CSC 411

Computer Organization (Spring 2023)
Lecture 8: Integer Representation

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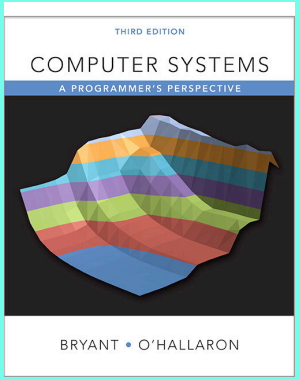
```c
#include <stdio.h>

int main() {
    char a = 254;
    unsigned char b = 254;
    unsigned int c = 0;

    printf("%d %d\n", a, b);

    if (-1 < c) {
        printf("yay\n");
    } else {
        printf("!!!???\n");
    }
}
```

Disclaimer

The following slides are from:

Computer Systems (Bryant and O'Hallaron)

A Programmer's Perspective



Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C does not mandate using two's complement**
  - But, most machines do, and we will assume so
- **C `short` 2 bytes long**

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two-complement: Simple Example

```
          -16   8   4   2   1
10 =       0   1   0   1   0      8+2 = 10

          -16   8   4   2   1
-10 =      1   0   1   1   0      -16+4+2 = -10
```


Two-complement Encoding Example (Cont.)

```
x =     15213: 00111011 01101101
y =    -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1
- **Two's Complement Values**
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
  - Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	**65535**	**FF FF**	11111111 11111111
TMax	**32767**	**7F FF**	01111111 11111111
TMin	**-32768**	**80 00**	10000000 00000000
-1	**-1**	**FF FF**	11111111 11111111
0	**0**	**00 00**	00000000 00000000

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- **Observations**
  - |*TMin*| = *TMax* + 1
    - Asymmetric range
  - *UMax* = 2 * *TMax* + 1
  - Question: abs(TMin)?
- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **⇒ Can Invert Mappings**
  - $U2B(x) = B2U^{-1}(x)$
    - Bit pattern for unsigned integer
  - $T2B(x) = B2T^{-1}(x)$
    - Bit pattern for two's comp integer

Question

- Using 2's complement and words of n bits …
  - can -2^{n-1} be represented?
  - can 2^{n-1} be represented?

Mapping Signed ↔ Unsigned

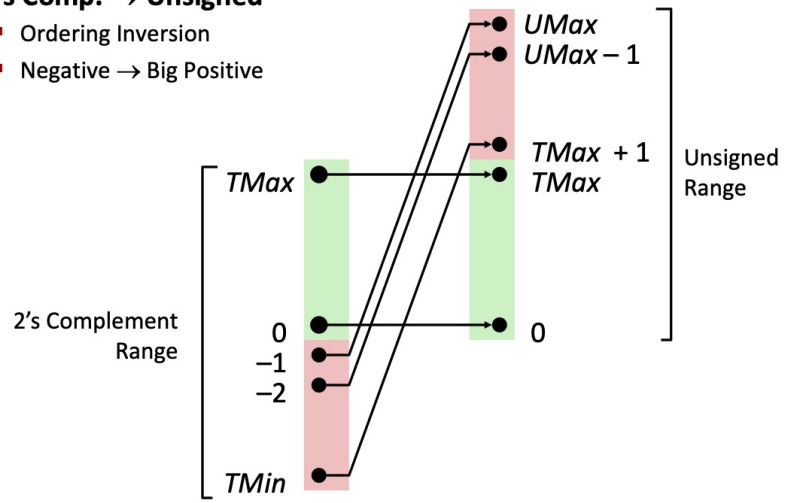
Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5	T2U →	5
0110	6	U2T ←	6
0111	7		7
1000	–8		8
1001	–7		9
1010	–6		10
1011	–5		11
1100	–4		12
1101	–3		13
1110	–2		14
1111	–1		15

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4	↔	4
0101	5		5
0110	6		6
0111	7		7
1000	–8		8
1001	–7		9
1010	–6		10
1011	–5	+/- 16	11
1100	–4	↔	12
1101	–3	$\pm 2^n$	13
1110	–2		14
1111	–1		15

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Negating 2's complement numbers

- Shortcut
  - invert all bits and add 1
  - works on both directions (negative to positive and vice-versa)
- Examples with 8-bit words:
  - negate 10

```
00001010
```

  - negate -20

```
11101100
```

Question

- What is the decimal value of this 64-bit number represented using 2's complement?

```
0xFFFFFFFFFFFFFFF8
```

```
11111111 11111111 11111111 11111111 11111111 11111111 11111111 11111000
```


Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

```
0U, 4294967259U
```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

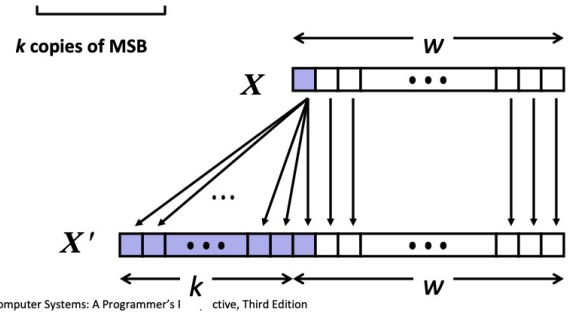
Constant$_1$	**Constant$_2$**	**Relation**	**Evaluation**
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

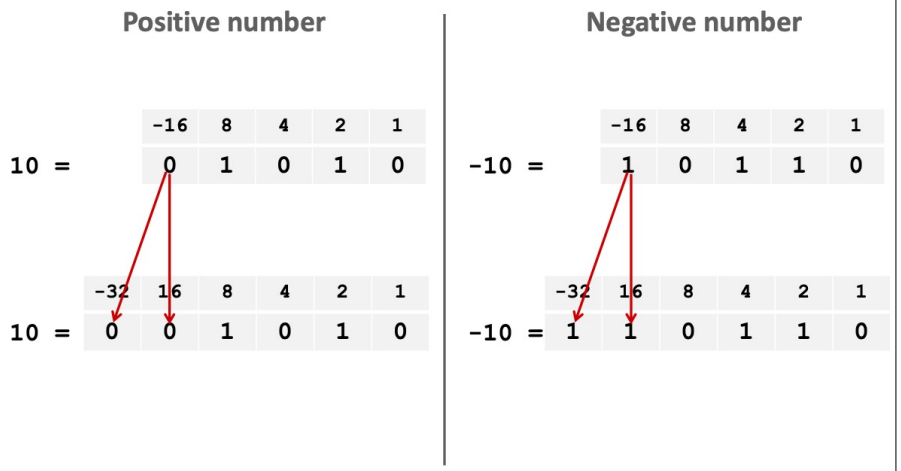
- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**
- **Expression containing signed and unsigned int**
  - `int` is cast to `unsigned`!!

Sign Extension

- **Task:**
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- **Rule:**
  - Make *k* copies of sign bit:
  - $X' = x_{w-1}, \ldots, x_{w-1}, x_{w-1}, x_{w-2}, \ldots, x_0$



Sign Extension: Simple Example



Larger Sign Extension Example

```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- **Converting from smaller to larger integer data type**
- **C automatically performs sign extension**

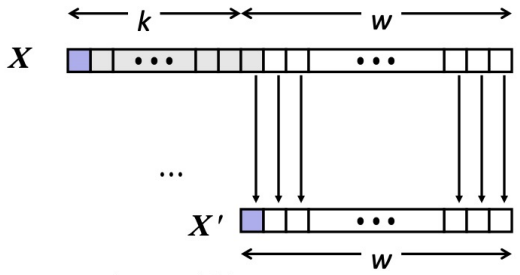
Loading bytes into registers

- Loading 32-bit words into 32-bit registers
  - signed and unsigned loads are identical
- Loading smaller words requires **bit-extension**

Load word	`lw x5, 40(x6)`
Load word, unsigned	`lwu x5, 40(x6)`
Store word	`sw x5, 40(x6)`
Load halfword	`lh x5, 40(x6)`
Load halfword, unsigned	`lhu x5, 40(x6)`
Store halfword	`sh x5, 40(x6)`
Load byte	`lb x5, 40(x6)`
Load byte, unsigned	`lbu x5, 40(x6)`
Store byte	`sb x5, 40(x6)`
Load reserved	`lr.d x5, (x6)`
Store conditional	`sc.d x7, x5, (x6)`
Load upper immediate	`lui x5, 0x12345`

Truncation

- **Task:**
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X' with same value for "small enough" X
- **Rule:**
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6