CSC 411

Computer Organization (Spring 2024)
Lecture 3: Bitwise Operations

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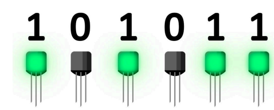
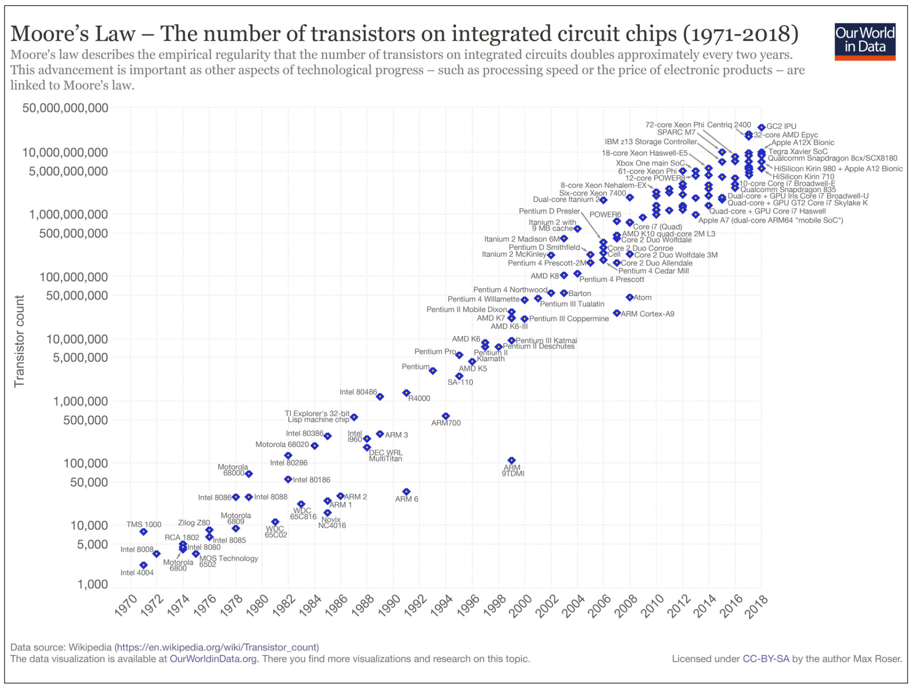
Bits

- Computers use the binary system to represent data
- A **bit** is the smallest unit of data in computing
  - can have a value of 0 or 1, easy to embed in digital devices
  - bits are the building blocks, forming the foundation for all digital information
- Bit Representation
  - bits are often represented by electrical voltages, where high voltage corresponds to 1 and low voltage to 0





Bytes

- A group of 8 bits is called a **byte**
  - bytes are commonly used to represent characters, numbers, and other data in computer systems

1	0	0	0	1	0	0	1
MSB most significant bit							LSB least significant bit

- How many different values can be stored in 1 byte?
- How many different values can be stored in n bits?

Basic data types in C

The C language does not explicitly define data sizes. The actual sizes can vary depending on the compiler and the system architecture.

C declaration		Bytes	
Signed	Unsigned	32-bit	64-bit
[signed] char	unsigned char	1	1
short	unsigned short	2	2
int	unsigned	4	4
long	unsigned long	4	8
int32_t	uint32_t	4	4
int64_t	uint64_t	8	8
char *		4	8
float		4	4
double		8	8

Boolean algebra

- Developed by George Boole in the 19th century
  - branch of mathematics that deals with binary variables and logic operations
  - binary values represent logical states true or false
- Three fundamental logic operations
  - **AND**: output is 1 only if both inputs are 1 — **conjunction**
  - **OR**: output is 1 if at least one input is 1 — **disjunction**
  - **NOT** output is the opposite of the input — **negation**
- Boolean expressions
  - formed by combining variables and logic operations

Basic boolean operations (AND, OR, NOT)

Logical operation	Operator	Notation	Alternative notations	Definition
Conjunction	AND	$x \wedge y$	x AND y, Kxy	$x \wedge y = 1$ if $x = y = 1$, $x \wedge y = 0$ otherwise
Disjunction	OR	$x \vee y$	x OR y, Axy	$x \vee y = 0$ if $x = y = 0$, $x \vee y = 1$ otherwise
Negation	NOT	$\neg x$	NOT x, Nx, $\bar{x}$, x', $!x$	$\neg x = 0$ if $x = 1$, $\neg x = 1$ if $x = 0$

x	y	$x \wedge y$	$x \vee y$
0	**0**	0	0
1	**0**	0	1
0	**1**	0	1
1	**1**	1	1

x	$\neg x$
0	1
1	0

Bit vectors

- Bit vectors are just sequences of bits
  - boolean algebra can be extended to operate on bit vectors
- Applications in Computer Science
  - representing sets of elements efficiently
  - implementing data structures
  - performing bitwise manipulation in low-level programming

```
Understanding boolean algebra with bit vectors is
essential for working with binary data in computer
science and digital design
```

Bitwise operators in C

- Used with “integer” data types
  - long, int, short, chart, unsigned
  - arguments are treated as bit vectors
  - corresponding binary logic operators are applied bitwise to operands
- Bitwise operators are commonly used to manipulate sets and masks

`~`	bitwise NOT	`~a`	the bitwise NOT of **a**
`&`	bitwise AND	`a & b`	the bitwise AND of **a** and **b**
`|`	bitwise OR	`a | b`	the bitwise OR of **a** and **b**
`^`	bitwise XOR	`a ^ b`	the bitwise XOR of **a** and **b**
`<<`	bitwise left shift	`a << b`	**a** left shifted by **b**
`>>`	bitwise right shift	`a >> b`	**a** right shifted by **b**

Bitwise operators in C

bit a	bit b	`a & b` (a AND b)
0	0	0
0	1	0
1	0	0
1	1	1

bit a	bit b	a | b (a OR b)
0	0	0
0	1	1
1	0	1
1	1	1

bit a	bit b	`a ^ b` (a XOR b)
0	0	0
0	1	1
1	0	1
1	1	0

~a (NOT a) is trivial

Examples

```
  1 0 1 1 1 0 0 1
& 0 1 1 1 0 1 1 0
  ---------------
  0 0 1 1 0 0 0 0
```

```
  1 0 1 1 1 0 0 1
| 0 1 1 1 0 1 1 0
  ---------------
  1 1 1 1 1 1 1 1
```

```
  1 0 1 1 1 0 0 1
^ 0 1 1 1 0 1 1 0
  ---------------
  1 1 0 0 1 1 1 1
```

```
~ 0 1 1 1 0 1 1 0
  ---------------
  1 0 0 0 1 0 0 1
```

Practice

```
~0x102
```

```
0xABC & 0x411
```

Practice

```
0xABC | 0x411
```

```
0x102030 & 0x00FF00
```

Shift operations

- Left shift (x << y)
  - shifts each bit in **x** to the left by the number of positions indicated by **y**
    - throw away **y** bits on left
  - blank spaces on right are filled up by zeroes

1	0	1	1	1	0	0	1	<< 2
1	1	1	0	0	1	0	0	

Shift operations

- Right shift (x >> y)
  - shifts each bit in **x** to the right by the number of positions indicated by **y**
    - throw away **y** bits on right

Logical shift: fill blank spaces on left with zeroes

Arithmetic shift: fill blank spaces by replicating original MSB (most compilers implement it — preserves sign bit)

1	0	1	1	1	0	0	1	>> 2
0	0	1	0	1	1	1	0	

logical

MSB

1	0	1	1	1	0	0	1	>> 2
1	1	1	0	1	1	1	0	

arithmetic

Practice

```
0xF3 << 2

0x9A >> 3    (logical)

0x9A >> 3    (arithmetic)
```

Example: bit masking

- Assume unsigned an integer j that stores the value 0x1A35B127
  - define a mask to extract the most significant byte
  - write C code to store the extracted value in another variable (unsigned int)

Example: bit masking

- Assume an integer j that stores the value 0x1A35B127
  - write C code to set the least significant byte of j to all ones leaving all other bytes unchanged

Example: encoding sets

- Instead of using arrays, we can store information more efficiently using bits
- Example:
  - assume we are encoding sets (8 different objects)
    - we can use a char variable, such that each bit represents 1 object

1	0	0	0	1	0	0	1
Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune

  - How to add, remove, or flip individual objects from the set?
  - How to check whether an object is in the set?
  - How to perform intersection, union, symmetric difference, and complement?

Practice

- Assume 4 DNA bases: A C T G
- How many bits are necessary per base? Write a possible encoding.
- If we store bases using integers (4 bytes), how many bases can we store in a single integer?
- Write the DNA sequence stored in 0x10012001

Bitwise vs logical operators in C

- Logical operators
  - NOT, AND, OR
  - apply to boolean values operands (true or false)
    - zero is considered false, and any non-zero value is considered true
  - always return a boolean value (true or false)

```
  !a
a && b
a || b
```

Practice

```
!0xF3

!0x00

!!0xF3

0xF3 && 0xF1

0xF3 || 0xF1
```