CSC 411

Computer Organization (Spring 2024)
Lecture 4: Integers

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TIOBE Index for January 2024

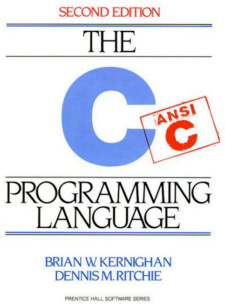
- Indicator of the popularity of programming languages
  - popular search engines such as Google, Bing, Yahoo!, Wikipedia, Amazon, YouTube and Baidu are used to calculate the ratings.

Jan 2024	Jan 2023	Change	Programming Language	Ratings	Change
1	1		Python	13.97%	-2.39%
2	2		C	11.44%	-4.81%
3	3		C++	9.96%	-2.95%
4	4		Java	7.87%	-4.34%
5	5		C#	7.16%	+1.43%
6	7	↑	JavaScript	2.77%	-0.11%
7	**10**	↑	**PHP**	**1.79%**	**+0.40%**
8	6	↓	Visual Basic	1.60%	-3.04%
9	8	↓	SQL	1.46%	-1.04%
10	20	⇈	Scratch	1.44%	+0.86%

https://www.tiobe.com/tiobe-index/

The C Language

- Developed by Dennis Ritchie at Bell Labs in the early 1970s
- Many operating systems, including Unix and its variants (Linux), are written in C
- Allows low-level access to memory, making it efficient for system programming
- C programs are generally portable across different platforms with minimal modification
- C follows a traditional compilation process, where the source code is translated into machine code by a compiler



Representing data

Representing data

- In memory, **all values** are stored as "bit-vectors"
  - data types are used to interpret the bits (provide meaning)
  - each possible bit-vector assigned exclusively to one meaning
- In a bit sequence of length w bits, we can represent 2^w different values
  - number of permutations with repetition — given w digits, there are two ways to choose each digit
  - example: how many different sequences can be represented in 4 bits?

0 0 0 0	1 0 0 0
0 0 0 1	1 0 0 1
0 0 1 0	1 0 1 0
0 0 1 1	1 0 1 1
0 1 0 0	1 1 0 0
0 1 0 1	1 1 0 1
0 1 1 0	1 1 1 0
0 1 1 1	1 1 1 1

$$2^4 = 16$$

What is this program doing?

```c
#include <stdio.h>

int main() {
    unsigned hex = 0x7e313134;

    float *f = (float *) &hex;
    char *s = (char *) &hex;

    printf("%u %d %.4ef %c%c%c%c",
           hex, hex, *f, s[0], s[1], s[2], s[3]);

    return 0;
}
```

```
2117153076   2117153076   5.8882e+37f   411~
```

Interlude: addition and multiplication

Binary addition

```
  1 1 1
  0 0 1 1 1 0 0 1
+ 0 1 1 1 0 1 1 0
  1 0 1 0 1 1 1 1
```

```
  1       1 1 1
  1 1 0 0 0 1 1 1
+ 0 1 0 1 0 1 1 1
1 0 0 0 1 1 1 1 0
```

Binary multiplication

```
      1 0 0 1
    x 0 1 1 0
    ---------
      0 0 0 0
    1 0 0 1
  1 0 0 1
0 0 0 0
---------
0 1 1 0 1 1 0
```

```
        1 1 1 1
      x 1 1 1 1
      ---------
        1 1 1 1
      1 1 1 1
    1 1 1 1
  1 1 1 1
---------
1 1 1 0 0 0 0 1
```

Tricky? the addition is performed row-by-row

Integer Representation

Unsigned integers

- Bits represent the number directly
  - same as binary-to-decimal conversion
  - w bits can represent 2^w unsigned integers ranging from 0 to $2^w - 1$

0	0	0	0	=	0
0	0	0	1	=	1
0	0	1	0	=	2
0	0	1	1	=	3
0	1	0	0	=	4
0	1	0	1	=	5
0	1	1	0	=	6
0	1	1	1	=	7

1	0	0	0	=	8
1	0	0	1	=	9
1	0	1	0	=	10
1	0	1	1	=	11
1	1	0	0	=	12
1	1	0	1	=	13
1	1	1	0	=	14
1	1	1	1	=	15

Overflow

- Assume $w = 8$ is the bit-length
  - what happens if you try to add 1 to 11111111? => **Overflow**
  - there's no room for a number larger than 255 !
- The arithmetic "wraps around" back to the beginning of the range
  - adding 1 to 255 in an 8-bit system results in 0 (leading bit is discarded)
  - this wrapping around behavior can be useful in certain situations
  - applies to unsigned integers
    - basically taking the result $\mod 2^n$ or taking the lowest n bits

Overflow

- Occurs when the result of an operation is too large or too small to be represented within the allocated data type
- Can have consequences if not handled properly
  - incorrect calculations
  - program crashes due to unexpected behavior
  - security vulnerabilities
- To prevent overflow
  - choose appropriate data types with sufficient range
  - implement checks and validations within the code

Signed integers

- Trivial approach (not used)
  - use MSB as the sign bit, and the remaining bits to represent the number
  - all possibilities using 3 bits:

```
0  0  0  =  0
0  0  1  =  1
0  1  0  =  2
0  1  1  =  3
1  0  0  =  -0
1  0  1  =  -1
1  1  0  =  -2
1  1  1  =  -3
```

```
- how is zero represented?

- would addition still work?

  - try adding 001 and 110
```

Going from x to $-x$ (and vice-versa)

- Assume x non-negative and represented by n bits
  - **one's complement** of x:
    - flip all bits in x
    - called complement because $x + -x = 11\ldots11$
  - **two's complement** of x
    - flip all bits in x and add 1 to the result
    - $\sim x + 1$ (bitwise)

Using 4 bits

Binary	Unsigned	One's complement	Two's complement
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	-7	-8
1001	9	-6	-7
1010	10	-5	-6
1011	11	-4	-5
1100	12	-3	-4
1101	13	-2	-3
1110	14	-1	-2
1111	15	-0	-1

Practice

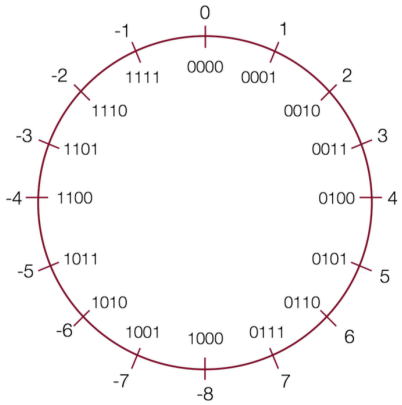
- Convert from x to -x using two's complement

```
0  0  1  1  =

1  1  0  0  =

1  0  0  0  =

1  1  1  1  =
```



Two's complement

- Positive numbers
  - no changes from the unsigned representation
- Negative numbers
  - two's complement of their positive counterparts (or their one's complement plus one)

unsigned	0 1 0 1 0 1 0 1 =	85
two's complement	1 0 1 0 1 0 1 1 =	-85

Two's complement

- Sign bit
  - MSB 0 for nonnegative
  - MSB 1 for negative
- Advantages
  - addition, subtraction, and multiplication of signed integers can be performed using the same hardware as their unsigned counterparts, without the need for special circuitry
  - there is a single representation for zero (no negative zero)
  - extending the sign bit when increasing the bit-length of a number is a simple operation
  - widespread adoption

From two's complement to decimal

$$\sum_{i=0}^{n-1} x_i 2^i$$

Conversion to decimal for unsigned representations

$$-x_{n-1}2^{n-1} + \sum_{i=0}^{n-2} x_i 2^i$$

Conversion to decimal for two's complement representations

Although C does not mandate using two's complement, in practice, two's complement is the most widely used representation for signed integers in modern computer systems.

Practice

- Convert from two's complement to decimal

−32	16	8	4	2	1	
0	0	1	1	1	0	=

−32	16	8	4	2	1	
1	1	1	1	1	0	=

−32	16	8	4	2	1	
1	1	0	0	0	1	=

−32	16	8	4	2	1	
0	0	0	0	0	0	=

−32	16	8	4	2	1	
1	0	0	0	0	0	=

−32	16	8	4	2	1	
0	0	0	0	0	1	=

−32	16	8	4	2	1	
1	1	1	1	1	1	=

−32	16	8	4	2	1	
0	1	1	1	1	1	=

Numeric ranges

- Unsigned representation
  - min => 00…00 0
  - max => 11…11 $2^n - 1$
- Signed representation (two's complement)
  - min => 10…00 -2^{n-1}
  - max => 01…11 $2^{n-1} - 1$

short int	Decimal	Hex	Binary
Unsigned min			
Unsigned max			
Signed min			
Signed max			

Range of values

Data type	Size in bits	Format	Value range
character	8	signed	–128 to 127
		unsigned	0 to 255
integer	16	signed	–32768 to 32767
		unsigned	0 to 65535
	32	signed	–2,147,483,648 to 2,147,483,647
		unsigned	0 to 4,294,967,295
	64	signed	–9,223,372,036,854,775,808 to 9,223,372,036,854,775,807
		unsigned	0 to 18,446,744,073,709,551,615

https://en.cppreference.com/w/cpp/language/types