COMP3411 Artificial Intelligence
Session 1, 2018

Tutorial Solutions - Week 11

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Activity 11.1 Conditional Probability

Only 4% of the population are color blind, but 7% of men are color blind. What percentage of color blind people are men?

> If we assume 50% of the population are men, then the fraction of "color blind men" is 0.5 * 0.07 = 0.035
> This means the fraction of "color blind women" is 0.04 - 0.035 = 0.005
> Therefore, the fraction of color blind people who are men is 0.035 / 0.04 = 87.5%.

Activity 11.2 Enumerating Probabilities

(Exercise 13.8 from Russell & Norvig)

	toothache		**¬toothache**	
	catch	**¬catch**	**catch**	**¬catch**
cavity	.108	.012	.072	.008
¬cavity	.016	.064	.144	.576

1. Given the full joint distribution shown in Figure 13.3 (also on page 17 of the Uncertainty lecture slides), calculate the following:

   a. P(toothache ∧ ¬ catch) = 0.012 + 0.064 = 0.076

   b. P(catch) = 0.108 + 0.016 + 0.072 + 0.144 = 0.34

   c. P(cavity | catch) = P(cavity ∧ catch) / P(catch)
   = (0.108 + 0.072)/(0.108 + 0.072 + 0.016 + 0.144) = 0.18 / 0.34 = 0.53

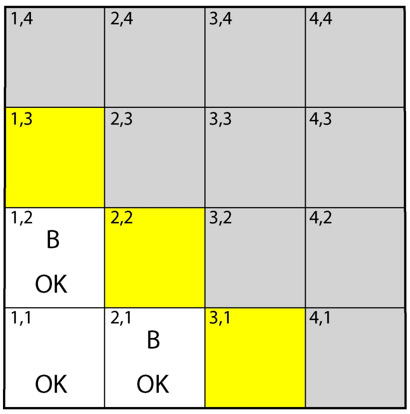
   d. P(cavity | toothache ∨ catch) =
   P(cavity ∧ (toothache ∨ catch)) / P(toothache ∨ catch)
   = (0.108 + 0.012 + 0.072)/(0.108 + 0.012 + 0.072 + 0.016 + 0.064 + 0.144)
   = 0.192 / 0.416 = 0.46

2. Verify the conditional independence claimed in the lecture slides by showing that P(catch | toothache, cavity) = P(catch | cavity)

   P(catch | toothache ∧ cavity) =
   P(catch ∧ toothach ∧ cavity) / P(toothache ∧ cavity)
   = 0.108 /(0.108 + 0.012) = 0.108 / 0.12 = 0.9

   P(catch | cavity) = P(catch ∧ cavity) / P(cavity)
   = (0.108 + 0.072)/(0.108 + 0.012 + 0.072 + 0.008) = 0.18 / 0.2 = 0.9

Activity 11.3 Wumpus World with Three Pits



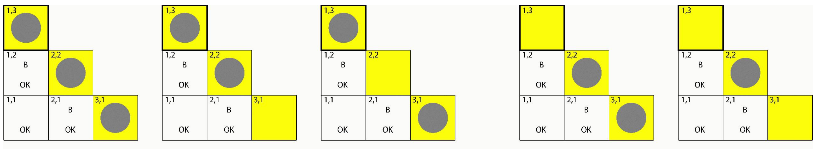
Consider the same Wumpus World situation shown above, but this time making a different prior assumption about the placement of the Pits - namely, that exactly three Pits have been placed randomly among the sixteen squares at the beginning of the game. Using this new prior, calculate:

1. the probability of a Pit in (1,3)
2. the probability of a Pit in (2,2)

   We start with these equations from the Lecture Notes:

$$P(Pit_{1,3} \mid known) = \Sigma_{fringe \in F}\, P(Pit_{1,3} \mid fringe)\, \Sigma_{other}\, P(known \mid fringe,other)P(fringe,other) \,/\, P(known)$$

$$P(known) = \Sigma_{fringe \in F}\, \Sigma_{other}\, P(known \mid fringe,other)\, P(fringe,other)$$



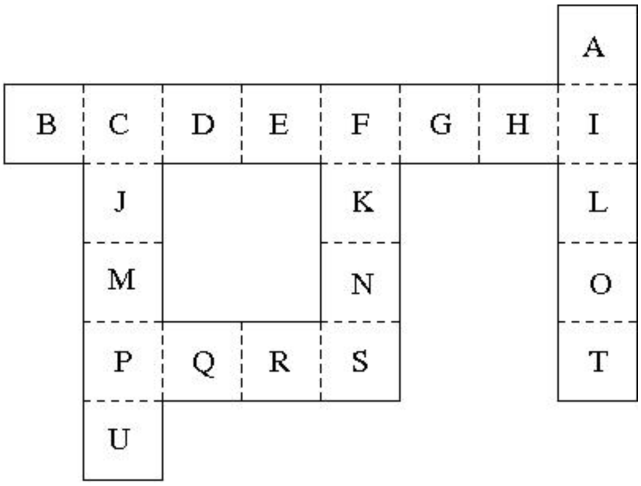
In this case, $P(known \mid fringe,other) = 1$ if and only if $fringe \in F$ and *fringe, other* between them contain exactly three Pits. For these configurations, $P(fringe,other)$ becomes a constant that can be cancelled from the numerator and denominator. If there are two pits in the fringe, the other pit can be placed in 10 different squares. If there is only one pit in the fringe, the other two pits can be placed in 45 different ways. Therefore,

$P(Pit_{1,3} \mid known) = (1+10+10)/(1+10+10+10+45) = 21/76 \approx 0.276$

$P(Pit_{2,2} \mid known) = (1+10+10+45)/(1+10+10+10+45) = 66/76 \approx 0.868$

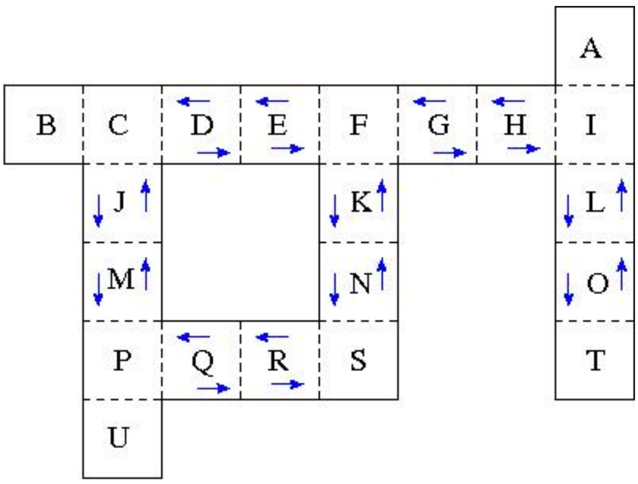
Activity 11.4 Robot Navigation

Consider a robot which has an accurate map of its environment (shown below), but is faced with the task of determining its own location within that environment. The robot has limited sensors which only enable it to perceive up to the boundaries of the square it currently occupies (i.e. it can tell whether there is a wall or another square, in each direction).



1. The robot actually begins in square N, facing Up (but does not yet know it). Based on its limited sensors, how many possibilities are there for its initial position and orientation, consistent with what it currently perceives?

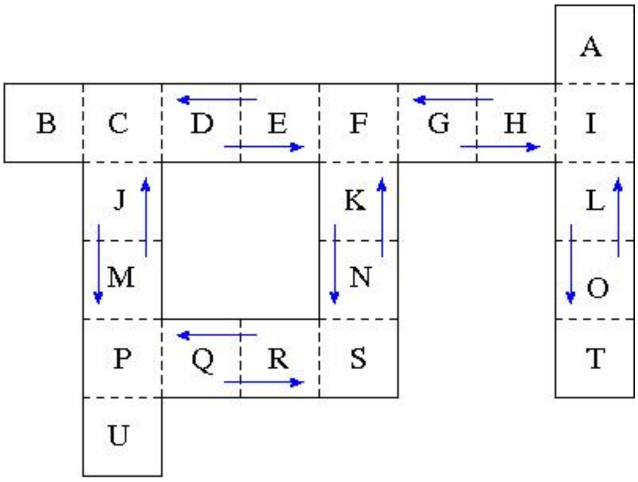
   There are 24 possibilities:



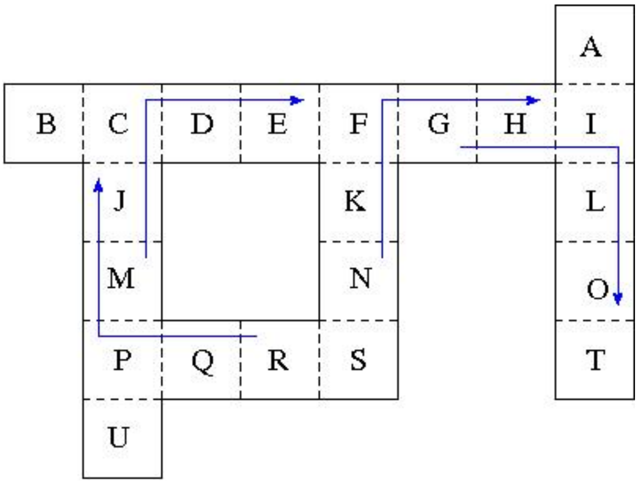
It could be in squares J, K, L, M, N, O facing Up or Down, or in squares D, E, G, H, Q, R facing Left or Right.

2. The robot then moves through the squares in this order: K, F, G, H, I, L, O, T. How many possibilities are there, at each step, for its position and orientation, consistent with its entire sequence of observations? At what point does it know its location with (effective) certainty?

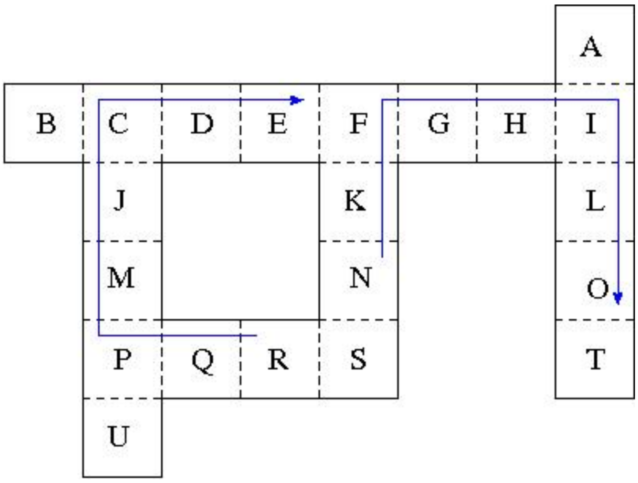
   After moving to K, only 12 possibilities remain:



   J, K, L facing Up; M, N, O facing Down; D, G, Q facing Left; E, H, R facing Right.
   After moving to F, only 4 possibilities remain, and these persist through G and H:



   Upon reaching I, only 2 possibilities remain, and these persist through L and O:



   Upon reaching T, the robot finally knows its position with (effective) certainty.