CSC 411

Computer Organization (Spring 2024)
Lecture 8: Floating Point (part 2)

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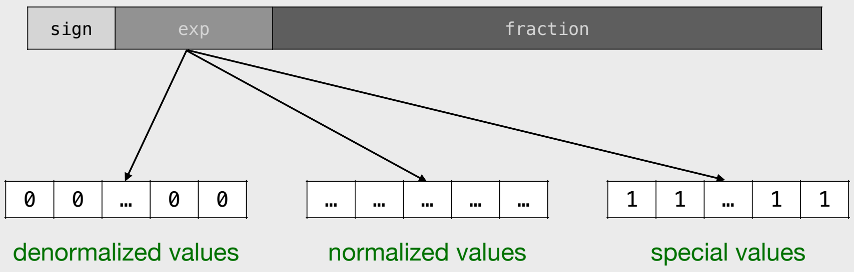
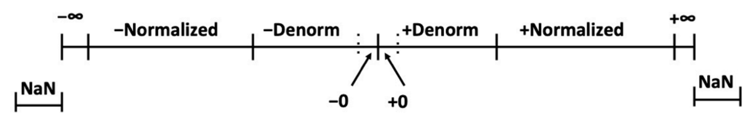
Floating point encodings





Denormalized values

$$(-1)^s M 2^E$$

- **E = 1 - bias**
  - exp field is 00…00
- **M = 0.bb…bb**
  - bb…bb are the bits in the fraction field
  - M is the decimal that corresponds to 0.bb…bb
    - note that M has an implied leading 0
- Comments
  - note that if the fraction field is 00…00, we can represent +0 and -0
  - for all other cases, denormalized values are numbers close to 0.0

Practice (decoding)

$$(-1)^s M 2^E$$

- Assume a float F = 0x00580000
  - write the binary
    - divide into **s**, **exp**, **frac**
    - 0 00000000 10110000000000000000000
  - calculate M
    - M = 0.10110000000000000000000 = 0.6875
  - calculate E
    - E = 1 - bias = 1 - 127 = -126
  - write final number
    - $(-1)^0 * 0.6875 * 2^{-126} = 8.0815236619 * 10^{-39}$

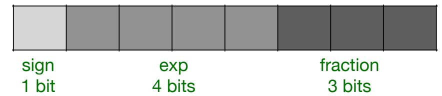
Special values

$$(-1)^s M 2^E$$

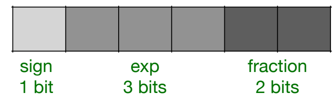
- Infinity
  - fraction field is 00…00
  - we can represent $+\infty$ and $-\infty$
- Not-a-number
  - fraction field != 00…00
  - no numeric value can be determined
  - e.g., $0/0, \sqrt{-1}, \pm\infty/\pm\infty$

IEEE-like example formats

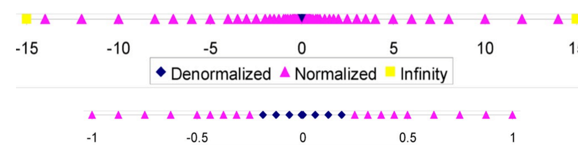
- Same general form can be extended to new formats
  - normalized, denormalized, and special values
- Using 8 bits
  - bias = 7



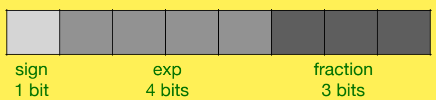
- Using 6 bits
  - bias = 3



  - distribution of values



Practice



- Using the 8-bit representation from the previous slide, provide the bit representation for the following **interesting numbers**:
  - zero
  - one
  - smallest positive denormalized
  - largest positive denormalized
  - smallest positive normalized
  - largest positive normalized

FP operations and rounding

- Operations using floating point numbers might produce results that can't be represented
  - e.g., x + y, x * y ⇐ will cover FP addition and multiplication later in the course
- Steps
  - perform the operation and compute the exact result
  - fit into the desired precision
    - may **overflow**
    - may need **rounding**
- Rounding (to nearest even)
  - default rounding mode
  - when exactly halfway between two values, round so that the least significant digit is **even**

Practice

- Rounding decimals
  - halfway when decimal digits to right of rounding position = 500…

4.32313333	4.32313333	4.32	
4.32500001	4.32500001	4.33	
4.31500000	4.31500000	4.3**2**	to nearest even
4.34500000	4.34500000	4.3**4**	to nearest even

- Rounding fractional binary numbers
  - halfway when binary digits to right of rounding position = 100…

10.00011	10.00011	10.00	
10.00110	10.00110	10.01	
10.11100	10.11100	11.0**0**	to nearest even
10.10100	10.10100	10.1**0**	to nearest even

Caution

- Rounding breaks associativity and other properties

```
#include <stdio.h>

int main() {
    float a = 1e20;
    float b = -1e20;
    float c = 1;

    if (((a + b) + c) == (a + (b + c))) {
        printf("equal\n");
    } else {
        printf("different\n");
    }

    return 0;
}
```

- When comparing floating point values:
  - use `(abs(a-b) < eps)` with a small value in `eps`
  - avoid `(a == b)`

Floating point in C

- Single (`float`) and double (`double`) precision
- Casting and conversions
  - casting between two's complement signed/unsigned integer types **does not change** the bit representation
    - bit-extension, truncating may be applied
  - casting between integer and floating point types **does change** the bit representation

From	To	Action
double/float	integer	truncate fractional part
integer	float	can't guarantee exact conversion, possibly rounding
integer	double	exact conversion, as long as integer size < 53 bits

Practice

- Will the following statements always be **`true`**?
  - `i == (int) ((float) i)`
    - assume i is an int
  - `f == (float) ((int) f))`
    - assume f is a float

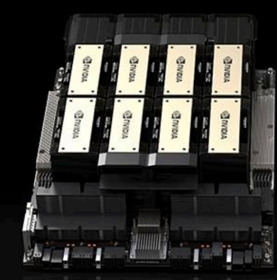
Floating point in C

- Consider an `int x`, a `float f`, and a `double d`
  - assuming d and f are not special values, what is the output of the following expressions?

Expression	Output
`x == (int) (float) x`	False
`x == (int) (double) x`	True
`f == (float) (double) f`	True
`d == (double) (float) d`	False
`f == -(-f)`	True
`2/3 == 2/3.0`	False
`(d < 0.0) then ((d*2) < 0.0)`	True
`(d > f) then (-f > -d)`	True
`d * d >= 0.0`	True
`(d + f) - d == f`	False

NVIDIA H200 Tensor Core GPU

The world's most powerful GPU for supercharging AI and HPC workloads.



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NVIDIA H200 Tensor Core GPU Quick Specs

GPU Memory	141GB
GPU Memory Bandwidth	4.8TB/s
FP8 Tensor Core Performance	4 PetaFLOPS

H100 GPU introduced support for a new datatype, FP8 (8-bit floating point), enabling higher throughput of matrix multiplies and convolutions.

NVIDIA H200 Tensor Core GPU

	sign	exponent	mantissa	
FP16	0	0 1 1 0 1	1 0 0 1 0 1 0 0 1 1	= 0.395264
BF16	0	0 1 1 1 1 1 0 1	1 0 0 1 0 1 0	= 0.394531
FP8 E4M3	0	0 1 0 1	1 0 1	= 0.40625
FP8 E5M2	0	0 1 1 0 1	1 0	= 0.375

Structure of the floating point datatypes. All of the values shown are the closest representations of value 0.3952.