CSC 411

Computer Organization (Spring 2024)
Lecture 4: Integers (signed, unsigned)

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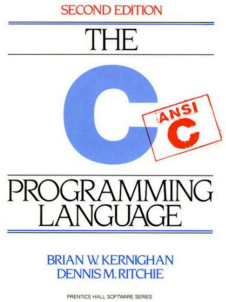
The C Language

- Developed by Dennis Ritchie at Bell Labs in the early 1970s
- Many operating systems, including Unix and its variants (Linux), are written in C
- Allows low-level access to memory, making it efficient for system programming
- C programs are generally portable across different platforms with minimal modification
- C follows a traditional compilation process, where the source code is translated into machine code by a compiler



TIOBE Index for January 2024

- Indicator of the popularity of programming languages
  - popular search engines such as Google, Bing, Yahoo!, Wikipedia, Amazon, YouTube and Baidu are used to calculate the ratings.

Jan 2024	Jan 2023	Change	Programming Language	Ratings	Change
1	1		Python	13.97%	-2.39%
2	2		C	11.44%	-4.81%
3	3		C++	9.96%	-2.95%
4	4		Java	7.87%	-4.34%
5	5		C#	7.16%	+1.43%
6	7	↑	JavaScript	2.77%	-0.11%
7	**10**	↑	**PHP**	**1.79%**	**+0.40%**
8	6	↓	Visual Basic	1.60%	-3.04%
9	8	↓	SQL	1.46%	-1.04%
10	20	⇈	Scratch	1.44%	+0.86%

https://www.tiobe.com/tiobe-index/

Representing data

Representing data

- In memory, **all values** are stored as "bit-vectors"
  - data types are used to interpret the bits (provide meaning)
  - each possible bit-vector assigned exclusively to one meaning
- In a bit sequence of w bits, we can represent 2^w different values
  - number of permutations with repetition — given w digits, there are two ways to choose each digit
  - example: how many different sequences can be represented in 4 bits?

0 0 0 0	1 0 0 0
0 0 0 1	1 0 0 1
0 0 1 0	1 0 1 0
0 0 1 1	1 0 1 1
0 1 0 0	1 1 0 0
0 1 0 1	1 1 0 1
0 1 1 0	1 1 1 0
0 1 1 1	1 1 1 1

$$2^4 = 16$$

What is this program doing?

```c
#include <stdio.h>

int main() {
    unsigned hex = 0x7e313134;

    float *f = (float *) &hex;
    char *s = (char *) &hex;

    printf("%u %d %.4ef %c%c%c%c",
           hex, hex, *f, s[0], s[1], s[2], s[3]);

    return 0;
}
```

```
2117153076   2117153076   5.8882e+37f   411~
```

Interlude: addition and multiplication

Binary addition

```
  1 1 1
  0 0 1 1 1 0 0 1
+ 0 1 1 1 0 1 1 0
  1 0 1 0 1 1 1 1
```

```
  1       1 1 1
  1 1 0 0 0 1 1 1
+ 0 1 0 1 0 1 1 1
1 0 0 0 1 1 1 1 0
```

Binary multiplication

```
      1 0 0 1
    x 0 1 1 0
    ---------
      0 0 0 0
    1 0 0 1
  1 0 0 1
0 0 0 0
---------
0 1 1 0 1 1 0
```

```
        1 1 1 1
      x 1 1 1 1
      ---------
        1 1 1 1
      1 1 1 1
    1 1 1 1
  1 1 1 1
---------
1 1 1 0 0 0 0 1
```

Tricky? perform the addition row-by-row

Integer Representation (unsigned, signed)

Unsigned integers

- Bits represent the number directly
  - same as binary-to-decimal conversion
  - w bits can represent 2^w unsigned integers ranging from 0 to $2^w - 1$

0 0 0 0 = 0	1 0 0 0 = 8
0 0 0 1 = 1	1 0 0 1 = 9
0 0 1 0 = 2	1 0 1 0 = 10
0 0 1 1 = 3	1 0 1 1 = 11
0 1 0 0 = 4	1 1 0 0 = 12
0 1 0 1 = 5	1 1 0 1 = 13
0 1 1 0 = 6	1 1 1 0 = 14
0 1 1 1 = 7	1 1 1 1 = 15

Overflow

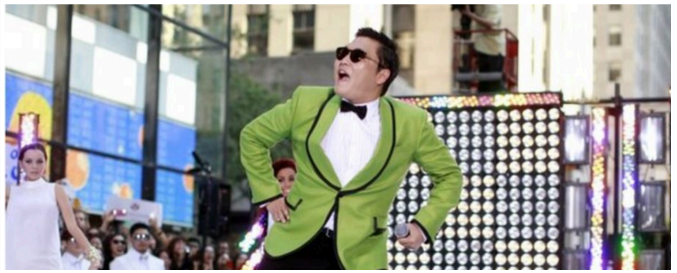
- Assume $w = 8$ is the bit-width
  - what happens if you try to add 1 to 11111111? => **Overflow**
  - there's no room for a number larger than 255 !
- The arithmetic "wraps around" back to the beginning of the range
  - adding 1 to 255 in an 8-bit system results in 0 (leading bit is discarded)
  - this wrapping around behavior can be useful in certain situations
  - applies to unsigned integers
    - basically taking the result $\mod 2^w$ or taking the lowest w bits

Overflow

- Occurs when the result of an operation is too large or too small to be represented within the allocated data type
  - in C, the runtime does not produce errors, values just **"wrap"**
- Can have consequences if not handled properly
  - incorrect calculations
  - program crashes due to unexpected behavior
  - security vulnerabilities
- To prevent overflow
  - choose appropriate data types with sufficient range
  - implement checks and validations within the code

Gangnam Style music video 'broke' YouTube view limit

4 December 2014



YouTube said the video - its most watched ever - has been viewed more than **2,147,483,647** times. It has now changed the maximum view limit to **9,223,372,036,854,775,808**, or more than nine quintillion.

Zero-Day Alert: Google Chrome Under Active Attack, Exploiting New Vulnerability

Nov 29, 2023 Newsroom

Google has rolled out security updates to fix seven security issues in its Chrome browser, including a zero-day that has come under active exploitation in the wild.

Tracked as **CVE-2023-6345**, the high-severity vulnerability has been described as an integer overflow bug in Skia, an open source 2D graphics library.

A zero-day attack takes place when hackers exploit the flaw before developers have a chance to address it.

Signed integers

- Trivial approach (**not used**)
  - use MSB as the sign bit, and the remaining bits to represent the number
  - all possibilities using $w = 3$ bits:

```
0 0 0  =  0
0 0 1  =  1
0 1 0  =  2
0 1 1  =  3
1 0 0  =  -0
1 0 1  =  -1
1 1 0  =  -2
1 1 1  =  -3
```

how is zero represented?

would addition still work?
(try adding 001 and 110)

Going from x to $-x$ (and vice-versa)

- **One's complement** of x:
  - flip all bits in x
  - called complement because $x + -x = 11\ldots11$
- **Two's complement** of x
  - flip all bits in x and add 1 to the result
  - $\sim x + 1$ (apply bitwise)

Using $w = 4$ bits

Binary	Unsigned	One's complement	Two's complement
0000	0	+0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

Practice

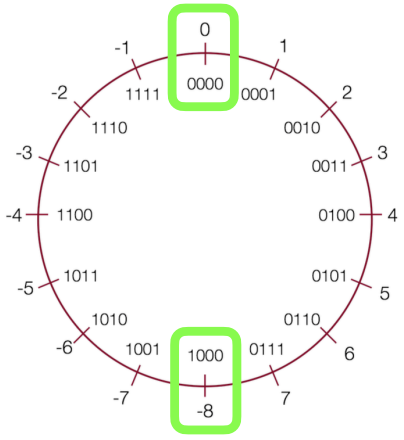
- Convert from x to -x using two's complement

`0 0 1 1 =`

`1 1 1 1 =`

`1 0 0 0 =`

`1 1 0 1 =`



The most negative number

- Exceptions to $\sim x + 1$
  - zero becomes zero (overflow)
  - the most negative number does not have a positive counterpart — impossible to represent (overflow)
- Can lead to unexpected programming bugs
  - the unary negation operator, -(-128) becomes -128
  - the absolute value may return a negative number, abs(-128) becomes -128
  - multiplication by -1 may fail, -128 * -1 becomes -128
  - division by -1 may cause an exception, -128 / -1 crashes
- In C the above behaviors are undefined

From two's complement to decimal

$$\sum_{i=0}^{w-1} x_i 2^i \qquad -x_{w-1}2^{w-1} + \sum_{i=0}^{w-2} x_i 2^i$$

Conversion to decimal for unsigned representations

Conversion to decimal for two's complement representations

Although C does not mandate using two's complement, in practice, two's complement is the **most widely used** representation for signed integers in modern computer systems.

Practice

- Convert from two's complement to decimal

```
-32 16 8 4 2 1
 0  0 1 1 1 0 =
 1  1 0 0 0 1 =
 1  0 0 0 0 0 =
 1  1 1 1 1 1 =
 1  1 1 1 1 0 =
 0  0 0 0 0 0 =
 0  0 0 0 0 1 =
 0  1 1 1 1 1 =
```

Two's complement (summary)

- Sign bit (MSB)
  - 0 for nonnegative, 1 for negative
- Non-negative numbers
  - no changes from the unsigned representation
- Negative numbers
  - two's complement of their positive counterparts
    - equivalent to their one's complement plus one

Binary	Unsigned	Signed (two's complement)
01010101	85	85
10101011	171	-85

Two's complement (advantages)

- Addition and subtraction of signed integers
  - use the same hardware as their unsigned counterparts
  - no need for special/additional circuitry
- Single representation for zero
  - no negative zero
- Trivial operation for extending the sign bit
  - e.g. increasing the bit-width of a number (**casting**)
- Widespread adoption

Numeric ranges

- Unsigned representation
  - min => 00…00 0
  - max => 11…11 $2^w - 1$
- Signed representation (two's complement)
  - min => 10…00 -2^{w-1}
  - max => 01…11 $2^{w-1} - 1$

data type	binary	hexadecimal	decimal
unsigned short int (min)			
unsigned short int (max)			
short int (min)			
short int (max)			

Basic data types in C

- The C standard does not define the size of "integer" types, except `char`
  - much safer to use `intN_t` and `uintN_t` for signed and unsigned integers of different sizes (stdint.h)
- The type of each variable tells the compiler how many bytes are necessary in memory
  - necessary for translation of high level code into machine code

```c
#include <stdint.h>

int main() {
    // unsigned integer using 4 bytes ==> 0x000000C8
    // 0000 0000 0000 0000 0000 0000 1100 1000
    uint32_t myvar = 200;
    // ....
    return 0;
}
```

Range of values

Data type	Size	Format	Value range
character	8	signed	–128 to 127
		unsigned	0 to 255
integer	16	signed	–32768 to 32767
		unsigned	0 to 65535
	32	signed	–2,147,483,648 to 2,147,483,647
		unsigned	0 to 4,294,967,295
	64	signed	–9,223,372,036,854,775,808 to 9,223,372,036,854,775,807
		unsigned	0 to 18,446,744,073,709,551,615

https://en.cppreference.com/w/cpp/language/types