**1.** (a) Because $Y = \begin{pmatrix} -9 & -3 & 5 \\ 3 & 8 & -5 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} -9 & 3 \\ -3 & 8 \\ 5 & -5 \end{pmatrix}$.

Hence, $X = \begin{pmatrix} -9 & 3 \\ -3 & 8 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} -9 & -3 & 5 \\ 3 & 8 & -5 \end{pmatrix}$

$= \begin{pmatrix} -9 \times (-9) + 3 \times 3 & -9 \times (-3) + 3 \times 8 & -9 \times 5 + 3 \times (-5) \\ -3 \times (-9) + 8 \times 3 & -3 \times (-3) + 8 \times 8 & -3 \times 5 + 8 \times (-5) \\ 5 \times (-9) - 5 \times 3 & 5 \times (-3) - 5 \times 8 & 5 \times 5 - 5 \times (-5) \end{pmatrix}$

$= \begin{pmatrix} 90 & 51 & -60 \\ 51 & 73 & -55 \\ -60 & -55 & 50 \end{pmatrix}$.

(b) Because $Y = \begin{pmatrix} 7 & -4 \\ -2 & -3 \\ -8 & 4 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 7 & -2 & -8 \\ -4 & -3 & 4 \end{pmatrix}$.

Hence, $X = \begin{pmatrix} 7 & -2 & -8 \\ -4 & -3 & 4 \end{pmatrix} \begin{pmatrix} 7 & -4 \\ -2 & -3 \\ -8 & 4 \end{pmatrix}$

$= \begin{pmatrix} 7 \times 7 - 2 \times (-2) - 8 \times (-8) & 7 \times (-4) - 2 \times (-3) - 8 \times 4 \\ -4 \times 7 - 3 \times (-2) + 4 \times (-8) & -4 \times (-4) - 3 \times (-3) + 4 \times 4 \end{pmatrix}$

$= \begin{pmatrix} 117 & -54 \\ -54 & 41 \end{pmatrix}$.

(c) Because $Y = \begin{pmatrix} 8 & 0 & -6 \\ -3 & -3 & -5 \\ 2 & 7 & -5 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 8 & -3 & 2 \\ 0 & -3 & 7 \\ -6 & -5 & -5 \end{pmatrix}$.

Hence, $X = \begin{pmatrix} 8 & -3 & 2 \\ 0 & -3 & 7 \\ -6 & -5 & -5 \end{pmatrix} \begin{pmatrix} 8 & 0 & -6 \\ -3 & -3 & -5 \\ 2 & 7 & -5 \end{pmatrix}$

$= \begin{pmatrix} 8 \times 8 - 3 \times (-3) + 2 \times 2 & 8 \times 0 - 3 \times (-3) + 2 \times 7 & 8 \times (-6) - 3 \times (-5) + 2 \times (-5) \\ 0 \times 8 - 3 \times (-3) + 7 \times 2 & 0 \times 0 - 3 \times (-3) + 7 \times 7 & 0 \times (-6) - 3 \times (-5) + 7 \times (-5) \\ -6 \times 8 - 5 \times (-3) - 5 \times 2 & -6 \times 0 - 5 \times (-3) - 5 \times 7 & -6 \times (-6) - 5 \times (-5) - 5 \times (-5) \end{pmatrix}$

$= \begin{pmatrix} 77 & 23 & -43 \\ 23 & 58 & -20 \\ -43 & -20 & 86 \end{pmatrix}$.

**2.** (a) Because $Y = \begin{pmatrix} -2 & -1 & -8 \\ 6 & 8 & 5 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} -2 & 6 \\ -1 & 8 \\ -8 & 5 \end{pmatrix}$.

Hence, $X = \begin{pmatrix} -2 & 6 \\ -1 & 8 \\ -8 & 5 \end{pmatrix} \begin{pmatrix} -2 & -1 & -8 \\ 6 & 8 & 5 \end{pmatrix}$

$= \begin{pmatrix} -2 \times (-2) + 6 \times 6 & -2 \times (-1) + 6 \times 8 & -2 \times (-8) + 6 \times 5 \\ -1 \times (-2) + 8 \times 6 & -1 \times (-1) + 8 \times 8 & -1 \times (-8) + 8 \times 5 \\ -8 \times (-2) + 5 \times 6 & -8 \times (-1) + 5 \times 8 & -8 \times (-8) + 5 \times 5 \end{pmatrix}$

$= \begin{pmatrix} 40 & 50 & 46 \\ 50 & 65 & 48 \\ 46 & 48 & 89 \end{pmatrix}.$

(b) Because $Y = \begin{pmatrix} 3 & 8 \\ -2 & -5 \\ 1 & 3 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 3 & -2 & 1 \\ 8 & -5 & 3 \end{pmatrix}.$

Hence, $X = \begin{pmatrix} 3 & -2 & 1 \\ 8 & -5 & 3 \end{pmatrix} \begin{pmatrix} 3 & 8 \\ -2 & -5 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 3 \times 3 - 2 \times (-2) + 1 \times 1 & 3 \times 8 - 2 \times (-5) + 1 \times 3 \\ 8 \times 3 - 5 \times (-2) + 3 \times 1 & 8 \times 8 - 5 \times (-5) + 3 \times 3 \end{pmatrix}$

$= \begin{pmatrix} 14 & 37 \\ 37 & 98 \end{pmatrix}.$

(c) Because $Y = \begin{pmatrix} 3 & -2 & -8 \\ -4 & -9 & -5 \\ -2 & -3 & -2 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 3 & -4 & -2 \\ -2 & -9 & -3 \\ -8 & -5 & -2 \end{pmatrix}.$

Hence, $X = \begin{pmatrix} 3 & -4 & -2 \\ -2 & -9 & -3 \\ -8 & -5 & -2 \end{pmatrix} \begin{pmatrix} 3 & -2 & -8 \\ -4 & -9 & -5 \\ -2 & -3 & -2 \end{pmatrix}$

$= \begin{pmatrix} 3 \times 3 - 4 \times (-4) - 2 \times (-2) & 3 \times (-2) - 4 \times (-9) - 2 \times (-3) & 3 \times (-8) - 4 \times (-5) - 2 \times (-2) \\ -2 \times 3 - 9 \times (-4) - 3 \times (-2) & -2 \times (-2) - 9 \times (-9) - 3 \times (-3) & -2 \times (-8) - 9 \times (-5) - 3 \times (-2) \\ -8 \times 3 - 5 \times (-4) - 2 \times (-2) & -8 \times (-2) - 5 \times (-9) - 2 \times (-3) & -8 \times (-8) - 5 \times (-5) - 2 \times (-2) \end{pmatrix}$

$= \begin{pmatrix} 29 & 36 & 0 \\ 36 & 94 & 67 \\ 0 & 67 & 93 \end{pmatrix}.$

**3.** (a) Because $Y = \begin{pmatrix} 6 & 4 & 7 \\ -5 & 1 & -1 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 6 & -5 \\ 4 & 1 \\ 7 & -1 \end{pmatrix}.$

$$\text{Hence, } X = \begin{pmatrix} 6 & -5 \\ 4 & 1 \\ 7 & -1 \end{pmatrix} \begin{pmatrix} 6 & 4 & 7 \\ -5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \times 6 - 5 \times (-5) & 6 \times 4 - 5 \times 1 & 6 \times 7 - 5 \times (-1) \\ 4 \times 6 + 1 \times (-5) & 4 \times 4 + 1 \times 1 & 4 \times 7 + 1 \times (-1) \\ 7 \times 6 - 1 \times (-5) & 7 \times 4 - 1 \times 1 & 7 \times 7 - 1 \times (-1) \end{pmatrix}$$

$$= \begin{pmatrix} 61 & 19 & 47 \\ 19 & 17 & 27 \\ 47 & 27 & 50 \end{pmatrix}.$$

(b) Because $Y = \begin{pmatrix} 3 & 6 \\ -4 & 6 \\ 2 & -2 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 3 & -4 & 2 \\ 6 & 6 & -2 \end{pmatrix}$.

$$\text{Hence, } X = \begin{pmatrix} 3 & -4 & 2 \\ 6 & 6 & -2 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ -4 & 6 \\ 2 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \times 3 - 4 \times (-4) + 2 \times 2 & 3 \times 6 - 4 \times 6 + 2 \times (-2) \\ 6 \times 3 + 6 \times (-4) - 2 \times 2 & 6 \times 6 + 6 \times 6 - 2 \times (-2) \end{pmatrix}$$

$$= \begin{pmatrix} 29 & -10 \\ -10 & 76 \end{pmatrix}.$$

(c) Because $Y = \begin{pmatrix} 9 & -1 & 9 \\ 4 & 4 & -8 \\ 2 & 8 & -5 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 9 & 4 & 2 \\ -1 & 4 & 8 \\ 9 & -8 & -5 \end{pmatrix}$.

$$\text{Hence, } X = \begin{pmatrix} 9 & 4 & 2 \\ -1 & 4 & 8 \\ 9 & -8 & -5 \end{pmatrix} \begin{pmatrix} 9 & -1 & 9 \\ 4 & 4 & -8 \\ 2 & 8 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \times 9 + 4 \times 4 + 2 \times 2 & 9 \times (-1) + 4 \times 4 + 2 \times 8 & 9 \times 9 + 4 \times (-8) + 2 \times (-5) \\ -1 \times 9 + 4 \times 4 + 8 \times 2 & -1 \times (-1) + 4 \times 4 + 8 \times 8 & -1 \times 9 + 4 \times (-8) + 8 \times (-5) \\ 9 \times 9 - 8 \times 4 - 5 \times 2 & 9 \times (-1) - 8 \times 4 - 5 \times 8 & 9 \times 9 - 8 \times (-8) - 5 \times (-5) \end{pmatrix}$$

$$= \begin{pmatrix} 101 & 23 & 39 \\ 23 & 81 & -81 \\ 39 & -81 & 170 \end{pmatrix}.$$

**4.** (a) Because $Y = \begin{pmatrix} -3 & 5 & 1 \\ -7 & -2 & 1 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} -3 & -7 \\ 5 & -2 \\ 1 & 1 \end{pmatrix}$.

Hence, $X = \begin{pmatrix} -3 & -7 \\ 5 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -3 & 5 & 1 \\ -7 & -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} -3 \times (-3) - 7 \times (-7) & -3 \times 5 - 7 \times (-2) & -3 \times 1 - 7 \times 1 \\ 5 \times (-3) - 2 \times (-7) & 5 \times 5 - 2 \times (-2) & 5 \times 1 - 2 \times 1 \\ 1 \times (-3) + 1 \times (-7) & 1 \times 5 + 1 \times (-2) & 1 \times 1 + 1 \times 1 \end{pmatrix}$

$= \begin{pmatrix} 58 & -1 & -10 \\ -1 & 29 & 3 \\ -10 & 3 & 2 \end{pmatrix}.$

(b) Because $Y = \begin{pmatrix} 2 & -4 \\ 1 & 4 \\ 8 & -5 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 2 & 1 & 8 \\ -4 & 4 & -5 \end{pmatrix}.$

Hence, $X = \begin{pmatrix} 2 & 1 & 8 \\ -4 & 4 & -5 \end{pmatrix} \begin{pmatrix} 2 & -4 \\ 1 & 4 \\ 8 & -5 \end{pmatrix}$

$= \begin{pmatrix} 2 \times 2 + 1 \times 1 + 8 \times 8 & 2 \times (-4) + 1 \times 4 + 8 \times (-5) \\ -4 \times 2 + 4 \times 1 - 5 \times 8 & -4 \times (-4) + 4 \times 4 - 5 \times (-5) \end{pmatrix}$

$= \begin{pmatrix} 69 & -44 \\ -44 & 57 \end{pmatrix}.$

(c) Because $Y = \begin{pmatrix} 9 & 4 & 5 \\ 8 & 9 & 1 \\ -8 & -4 & 9 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 9 & 8 & -8 \\ 4 & 9 & -4 \\ 5 & 1 & 9 \end{pmatrix}.$

Hence, $X = \begin{pmatrix} 9 & 8 & -8 \\ 4 & 9 & -4 \\ 5 & 1 & 9 \end{pmatrix} \begin{pmatrix} 9 & 4 & 5 \\ 8 & 9 & 1 \\ -8 & -4 & 9 \end{pmatrix}$

$= \begin{pmatrix} 9 \times 9 + 8 \times 8 - 8 \times (-8) & 9 \times 4 + 8 \times 9 - 8 \times (-4) & 9 \times 5 + 8 \times 1 - 8 \times 9 \\ 4 \times 9 + 9 \times 8 - 4 \times (-8) & 4 \times 4 + 9 \times 9 - 4 \times (-4) & 4 \times 5 + 9 \times 1 - 4 \times 9 \\ 5 \times 9 + 1 \times 8 + 9 \times (-8) & 5 \times 4 + 1 \times 9 + 9 \times (-4) & 5 \times 5 + 1 \times 1 + 9 \times 9 \end{pmatrix}$

$= \begin{pmatrix} 209 & 140 & -19 \\ 140 & 113 & -7 \\ -19 & -7 & 107 \end{pmatrix}.$

**5.** (a) Because $Y = \begin{pmatrix} -4 & -3 & 0 \\ -1 & -7 & 2 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} -4 & -1 \\ -3 & -7 \\ 0 & 2 \end{pmatrix}.$

$$
\text{Hence, } X = \begin{pmatrix} -4 & -1 \\ -3 & -7 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -4 & -3 & 0 \\ -1 & -7 & 2 \end{pmatrix}
$$

$$
= \begin{pmatrix} -4 \times (-4) - 1 \times (-1) & -4 \times (-3) - 1 \times (-7) & -4 \times 0 - 1 \times 2 \\ -3 \times (-4) - 7 \times (-1) & -3 \times (-3) - 7 \times (-7) & -3 \times 0 - 7 \times 2 \\ 0 \times (-4) + 2 \times (-1) & 0 \times (-3) + 2 \times (-7) & 0 \times 0 + 2 \times 2 \end{pmatrix}
$$

$$
= \begin{pmatrix} 17 & 19 & -2 \\ 19 & 58 & -14 \\ -2 & -14 & 4 \end{pmatrix}.
$$

(b) Because $Y = \begin{pmatrix} -6 & 1 \\ 2 & -9 \\ -9 & -7 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} -6 & 2 & -9 \\ 1 & -9 & -7 \end{pmatrix}$.

$$
\text{Hence, } X = \begin{pmatrix} -6 & 2 & -9 \\ 1 & -9 & -7 \end{pmatrix} \begin{pmatrix} -6 & 1 \\ 2 & -9 \\ -9 & -7 \end{pmatrix}
$$

$$
= \begin{pmatrix} -6 \times (-6) + 2 \times 2 - 9 \times (-9) & -6 \times 1 + 2 \times (-9) - 9 \times (-7) \\ 1 \times (-6) - 9 \times 2 - 7 \times (-9) & 1 \times 1 - 9 \times (-9) - 7 \times (-7) \end{pmatrix}
$$

$$
= \begin{pmatrix} 121 & 39 \\ 39 & 131 \end{pmatrix}.
$$

(c) Because $Y = \begin{pmatrix} 7 & -6 & -2 \\ 8 & 4 & -8 \\ -8 & 6 & -4 \end{pmatrix}$, we have $Y^T = \begin{pmatrix} 7 & 8 & -8 \\ -6 & 4 & 6 \\ -2 & -8 & -4 \end{pmatrix}$.

$$
\text{Hence, } X = \begin{pmatrix} 7 & 8 & -8 \\ -6 & 4 & 6 \\ -2 & -8 & -4 \end{pmatrix} \begin{pmatrix} 7 & -6 & -2 \\ 8 & 4 & -8 \\ -8 & 6 & -4 \end{pmatrix}
$$

$$
= \begin{pmatrix} 7 \times 7 + 8 \times 8 - 8 \times (-8) & 7 \times (-6) + 8 \times 4 - 8 \times 6 & 7 \times (-2) + 8 \times (-8) - 8 \times (-4) \\ -6 \times 7 + 4 \times 8 + 6 \times (-8) & -6 \times (-6) + 4 \times 4 + 6 \times 6 & -6 \times (-2) + 4 \times (-8) + 6 \times (-4) \\ -2 \times 7 - 8 \times 8 - 4 \times (-8) & -2 \times (-6) - 8 \times 4 - 4 \times 6 & -2 \times (-2) - 8 \times (-8) - 4 \times (-4) \end{pmatrix}
$$

$$
= \begin{pmatrix} 177 & -58 & -46 \\ -58 & 88 & -44 \\ -46 & -44 & 84 \end{pmatrix}.
$$