Dear Dr. Becker,

We thank you and the anonymous reviewers for the thoughtful and insightful suggestions regarding our manuscript. Below, please find a point-by-point accounting of our responses to the reviews.

Associate Editor comments (Remarks to Author): I have sent the manuscript out to two referees and received those reports back, and I have also looked at your paper myself. Both referees have identified quite a few items they would like to see addressed, but overall I'm pleased to report that I think the manuscript has merit and impact, and so I am recommending a major revision. In addition to rephrasing, reorganizing and clarifying the paper (per the referees' comments), the two major items to address are (1) details about the real world examples (cf comments from referee 1), and (2) referee 2 raises an issue about different ground truths.

Referee #2 (Remarks to the Author):

In "Let them have CAKES [...]", the authors describe a new method to perform variants of the nearest neighbor search. The Rust implementation of this new algorithm is relatively (comparable or somewhat slower than the competition) but has a much better recall across many different tasks.

The description of the method is clear, although the description of the run time costs is ambiguous. Overall, CAKES is an interesting contribution to the field of NN-search.

Major comments

The big O notation by default suggest worst-case run time complexity. In this manuscript, it seems that complexity is a mixture of expected run time complexity and under the loose assumption that the data has particular characteristics (e.g., low fractal dimension, the tree is balanced). These assumptions should be made clear in the intro and when giving the run time expressions.

this is easy to address, and we will do so

For example, line 215, does the cost of partitioning of $O(n \log(n))$ depend on the tree being balanced? It is an important hypothesis because line 197 states that "the imbalance in the tree is a feature, not a bug, as it reflects the underlying structure of the data". That may look like a contradiction.

we can address this with the plots that include a balanced tree – expand 2.1.3, balanced vs unbalanced, dropping off points faster with an unbalanced tree?

The cost on line 260 uses \hat{r} , which is a mean radius. Now that cost is in expectation (in addition to the balanced tree hypothesis).

this is restating the complexity from the CHESS paper, but it's correct that it's an expected cost. Worst-case analysis isn't really useful here.

Another example, line 326, comes from the minimum with value 2 on line 284. If the assumption on the LFD is not satisfied, then an extra log factor would appear in the cost.

write in plain english right after eq 2.3, explaining in plain english the rationale for the $\min(2,x)$ expansion: namely, we don't think the very low LFD would be representative of what we'd see if we massively increased the radius

The discussion on LFD after its definition line 141 is hard to understand: there are 2 parameters in the definition of LFD $(r_1 \text{ and } r_2)$, but the discussion seems to imply there is only 1 (a radius r). That discussion only seems to make sense after line 163 and the rewrite of the definition 2.1.

Right after or in eq 2.1, immediately follow up with the simplified form where $r_1 = 2r_2$. Also drop subscript X, since we're dealing with only one dataset. Explain.

Smith-Waterman and Needleman-Wunsch are not distances (unlike Levenshtein and Hamming distances). They are dynamic programming algorithms to compute distances like the Levenshtein distance (and others depending on the exact parameters of the algorithms).

Correct; fix in language.

The explanation that the augmentation process (starting at line 418) preserves the topological structure of the underlying data is not very convincing. This process adds small clouds of points extending in every dimension around existing points. That is not quite the same as generating new points that belong to a small dimension manifold in the sample space.

We should acknowledge this limitation, and suggest that in future work we would do something like favor augmentation along the first few PCs of the local manifold, but not in scope here. This goes in discussion.

Line 467-473: Why are there multiple linear search implementations used as ground truth? These different implementation are apparently used to evaluate different algorithms (ANNOY, FAISS-IVF vs CAKES). This is troubling: why would the ground truth differ based on which tool is evaluated or which language it is written in?

Do these 2 tools return the same result? If so, why use multiple tools? If not, then the evaluation is flawed.

We don't have a python wrapper for cakes, and the other tools were in python. There is only one ground truth. Fix language starting in Results. Najib, can you handle this one?

Minor comments

Line 421: The points are created with distance ϵ of x, not $\epsilon||x||$. r is chosen in the sphere of radius ϵ and added to x, and, as stated correctly line 424, x' is within ϵ of x.

We have addressed this, by clarifying in notation, particularly by making all vectors boldfaced.

There are many small LaTeX issues: wrong space after "i.e." and "e.g." (add a backslash), large spaces around formulas (e.g., line 342), em-dashes that look too short (e.g., line 594).

We have addressed much of this, including replacing en-dashes with em-dashes where appropriate. The space around formulas seems to be something that, if accepted, the production team could address.

The figures should not be shrank so much: the labels on the axis and the keys are not readable. Regenerate the pictures at the correct size to avoid scaling.

fix some font sizes and aspect ratios

Put the keys outside of the figures, not over the data lines.

need to fix this (NI)

Many horizontal and vertical lines in Table~2 should be removed to help with readability.

Per the suggestion of reviewer 2, we have broken this table up into four tables (one per dataset) which we helieve helps readability.

Line 585: "we observe performs quite slowly". The sentence seems incomplete.

This has been fixed.

Many of the explicit algorithms do not serve any real purpose: they are not more readable in pseudo-code form and are just fine in explained in the text. (E.g., Algorithm 2.4).

get rid of alg 2.4

The Github repository gives very little instructions on how to compile the code, run it or reproduce the results from this manuscript.

will fix (NI)

Review of Let them have CAKES: A Cutting-Edge Algorithm for Scalable, Efficient, and Exact Search on Big Data

SIMODS manuscript M166272

The CAKES manuscript proposes an entropy scaling approach to the search problems of k Nearest Neighbors (k-NN) and rho Nearest Neighbors $(\rho - NN)$ in a data set X. The general approach is to construct a hierarchical data structure of clusters to assist in the searching. While there are interesting ideas in the manuscript, there are major issues both technically and expositionally, as well as some minor issues, that need to be addressed before another round of reviewing.

While there is implicitly a tree that is the key data structure, the tree itself is never named or given a notation; the same is true of the root of the tree. The authors should review the requirements for defining proper terminology and notation for a data structure and apply it consistently to their data structure.

we fixed this in pancakes, just backport it

The authors appear to use the terms similarity and distance interchangeably, though they are certainly different. In particular, a similarity is a measure (nominally, non-negative) that is (ideally) larger the closer two data points are and, in some biological contexts, has no upper bound. Moreover, the use of a distance measure alone is inadequate for the described algorithms. On line 155, there is mention of the geometric median of the points, but there may be no such median depending on the space (set) containing the points of the data set. For example, there is no obvious median for a set of biological sequences. The authors must identify the actual spaces from which data sets can be extracted for CAKES to work.

add def in paper for geometric median, extended from continues vector spaces into any metric space. line 43, use distance, not similarity

On line 141, there is a function defined for the local fractal dimension (LFD), but it is not expressed in functional notation. In other words, it should start with $LFD(X, r_1, r_2)$, since it is clearly a function of all three. This functional notation should propagate throughout the rest of the manuscript. Much the same is true on line 142, where it should read B(X, q, r), not $B_X(q, r)$. If makes little sense to put X as a subscript in this context.

similar to complaint from reviewer 2, address with same language

In the algorithms, starting with Algorithm 2.1, the pseudocode style is ugly. Much better would be the pseudocode style from the fourth edition of Introduction to Algorithms, CLRS. Also, the description of criteria is as a "stopping criteria", while the logic of the code has it as a continuing criteria. Please fix the pseudocode throughout.

We evaluated the CLRS package, but did not see any aesthetic differences. We have, however, increased the font size for the algorithm environments, so that it should be more legible. call it continuation criteria (NI)

Section 2.2 is meant to be the definition of the search problem(s), but that is really only contained in the paragraph from lines 218 to 223. In any case, it is essential to put the problem(s) to be solved earlier, that is, first, before any

description of the solutions with ideas such as clusters. This movement of text will require a bit of text refactoring at a minimum. Also, the sentence at lines 221 to 223 should be expanded in full detail into a separate paragraph. Finally, one assumes that the integer k or the number ρ is a parameter to the corresponding problem and should be mentioned as such.

move introduction of the three deltas to later, to start of 2.4.3, and expand definition of rnn. (NI)

There are no theorems in the manuscript, yet there are arguments for quantities like the quality of the results or the time complexity that are given without much precision. It is better to give precise theorems or propositions for each thing to be proved, with appropriate assumptions, and to give precise proofs of same.

state theorem, which is the time complexity, and formalize the argument into a proof

The manuscript makes too much use of unfamiliar terminology taken from earlier papers without defining terms. For example, see line 233 and Reference [18]. See also the paragraph at lines 306 to 310. All terms used must be precisely defined in the manuscript to allow proper assessment of the manuscript.

remind the reader what rnn search is, and in 306-310 reword to refer to previous section rather than external refs

The Results section is a slog to read and parts of the Discussion seem redundant. Take lines 651 to 668 as an example. Tightening up the Results and the Discussion can only help the exposition.

offload to figure captions where possible, don't spell out everything

I led a bioinformatics project that used an entropy scaling approach to search a database of protein sequences. See Reference [1]. As a result, I am confused about some of the real world examples. I know that protein sequences are challenging to search for reasons such as their having widely varying lengths and there being no obvious notion of a center of a set of protein sequences. In any case, I would like the authors to reference [1] and discuss in more depth how they address the challenges of the various real world examples.

fine, cite 1, and discuss (NI)

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"Data set" is pronounced as two words and should be written as such throughout the manuscript.

We respectfully disagree; this appears as a term of art in existing literature, and is in the Merriam-Webster Dictionary. "Dataset" is listed by Merriam-Webster as the primary form, with "data set" being the less common form (https://www.merriam-webster.com/dictionary/dataset). Unless this is an editorial policy, we would rather keep it as "dataset."

In line 22, use "rate" instead of "scale".

This has been fixed.

Throughout, the word "which" is used when "that" is the correct word. If "which" is used, then it is not defining and is preceded by a comma. Change all "which" is to "that" is.

We somewhat agree, but disagree that *all* instances should be changed. We have made selected changes where it seems to fit the pattern suggested by this reviewer.

On line 75, there is a Subsection 1.1, but there is no Subsection 1.2. This is not a good practice. Just refactor the text without Section 1.1 or add a Subsection 1.2.

We added Subsection 1.2 (Entropy-Scaling Search).

The use of \forall , \exists , \Leftrightarrow , and other similar logical operators is not appropriate in a manuscript that is not about logic. Spell out the meaning in English, and be creative.

We respectfully disagree. These operators are also standard in set theory and functional analysis, and it is in this context that we use them. They are also standard in the algorithms literature, including CLRS. Unless this is editorial policy, we would prefer to keep them. We believe they lend precision.

Near line 154, it is important to give an overview of your solution strategy and, in particular, why clustering is used first. Clustering is a heavily overloaded word. I would have chosen a more neutral term instead of "cluster".

We do define our clustering approach, and moreover, we build on prior work (some ours, some others) that relies on clustering. We respectfully would prefer to stay with the language we have here.

On line 155, there is the parenthetical comment about "a smaller sample of the points". What does that mean precisely and where does it occur later in the manuscript?

We clarified this by changing the language about "a smaller sample of the points" to "a random subsample of $\sqrt{|C|}$ points."

On line 156, there is the claim "so it is a real data point". What is a real data point? Please explain.

We removed the language "real data point" and clarified that the center of a cluster is the point which is its geometric median.

On line 161 and elsewhere, "i.e." is always preceded and followed by commas.

Fixed—"i.e." is now always followed by a comma.

On line 178, there is the phrase "find the point ... in the sample". Can you make the entire paragraph more precise by giving notation for the sample?

We now refer to the sample as S.

Section 2.1.1 overall is lacking in mathematical rigor.

we will refactor 2.1.1 to be more rigorous (NI)

In Section 2.1.3, the cost of assessing "criteria' is not counted in the time complexity, but it must be.

add some explanation: these can be as complex as a user wants, but for us it is a constant-time check (NI)

In line 252, there is again mention of "real points" without any indication of the meaning of "real".

We removed the section which contained this from the paper.

The fonts in Algorithms 2.2 through 2.7 are simply too small. Make them the font size of the remainder of the text.

We have made this larger to improve readability.

The numbers in Table 2 are too small. Refactor into multiple tables perhaps.

We have broken this table up into four tables (one per dataset) which we believe helps readability.

In line 641, what is "radial increase 2.3"?

We added that 2.3 is an equation.

In line 642, why is "harmonic" italicized?

This has been fixed.

The References section needs a through going over for proper formatting and capitalization. For example, check the title capitalization in [2], [4], and [8]. Check the journal name capitalization in [9] and [21], among others.

We have capitalized titles as published (confirmed for all of the references mentioned). We have corrected the journal capitalization where appropriate.

Several references have repeated URLs. Trim these up.

We believe these are now fixed.

References [2] and [3] appear to be duplicates.

This has been fixed.

Reference [39] is incomplete and inadequate.

This has been fixed.

[1] Yoonjin Kim, Zhen Guo, Jeffrey A. Robertson, Benjamin Reidys, Ziyan Zhang, Lenwood S. Heath, EnTrance: Exploration of Entropy Scaling Ball Cover Search

 $in\ Protein\ Sequences,\ bioRxiv\ https://urldefense.com/v3/https://www.biorxiv.org/content/10.1101/2021\\ q3TzzRIBjJpAmGsw1kVTB7zosA7khu71pJpSxxrEqcYu6KceYMTg_VZ_MJ8B-PoPqf\$\ ,\ 2021.$