## Samples Simultaneous equations SOLUTIONS

**1.** Let $z = \tan y$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} -9x - 9z &= -81 \qquad (1) \\ 4x - 2z &= 36 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 2 and equation (2) by $-9$ , giving

$$\begin{aligned} -18x - 18z &= -162 \qquad (3) \\ -36x + 18z &= -324 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-36x - 18x + 18z - 18z = -324 - 162 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -54x &= -486 \qquad (6) \\ x &= 9 \qquad (7) \end{aligned}$$

Next we substitute the value for $x$ into equation (1) to obtain the value for $z$, giving

$$\begin{aligned} -9 \times 9 - 9z &= -81 \\ -9z &= 0 \qquad \text{so} \\ z &= 0 \end{aligned}$$

Now we can find the value of $y$: $\tan y = 0$, so $y = 0; \pi$

Hence the simultaneous solution to equations (1) and (2) is $x = 9$; $y = 0; \pi$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1) \quad -9 \times 9 - 9 \times \tan 0 &= -81 \\ -9 \times 9 - 9 \times 0 &= -81 \\ -81 &= -81 \end{aligned}$$

$$\begin{aligned} (2) \quad 4 \times 9 - 2 \times \tan 0 &= 36 \\ 4 \times 9 - 2 \times 0 &= 36 \\ 36 &= 36 \end{aligned}$$

We have checked one value of $y$, you do the other! )

**2.** Let $z = \ln x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} 13z + 4y &= 0 \qquad (1) \\ -5z + 7y &= 0 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by $-7$ and equation (2) by 4 , giving

$$\begin{aligned} -91z - 28y &= 0 \qquad (3) \\ -20z + 28y &= 0 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-20z - 91z + 28y - 28y = 0 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -111z &= 0 \qquad (6) \\ z &= 0 \qquad (7) \end{aligned}$$

Next we substitute the value for $z$ into equation (1) to obtain the value for $y$, giving

$$\begin{aligned} 13 \times 0 + 4y &= 0 \\ 4y &= 0 \qquad \text{so} \\ y &= 0 \end{aligned}$$

Now we can find the value of $x$: $\ln x = 0$, so $x = 1$

Hence the simultaneous solution to equations (1) and (2) is $x = 1$; $y = 0$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 13 \times \ln 1 + 4 \times 0 &= 0 \\ 13 \times 0 + 4 \times 0 &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} (2)\ \ -5 \times \ln 1 + 7 \times 0 &= 0 \\ -5 \times 0 + 7 \times 0 &= 0 \\ 0 &= 0 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

**3.** Let $z = \sin x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} 13z + 6y &= -18 \qquad (1) \\ -3z - 9y &= 27 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (1) by 3 and equation (2) by 2 , giving

$$\begin{aligned} 39z + 18y &= -54 \qquad (3) \\ -6z - 18y &= 54 \qquad (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-6z + 39z - 18y + 18y = 54 - 54 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} 33z &= 0 \qquad (6) \\ z &= 0 \qquad (7) \end{aligned}$$

Next we substitute the value for $z$ into equation (1) to obtain the value for $y$, giving

$$\begin{aligned} 13 \times 0 + 6y &= -18 \\ 6y &= -18 \qquad \text{so} \\ y &= -3 \end{aligned}$$

Now we can find the value of $x$: $\sin x = 0$, so $x = 0; \pi$

Hence the simultaneous solution to equations (1) and (2) is $x = 0; \pi$; $y = -3$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ \ 13 \times \sin 0 + 6 \times (-3) &= -18 \\ 13 \times 0 + 6 \times (-3) &= -18 \\ -18 &= -18 \end{aligned} \qquad \begin{aligned} (2)\ \ -3 \times \sin 0 - 9 \times (-3) &= 27 \\ -3 \times 0 - 9 \times (-3) &= 27 \\ 27 &= 27 \end{aligned}$$

We have checked one value of $x$, you do the other! )

**4.** Let $z = \sqrt{x}$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} 5z - 9y &= 32 \qquad (1) \\ 5z + 2y &= -1 \qquad (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (2) by $-1$ , giving

$$\begin{aligned} 5z - 9y &= 32 && (3) \\ -5z - 2y &= 1 && (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$-5z + 5z - 2y - 9y = 1 + 32 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -11y &= 33 && (6) \\ y &= -3 && (7) \end{aligned}$$

Next we substitute the value for $y$ into equation (1) to obtain the value for $z$, giving

$$\begin{aligned} 5z - 9 \times (-3) &= 32 \\ 5z &= 5 \qquad \text{so} \\ z &= 1 \end{aligned}$$

Now we can find the value of $x$: $\sqrt{x} = 1$, so $x = 1$

Hence the simultaneous solution to equations (1) and (2) is $x = 1$; $y = -3$.

(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$\begin{aligned} (1)\ 5 \times \sqrt{1} - 9 \times (-3) &= 32 \\ 5 \times 1 - 9 \times (-3) &= 32 \\ 5 + 27 &= 32 \\ 32 &= 32 \end{aligned}$$

$$\begin{aligned} (2)\ 5 \times \sqrt{1} + 2 \times (-3) &= -1 \\ 5 \times 1 + 2 \times (-3) &= -1 \\ 5 - 6 &= -1 \\ -1 &= -1 \end{aligned}$$

Both equations turned into true statements, as required. Hence the answer is correct.)

**5.** Let $z = \tan x$. Now we have two linear simultaneous equations, which we also number for convenience:

$$\begin{aligned} -12z + 3y &= 18 && (1) \\ -4z + 3y &= 26 && (2) \end{aligned}$$

It's probably easier to solve these using elimination. Multiply equation (2) by $-1$ , giving

$$\begin{aligned} -12z + 3y &= 18 && (3) \\ 4z - 3y &= -26 && (4) \end{aligned}$$

We add both sides of equations (3) and (4) , giving

$$4z - 12z - 3y + 3y = -26 + 18 \qquad (5)$$

Simplifying equation (5) gives

$$\begin{aligned} -8z &= -8 && (6) \\ z &= 1 && (7) \end{aligned}$$

Next we substitute the value for $z$ into equation (1) to obtain the value for $y$, giving

$$\begin{aligned} -12 \times 1 + 3y &= 18 \\ 3y &= 30 \qquad \text{so} \\ y &= 10 \end{aligned}$$

Now we can find the value of $x$: $\tan x = 1$, so $x = \frac{\pi}{4}; \frac{5\pi}{4}$

Hence the simultaneous solution to equations (1) and (2) is $x = \frac{\pi}{4}; \frac{5\pi}{4}$; $y = 10$.
(As good boys and girls always do, check your answers by substituting into equations (1) and (2):

$$
\begin{aligned}
(1) \quad -12 \times \tan\frac{\pi}{4} + 3 \times 10 &= 18 \\
-12 \times 1 + 3 \times 10 &= 18 \\
-12 + 30 &= 18 \\
18 &= 18
\end{aligned}
\qquad\qquad
\begin{aligned}
(2) \quad -4 \times \tan\frac{\pi}{4} + 3 \times 10 &= 26 \\
-4 \times 1 + 3 \times 10 &= 26 \\
-4 + 30 &= 26 \\
26 &= 26
\end{aligned}
$$

We have checked one value of $x$, you do the other! )