Samples **Matrix Arithmetic SOLUTIONS**

1. (a)

$$
\begin{aligned}
&\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix} \\
=&\begin{pmatrix} 5\times(-28)+7\times 45+9\times(-19) & 5\times 44+7\times(-70)+9\times 30 & 5\times 20+7\times(-31)+9\times 13 \\ 1\times(-28)+4\times 45+8\times(-19) & 1\times 44+4\times(-70)+8\times 30 & 1\times 20+4\times(-31)+8\times 13 \\ 5\times(-28)+1\times 45-5\times(-19) & 5\times 44+1\times(-70)-5\times 30 & 5\times 20+1\times(-31)-5\times 13 \end{pmatrix} \\
=&\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}
\end{aligned}
$$

(b) From Part (a) we have that

$$\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix} = 4\mathbf{I}.$$

After a slight rearrangement we have that

$$\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}\times\frac{1}{4}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix} = \mathbf{I}.$$

The above allows us to deduce that

$$\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}^{-1} = \frac{1}{4}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix}.$$

(c) The set of simultaneous equations can be written in matrix form as

$$\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \\ 8 \end{pmatrix}.$$

From Part (b) we know that

$$\begin{pmatrix} 5 & 7 & 9 \\ 1 & 4 & 8 \\ 5 & 1 & -5 \end{pmatrix}^{-1} = \frac{1}{4}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix}.$$

So,

$$
\begin{aligned}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \frac{1}{4}\begin{pmatrix} -28 & 44 & 20 \\ 45 & -70 & -31 \\ -19 & 30 & 13 \end{pmatrix}\begin{pmatrix} -1 \\ 6 \\ 8 \end{pmatrix} \\
&= \frac{1}{4}\begin{pmatrix} -28\times(-1)+44\times 6+20\times 8 \\ 45\times(-1)-70\times 6-31\times 8 \\ -19\times(-1)+30\times 6+13\times 8 \end{pmatrix} \\
&= \frac{1}{4}\begin{pmatrix} 452 \\ -713 \\ 303 \end{pmatrix}.
\end{aligned}
$$

So the solution to the simultaneous equations is $x = 113$, $y = \dfrac{-713}{4}$ and $z = \dfrac{303}{4}$.

2. (a)

$$
\begin{aligned}
&\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix} \\
= &\begin{pmatrix} -8\times 9-3\times 20-4\times(-32) & -8\times(-17)-3\times(-40)-4\times 64 & -8\times 14-3\times 32-4\times(-52) \\ -4\times 9+5\times 20+2\times(-32) & -4\times(-17)+5\times(-40)+2\times 64 & -4\times 14+5\times 32+2\times(-52) \\ 0\times 9+8\times 20+5\times(-32) & 0\times(-17)+8\times(-40)+5\times 64 & 0\times 14+8\times 32+5\times(-52) \end{pmatrix} \\
= &\begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}
\end{aligned}
$$

(b) From Part (a) we have that

$$
\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix} = -4\mathbf{I}.
$$

After a slight rearrangement we have that

$$
\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix} \times \frac{1}{-4}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix} = \mathbf{I}.
$$

The above allows us to deduce that

$$
\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix}^{-1} = \frac{1}{-4}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix}.
$$

(c) The set of simultaneous equations can be written in matrix form as

$$
\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ -6 \end{pmatrix}.
$$

From Part (b) we know that

$$\begin{pmatrix} -8 & -3 & -4 \\ -4 & 5 & 2 \\ 0 & 8 & 5 \end{pmatrix}^{-1} = \frac{1}{-4}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix}.$$

So,

$$\begin{aligned}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \frac{1}{-4}\begin{pmatrix} 9 & -17 & 14 \\ 20 & -40 & 32 \\ -32 & 64 & -52 \end{pmatrix}\begin{pmatrix} -3 \\ -6 \\ -6 \end{pmatrix} \\
&= \frac{1}{-4}\begin{pmatrix} 9\times(-3) - 17\times(-6) + 14\times(-6) \\ 20\times(-3) - 40\times(-6) + 32\times(-6) \\ -32\times(-3) + 64\times(-6) - 52\times(-6) \end{pmatrix} \\
&= \frac{1}{-4}\begin{pmatrix} -9 \\ -12 \\ 24 \end{pmatrix}.
\end{aligned}$$

So the solution to the simultaneous equations is $x = \frac{9}{4}$, $y = -3$ and $z = 6$.

3. (a)

$$\begin{aligned}
&\begin{pmatrix} 2 & 3 & -1 \\ 4 & 7 & 8 \\ -5 & -7 & 8 \end{pmatrix}\begin{pmatrix} 112 & -17 & 31 \\ -72 & 11 & -20 \\ 7 & -1 & 2 \end{pmatrix} \\
= &\begin{pmatrix} 2\times 112 + 3\times(-72) - 1\times 7 & 2\times(-17) + 3\times 11 - 1\times(-1) & 2\times 31 + 3\times(-20) - 1\times 2 \\ 4\times 112 + 7\times(-72) + 8\times 7 & 4\times(-17) + 7\times 11 + 8\times(-1) & 4\times 31 + 7\times(-20) + 8\times 2 \\ -5\times 112 - 7\times(-72) + 8\times 7 & -5\times(-17) - 7\times 11 + 8\times(-1) & -5\times 31 - 7\times(-20) + 8\times 2 \end{pmatrix} \\
= &\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

(b) From Part (a) we have that

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 7 & 8 \\ -5 & -7 & 8 \end{pmatrix}\begin{pmatrix} 112 & -17 & 31 \\ -72 & 11 & -20 \\ 7 & -1 & 2 \end{pmatrix} = \mathbf{I}.$$

The above allows us to deduce that

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 7 & 8 \\ -5 & -7 & 8 \end{pmatrix}^{-1} = \begin{pmatrix} 112 & -17 & 31 \\ -72 & 11 & -20 \\ 7 & -1 & 2 \end{pmatrix}.$$

(c) The set of simultaneous equations can be written in matrix form as

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 7 & 8 \\ -5 & -7 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix}.$$

From Part (b) we know that

$$\begin{pmatrix} 2 & 3 & -1 \\ 4 & 7 & 8 \\ -5 & -7 & 8 \end{pmatrix}^{-1} = \begin{pmatrix} 112 & -17 & 31 \\ -72 & 11 & -20 \\ 7 & -1 & 2 \end{pmatrix}.$$

So,

$$\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 112 & -17 & 31 \\ -72 & 11 & -20 \\ 7 & -1 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix} \\ &= \begin{pmatrix} 112 \times 7 - 17 \times 7 + 31 \times 8 \\ -72 \times 7 + 11 \times 7 - 20 \times 8 \\ 7 \times 7 - 1 \times 7 + 2 \times 8 \end{pmatrix} \\ &= \begin{pmatrix} 913 \\ -587 \\ 58 \end{pmatrix}. \end{aligned}$$

So the solution to the simultaneous equations is $x = 913$, $y = -587$ and $z = 58$.