COMP3411 Artificial Intelligence
Session 1, 2018

Tutorial Solutions - Week 8

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Activity 8.1 Three Godesses in a Temple

Three goddesses were sitting in an old Indian temple. Their names were Truth, Lie and Wisdom. Truth always told the truth, Lie always lied and Wisdom sometimes told the truth and sometimes lied. A man entered the temple. He first asked the goddess on the left: "Who is sitting next to you?" "Truth," she answered. He then asked the middle one: "Who are you?" "Wisdom." Finally he asked the one on the right: "Who is your neighbor?" "Lie," she replied. Can you say which goddess was which?

The goddess on left cannot be True because she said someone else was True. The middle one cannot be True either, so the one on the right must be True. This means the middle one is Lie and the left goddess is Wisdom.

Activity 8.2 Validity and Satisfiability

(Exercise 7.10 from R & N)

Decide whether each of the following sentences is valid, satisfiable, or neither. Verify your decisions using truth tables or equivalence and inference rules. For those that are satisfiable, list all the models that satisfy them.

a. Smoke ⇒ Smoke

Valid   [implication, excluded middle]

b. Smoke ⇒ Fire

Satisfiable

Smoke	Fire	Smoke ⇒ Fire
T	T	T
T	F	F
F	T	T
F	F	T

Models are: {Smoke, Fire}, {Fire}, {}

c. (Smoke ⇒ Fire) ⇒ (¬ Smoke ⇒ ¬ Fire)

Satisfiable

Smoke	Fire	Smoke ⇒ Fire	¬ Smoke ⇒ ¬ Fire	KB
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Models are: {Smoke, Fire}, {Smoke}, {}

d. Smoke ∨ Fire ∨ ¬ Fire

Valid

e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Valid

((S ∧ H) ⇒ F)	⇔ (F ∨ ¬(S ∧ H))	[implication]
	⇔ (F ∨ ¬ S ∨ ¬ H)	[de Morgan]
	⇔ (F ∨ ¬ S ∨ F ∨ ¬ H)	[idempotent, commutativity]
	⇔ (S ⇒ F) ∨ (H ⇒ F)	[implication]

f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

Valid

$(S \Rightarrow F) \Leftrightarrow (F \vee \neg S)$ [implication]

$\Rightarrow (F \vee \neg S \vee \neg H)$ [generalization]

$\Rightarrow (F \vee \neg (S \wedge H))$ [de Morgan]

$\Rightarrow ((S \wedge H) \Rightarrow F)$ [conditional]

g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Valid

Big ∨ Dumb ∨ (Big ⇒ Dumb) ⇔ Big ∨ Dumb ∨ Dumb ∨ ¬ Big [implication]

⇔ Big ∨ ¬ Big ∨ Dumb [idempotent]

⇔ TRUE ∨ Dumb [excluded middle]

⇔ TRUE

h. (Big ∧ Dumb) ∨ ¬ Dumb

Satisfiable

Big	**Dumb**	**(Big ∧ Dumb)**	**(Big ∧ Dumb) ∨ ¬ Dumb**
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	T

Models are: {Big, Dumb}, {Big}, {}

Activity 8.3 Resolution and Conjunctive Normal Form

(Exercise 7.2 from R & N)

Consider the following Knowledge Base of facts:

If the unicorn in mythical, then it is immortal, but if it is not mythical, then it is mortal and a mammal. If the unicorn in either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

1. Translate the above statements into Propositional Logic.

   Myth ⇒ ¬ Mortal
   ¬Myth ⇒ (Mortal ∧ Mammal)
   ¬Mortal ∨ Mammal ⇒ Horned
   Horned ⇒ Magic

2. Convert this Knowledge Base into Conjunctive Normal Form.

   (¬Myth ∨ ¬ Mortal) ∧ (Myth ∨ Mortal) ∧ (Myth ∨ Mammal) ∧ (Mortal ∨ Horned) ∧ (¬Mammal ∨ Horned) ∧ (¬Horned ∨ Magic)

3. Use a series of resolutions to prove that the unicorn is Horned.

   Using Proof by Contradiction, we add to the database the negative of what we are trying to prove:

   ¬Horned

   We then try to derive the "empty clause" by a series of Resolutions:

   <u>¬Horned ∧ (Mortal ∨ Horned)</u>
   Mortal

   <u>¬Horned ∧ (¬Mammal ∨ Horned)</u>
   ¬Mammal

   <u>Mortal ∧ (¬Myth ∨ ¬Mortal)</u>
   ¬Myth

   <u>¬Myth ∧ (Myth ∨ Mammal)</u>
   Mammal

   <u>Mammal ∧ ¬Mammal</u>

   Having derived the empty clause, the proof (of Horned) is complete.

4. Give all models that satisfy the Knowledge Base. Can you prove that the unicorn is Mythical? How about Magical?

   Because of the rule (Horned ⇒ Magic), Magic must also be True.

   We can construct a truth table for the remaining three variables:

Myth	**Mortal**	**Mammal**	**Myth ⇒ ¬ Mortal**	**¬ Myth ⇒ (Mortal ∧ Mammal)**	**KB**
T	T	T	F	T	F
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

   There are three models which satisfy the entire Knowledge Base:
   {Horned, Magic, Myth, Mammal}, {Horned, Magic, Myth}, {Horned, Magic, Mortal, Mammal}.
   We cannot prove that the unicorn is Mythical, because of the third model where Mythical is False.

Activity 8.4 Sentences in First Order Logic

Represent the following sentences in first-order logic, using a consistent vocabulary.

a. Some students studied French in 2016.

   $\exists x\ Student(x) \wedge Study(x,French,2016)$

b. Only one student studied Greek in 2015.

   $\exists x\ Study(x,Greek,2015) \wedge \forall y\ (Study(y,Greek,2015) \Rightarrow y = x\ )$

   sometimes written as

   $\exists! x\ Study(x,Greek,2015)$

c. The highest score in Greek is always higher than the highest score in French.

   $\forall t\ \exists x\ \forall y\ Score(x,Greek,t) > Score(y,French,t)$

d. Every person who buys a policy is smart.

   $\forall x,p\ Person(x) \wedge Policy(p) \wedge Buy(x,p) \Rightarrow Smart(x)$

e. No person buys an expensive policy.

   $\neg \exists x,p\ Person(x) \wedge Policy(p) \wedge Expensive(p) \wedge Buy(x,p)$

f. There is a barber who shaves all men in town who do not shave themselves.

   $\exists b\ Barber(b) \wedge \forall m\ (Man(m) \wedge InTown(m) \wedge \neg Shave(m,m) \Rightarrow Shave(b,m))$

g. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

   $\forall p\ (\ Politician(p) \Rightarrow ((\exists x \forall t\ Fool(p,x,t)) \wedge (\exists t \forall x\ Fool(p,x,t)) \wedge (\neg \forall x \forall t\ Fool(p,x,t))))$

Activity 8.5 The Interplanetary Visitor

On a certain planet there are 100 highly intelligent but also highly religious inhabitants who all know and see each other on a daily basis. Some number *n* of them have blue eyes, the rest have brown eyes. The religious laws of the planet dictate that anybody who is able to prove that their own eyes must be blue has to ritually commit suicide the same evening, at midnight. Everybody knows that everybody else will obey this law without question. For this reason, it has become forbidden on the planet to discuss eye colour, and all mirrors, cameras and other such devices have long since been destroyed. One day a visitor comes from a neighboring planet whose inhabitants are known to always speak the truth. Everyone has gathered for his departure. Just before closing the door of his spaceship, he says in a voice loud enough for everyone to hear: "At least one of you has blue eyes." What will happen in the days (and nights) that follow?
(Hint: consider first the case *n*=1, then *n*=2, *n*=3, etc.)

   First consider the case where there is only one inhabitant with blue eyes. When he looks around and sees that everyone else has brown eyes, he will conclude that he must have blue eyes and therefore commit suicide at midnight on the day of the visitor's departure.

If two inhabitants have blue eyes, each of them will see one other person with blue eyes. Seeing the other person still alive the next day, each of them will conclude that they must have blue eyes as well, so they will both commit suicide at midnight on the second day. We can proceed inductively to conclude that, if n inhabitants have blue eyes, all of them will commit suicide at midnight on the n^{th} day.

This result is somewhat surprising because, in the case where more than two inhabitants have blue eyes, everyone already knew that there was at least one inhabitant with blue eyes, and everyone knew that everyone else knew it; so it may at first appear that the visitor is not providing any new information. But this is not the case. For example, if exactly three inhabitants A, B and C had blue eyes, then B knew that C knew that somebody had blue eyes, but A didn't know that B knew that C knew that somebody had blue eyes. So there is always some new information provided by the visitor.