This exam has 4 questions, for a total of 28 points. You have 2.5 hours to complete the exam. Please read carefully the guidelines below:

- Your submission to Gradescope must include the following files:

  1. A text file named `XXXX.txt`, where `XXXX` are the last four digits of your **student ID**. This file is the most important as it will be used for **grading** your work. This file must contain your final answers to all questions, one per line. If you don't have an answer, you can leave the line empty. A template is provided at: `https://homepage.cs.uri.edu/~malvarez/stationary/exam/ans.txt`.
  2. A PDF file named `XXXX.pdf`, where `XXXX` are the last four digits of your **student ID**. This file will contain your work. You can write your solutions on your own paper(s) and then scan or photograph them into a single PDF. Do not worry about alignment or format, as long as your work is readable. In this file, your work on each question can be in any order.

- If the question is multiple choice, the answer **must be** the corresponding letter (A, B, C, ...). If the question is open, the answer will be a single number, or as otherwise specified in the question.

- You may use any of our lecture notes, books, or additional written/online references. However, when solving the questions, your solution must follow the algorithms and formulas introduced in our lectures.

By submitting my solutions to this exam I acknowledge that I have read and understood the guidelines above, that all answers are my own, and that I have neither gained unfairly from others nor have I assisted others in obtaining an unfair advantage.

1. (7 points) Consider a BST where keys can be integers (no repeats) between 10 and 50, inclusive. The root node is 13 and the left and right subtrees have the same height, which is greater or equal to 1. That is, the tree's height is at least equal to 2. Of all possible trees, consider the tree with the **minimum sum** over all its keys. What is this minimum sum?

1. _____

2. (7 points) Consider an empty hash table of length 11, in which keys $21, 12, 5, 32, 7, 10, 16, 24$ are inserted using the hash function $h(x) = (x + 3) \mod 11$ and separate chaining. What is the total number of *collisions*?

2. _____

3. (7 points) Assuming you have a valid max-heap with 7 elements such that a post-order traversal outputs the sequence $1, 2, \ldots, 6, 7$. What is the sum of all nodes of height $h = 1$?

3. _____

4. (7 points) Consider inserting the keys $31, 53, 1, 4, 47, 39, 67, 82$ into an empty hash table of length $m = 11$. Assume collisions are handled using a technique called *double hashing* which uses the function $h(k) = (h_1(k) + jh_2(k)) \mod m$, where $h_1(k) = k \mod m$ and $h_2(k) = 5 - (k \mod 5)$, for $j = 0, 1, 2, \ldots, m - 1$. Indicate the sum of the values corresponding to all statements that are `True`. If none are `True`, the answer should be 0. Note that you must insert all values into the table and deal with collisions before answering the question.

   (1) 39 will be stored at index 1

   (2) 82 will be stored at index 0

   (4) 31 will be stored at index 9

   (8) 53 will be stored at index 0

4. _____