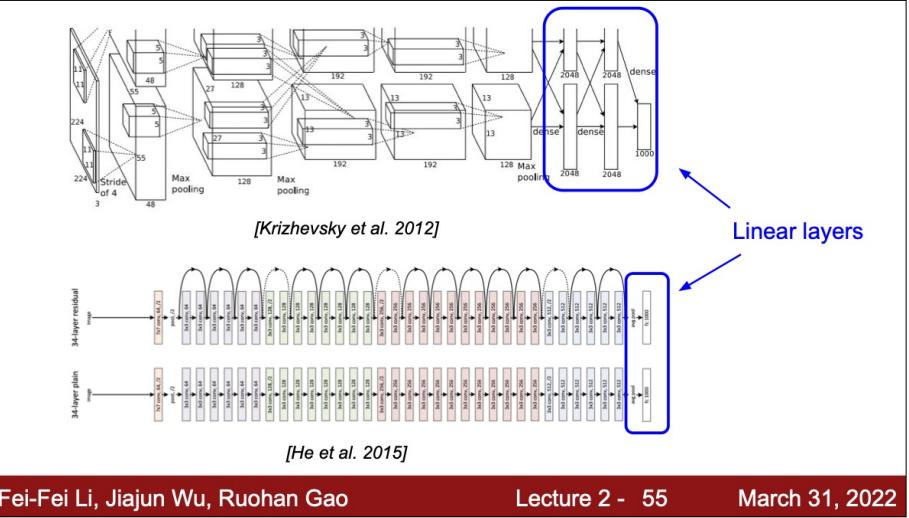


Lecture 4: Neural Networks and Backpropagation

Fei-Fei Li, Jiajun Wu, Ruohan Gao

Lecture 4 - 1

April 07, 2022



Fei-Fei Li, Jiajun Wu, Ruohan Gao

Lecture 2 - 55

March 31, 2022

Neural networks: the original linear classifier

$$\begin{aligned} \text{(Before) Linear score function: } f &= Wx \\ x \in \mathbb{R}^D, W &\in \mathbb{R}^{C \times D} \end{aligned}$$

Neural networks: 2 layers

$$\begin{aligned} \text{(Before) Linear score function: } f &= Wx \\ \text{(Now) 2-layer Neural Network } f &= W_2 \max(0, W_1 x) \\ x \in \mathbb{R}^D, W_1 &\in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H} \end{aligned}$$

(In practice we will usually add a learnable bias at each layer as well)

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Neural networks: also called fully connected network

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

“Neural Network” is a very broad term; these are more accurately called “fully-connected networks” or sometimes “multi-layer perceptrons” (MLP)

(In practice we will usually add a learnable bias at each layer as well)

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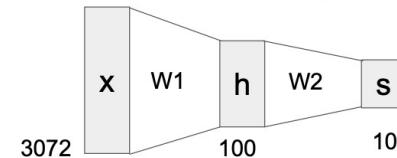
Lecture 4 - 19

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Neural networks: hierarchical computation

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



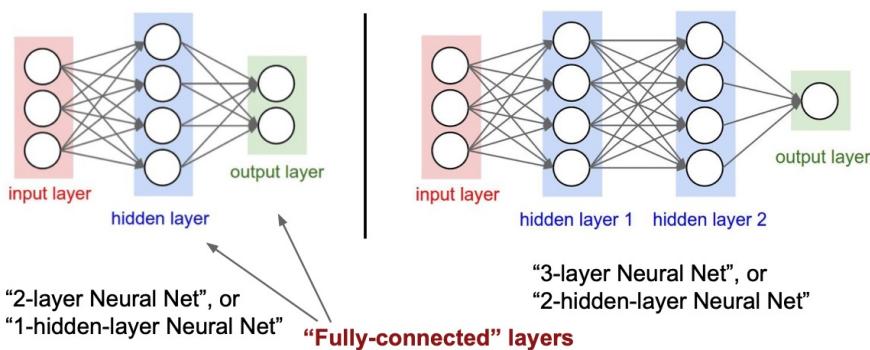
$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

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Neural networks: Architectures



Neural networks: 3 layers

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$
or 3-layer Neural Network

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H_1 \times D}, W_2 \in \mathbb{R}^{H_2 \times H_1}, W_3 \in \mathbb{R}^{C \times H_2}$$

(In practice we will usually add a learnable bias at each layer as well)

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Lecture 4 - 20

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Neural networks: why is max operator important?

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

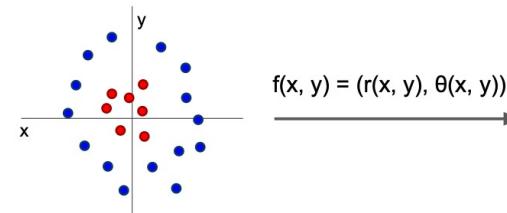
The function $\max(0, z)$ is called the **activation function**.

Q: What if we try to build a neural network without one?

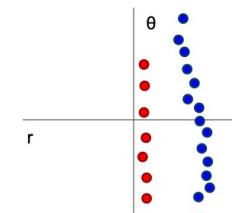
$$f = W_2 W_1 x \quad W_3 = W_2 W_1 \in \mathbb{R}^{C \times H}, f = W_3 x$$

A: We end up with a linear classifier again!

Why do we want non-linearity?



Cannot separate red and blue points with linear classifier

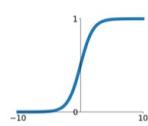


After applying feature transform, points can be separated by linear classifier

Activation functions

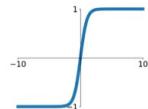
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



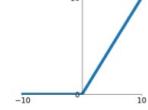
tanh

$$\tanh(x)$$



ReLU

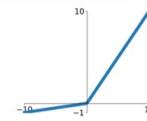
$$\max(0, x)$$



ReLU is a good default choice for most problems

Leaky ReLU

$$\max(0.1x, x)$$



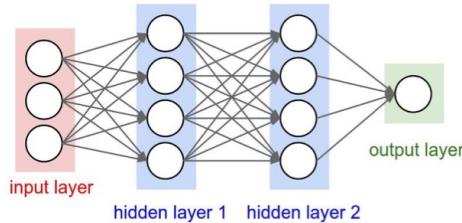
Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$

Example feed-forward computation of a neural network



```
# forward-pass of a 3-layer neural network:
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Full implementation of training a 2-layer Neural Network needs ~20 lines:

```
1 import numpy as np
2 from numpy.random import randn
3
4 N, D_in, H, D_out = 64, 1000, 100, 10
5 x, y = randn(N, D_in), randn(N, D_out)
6 w1, w2 = randn(D_in, H), randn(H, D_out)
7
8 for t in range(2000):
9     h = 1 / (1 + np.exp(-x.dot(w1)))
10    y_pred = h.dot(w2)
11    loss = np.square(y_pred - y).sum()
12    print(t, loss)
13
14    grad_y_pred = 2.0 * (y_pred - y)
15    grad_w2 = h.T.dot(grad_y_pred)
16    grad_h = grad_y_pred.dot(w2.T)
17    grad_w1 = x.T.dot(grad_h * h * (1 - h))
18
19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

Define the network

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Lecture 4 - 29

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Full implementation of training a 2-layer Neural Network needs ~20 lines:

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19    w1 -= 1e-4 * grad_w1
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```

Define the network

Forward pass

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Lecture 4 - 30

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```

Define the network

Forward pass

Calculate the analytical gradients

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Lecture 4 - 31

April 07, 2022

Full implementation of training a 2-layer Neural Network needs ~20 lines:

```
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```

Define the network

Forward pass

Calculate the analytical gradients

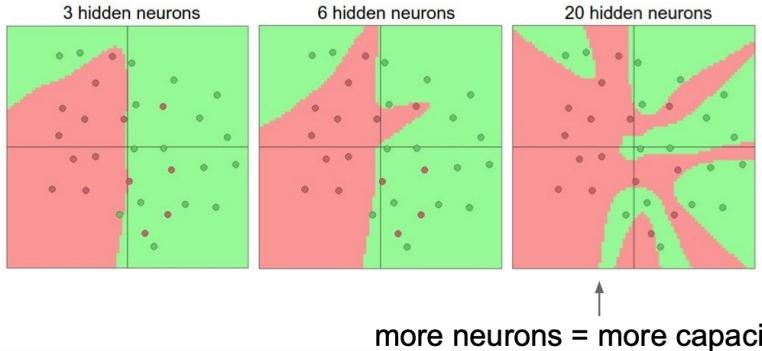
Gradient descent

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Lecture 4 - 32

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Setting the number of layers and their sizes



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Lecture 4 - 33

April 07, 2022

Problem: How to compute gradients?

$$s = f(x; W_1, W_2) = W_2 \max(0, W_1 x) \quad \text{Nonlinear score function}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \quad \text{SVM Loss on predictions}$$

$$R(W) = \sum_k W_k^2 \quad \text{Regularization}$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda R(W_1) + \lambda R(W_2) \quad \text{Total loss: data loss + regularization}$$

If we can compute $\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}$ then we can learn W_1 and W_2

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Lecture 4 - 44

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(Bad) Idea: Derive $\nabla_W L$ on paper

$$s = f(x; W) = Wx$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$= \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda \sum_k W_k^2$$

$$= \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1) + \lambda \sum_k W_k^2$$

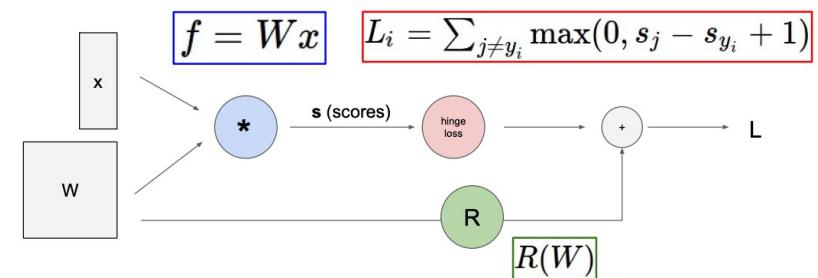
$$\nabla_W L = \nabla_W \left(\frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, W_{j,:} \cdot x + W_{y_i,:} \cdot x + 1) + \lambda \sum_k W_k^2 \right)$$

Problem: Very tedious: Lots of matrix calculus, need lots of paper

Problem: What if we want to change loss? E.g. use softmax instead of SVM? Need to re-derive from scratch =(

Problem: Not feasible for very complex models!

Better Idea: Computational graphs + Backpropagation



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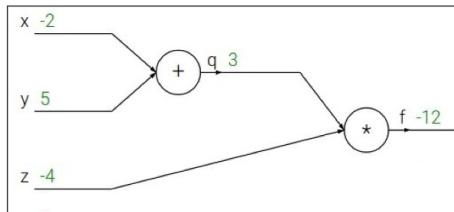
Lecture 4 - 46

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Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$

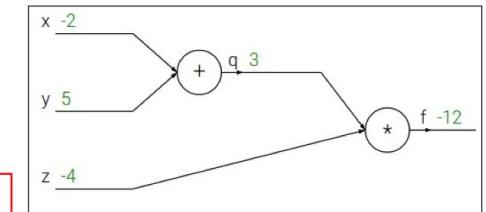


Backpropagation: a simple example

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$$q = x + y \quad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$



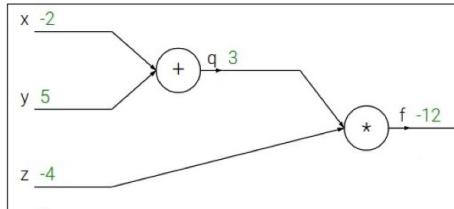
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Backpropagation: a simple example

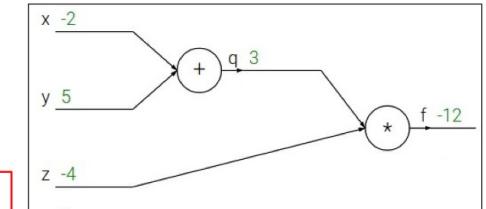
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Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



Backpropagation: a simple example

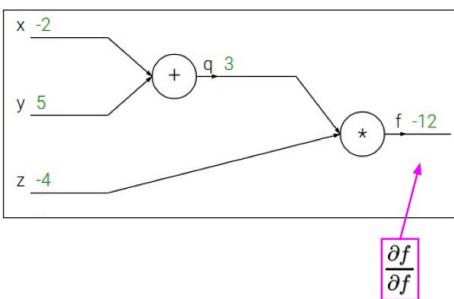
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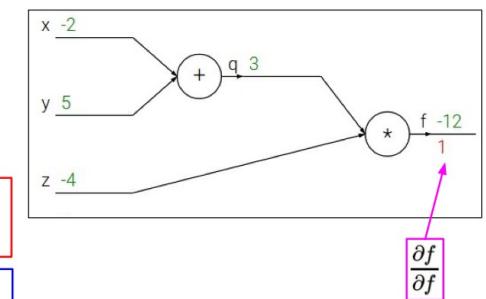
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Backpropagation: a simple example

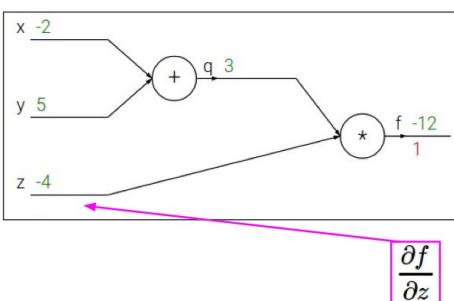
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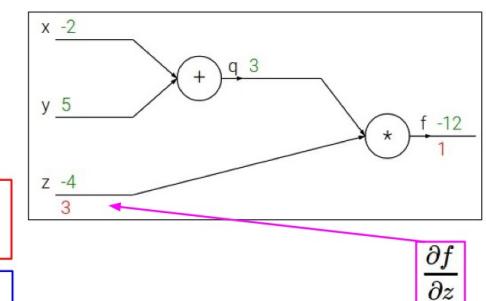
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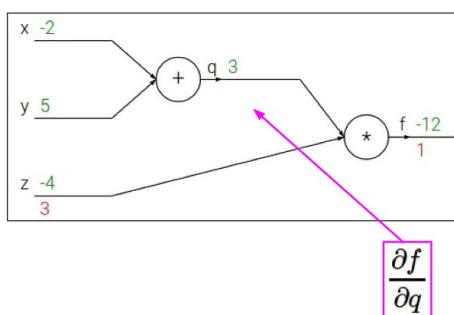
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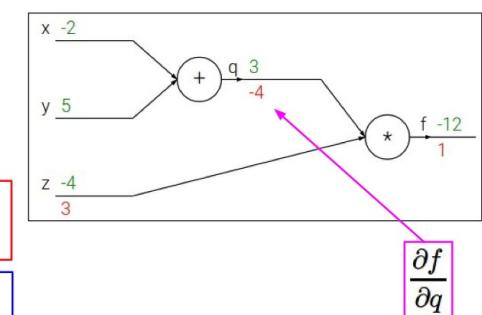
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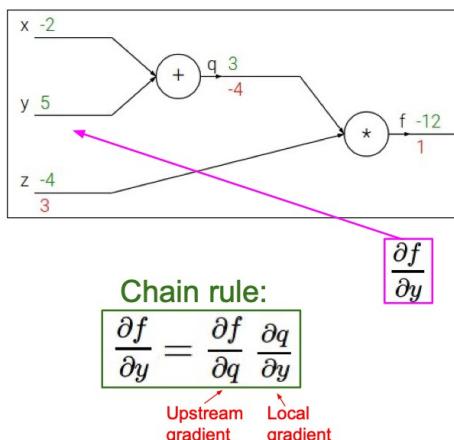
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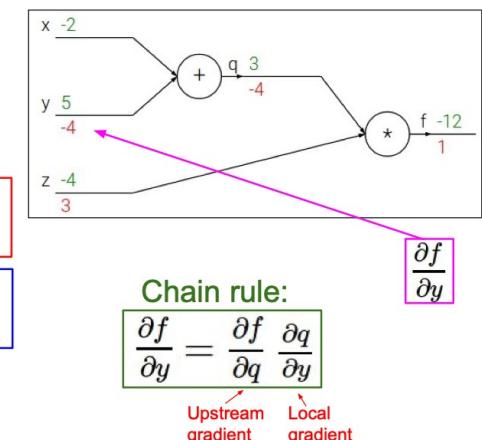
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Backpropagation: a simple example

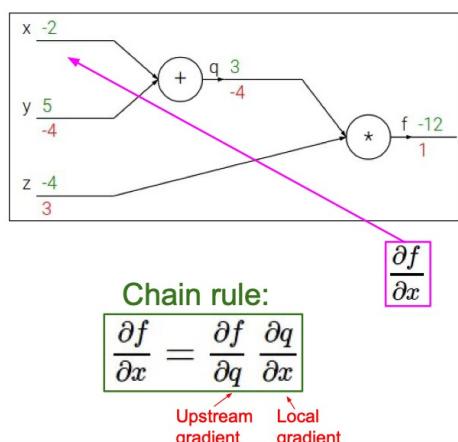
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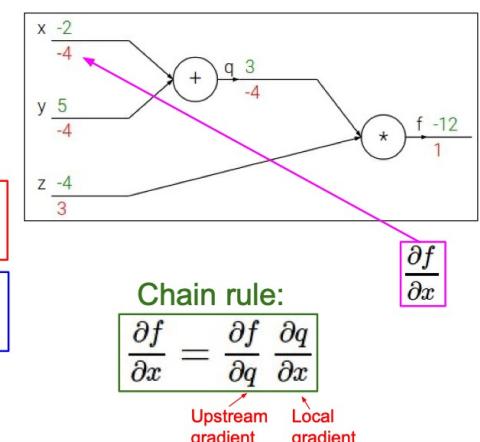
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Fei-Fei Li, Jiajun Wu, Ruohan Gao

Lecture 4 - 65

April 07, 2022

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Lecture 4 - 66

April 07, 2022

Neural Networks

DALL-E 2



"Teddy bears working on new AI research on the moon in the 1980s."

"Rabbits attending a college seminar on human anatomy."

"A wise cat meditating in the Himalayas searching for enlightenment."

Fei-Fei Li, Ehsan Adeli

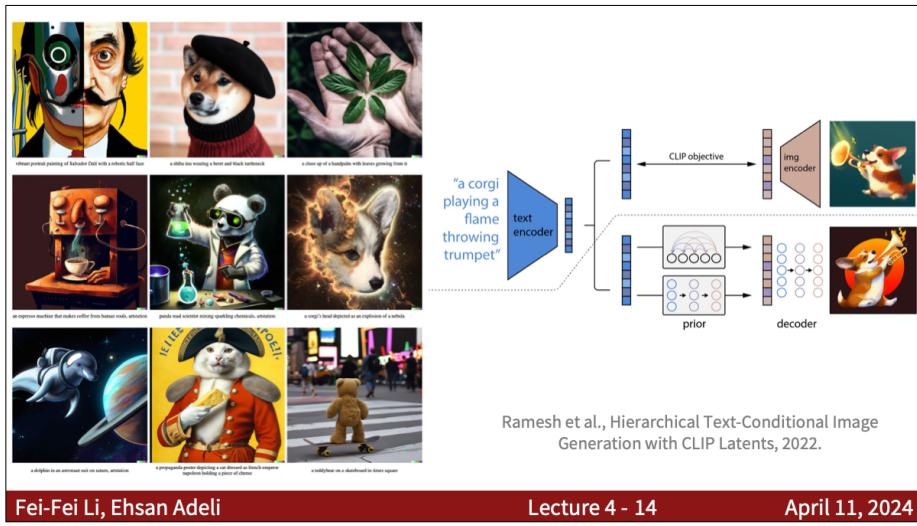
Lecture 4 - 13

April 11, 2024

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Lecture 4 - 22

April 11, 2024



Fei-Fei Li, Ehsan Adeli

Lecture 4 - 14

April 11, 2024

DALL-E 3

An illustration from a graphic novel. A bustling city street under the shine of a full moon. The sidewalks busting with pedestrians enjoying the nightlife. At the corner stall, a young woman with fiery red hair, dressed in a signature velvet cloak, is haggling with the grumpy old vendor. The grumpy vendor, a tall, sophisticated man wearing a sharp suit, who sports a noteworthy mustache is animatedly conversing on his steampunk telephone.



Betker, James, et al. "Improving image generation with better captions." Computer Science. <https://cdn.openai.com/papers/dall-e-3.pdf> (2023).

Fei-Fei Li, Ehsan Adeli

Lecture 4 - 16

April 11, 2024

Segment Anything Model (SAM)



Fei-Fei Li, Ehsan Adeli

Lecture 4 - 18

April 11, 2024

Sora

- Animating Images (generated by DALL-E)
- Video-to-video editing



A Shiba Inu dog wearing a beret and black turtleneck.



put the video in space with a rainbow road
change the video setting to be different than a mountain? perhaps joshua tree

<https://openai.com/research/video-generation-models-as-world-simulators>

Fei-Fei Li, Ehsan Adeli

Lecture 4 - 20

April 11, 2024

Sora

- More compute



Base Compute



4x Compute



32x Compute

<https://openai.com/research/video-generation-models-as-world-simulators>