

Let $\mathbf{u} = \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

(a) Determine $\mathbf{u} \cdot \mathbf{v}$.

(b) Determine the angle in degrees (to the nearest degree) between \mathbf{u} and \mathbf{v} .

Solution

(a) $\mathbf{u} \cdot \mathbf{v} = 5 \cdot 2 + (-1) \cdot (-1) + (-3) \cdot 3$
 $= 10 + 1 - 9$
 $= 2$

(b) From (a) $\mathbf{u} \cdot \mathbf{v} = 2$.

$$\|\mathbf{u}\| = \sqrt{5^2 + (-1)^2 + (-3)^2}$$

$$= \sqrt{35}$$

$$\|\mathbf{v}\| = \sqrt{2^2 + (-1)^2 + 3^2}$$

$$= \sqrt{14}$$

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

$$\text{So } 2 = \sqrt{35} \sqrt{14} \cos \theta.$$

$$\text{Therefore } \theta = \arccos \frac{2}{\sqrt{35} \sqrt{14}}$$

$$\text{So } \theta \approx 78^\circ$$