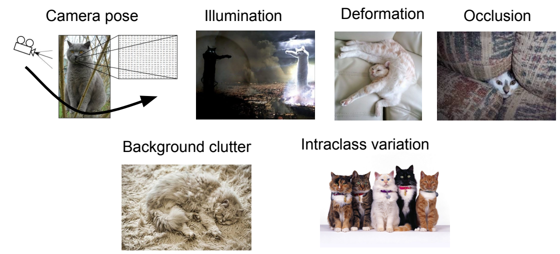
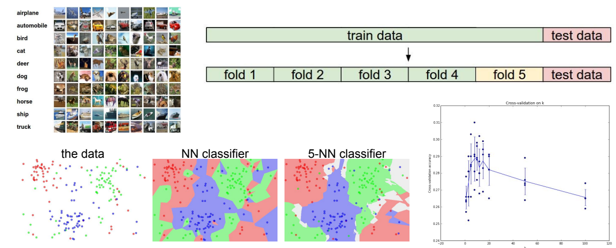
Lecture 3:
Loss functions and Optimization

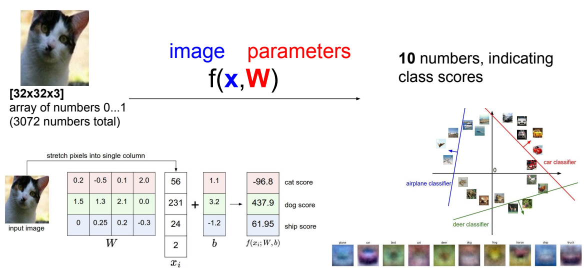
Recall from last time… Challenges in Visual Recognition



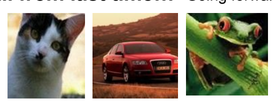
Recall from last time… data-driven approach, kNN



Recall from last time… Linear classifier



Recall from last time… Going forward: Loss function/Optimization

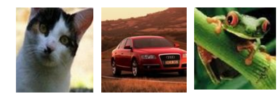


airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.
2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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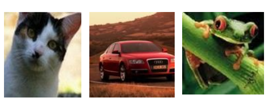

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

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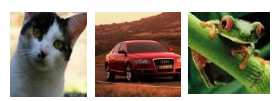

cat	**3.2**	1.3	2.2
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Losses:	2.9		

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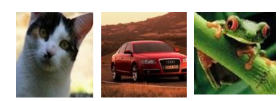
the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



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Losses:	2.9	0	

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 5.3) + max(0, 5.6)
= 5.3 + 5.6
= 10.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
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where x_i is the image and
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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

and the full training loss is the mean over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 10.9)/3
= 4.6

Suppose: 3 training examples, 3 classes.
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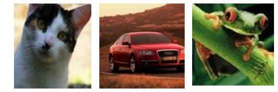

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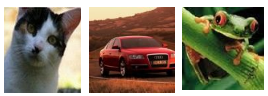
and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

Q: what if the sum was instead over all classes? (including j = y_i)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



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Losses:	2.9	0	10.9

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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
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Q2: what if we used a mean instead of a sum here?

Suppose: 3 training examples, 3 classes.
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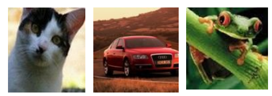

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Multiclass SVM loss:

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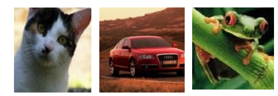
and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

Q3: what if we used

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



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Multiclass SVM loss:

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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

Q4: what is the min/max possible loss?

Suppose: 3 training examples, 3 classes.
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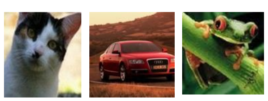

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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:
$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

Q5: usually at initialization W are small numbers, so all s ~= 0. What is the loss?

Example numpy code:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

```python
def L_i_vectorized(x, y, W):
  scores = W.dot(x)
  margins = np.maximum(0, scores - scores[y] + 1)
  margins[y] = 0
  loss_i = np.sum(margins)
  return loss_i
```


$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

There is a bug with the loss:

$f(x,W) = Wx$

$L = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq y_i}\max(0, f(x_i;W)_j - f(x_i;W)_{y_i} + 1)$

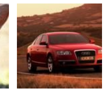
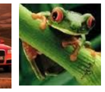
There is a bug with the loss:

$f(x,W) = Wx$

$L = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq y_i}\max(0, f(x_i;W)_j - f(x_i;W)_{y_i} + 1)$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x,W) = Wx$ are:

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Losses:	2.9	0	

$L_i = \sum_{j\neq y_i}\max(0, s_j - s_{y_i} + 1)$

Before:
= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

With W twice as large:
= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

Weight Regularization

\lambda = regularization strength
(hyperparameter)

$L = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq y_i}\max(0, f(x_i;W)_j - f(x_i;W)_{y_i} + 1) + \boxed{\lambda R(W)}$

In common use:

L2 regularization $R(W) = \sum_k\sum_l W_{k,l}^2$
L1 regularization $R(W) = \sum_k\sum_l |W_{k,l}|$
Elastic net (L1 + L2) $R(W) = \sum_k\sum_l \beta W_{k,l}^2 + |W_{k,l}|$
Max norm regularization (might see later)
Dropout (will see later)

L2 regularization: motivation

$x = [1,1,1,1]$

$w_1 = [1,0,0,0]$
$w_2 = [0.25, 0.25, 0.25, 0.25]$

$$w_1^T x = w_2^T x = 1$$

Softmax Classifier (Multinomial Logistic Regression)

cat **3.2**
car 5.1
frog -1.7

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$s = f(x_i; W)$

cat **3.2**
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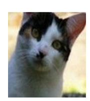
Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$ where $s = f(x_i; W)$

cat **3.2**
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Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

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Softmax function

cat **3.2**
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Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$ where $s = f(x_i; W)$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

cat	**3.2**
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Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

in summary: $L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

cat	**3.2**
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unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

unnormalized probabilities

		exp	
cat	**3.2**		**24.5**
car	5.1		164.0
frog	-1.7		0.18

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

unnormalized probabilities

		exp		normalize	
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

unnormalized probabilities

		exp		normalize	
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
= **0.89**

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

Q: What is the min/max possible loss L_i?

unnormalized probabilities

		exp		normalize	
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
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unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$

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unnormalized probabilities

		exp		normalize	
cat	**3.2**		**24.5**		**0.13**
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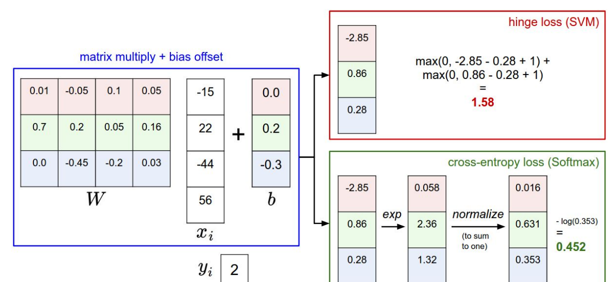
unnormalized log probabilities

probabilities



Softmax vs. SVM

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}) \qquad L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

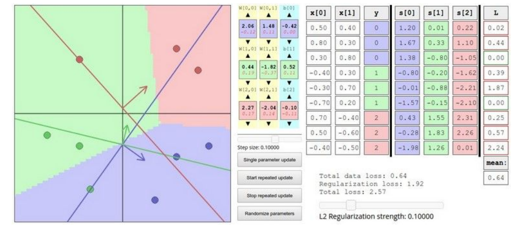
Softmax vs. SVM

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}) \qquad L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:
[10, -2, 3]
[10, 9, 9]
[10, -100, -100]
and $\boxed{y_i = 0}$

Q: Suppose I take a datapoint and I jiggle a bit (changing its score slightly). What happens to the loss in both cases?

Interactive Web Demo time....



http://vision.stanford.edu/teaching/cs231n/linear-classify-demo/

Optimization

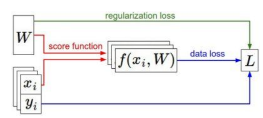
Recap

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

$$L_i = -\log(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}})$$ Softmax

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$ SVM

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



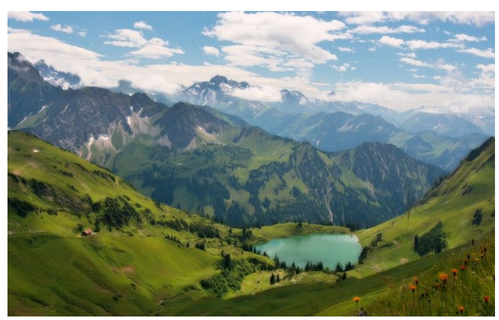
Strategy #1: A first very bad idea solution: **Random search**

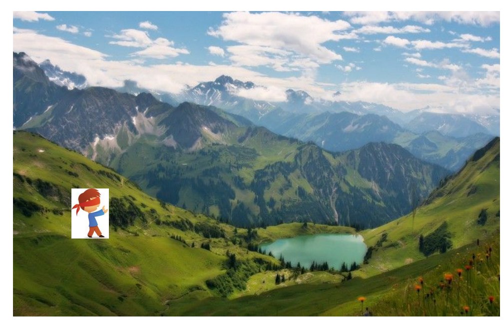
```
# assume X_train is the data where each column is an example (e.g. 3073 x 50,000)
# assume Y_train are the labels (e.g. 1D array of 50,000)
# assume the function L evaluates the loss function

bestloss = float("inf") # Python assigns the highest possible float value
for num in xrange(1000):
  W = np.random.randn(10, 3073) * 0.0001 # generate random parameters
  loss = L(X_train, Y_train, W) # get the loss over the entire training set
  if loss < bestloss: # keep track of the best solution
    bestloss = loss
    bestW = W
  print 'in attempt %d the loss was %f, best %f' % (num, loss, bestloss)

# prints:
# in attempt 0 the loss was 9.401632, best 9.401632
# in attempt 1 the loss was 8.959668, best 8.959668
# in attempt 2 the loss was 9.044034, best 8.959668
# in attempt 3 the loss was 9.278948, best 8.959668
# in attempt 4 the loss was 8.857370, best 8.857370
# in attempt 5 the loss was 8.943151, best 8.857370
# in attempt 6 the loss was 8.605604, best 8.605604
# ... (trunctated: continues for 1000 lines)
```


Lets see how well this works on the test set...

```
# Assume X_test is [3073 x 10000], Y_test [10000 x 1]
scores = Wbest.dot(Xte_cols) # 10 x 10000, the class scores for all test examples
# find the index with max score in each column (the predicted class)
Yte_predict = np.argmax(scores, axis = 0)
# and calculate accuracy (fraction of predictions that are correct)
np.mean(Yte_predict == Yte)
# returns 0.1555
```

15.5% accuracy! not bad!
(SOTA is ~95%)





Strategy #2: **Follow the slope**

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives).

current W:		**gradient dW:**
[0.34,		[?,
-1.11,		?,
0.78,		?,
0.12,		?,
0.55,		?,
2.81,		?,
-3.1,		?,
-1.5,		?,
0.33,…]		?,…]
loss 1.25347		

current W:	**W + h** (first dim):	**gradient dW:**
[0.34,	[0.34 + **0.0001**,	[?,
-1.11,	-1.11,	?,
0.78,	0.78,	?,
0.12,	0.12,	?,
0.55,	0.55,	?,
2.81,	2.81,	?,
-3.1,	-3.1,	?,
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25322**	

current W:	**W + h** (first dim):	**gradient dW:**
[0.34,	[0.34 + **0.0001**,	[**-2.5**,
-1.11,	-1.11,	?,
0.78,	0.78,	?,
0.12,	0.12,	
0.55,	0.55,	
2.81,	2.81,	
-3.1,	-3.1,	
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25322**	

(1.25322 - 1.25347)/0.0001
= -2.5

$$\boxed{\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

current W:	**W + h** (second dim):	**gradient dW:**
[0.34,	[0.34,	[-2.5,
-1.11,	-1.11 + **0.0001**,	?,
0.78,	0.78,	?,
0.12,	0.12,	?,
0.55,	0.55,	?,
2.81,	2.81,	?,
-3.1,	-3.1,	?,
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25353**	

current W:	**W + h** (second dim):	**gradient dW:**
[0.34,	[0.34,	[-2.5,
-1.11,	-1.11 + **0.0001**,	**0.6**,
0.78,	0.78,	?,
0.12,	0.12,	?,
0.55,	0.55,	
2.81,	2.81,	
-3.1,	-3.1,	
-1.5,	-1.5,	
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25353**	

(1.25353 - 1.25347)/0.0001
= 0.6

$$\boxed{\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

current W:	**W + h** (third dim):	**gradient dW:**
[0.34,	[0.34,	[-2.5,
-1.11,	-1.11,	0.6,
0.78,	0.78 + **0.0001**,	?,
0.12,	0.12,	?,
0.55,	0.55,	?,
2.81,	2.81,	?,
-3.1,	-3.1,	?,
-1.5,	-1.5,	?,
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25347**	

current W:	**W + h** (third dim):	**gradient dW:**
[0.34,	[0.34,	[-2.5,
-1.11,	-1.11,	0.6,
0.78,	0.78 + **0.0001**,	**0**,
0.12,	0.12,	?,
0.55,	0.55,	
2.81,	2.81,	
-3.1,	-3.1,	
-1.5,	-1.5,	
0.33,…]	0.33,…]	?,…]
loss 1.25347	**loss 1.25347**	

(1.25347 - 1.25347)/0.0001
= 0

$$\boxed{\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Evaluation the gradient numerically

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

```
def eval_numerical_gradient(f, x):
  """
  a naive implementation of numerical gradient of f at x
  - f should be a function that takes a single argument
  - x is the point (numpy array) to evaluate the gradient at
  """

  fx = f(x) # evaluate function value at original point
  grad = np.zeros(x.shape)
  h = 0.00001

  # iterate over all indexes in x
  it = np.nditer(x, flags=['multi_index'], op_flags=['readwrite'])
  while not it.finished:

    # evaluate function at x+h
    ix = it.multi_index
    old_value = x[ix]
    x[ix] = old_value + h # increment by h
    fxh = f(x) # evalute f(x + h)
    x[ix] = old_value # restore to previous value (very important!)

    # compute the partial derivative
    grad[ix] = (fxh - fx) / h # the slope
    it.iternext() # step to next dimension

  return grad
```

Evaluation the gradient numerically

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- approximate
- very slow to evaluate

```python
def eval_numerical_gradient(f, x):
  """
  a naive implementation of numerical gradient of f at x
  - f should be a function that takes a single argument
  - x is the point (numpy array) to evaluate the gradient at
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    old_value = x[ix]
    x[ix] = old_value + h # increment by h
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    # compute the partial derivative
    grad[ix] = (fxh - fx) / h # the slope
    it.iternext() # step to next dimension

  return grad
```


This is silly. The loss is just a function of W:

$L = \frac{1}{N} \sum_{i=1}^{N} L_i + \sum_k W_k^2$

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

$s = f(x; W) = Wx$

want $\nabla_W L$

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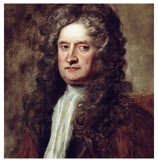

This is silly. The loss is just a function of W:

$L = \frac{1}{N} \sum_{i=1}^{N} L_i + \sum_k W_k^2$

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$s = f(x; W) = Wx$

want $\nabla_W L$

Calculus

This is silly. The loss is just a function of W:

$L = \frac{1}{N} \sum_{i=1}^{N} L_i + \sum_k W_k^2$

$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$

$s = f(x; W) = Wx$

$\nabla_W L$ = ...

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,…]

loss 1.25347

dW = ...
(some function
data and W)

gradient dW:

[-2.5,
0.6,
0,
0.2,
0.7,
-0.5,
1.1,
1.3,
-2.1,…]

In summary:

- Numerical gradient: approximate, slow, easy to write
- Analytic gradient: exact, fast, error-prone

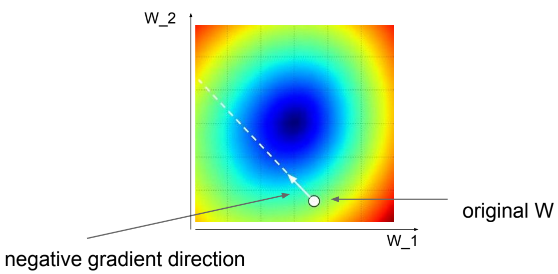
=>

In practice: Always use analytic gradient, but check implementation with numerical gradient. This is called a **gradient check.**

Gradient Descent

```python
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

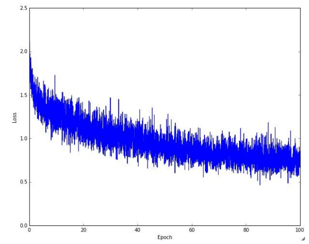
Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

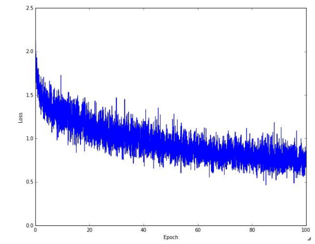
Common mini-batch sizes are 32/64/128 examples
e.g. Krizhevsky ILSVRC ConvNet used 256 examples

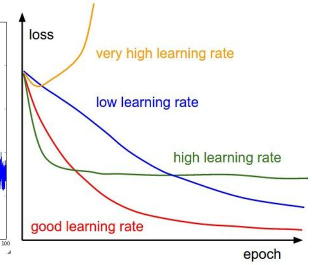


Example of optimization progress while training a neural network.

(Loss over mini-batches goes down over time.)

The effects of step size (or “learning rate”)





Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

we will look at more fancy update formulas (momentum, Adagrad, RMSProp, Adam, …)

Common mini-batch sizes are 32/64/128 examples
e.g. Krizhevsky ILSVRC ConvNet used 256 examples

Next class:

Becoming a backprop ninja
and
Neural Networks (part 1)