

## 2. The Global Tea and Organic Juice companies have merged.

### 1 Compute the expected time for each activity.

| Activity | Predecessor | Opt.<br>(a) | Ml.<br>(m) | Pess.<br>(b) | $t_e$ |
|----------|-------------|-------------|------------|--------------|-------|
| 1        | None        | 16          | 19         | 28           | 20    |
| 2        | None        | 30          | 30         | 30           | 30    |
| 3        | None        | 60          | 72         | 90           | 73    |
| 4        | None        | 18          | 27         | 30           | 26    |
| 5        | 1           | 17          | 29         | 47           | 30    |
| 6        | 1           | 4           | 7          | 10           | 7     |
| 7        | 5           | 12          | 15         | 18           | 15    |
| 8        | 6,7         | 6           | 12         | 24           | 13    |
| 9        | 2           | 18          | 27         | 30           | 26    |
| 10       | 3           | 20          | 35         | 50           | 35    |
| 11       | 4           | 40          | 55         | 100          | 60    |
| 12       | 8           | 11          | 20         | 29           | 20    |
| 13       | 11          | 14          | 23         | 26           | 22    |
| 14       | 9,12        | 13          | 16         | 19           | 16    |
| 15       | 10,13,14    | 0           | 0          | 0            | 0     |

The weighted average activity time is computed by the following formula:

$$t_e = \frac{a + 4m + b}{6} \quad (7.1)$$

where  $t_e$  = weighted average activity time  
 $a$  = optimistic activity time (1 chance in 100 of completing the activity earlier under *normal* conditions)  
 $b$  = pessimistic activity time (1 chance in 100 of completing the activity later under *normal* conditions)  
 $m$  = most likely activity time

## 2 Compute the variance for each activity.

The variability in the activity time estimates is approximated by the following equations:

**The standard deviation for the activity:**

$$\sigma_{t_e} = \left( \frac{b - a}{6} \right) \quad (7.2)$$

**The standard deviation for the project:**

$$\sigma_{T_E} = \sqrt{\sum \sigma_{t_e}^2} \quad (7.3)$$

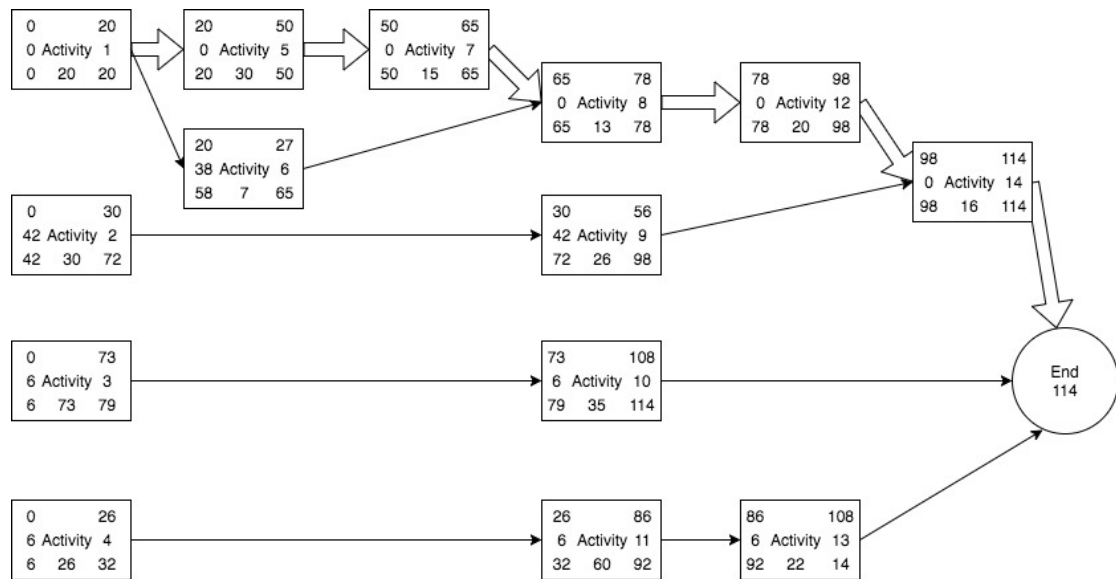
Note the standard deviation of the activity is squared in this equation; this is also called variance. This sum includes only activities on the critical path(s) or path being reviewed.

| Activity | Opt. (a) | ML. (m) | Pess. (b) | $t_e$ | Variance $[(b - a)/6]^2$ |
|----------|----------|---------|-----------|-------|--------------------------|
| 1        | 16       | 19      | 28        | 20    | 4                        |
| 2        | 30       | 30      | 30        | 30    | 0                        |
| 3        | 60       | 72      | 90        | 73    | 25                       |
| 4        | 18       | 27      | 30        | 26    | 4                        |
| 5        | 17       | 29      | 47        | 30    | 25                       |
| 6        | 4        | 7       | 10        | 7     | 1                        |
| 7        | 12       | 15      | 18        | 15    | 1                        |
| 8        | 6        | 12      | 24        | 13    | 9                        |
| 9        | 18       | 27      | 30        | 26    | 4                        |
| 10       | 20       | 35      | 50        | 35    | 25                       |
| 11       | 40       | 55      | 100       | 60    | 100                      |
| 12       | 11       | 20      | 29        | 20    | 9                        |
| 13       | 14       | 23      | 26        | 22    | 4                        |
| 14       | 13       | 16      | 19        | 16    | 1                        |
| 15       | 0        | 0       | 0         | 0     | 0                        |

### 3. Compute the expected project duration.

Duration = 114 days

| Activity | Opt.<br>(a) | ML.<br>(m) | Pess. (b) | $t_e$     | Variance<br>$[(b - a)/6]^2$ | Critical? |
|----------|-------------|------------|-----------|-----------|-----------------------------|-----------|
| 1        | 16          | 19         | 28        | <b>20</b> | 4                           | x         |
| 2        | 30          | 30         | 30        | <b>30</b> | 0                           |           |
| 3        | 60          | 72         | 90        | <b>73</b> | 25                          |           |
| 4        | 18          | 27         | 30        | <b>26</b> | 4                           |           |
| 5        | 17          | 29         | 47        | <b>30</b> | 25                          | x         |
| 6        | 4           | 7          | 10        | <b>7</b>  | 1                           |           |
| 7        | 12          | 15         | 18        | <b>15</b> | 1                           | x         |
| 8        | 6           | 12         | 24        | <b>13</b> | 9                           | x         |
| 9        | 18          | 27         | 30        | <b>26</b> | 4                           |           |
| 10       | 20          | 35         | 50        | <b>35</b> | 25                          |           |
| 11       | 40          | 55         | 100       | <b>60</b> | 100                         |           |
| 12       | 11          | 20         | 29        | <b>20</b> | 9                           | x         |
| 13       | 14          | 23         | 26        | <b>22</b> | 4                           |           |
| 14       | 13          | 16         | 19        | <b>16</b> | 1                           | x         |
| 15       | 0           | 0          | 0         | <b>0</b>  | 0                           |           |



Legend

|    |             |
|----|-------------|
| ES | EF          |
| SL | Activity ID |
| LS | DUR LF      |

#### 4. What is the probability of completing the project by day 112?

The equation below is used to compute the 'Z' value found in statistical tables (Z = number of standard deviations from the mean), which in turn tells the probability of completing the project in the time specified.

$$Z = \frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}} \quad (7.4)$$

where  $T_E$  = critical path duration  
 $T_S$  = scheduled project duration  
 $Z$  = probability (of meeting scheduled duration)

$$\frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}} = \frac{112 - 114}{\sqrt{4 + 25 + 1 + 9 + 9 + 1}} = \frac{-2}{\sqrt{49}} = \frac{-2}{7} = -.28 \quad \mathbf{P = 0.3897}$$

$$\mathbf{P \approx .39}$$

| Activity | Variance<br>[(b - a)/ 6] <sup>2</sup> |
|----------|---------------------------------------|
| 1        | 4                                     |
| 2        | 0                                     |
| 3        | 25                                    |
| 4        | 4                                     |
| 5        | 25                                    |
| 6        | 1                                     |
| 7        | 1                                     |
| 8        | 9                                     |
| 9        | 4                                     |
| 10       | 25                                    |
| 11       | 100                                   |
| 12       | 9                                     |
| 13       | 4                                     |
| 14       | 1                                     |
| 15       | 0                                     |

**Within 116 days?**

$$\frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}} = \frac{116 - 114}{\sqrt{4 + 25 + 1 + 9 + 9 + 1}} = \frac{+2}{\sqrt{49}} = \frac{+2}{7} = +.28 \quad \mathbf{P=0.6103}$$

$$\mathbf{P \approx .61}$$

**5. What is the probability of completing activity 11 by day 90?**

Critical path duration = 86 days

$$\frac{T_s - T_E}{\sqrt{\sum \sigma_{i_e}^2}} = \frac{90 - 86}{\sqrt{4 + 100}} = \frac{+4}{\sqrt{104}} = \frac{+4}{10} = +.40$$

Probability of within 90 days  $\approx .65$

| Activity | Variance<br>[(b - a)/ 6] <sup>2</sup> |
|----------|---------------------------------------|
| 1        | 4                                     |
| 2        | 0                                     |
| 3        | 25                                    |
| 4        | 4                                     |
| 5        | 25                                    |
| 6        | 1                                     |
| 7        | 1                                     |
| 8        | 9                                     |
| 9        | 4                                     |
| 10       | 25                                    |
| 11       | 100                                   |
| 12       | 9                                     |
| 13       | 4                                     |
| 14       | 1                                     |
| 15       | 0                                     |

## Standard Normal Probabilities

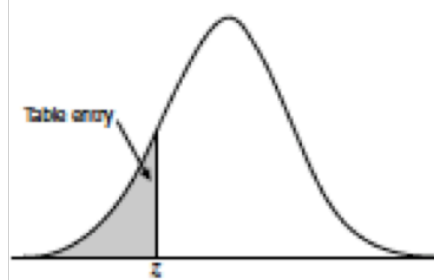


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

| $z$  | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |