Samples **Probabilities SOLUTIONS**

1. **(1)** $Prob\,(s_1 \text{ is odd}) = \dfrac{1}{2}$

(2) $Prob\,(s_1 = 7) = \dfrac{1}{2}$

(3) $Prob\,(s_1 \geq 6) = \dfrac{2}{2} = 1$

(4) $Prob\,(s_1 \text{ is odd and } s_1 \geq 6) = \dfrac{1}{2}$

(5) $Prob\,(s_1 \text{ is odd or } s_1 \geq 6) = \dfrac{2}{2} = 1$

(6) $Prob\,(s_1 \text{ is odd given that } s_1 \geq 6) = \dfrac{1}{2}$

(7) $Prob\,(s_1 \text{ is odd}) = \dfrac{1}{2}$, and $Prob\,(s_2 \text{ is odd}) = \dfrac{1}{2}$.
Now s_1 and s_2 are chosen independently,
so $Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are odd }) = Prob\,(s_1 \text{ is odd}) \times Prob\,(s_2 \text{ is odd})$.
Hence $Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are odd }) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

(8) By the principle of inclusion\exclusion,
$Prob\,(s_1 \text{ is odd } \textbf{or } s_2 \text{ is odd}) = Prob\,(s_1 \text{ is odd}) + Prob\,(s_2 \text{ is odd}) - Prob\,(\text{both } s_1 \text{ and } s_2 \text{ are odd })$.
Hence $Prob\,(s_1 \text{ is odd } \textbf{or } s_2 \text{ is odd}) = \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{4} = \dfrac{3}{4}$

(9) Now s_1 and s_2 are chosen independently, so
$Prob\,(s_1 \text{ is odd } \textbf{given that } s_2 \text{ is even }) = Prob\,(s_1 \text{ is odd})$.
Hence $Prob\,(s_1 \text{ is odd } \textbf{given that } s_2 \text{ is even }) = \dfrac{1}{2}$

2. **(1)** $Prob\,(r_1 \text{ is odd}) = \dfrac{4}{8} = \dfrac{1}{2}$

(2) $Prob\,(r_1 = 6) = \dfrac{1}{8}$

(3) $Prob\,(r_1 > 6) = \dfrac{2}{8} = \dfrac{1}{4}$

(4) $Prob\,(r_1 \text{ is odd and } r_1 > 6) = \dfrac{1}{8}$

(5) $Prob\,(r_1 \text{ is odd or } r_1 > 6) = \dfrac{5}{8}$

(6) $Prob\,(r_1 \text{ is odd given that } r_1 > 6) = \dfrac{1}{2}$

(7) $Prob\,(r_1 \text{ is odd}) = \dfrac{1}{2}$, and $Prob\,(r_2 \text{ is odd}) = \dfrac{1}{2}$.
Now r_1 and r_2 are chosen independently,
so $Prob\,(\text{both } r_1 \text{ and } r_2 \text{ are odd }) = Prob\,(r_1 \text{ is odd}) \times Prob\,(r_2 \text{ is odd})$.

Hence $Prob\,(\text{both } r_1 \text{ and } r_2 \text{ are odd }) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(8) By the principle of inclusion\exclusion,
$Prob\,(r_1 \text{ is odd } \textbf{or } r_2 \text{ is odd}) = Prob\,(r_1 \text{ is odd}) + Prob\,(r_2 \text{ is odd}) - Prob\,(\text{both } r_1 \text{ and } r_2 \text{ are odd })$.
Hence $Prob\,(r_1 \text{ is odd } \textbf{or } r_2 \text{ is odd}) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

(9) Now r_1 and r_2 are chosen independently, so
$Prob\,(r_1 \text{ is odd } \textbf{given that } r_2 \text{ is even }) = Prob\,(r_1 \text{ is odd})$.
Hence $Prob\,(r_1 \text{ is odd } \textbf{given that } r_2 \text{ is even }) = \frac{1}{2}$

3. **(1)** $Prob\,(t_1 \text{ is even}) = \frac{3}{7}$

(2) $Prob\,(t_1 = 7) = \frac{1}{7}$

(3) $Prob\,(t_1 \leq 4) = \frac{4}{7}$

(4) $Prob\,(t_1 \text{ is even and } t_1 \leq 4) = \frac{2}{7}$

(5) $Prob\,(t_1 \text{ is even or } t_1 \leq 4) = \frac{5}{7}$

(6) $Prob\,(t_1 \text{ is even given that } t_1 \leq 4) = \frac{2}{4} = \frac{1}{2}$

(7) $Prob\,(t_1 \text{ is even}) = \frac{3}{7}$, and $Prob\,(t_2 \text{ is even}) = \frac{1}{3}$.
Now t_1 and t_2 are chosen independently,
so $Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are even }) = Prob\,(t_1 \text{ is even}) \times Prob\,(t_2 \text{ is even})$.
Hence $Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are even }) = \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$

(8) By the principle of inclusion\exclusion,
$Prob\,(t_1 \text{ is even } \textbf{or } t_2 \text{ is even}) = Prob\,(t_1 \text{ is even}) + Prob\,(t_2 \text{ is even}) - Prob\,(\text{both } t_1 \text{ and } t_2 \text{ are even })$.
Hence $Prob\,(t_1 \text{ is even } \textbf{or } t_2 \text{ is even}) = \frac{3}{7} + \frac{1}{3} - \frac{1}{7} = \frac{13}{21}$

(9) Now t_1 and t_2 are chosen independently, so
$Prob\,(t_1 \text{ is even } \textbf{given that } t_2 \text{ is even }) = Prob\,(t_1 \text{ is even})$.
Hence $Prob\,(t_1 \text{ is even } \textbf{given that } t_2 \text{ is even }) = \frac{3}{7}$