

Lecture 5: Image Classification with CNNs

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April 16, 2024

Image Classification: A core task in Computer Vision



(assume given a set of labels)
{dog, cat, truck, plane, ...}

cat
dog
bird
deer
truck

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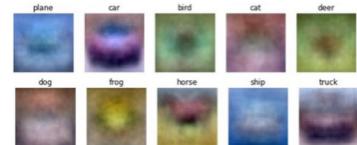
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Pixel space



$$f(x) = Wx$$

Class scores



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Image features



$$f(x) = Wx$$

Feature Representation

Class scores

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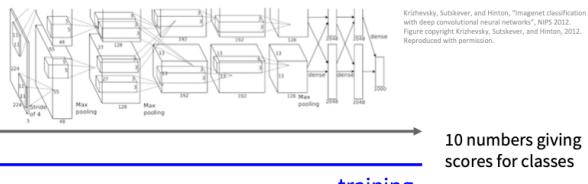
Image features vs. ConvNets



f

training

10 numbers giving scores for classes



10 numbers giving scores for classes

training

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Last Time: Neural Networks

Linear score function:

2-layer Neural Network

$$f = Wx$$

$$f = W_2 \max(0, W_1 x)$$

The spatial structure of images is destroyed!

32x32x3 \rightarrow 3072 \rightarrow 100 \rightarrow 10

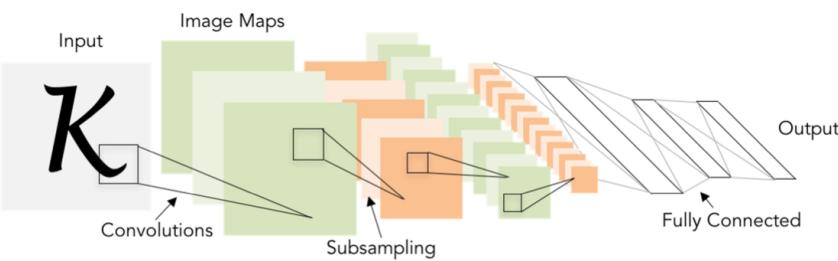


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Next: Convolutional Neural Networks



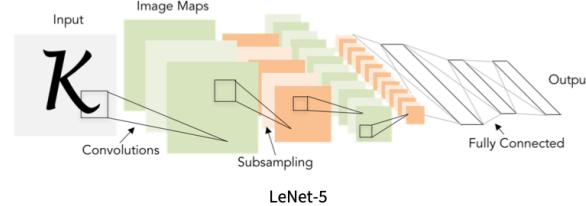
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A bit of history:

Gradient-based learning applied to document recognition
[LeCun, Bottou, Bengio, Haffner 1998]



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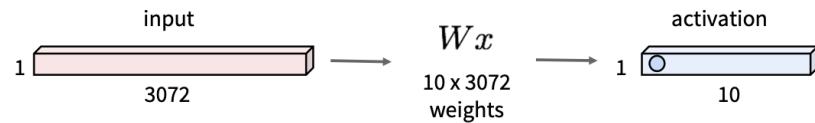
Convolutional Neural Networks

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Recap: Fully Connected Layer

32x32x3 image \rightarrow stretch to 3072 x 1

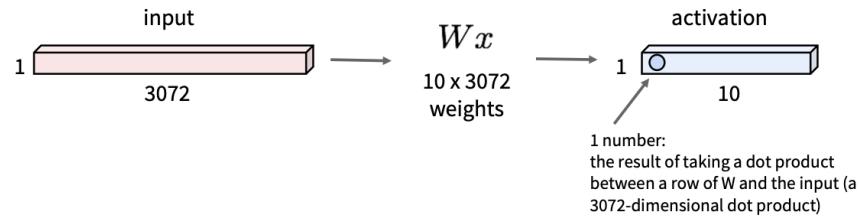


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Fully Connected Layer

32x32x3 image \rightarrow stretch to 3072 x 1

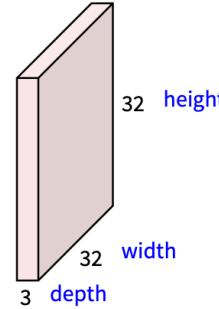


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Convolution Layer

32x32x3 image \rightarrow preserve spatial structure

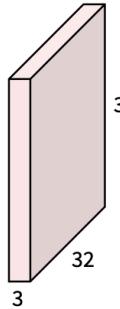


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Convolution Layer

32x32x3 image



5x5x3 filter



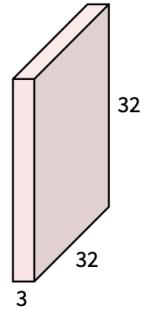
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

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Convolution Layer

32x32x3 image



5x5x3 filter



Filters always extend the full
depth of the input volume

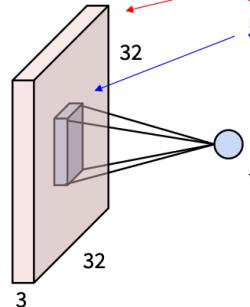
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

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Convolution Layer

32x32x3 image
5x5x3 filter w

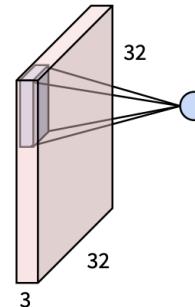


1 number:
the result of taking a dot product between the
filter and a small 5x5x3 chunk of the image
(i.e. $5 \times 5 \times 3 = 75$ -dimensional dot product + bias)
 $w^T x + b$

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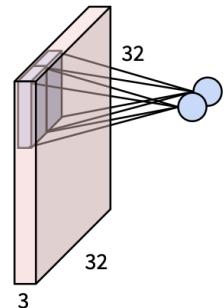
Convolution Layer



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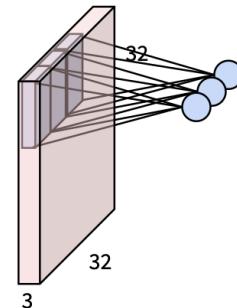
Convolution Layer



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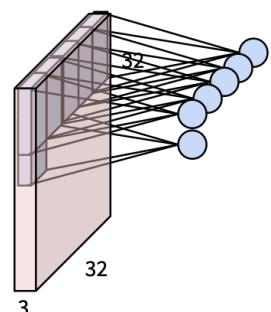
Convolution Layer



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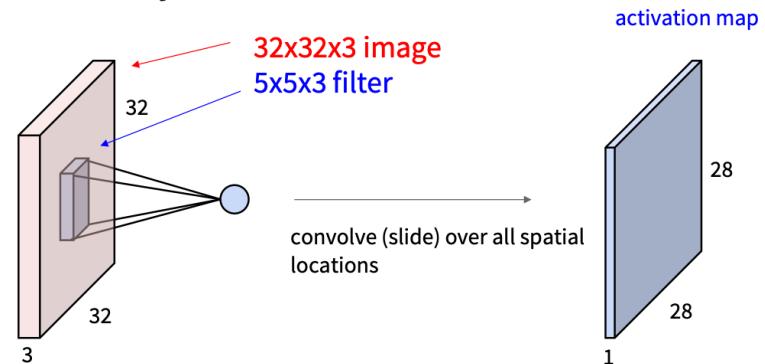
Convolution Layer



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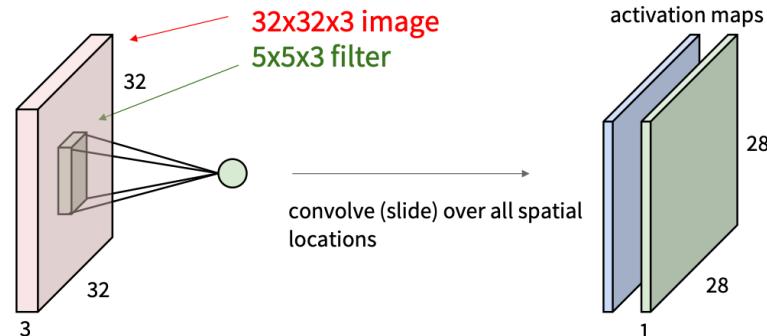
Convolution Layer



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Convolution Layer

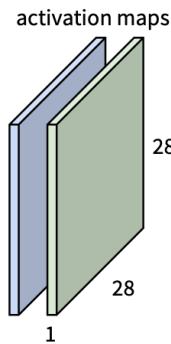


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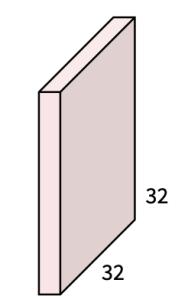
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consider a second, green filter

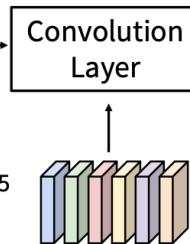


Convolution Layer

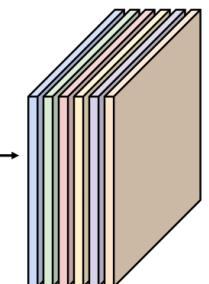
32x32x3 image



Consider 6 filters,
each 3x5x5



6 activation maps,
each 1x28x28



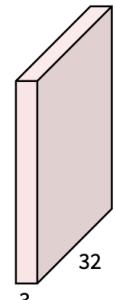
Slide inspiration: Justin Johnson
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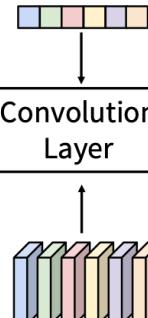
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Convolution Layer

3x32x32 image



Also 6-dim bias vector:

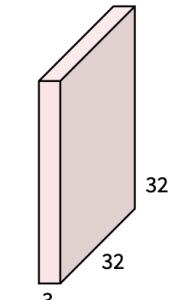


6 activation maps,
each 1x28x28

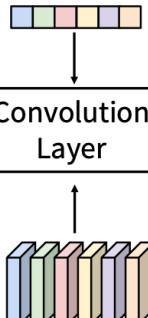
Stack activations to get a
6x28x28 output image!

Convolution Layer

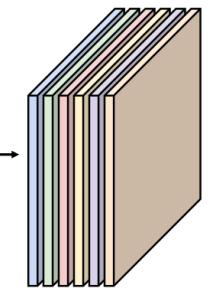
3x32x32 image



Also 6-dim bias vector:



28x28 grid, at each
point a 6-dim vector



Slide inspiration: Justin Johnson
Fei-Fei Li, Ehsan Adeli

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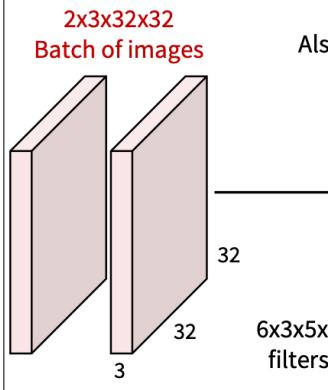
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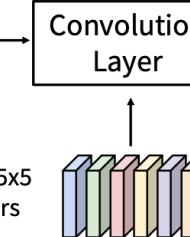
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Convolution Layer



Also 6-dim bias vector:



Slide inspiration: Justin Johnson

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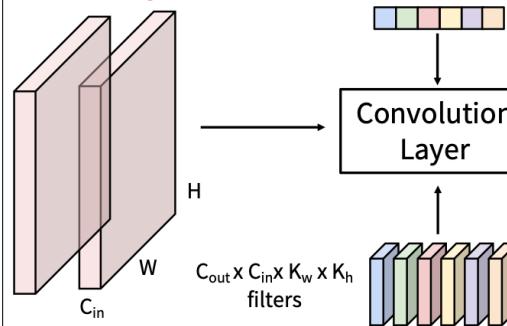
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Convolution Layer

N x C_{in} x H x W
Batch of images

Also C_{out}-dim bias vector:



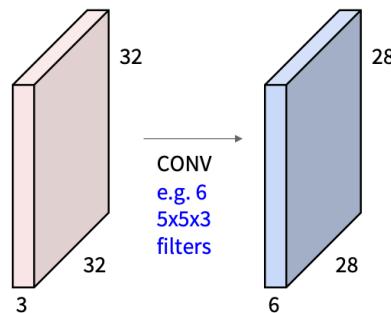
Slide inspiration: Justin Johnson

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Preview: ConvNet is a sequence of Convolution Layers

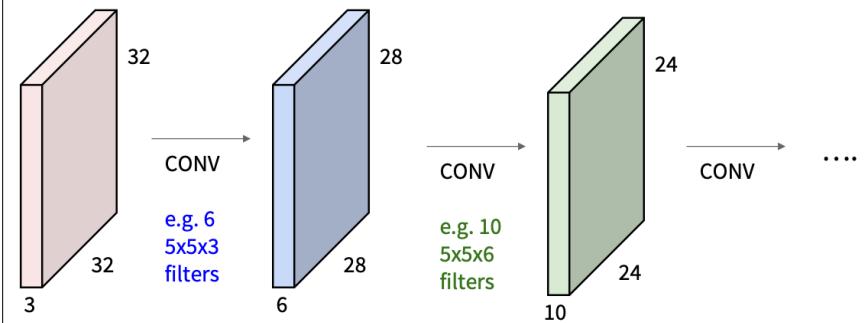


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Preview: ConvNet is a sequence of Convolution Layers

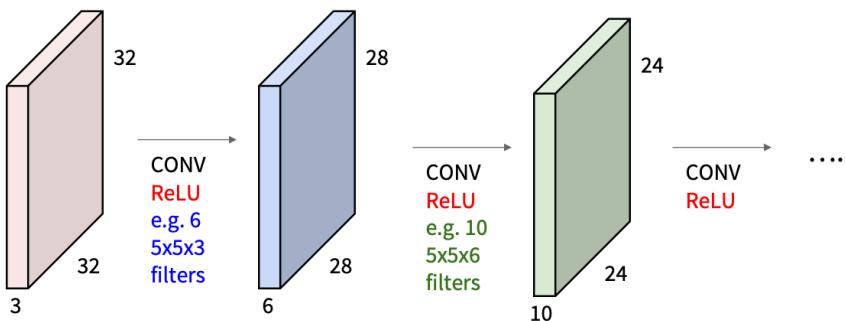


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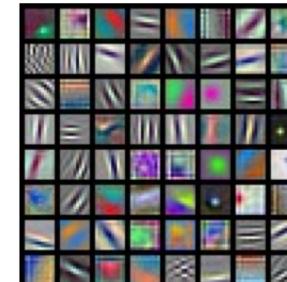
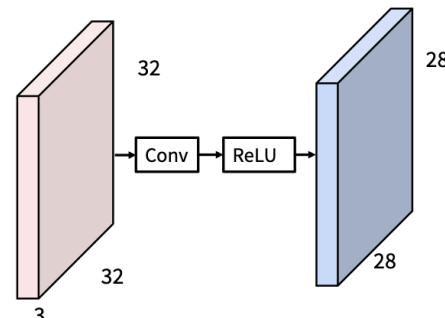
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



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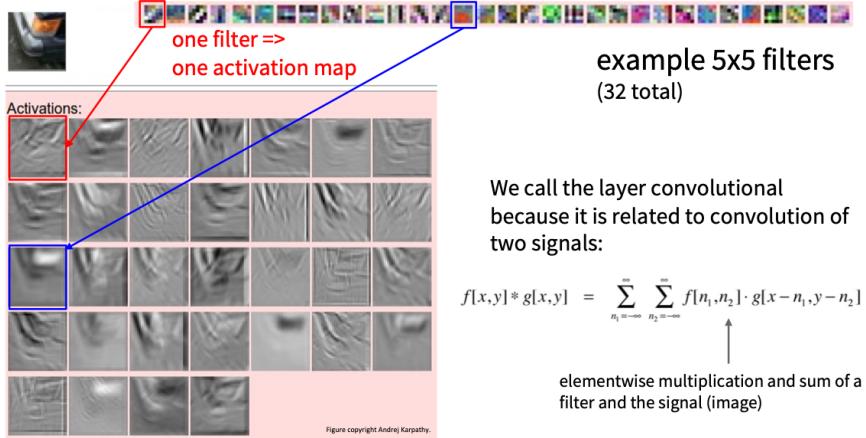
Preview: What do convolutional filters learn?



AlexNet: 64 filters, each $3 \times 11 \times 11$

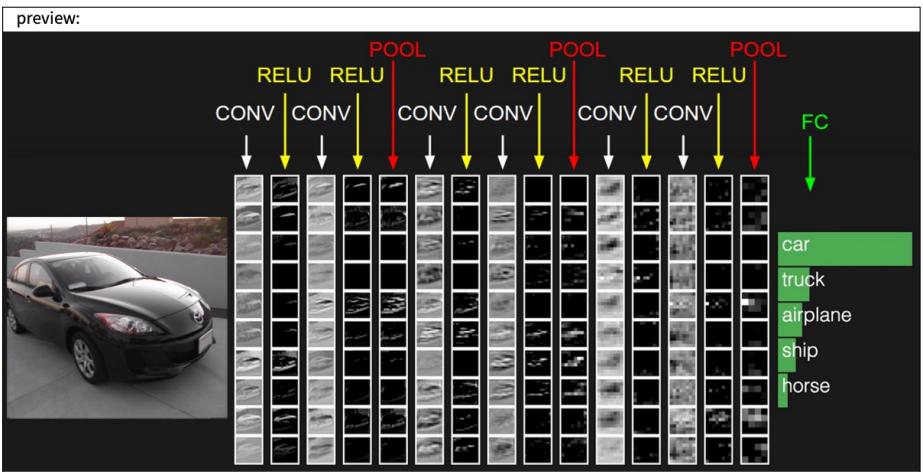
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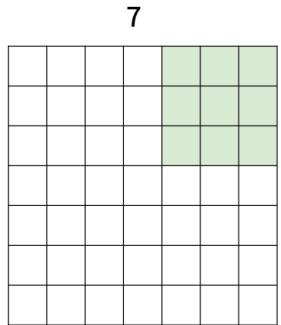
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A closer look at spatial dimensions:

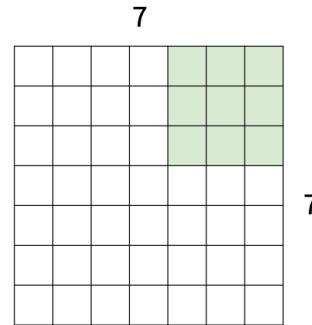


7x7 input (spatially)
assume 3x3 filter
=> 5x5 output

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A closer look at spatial dimensions:

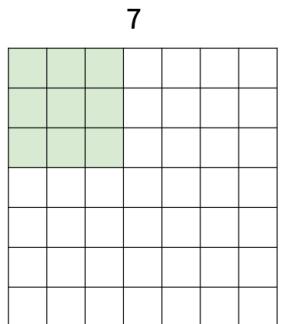


7x7 input (spatially)
assume 3x3 filter
applied with stride 2
=> 3x3 output!

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A closer look at spatial dimensions:



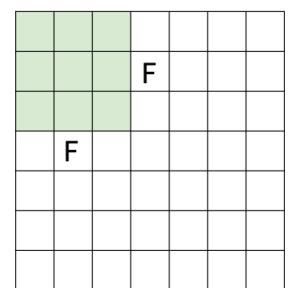
7x7 input (spatially)
assume 3x3 filter
applied with stride 3?

doesn't fit!
cannot apply 3x3 filter on 7x7
input with stride 3.

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N



Output size:
 $(N - F) / \text{stride} + 1$

e.g. N = 7, F = 3:
stride 1 => $(7 - 3)/1 + 1 = 5$
stride 2 => $(7 - 3)/2 + 1 = 3$
stride 3 => $(7 - 3)/3 + 1 = 2.33 : \backslash$

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In practice: Common to zero pad the border

0	0	0	0	0	0	0	
0							
0							
0							
0							

e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

(recall):
$$(N - F) / \text{stride} + 1$$

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In practice: Common to zero pad the border

0	0	0	0	0	0	0	
0							
0							
0							
0							

e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

7x7 output!

(recall):
$$(N + 2P - F) / \text{stride} + 1$$

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In practice: Common to zero pad the border

0	0	0	0	0	0	0	
0							
0							
0							
0							

e.g. input 7x7
3x3 filter, applied with stride 1
pad with 1 pixel border => what is the output?

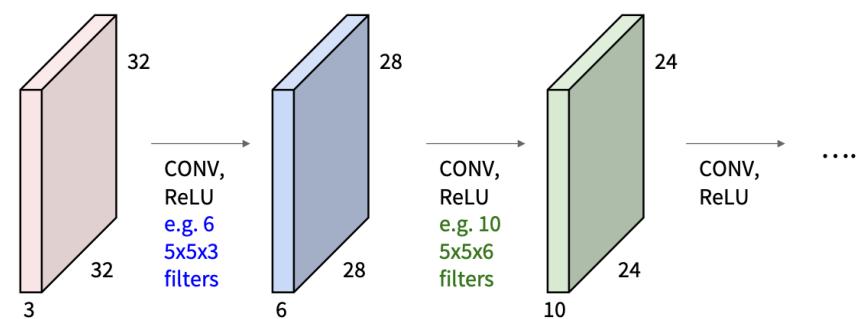
7x7 output!
in general, common to see CONV layers with stride 1,
filters of size $F \times F$, and zero-padding with $(F-1)/2$. (will
preserve size spatially)
e.g. $F=3 \Rightarrow$ zero pad with 1
 $F=5 \Rightarrow$ zero pad with 2
 $F=7 \Rightarrow$ zero pad with 3

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Remember back to...

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially!
(32 \rightarrow 28 \rightarrow 24 ...). Shrinking too fast is not good, doesn't work well.

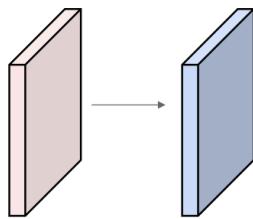


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Examples time:

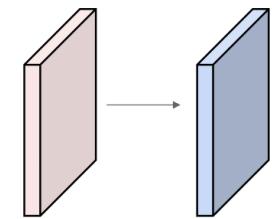
Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2



Output volume size: ?

Examples time:

Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2

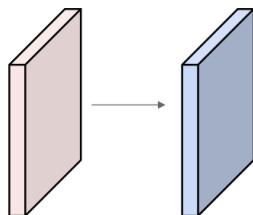


Output volume size:

$(32+2*2-5)/1+1 = 32$ spatially, so
32x32x10

Examples time:

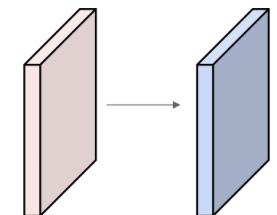
Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

Examples time:

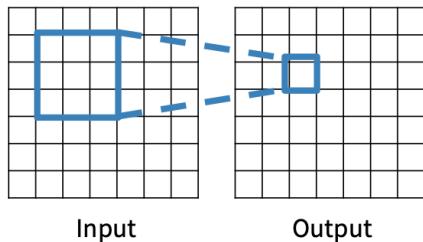
Input volume: 32x32x3
10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?
each filter has $5*5*3 + 1 = 76$ params
=> $76*10 = 760$ (+1 for bias)

Receptive Fields

For convolution with **kernel size K**, each element in the output depends on a $K \times K$ receptive field in the input



Slide inspiration: Justin Johnson

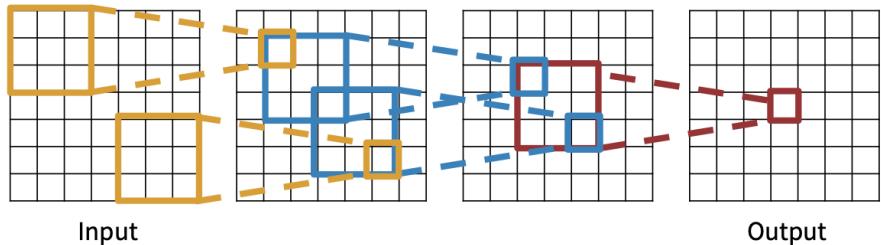
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Receptive Fields

Each successive convolution adds $K - 1$ to the receptive field size
With L layers the receptive field size is $1 + L * (K - 1)$



Be careful – “receptive field in the input” vs. “receptive field in the previous layer”

Slide inspiration: Justin Johnson

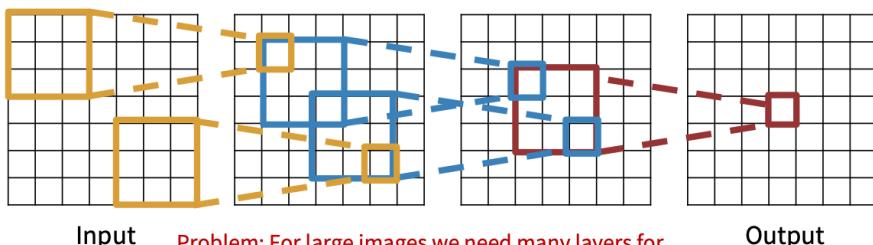
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Receptive Fields

Each successive convolution adds $K - 1$ to the receptive field size
With L layers the receptive field size is $1 + L * (K - 1)$



Slide inspiration: Justin Johnson

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Convolution layer: summary

Common settings:

Let's assume input is $W_1 \times H_1 \times C$
Conv layer needs 4 hyperparameters:

- Number of filters K
 - The filter size F
 - The stride S
 - The zero padding P
- K = (powers of 2, e.g. 32, 64, 128, 512)
- F = 3, S = 1, P = 1
 - F = 5, S = 1, P = 2
 - F = 5, S = 2, P = ? (whatever fits)
 - F = 1, S = 1, P = 0

This will produce an output of $W_2 \times H_2 \times K$
where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

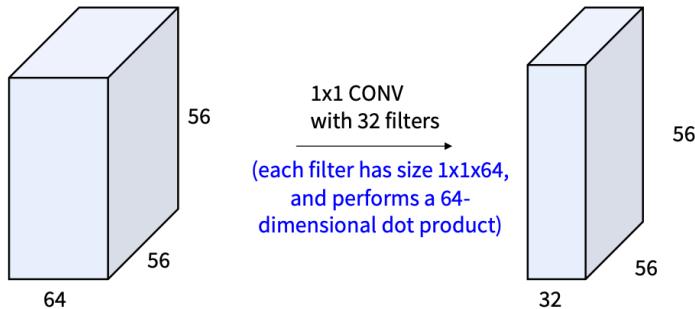
Number of parameters: F^2CK and K biases

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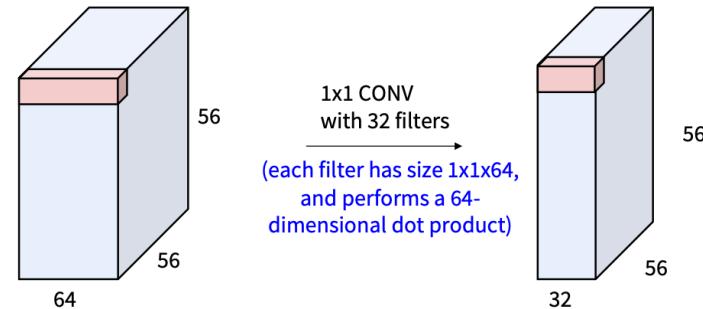
(btw, 1x1 convolution layers make perfect sense)



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(btw, 1x1 convolution layers make perfect sense)



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Example: CONV layer in PyTorch

```
Conv2d
CLASS torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0,
dilation=1, groups=1, bias=True)
[SOURCE]
```

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{out}, H_{out}, W_{out})$ can be precisely described:

$$\text{out}(N_i, C_{out,j}) = \text{bias}(C_{out,j}) + \sum_{k=0}^{C_{in}-1} \text{weight}(C_{out,j}, k) * \text{input}(N_i, k)$$

where $*$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is a width in pixels.

- stride controls the stride for the cross-correlation, a single number or a tuple.
- padding controls the amount of implicit zero-paddings on both sides for padding number of points for each dimension.
- dilation controls the spacing between the kernel points, also known as the a trous algorithm. It is harder to describe, but this [link](#) has a nice visualization of what dilation does.
- groups controls the connections between inputs and outputs. in_channels and out_channels must both be divisible by groups . For example:

 - At $\text{groups}=1$, all inputs are convolved to all outputs.
 - At $\text{groups}=2$, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
 - At $\text{groups}=\text{in_channels}$, each input channel is convolved with its own set of filters, of size $\lceil \frac{\text{out_channels}}{\text{in_channels}} \rceil$.

Conv layer needs 4 hyperparameters:

- Number of filters K
- The filter size F
- The stride S
- The zero padding P

The parameters kernel_size , stride , padding , dilation can either be:

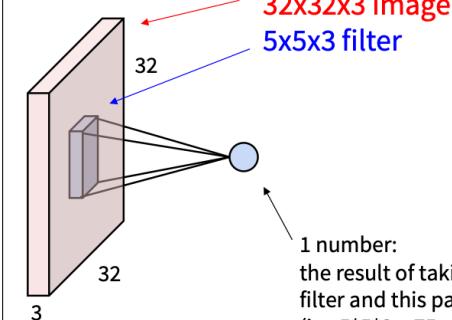
- a single int – in which case the same value is used for the height and width dimension
- a tuple of two ints – in which case, the first int is used for the height dimension, and the second int for the width dimension

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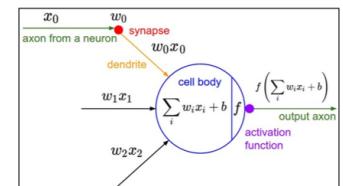
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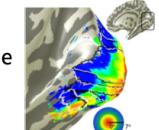
The brain/neuron view of CONV Layer



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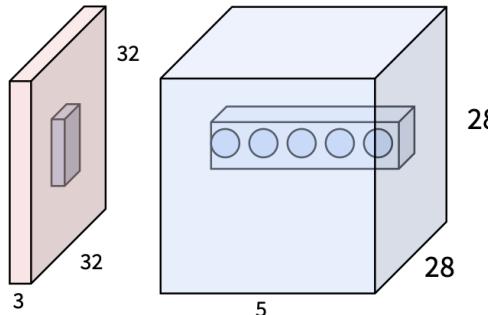
It's just a neuron with local connectivity...



1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. $5 \times 5 \times 3 = 75$ -dimensional dot product)

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The brain/neuron view of CONV Layer



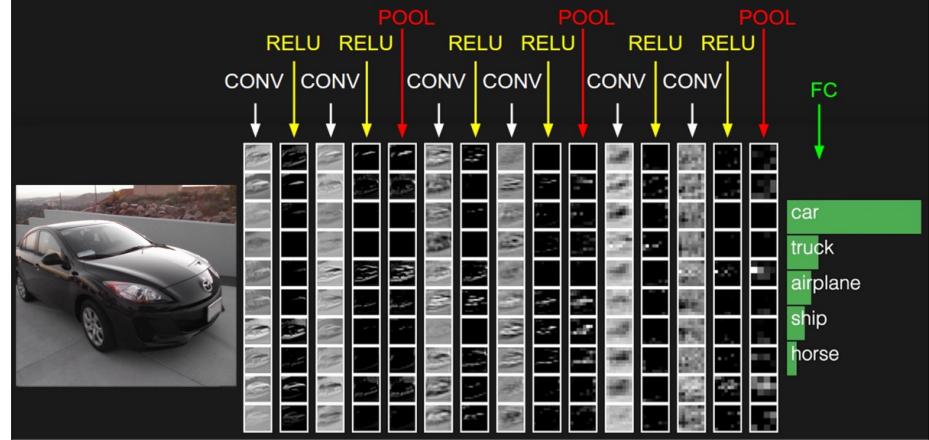
E.g. with 5 filters,
CONV layer consists of neurons
arranged in a 3D grid
($28 \times 28 \times 5$)

There will be 5 different neurons
all looking at the same region in
the input volume

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Neural Network Architecture

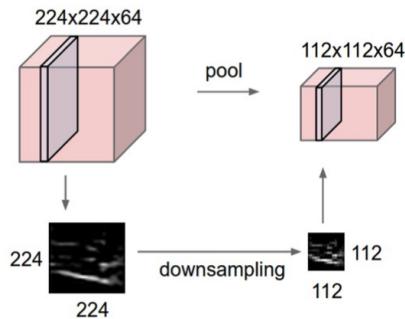


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Pooling layer

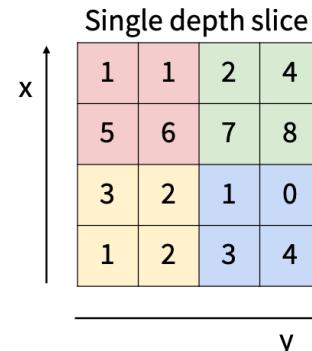
- makes the representations smaller and more manageable
- operates over each activation map independently



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MAX POOLING

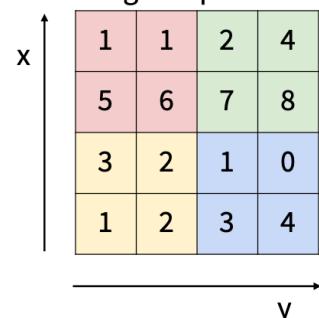


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MAX POOLING

Single depth slice



max pool with 2x2 filters
and stride 2

- No learnable parameters
- Introduces spatial invariance

Pooling layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 2 hyperparameters:

- The spatial extent F
- The stride S

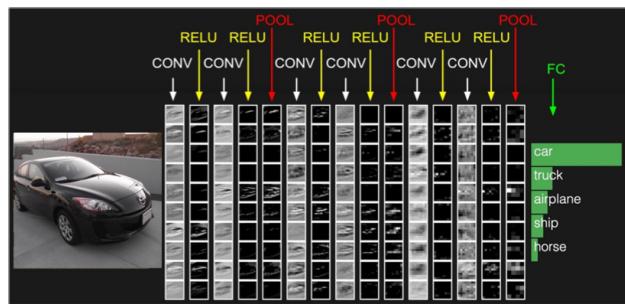
This will produce an output of $W_2 \times H_2 \times C$ where:

- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

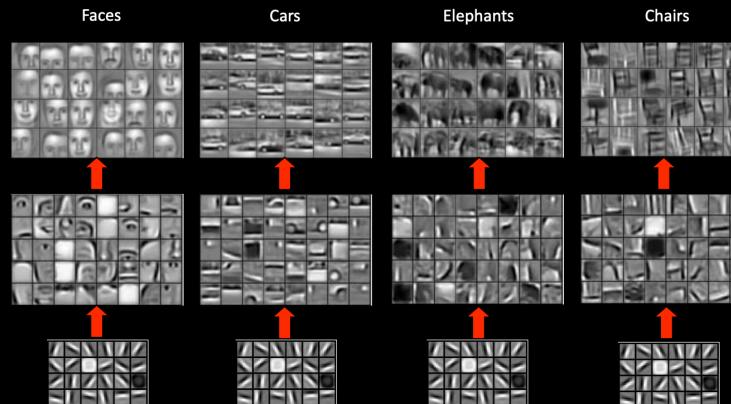
Number of parameters: 0

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Features learned from training on different object classes.



Alexnet

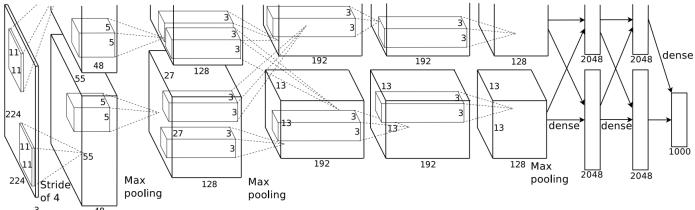


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

"ImageNet Classification with Deep Convolutional Neural Networks", NIPS, 2012

Pytorch

```
class AlexNet(nn.Module):
    def __init__(self, num_classes: int = 1000, dropout: float = 0.5) -> None:
        super().__init__()
        self.features = nn.Sequential(
            nn.Conv2d(3, 64, kernel_size=11, stride=4, padding=2),
            nn.ReLU(inplace=True),
            nn.MaxPool2d(kernel_size=3, stride=2),
            nn.Conv2d(64, 192, kernel_size=5, padding=2),
            nn.ReLU(inplace=True),
            nn.MaxPool2d(kernel_size=3, stride=2),
            nn.Conv2d(192, 384, kernel_size=3, padding=1),
            nn.ReLU(inplace=True),
            nn.Conv2d(384, 256, kernel_size=3, padding=1),
            nn.ReLU(inplace=True),
            nn.Conv2d(256, 256, kernel_size=3, padding=1),
            nn.ReLU(inplace=True),
            nn.MaxPool2d(kernel_size=3, stride=2),
        )
        self.avgpool = nn.AdaptiveAvgPool2d((6, 6))
        self.classifier = nn.Sequential(
            nn.Dropout(p=dropout),
            nn.Linear(256 * 6 * 6, 4096),
            nn.ReLU(inplace=True),
            nn.Dropout(p=dropout),
            nn.Linear(4096, 4096),
            nn.ReLU(inplace=True),
            nn.Linear(4096, num_classes),
        )

    def forward(self, x: torch.Tensor) -> torch.Tensor:
        x = self.features(x)
        x = self.avgpool(x)
        x = torch.flatten(x, 1)
        x = self.classifier(x)
        return x
```

AlexNet was originally introduced in "ImageNet Classification with Deep Convolutional Neural Networks". This implementation is based instead on the "One Weird Trick for Parallelizing Convolutional Neural Networks" paper.

single-column model with 64, 192, 384, 384, 256 filters in the five convolutional layers, respectively

```
import torch
from PIL import Image
from torchvision import transforms

model = torch.hub.load('pytorch/vision:v0.10.0', 'alexnet', pretrained=True)
model.eval()

input_image = Image.open(filename)
preprocess = transforms.Compose([
    transforms.Resize(256),
    transforms.CenterCrop(224),
    transforms.ToTensor(),
    transforms.Normalize(mean=[0.485, 0.456, 0.406], std=[0.229, 0.224, 0.225]),
])
input_tensor = preprocess(input_image)
# create a mini-batch as expected by the model
input_batch = input_tensor.unsqueeze(0)

if torch.cuda.is_available():
    input_batch = input_batch.to('cuda')
    model.to('cuda')

with torch.no_grad():
    output = model(input_batch)
    probabilities = torch.nn.functional.softmax(output[0], dim=0)

print(torch.topk(probabilities, 5))
# https://raw.githubusercontent.com/pytorch/hub/master/imagenet_classes.txt
```

Pytorch

[ConvNetJS demo: training on CIFAR-10]

ConvNetJS CIFAR-10 demo

Description

This demo trains a Convolutional Neural Network on the [CIFAR-10 dataset](#) in your browser, with nothing but Javascript. The state of the art on this dataset is about 90% accuracy and human performance is about 94% (not perfect as the dataset can be a bit ambiguous). I used this [python script](#) to parse the [original files](#) (python version) into batches of images that can be easily loaded into page DOM with img tags.

This dataset is more difficult and it takes longer to train a network. Data augmentation includes random flipping and random image shifts by up to 2px horizontally and vertically.

By default, in this demo we're using Adadelta which is one of per-parameter adaptive step size methods, so we don't have to worry about changing learning rates or momentum over time. However, I still included the text fields for changing these if you'd like to play around with SGD+Momentum trainer.

Report questions/bugs/suggestions to [@karpathy](#).



<http://cs.stanford.edu/people/karpathy/convnetjs/demo/cifar10.html>

Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Historically architectures looked like
 $[(CONV-RELU)^*N-POOL?]^*M-(FC-RELU)^*K, SOFTMAX$
 where N is usually up to ~5, M is large, $0 \leq K \leq 2$.
- But recent advances such as ResNet/GoogLeNet have challenged this paradigm

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Transfer learning

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You need a lot of data if you want to train/use CNNs?

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Transfer Learning with CNNs

1. Train on Imagenet

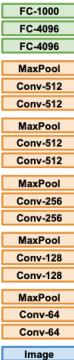


Donahue et al., "DeCAF: A Deep Convolutional Activation Feature for Scalable Visual Recognition", ICML 2014
Razavian et al., "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

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Transfer Learning with CNNs

1. Train on Imagenet



2. Small Dataset (C classes)

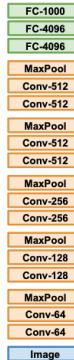


Reinitialize this and train
Freeze these

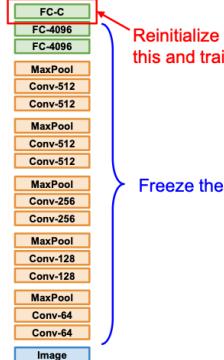
Donahue et al., "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
Razavian et al., "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

Transfer Learning with CNNs

1. Train on Imagenet



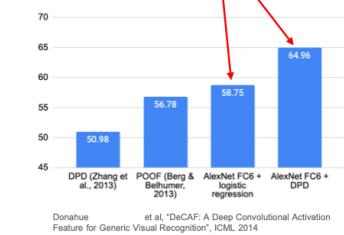
2. Small Dataset (C classes)



Reinitialize this and train
Freeze these

Donahue et al., "DeCAF: A Deep Convolutional Activation Feature for Generic Visual Recognition", ICML 2014
Razavian et al., "CNN Features Off-the-Shelf: An Astounding Baseline for Recognition", CVPR Workshops 2014

Finetuned from AlexNet



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Transfer Learning with CNNs

1. Train on Imagenet

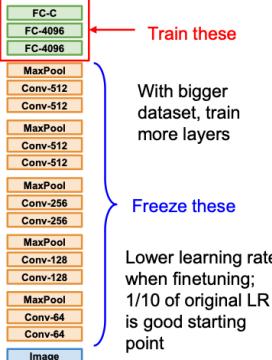


2. Small Dataset (C classes)



Reinitialize this and train
Freeze these

3. Bigger dataset



With bigger dataset, train more layers
Freeze these
Lower learning rate when finetuning;
1/10 of original LR is good starting point

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Batch Normalization

Consider a single layer $y = Wx$

The following could lead to tough optimization:

- Inputs x are not *centered around zero* (need large bias)
- Inputs x have different scaling per-element (entries in W will need to vary a lot)

Idea: force inputs to be “nicely scaled” at each layer!

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Batch Normalization

[Ioffe and Szegedy, 2015]

"you want zero-mean unit-variance activations? just make them so."

consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

this is a vanilla
differentiable function...

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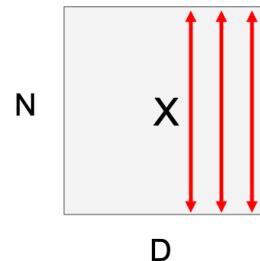
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Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

Problem: What if zero-mean, unit variance is too hard of a constraint?

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Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

Learning $\gamma = \sigma$,
 $\beta = \mu$ will recover the
identity function!

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is $N \times D$

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Batch Normalization: Test-Time

Estimates depend on minibatch;
can't do this at test-time!

Input: $x : N \times D$

$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j}$$

Per-channel mean,
shape is D

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

Learning $\gamma = \sigma$,
 $\beta = \mu$ will recover the
identity function!

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$

Per-channel var,
shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x,
Shape is $N \times D$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is $N \times D$

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Batch Normalization: Test-Time

Input: $x : N \times D$

$$\mu_j = \text{(Running) average of values seen during training}$$

Per-channel mean, shape is D

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

During testing batchnorm becomes a linear operator!
Can be fused with the previous fully-connected or conv layer

$$\sigma_j^2 = \text{(Running) average of values seen during training}$$

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

Normalized x,
Shape is $N \times D$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output,
Shape is $N \times D$

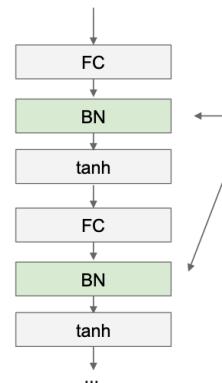
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[Ioffe and Szegedy, 2015]

Batch Normalization



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

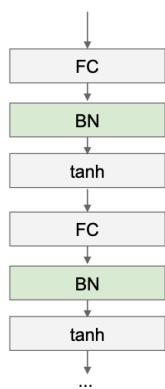
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Batch Normalization

[Ioffe and Szegedy, 2015]



- Makes deep networks **much** easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as a kind of regularization during training
- Behaves differently during training and testing: this is a very common source of bugs!

Batch Normalization for ConvNets

Batch Normalization for **fully-connected** networks

$$\mathbf{x} : N \times D$$

Normalize

$$\mu, \sigma : 1 \times D$$

$$\gamma, \beta : 1 \times D$$

$$\mathbf{y} = \gamma(\mathbf{x} - \mu) / \sigma + \beta$$

Batch Normalization for **convolutional** networks
(Spatial Batchnorm, BatchNorm2D)

$$\mathbf{x} : N \times C \times H \times W$$

Normalize

$$\mu, \sigma : 1 \times C \times 1 \times 1$$

$$\gamma, \beta : 1 \times C \times 1 \times 1$$

$$\mathbf{y} = \gamma(\mathbf{x} - \mu) / \sigma + \beta$$

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Layer Normalization

Batch Normalization for fully-connected networks

$$\mathbf{x}: N \times D$$

Normalize

$$\mu, \sigma: 1 \times D$$

$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x - \mu) / \sigma + \beta$$

Layer Normalization for fully-connected networks
Same behavior at train and test!
Can be used in recurrent networks

$$\mathbf{x}: N \times D$$

Normalize

$$\mu, \sigma: N \times 1$$

$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x - \mu) / \sigma + \beta$$

Ba, Kiros, and Hinton, "Layer Normalization", arXiv 2016

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Instance Normalization

Batch Normalization for convolutional networks

$$\mathbf{x}: N \times C \times H \times W$$

Normalize

$$\mu, \sigma: 1 \times C \times 1 \times 1$$

$$\gamma, \beta: 1 \times C \times 1 \times 1$$

$$y = \gamma(x - \mu) / \sigma + \beta$$

Instance Normalization for convolutional networks
Same behavior at train / test!

$$\mathbf{x}: N \times C \times H \times W$$

Normalize

$$\mu, \sigma: N \times C \times 1 \times 1$$

$$\gamma, \beta: 1 \times C \times 1 \times 1$$

$$y = \gamma(x - \mu) / \sigma + \beta$$

Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

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