CSC 411

Computer Organization (Fall 2024)
Lecture 8: Floating Point (part 2)

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"95% of folks out there are completely clueless about floating-point."

James Gosling (founder and lead designer behind the Java programming language)

Denormalized and special values

Denormalized values $(-1)^s M 2^E$

‣ **E = 1 - bias**

- exp field is 00…00

‣ **M = 0.bb…bb**

- bb…bb are the bits in the fraction field

- M is the decimal that corresponds to 0.bb…bb

  - note that <u>M has an implied leading 0</u>

‣ Key points:

- if fraction field is 00…00, represents +0 or -0) depending on sign bit

- for all other cases, denormalized values are numbers close to 0.0

sign	exp	fraction

Practice (decoding) $(-1)^s M 2^E$

- ‣ Assume a float F = 0x00580000
  - write the binary
    - divide into **s, exp, frac**
    - 0 00000000 10110000000000000000000
  - calculate M
    - M = 0.10110000000000000000000 = 0.6875
  - calculate E
    - E = 1 - bias = 1 - 127 = -126
  - write final number
    - $(-1)^0 * 0.6875 * 2^{-126} = 8.0815236619 * 10^{-39}$

Special values $(-1)^s M 2^E$

- ‣ Infinity
  - fraction field is 00…00
  - sign bit determines $+\infty$ or $-\infty$
- ‣ Not-a-number (NaN)
  - fraction field != 00…00
  - represents undefined or unrepresentable values (e.g., $0/0, \sqrt{-1}$)

sign	exp	fraction

Final considerations

IEEE-like example formats

- ‣ Same general form can be extended to new formats
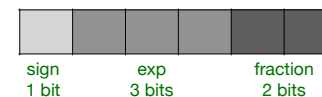 - smaller formats trade precision and range for memory efficiency
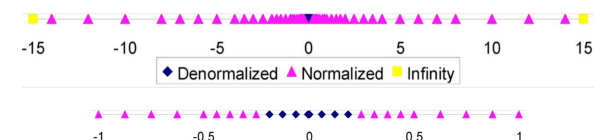- ‣ Using 8 bits
  - bias = 7

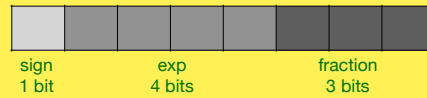sign 1 bit   exp 4 bits   fraction 3 bits

- ‣ Using 6 bits
  - bias = 3

sign 1 bit   exp 3 bits   fraction 2 bits

- distribution of values

-15   -10   -5   0   5   10   15

◆ Denormalized ▲ Normalized ■ Infinity

-1   -0.5   0   0.5   1

Practice

sign 1 bit | exp 4 bits | fraction 3 bits

‣ Using the 8-bit representation from the previous slide, provide the bit representation for the following numbers:

- zero

- one

- smallest positive denormalized

- largest positive denormalized

- smallest positive normalized

- largest positive normalized

FP operations and rounding

‣ Operations using floating point numbers might produce results that can't be exactly represented

- e.g., x + y, x * y   ⇐ will cover FP addition and multiplication later in the course

‣ Steps

- perform the operation and compute the exact result

- fit into the desired precision (may involve **overflow** or **rounding**)

‣ Round to nearest, ties to even (default)

- when exactly halfway between two values, round so that the least significant digit is **even**

Practice

‣ Rounding decimals

- halfway when decimal digits to right of rounding position = 500…

4.32313333	4.32313333	4.32	
4.32500001	4.32500001	4.33	
4.31500000	4.31500000	4.3**2**	to nearest even
4.34500000	4.34500000	4.3**4**	to nearest even

‣ Rounding fractional binary numbers

- halfway when binary digits to right of rounding position = 100…

10.00011	10.00011	10.00	
10.00110	10.00110	10.01	
10.11100	10.11100	11.0**0**	to nearest even
10.10100	10.10100	10.1**0**	to nearest even

FP arithmetic cautions

‣ Rounding can break <u>associativity</u> and other mathematical properties

```c
#include <stdio.h>

int main() {
    float a = 1e20;
    float b = -1e20;
    float c = 1;

    if (((a + b) + c) == (a + (b + c))) {
        printf("equal\n");
    } else {
        printf("different\n");
    }

    return 0;
}
```

‣ When comparing floating point values:

- use (abs(a-b) < eps) with a small value in eps

- **<u>avoid direct equality comparisons</u>** (a == b)

Floating point in C

‣ Single precision (`float`) and double precision (`double`)

‣ Casting and conversions

- casting between two's complement signed/unsigned integer types **does not change** the bit representation

  - bit-extension, truncating may be applied

- casting between integer and floating point types **does change** the bit representation

From	To	Action
double/float	integer	truncate fractional part
integer	float	can't guarantee exact conversion, possibly rounding
integer	double	exact conversion, as long as integer size < 53 bits

Practice

‣ Will the following statements always be **true**?

- `i == (int) ((float) i)`

  - assume i is an int

- `f == (float) ((int) f))`

  - assume f is a float

Floating point in C

‣ Consider an `int x`, a `float f`, and a `double d`

- assuming d and f are not special values, what is the output of the following expressions?

Expression	Output
`x == (int) (float) x`	False
`x == (int) (double) x`	True
`f == (float) (double) f`	True
`d == (double) (float) d`	False
`f == -(-f)`	True
`2/3 == 2/3.0`	False
`if (d < 0.0) then ((d*2) < 0.0)`	True
`if (d > f) then (-f > -d)`	True
`d * d >= 0.0`	True
`(d + f) - d == f`	False

Precision

Format	Smallest positive value (*)	Largest positive value (*)	Precision (**)
single	~$1.401 \cdot 10^{-45}$	~$3.403 \cdot 10^{+38}$	6-9 digits
double	~$4.941 \cdot 10^{-324}$	~$1.798 \cdot 10^{+308}$	15-17 digits

(∗) Smallest/largest negative values are the same as their positive counterparts, but negative.

(∗∗) Precision refers to to the number of significant digits that can be represented in a number.

In C, FLT_MIN and DBL_MIN are defined as the smallest normalized numbers, and FLT_TRUE_MIN and DBL_TRUE_MIN represent the smallest positive value that can be represented by the float type, including denormalized numbers.

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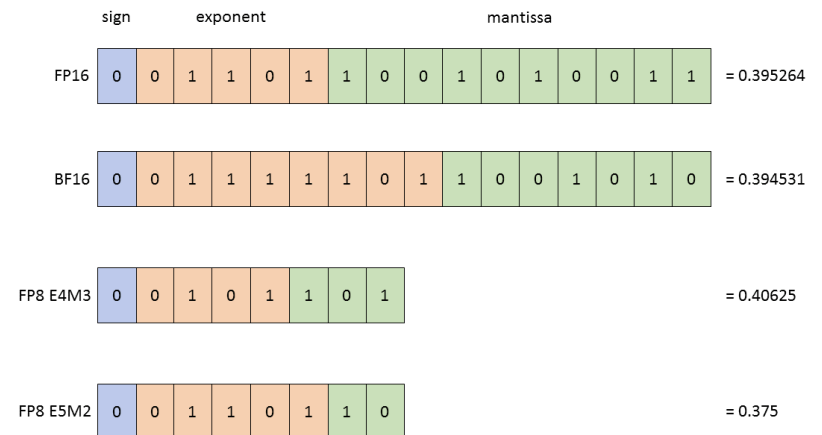## NVIDIA H200 Tensor Core GPU Quick Specs

GPU Memory	141GB
GPU Memory Bandwidth	4.8TB/s
FP8 Tensor Core Performance	4 PetaFLOPS

> H100 GPU introduced support for a new datatype, FP8 (8-bit floating point), enabling higher throughput of matrix multiplies and convolutions.

NVIDIA H200 Tensor Core GPU

sign   exponent   mantissa

FP16:   0 | 0 1 1 0 1 | 1 0 0 1 0 1 0 0 1 1   = 0.395264

BF16:   0 | 0 1 1 1 1 1 0 | 1 1 0 0 1 0 1 0   = 0.394531

FP8 E4M3:   0 | 0 1 0 1 | 1 0 1   = 0.40625

FP8 E5M2:   0 | 0 1 1 0 1 | 1 0   = 0.375

Structure of the floating point datatypes. All of the values shown are the closest representations of value 0.3952.