix} -6 & -8 \\ 4 & 2 \end{vmatrix} \\
 &= 8(-8 \times 9 - 2 \times 5) - (-5(-6 \times 9 - 4 \times 5)) + 4(-6 \times 2 - 4 \times (-8)) \\
 &= 8(-82) + 5(-74) + 4(20) \\
 &= -656 - 370 + 80 \\
 &= -946
 \end{aligned}$$

Since $|A|$ is non-zero the matrix is non-singular and hence invertible.

5. Since the matrix contains no zeroes we will expand about the first row.

$$\begin{aligned}
 & \begin{vmatrix} 8 & -6 & 1 \\ -6 & -3 & -3 \\ -7 & -1 & -1 \end{vmatrix} \\
 &= 8 \times \begin{vmatrix} -3 & -3 \\ -1 & -1 \end{vmatrix} - \left(-6 \times \begin{vmatrix} -6 & -3 \\ -7 & -1 \end{vmatrix} \right) + 1 \times \begin{vmatrix} -6 & -3 \\ -7 & -1 \end{vmatrix} \\
 &= 8(-3 \times (-1) - (-1 \times (-3))) - (-6(-6 \times (-1) - (-7 \times (-3)))) - 6 \times (-1) - (-7 \times (-3)) \\
 &= 8(0) + 6(-15) + (-15) \\
 &= -90 - 15 \\
 &= -105
 \end{aligned}$$

Since $|A|$ is non-zero the matrix is non-singular and hence invertible.